# **Computer Simulation of Weldment Fatigue Life**

by

C. Y. Hou, N. Chen and F. V. Lawrence

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# COMPUTER SIMULATION OF WELDMENT FATIGUE LIFE

C. Y. Hou\*, N. Chen\*, and F. V. Lawrence†

#### **Abstract**

Modeling strategies for predicting the fatigue crack propagation life and total fatigue life of weldments are compared with experimental results reported in the literature. Three categories of analytical model are considered: (1) Models based solely on fatigue crack growth but which do not explicitly consider the effects of crack closure - Linear Elastic Fracture Mechanics or LEFM models; (2) Models based on fatigue crack growth but which do consider the effects of crack closure and the peculiar behavior of short cracks - the Crack Closure at a Notch model or CCN model; (3) Models which include fatigue crack initiation - Initiation-Propagation or IP models.

The results of this comparison suggest that both the CCN and IP give good predictions; whereas, LEFM models which neglect crack initiation and the behavior of short cracks underestimate the total fatigue life particularly at long lives.

#### **List of Symbols**

a	Peterson's constant
b	Fatigue strength exponent
C, C', m	Paris law and modified Paris law material constants
C	Fatigue ductility exponent
$\epsilon_{ m f}'$	Fatigue ductility coefficient
<i>K'</i>	Cyclic strength coefficient
n'	Cyclic strain hardening exponent
$N_{ m I}$	Crack initiation life
$N_{ m N}$	Crack nucleation life

<sup>\*</sup>Department of Civil Engineering, 205 N. Mathews, University of Illinois, Urbana, IL 61801, U.S.A.

<sup>†</sup>Departments of Civil Engineering and Material Science and Engineering, 205 N. Mathews, Urbana, IL 61801, U.S.A.

 $N_{\rm Pl}$  Short crack propagation life

N<sub>P2</sub> Long crack propagation life

N<sub>T</sub> Total fatigue life

R Stress ratio

σ'<sub>f</sub> Fatigue strength coefficient

 $S_{\text{max}}$ ,  $S_{\text{min}}$  Applied maximum and minimum stress

 $S_{\text{open}}$  Crack-opening stress

t Plate thickness or base plate thickness of weldments

U Effective stress-intensity factor ratio

 $\Delta K$ ,  $\Delta K_{\rm eff}$  Stress-intensity range and effective stress-intensity range

 $S_{y}, S_{u}$  Yield and ultimate strength of material

#### **List of Abbreviations**

CTPS Crack-Tip Plastic Stretches
FEM Finite Element Analysis

LEFM Linear Elastic Fracture Mechanics

NPS Notch Plastic Stretches

OICC Oxide - Induced Crack Closure
PICC Plasticity - Induced Crack Closure
RICC Roughness - Induced Crack Closure

SYMNC Strip-Yield Model for Notched Components

#### Estimating the Fatigue Life of Weldments

There are two ways of estimating the fatigue life of weldments: from S-N diagrams generated by laboratory testing, or through computer simulations based on analytical models of fatigue-damage accumulation. Fatigue data banks are an invaluable resource and ultimately provide the basis of fatigue design codes. Information from S-N diagrams provides the designer with incontrovertible or at least credible experimental data. On the other hand, analytical models can help the designer interpret this data, interpolate and extrapolate such information to conditions for which there are no test data, perform sensitivity analyses to optimize fatigue life, and actually to predict weldment fatigue life or strength. Analytical models can also be used to estimate the uncertainty in

weldment fatigue life using Monte Carlo methods or other stochastic modeling concepts. Thus, these two rather different design methods are in fact complimentary, and both are useful tools for the designer.

Analytical modeling of fatigue behavior has many special advantages for the designer. For example, computer simulations permit one to assess the influence of one or several changed service conditions on weldment fatigue life, to play "what if" games. This use of computer simulations is very valuable because many of the parameters which affect the fatigue life of weldments do so in a non-linear way which leads to less-than-obvious and sometimes counter-intuitive dependencies. Analytical models can predict these non-linear effects. In the future, it should prove possible to predict the high-cycle regime fatigue life of weldments subjected to variable load histories and corrosive environments. Given the expense and time required to obtain such information experimentally, computer simulations are probably the only practical path to such knowledge.

# **Modeling Strategies**

#### Concepts

Recent improvements in welding technology and the shift toward Gas-Metal-Arc welding processes now permit the fabrication of weldments free from large, crack-like welding defects so that their fatigue life must be considered the sum of three periods: crack nucleation (N<sub>N</sub>), short crack propagation (N<sub>P1</sub>) in which the crack closure phenomenon is a function of crack length, and long crack propagation (N<sub>P2</sub>) in which the crack closure phenomenon is more or less independent of crack length.

$$N_{T} = N_{N} + N_{P1} + N_{P2} \tag{1}$$

In this discussion, modeling strategies for predicting the fatigue crack propagation life and total fatigue life of weldments will be compared with experimental results reported in the literature. The three categories of analytical model which will be considered are given below and are listed in **Table 1**:

 Models based solely on fatigue crack growth but which do not explicitly consider the effects of crack closure (Linear Elastic Fracture Mechanics or LEFM models);

- Models based on fatigue crack growth but which do consider the effects of crack closure and the peculiar behavior of short cracks (the Crack Closure at a Notch model or CCN model).
- Models which include fatigue crack initiation (Initiation-Propagation or IP models);

Table 1 ANALYTICAL MODELS CONSIDERED

Model Category	Description	Concept
LEFM	This model type uses the relationship between stress intensity factor and crack growth rate to estimate the crack propagation life. The fatigue crack initiation life and crack closure effects are neglected.	N <sub>T</sub> ≈ N <sub>P</sub> 2
CCN	This model type uses the relationship between <u>effective</u> stress intensity factor and crack growth rate to estimate the short and long crack propagation life. Crack nucleation life may be neglected.	$N_{T} = N_{N} + (N_{P1} + N_{P2})$ or $N_{T} = N_{N} + N_{F}$ or $N_{T} \approx N_{F}$
IP	This model sums estimates of the crack initiation and propagation life to determine the total fatigue life.	$N_{T} = (N_{N} + N_{P1}) + N_{P2}$ or $N_{T} = N_{I} + N_{P}$

#### The LEFM models

The LEFM model estimates the  $N_{\rm P2}$  of weldments, or estimates  $N_{\rm T}$  assuming that the crack initiation stage is negligible. A straight-front crack is frequently assumed for calculations of stress-intensity factors: see Fig. 1a. Since multiple crack initiation and coalescence of surface semi-elliptical cracks (see Fig. 1b) are commonly observed in the fatigue of shielded metal arc weldments, many researchers have measured the crack-shape development in weldments by the ink-staining technique. Bell and Vosikovsky [1] gave an empirical forcing function to account for the crack-shape development:

$$a/c = e^{-ka}$$
 (2)

where a/c is the aspect ratio (the ratio of minor axis length to the major axis length) of the seem-elliptical crack and  $k = 2.09 \times 10^{-6} (K_t S_{max})^{1.95}$ . The stress-intensity factor at the deepest point of a seem-elliptical crack can be calculated by the weight function procedure proposed by Niu and Glinka [2].

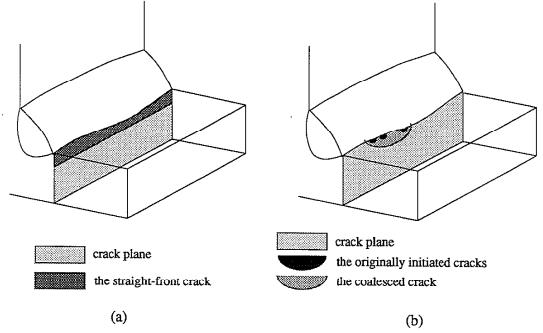


Fig. 1 (a) Straight-front crack at a weld toe, (b) Multiple crack initiation and crack coalescence.

#### Crack Closure at a Notch or CCN Model

The crack closure phenomenon was first reported by Elber [3] who found that the crack faces remain closed when the remote load is tensile. It has been found that various factors can induce crack-closure, e.g., oxidation-induced crack closure (OICC), roughness-induced crack closure (RICC). When the closure effect is caused by plastic deformation, i.e., when the plastic zone ahead of the crack tip and the plastic wake behind the crack tip keep the crack faces from opening, it is termed plasticity-induced crack closure (PICC). PICC has been used to explain the mean stress effect (R ratio effect), the retardation of crack growth after overloads, the short crack behavior and the existence of non-propagating cracks at notches. In the Crack-Closure-at-a-Notch or CCN model developed by Ting [4], PICC is the only closure effect considered since it is the easiest to simulate with elastic-plastic fracture mechanics approach and because, under most circumstances, it is the dominant effect.

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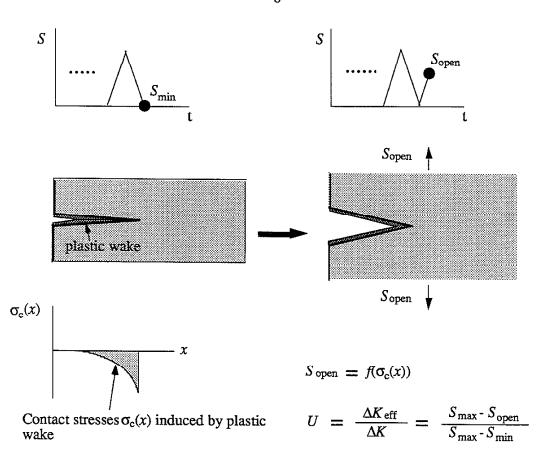


Fig. 2 An illustration of the modified Dugdale strip-yield model for crack closure.

Newman [5] first developed a numerical scheme which used the idea of Dugdale model and the wake of plastic deformation behind crack tip to simulate the PICC effect. Figure 2 schematically illustrates the Newman's Dugdale model. The load level at which the crack faces are opened is  $S_{open}$ . The value of  $S_{open}$  is estimated from the crack-face contact stresses by considering the effects of the plastic wake. The corresponding stress-intensity factor is defined as  $K_{open}$ . Thus, the effective stress intensity factor range  $\Delta K_{eff}$  is:

$$\Delta K_{\rm eff} = K_{\rm max} - K_{\rm open} \tag{3}$$

and the effective stress intensity factor ratio U is:

$$U(x) = \frac{\Delta Keff(x)}{\Delta K(x)} = \frac{S_{max} - S_{open}}{S_{max} - S_{min}}$$
(4)

From the modified Paris equation,

$$da/dN = C' (\Delta Keff)^{m}$$
(5)

we can calculate the total fatigue life as:

$$N_T = N_N + N_{P1} + N_{P2} = N_N + N_F,$$
 (6)

where:

$$N_{F} = \int_{a_{i}}^{a_{f}} \frac{da}{C' \Delta K_{eff}^{m}} = \int_{a_{i}}^{a_{f}} \frac{da}{C' (U \Delta K)^{m}}$$
(7)

If N<sub>N</sub> is small compared to N<sub>F</sub> (as is frequently the case), then:

$$N_{T} \approx N_{F} \tag{8}$$

Since in engineering applications most of the cracks emanate from the vicinity of a notch, the crack growth behavior at a notch is very important. Ting [4] first developed a Crack-Closure-at-a-Notch (CCN) model which used FEM results of Sun [6] to estimate U(x) in the presence of a notch. Later a Strip-Yield Model for Notched Component (SYMNC) was developed by Hou [7]. in which the basic approach of Newman's modified Dugdale model was applied, but in addition to the crack tip plastic stretch (CTPS) considered in Newman's model, the plastic deformation due to the presence of the notch (NPS, notch plastic stretch) was also estimated using FEM and summed up with the CTPS to form total plastic deformation, see Fig. 3. In this manner, the effect of the notch was incorporated into the crack-closure calculation. A weight-function method developed by Petroski [8-9] et al. was used to calculate stress intensity factor needed in the SYMNC. The SYMNC successfully explains some abnormal short crack effects in the vicinity of a notch, e.g., the "dip" of U factor at the early stage of crack growth near a notch [7].

When one applies the crack-closure concept to the fatigue life prediction of weldments, residual stresses must be considered. Figure 4 shows the residual stress distribution parallel and transverse to a weld. It is obvious that if a crack initiates at weld toe close to the center of the plate (location A), the crack is fully embedded in a tensile residual stress field. The tensile residual stresses easily open the crack; hence, crack closure is unlikely to develop under this condition. Of course, residual stress relaxation

or tensile overloads could decrease these tensile residual stress and create an environment for the occurrence of crack closure. As the crack propagates into a compressive residual stress field, crack closure is certain. If the crack starts from a location close to the edge of the plate, i.e., location B, which is in the compressive stress field of the residual stress distribution parallel to the weld, the compressive residual stresses reduce the nominal R values and will induce crack closure in the early stage of crack propagation. These observations suggest that crack closure in an as-welded weldment is a complex phenomenon because the crack initiation sites are unknown and the residual stress distributions are uncertain. To simplify the uncertain effects of residual stresses, two crack propagation models based limiting cases of crack-closure concept are used to bound the possible values of  $N_{\rm P2}$ :

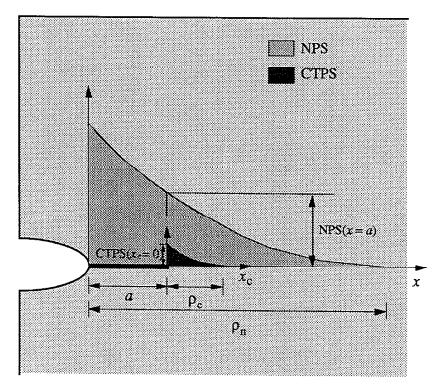


Fig. 3 The notch plastic stretches (NPS) and the crack-tip plastic stretches (CTPS)

#### The IP Model

While the transition from initiation to propagation is not easily defined, one definition of crack initiation is the portion of life spent developing an engineering size crack (0.25 mm). The initiation life ( $N_I$ ) is the portion of total life devoted to the nucleation ( $N_N$ ) and early growth ( $N_{P1}$ ) of small cracks. The crack Initiation-

Propagation model (IP model) proposed by Lawrence et al. [10-12] combines  $\,N_N\,$  and the  $\,N_{P1}\,$  together as the "crack initiation life" ( $\,N_I\,$ ), and  $\,N_T\,$  is written as:

$$N_T = (N_N + N_{P1}) + N_{P2} = N_I + N_{P2}$$
 (9)

 $N_I$  is commonly estimated by the Basquin-Morrow equation or the strain-life approach. The long crack propagation life  $N_{P2}$  is calculated by the LEFM approach, i.e., integration of the da/dN versus  $\Delta K$  relationship:

$$N_{P2} = \int_{a_i}^{a_f} \frac{\mathrm{d}a}{\mathrm{C}\Delta\mathrm{K}^{\mathrm{m}}} \tag{10}$$

An "engineering-size" crack ( $\approx 0.25$  mm) is assumed to be the end of the crack initiation stage. The long crack propagation life (Np<sub>2</sub>) is estimated by Eq. 10 using the "engineering-size" crack as the initial crack length  $a_i$ .

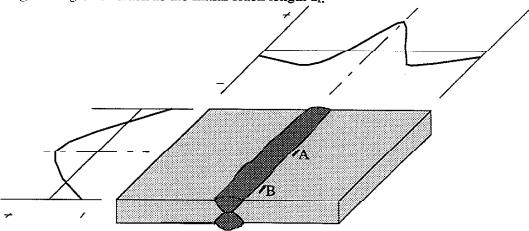


Fig. 4 Residual stress distribution in a welded plate.

## **Estimates of the Crack Propagation Life:**

For the crack propagation life predictions, the weld toe radius used in stress-intensity factor calculations is important when the weight function method is applied because the localized unflawed stress state around weld toe is affected by the weld toe radius. In this study, the reported average value of measured weld toe radius (0.5 mm [13]) was used for the weld-toe, crack-free stress distribution and the stress-intensity factor calculations.

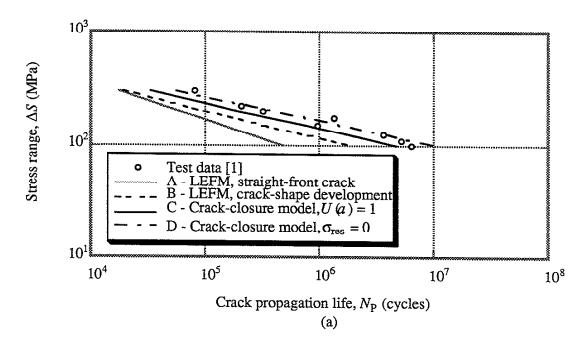
THE MATERIAL PROPERTIES FOR FATIGUE LIFE PREDICTION OF THE T-JOINTS Table 2 [1, 13].

Yield strength, Sy (MPa)	42.1
Ultimate strength, $S_u$ (MPa)	509
$C^*$ , m (for LEFM)	$5.36 \times 10^{-9}, 3.2$
C'*	1.5×10 <sup>-9</sup> [ <b>13</b> ]
	6.1×10-12** [14]
m	3.2 [ <b>13</b> ]
nt	5.2 [13] 6.3** [14]
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$\sigma_{\rm f}' = 1.5S_{\rm u} + 345 \text{ (MPa) } [15]$	
$h = \frac{1}{1000} \log 2(1 + \frac{345}{1000})$ [15]	-0.077
$b = -\frac{1}{6}\log 2(1 + \frac{345}{1.5S_0})$ [15]	
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<sup>\*</sup> All the C' values listed give the unit of da/dN in mm/cycle. \*\* Constants for the Stage I crack propagation.

Table 3 MODELS CONSIDERED IN FATIGUE CRACK PROPAGATION LIFE PREDICTIONS

Model	Model Category	Description
A	LEFM	Plane-front crack considered. No crack shape development. Initial crack length assumed to be $a_i = 0.5$ mm for consistency with the measurements of Bell et al. Residual stresses assumed to be zero, $\sigma_{res} = 0$ .
В	LEFM	Crack Shape Development considered using the Bell and Vosikovsky forcing function. Initial crack length assumed to be $a_i = 0.5$ mm for consistency with the measurements of Bell et al. Residual stresses assumed to be zero, $\sigma_{res} = 0$ .
С	CCN	Crack-closure model, crack-shape development, $U(a) = 1$ and $\sigma_{res} \neq 0$ assumed. Initial crack length assumed to be $a_i = 0.5$ mm for consistency with the measurements of Bell et al.
D	CCN	Crack-closure model, crack-shape development, $\sigma_{res} = 0$ assumed. Initial crack length assumed to be $a_i = 0.5$ mm for consistency with the measurements of Bell et al



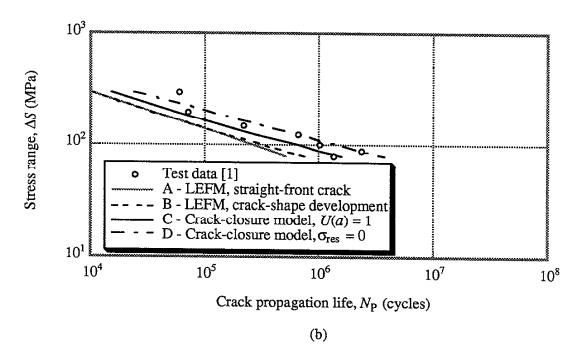


Fig. 5 Comparison of the model predictions with the observed [1] crack propagation life: (a) = 16 mm (b) = 78 mm.

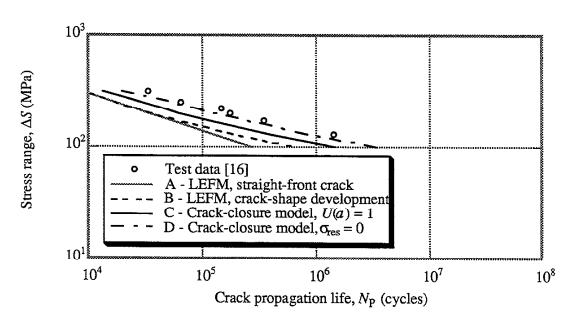


Fig. 6 Comparison of the model predictions with the observed [14] crack propagation life from Yagi et al [14], t = 22 mm.

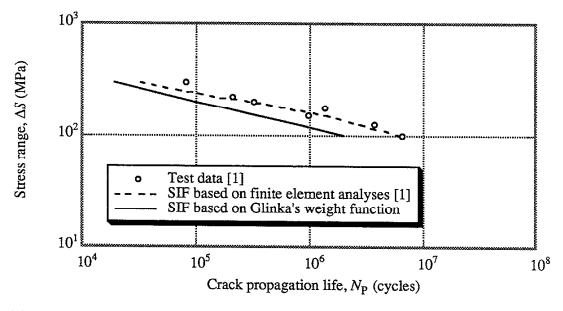


Fig. 7 Comparison of the results between different approaches for the SIF calculations for the case of t = 16 mm.

Bell et al. [1] performed a series of three-point bending tests on various base plate thickness of T-joints. The test results for base plate thickness t = 16 and 78 mm specimens were chosen for comparison with the model predictions. Using of the straingage technique, they claimed that cracks of 0.5 mm depth could be detected. The fatigue life from this crack size to the final failure and the total fatigue life were both recorded. An initial crack depth of 0.5 mm was used in Model A to D to estimate  $N_{P2}$  of these specimens. Material properties for crack-closure calculations and constants for Paris equation are listed in **Table 2**. **Table 3** summarizes the modeling assumptions. **Figure 5(a) and (b)** show the predicted and experimental results for  $N_{P2}$ .

Yagi et al. [16] also tested T-joints and cruciform welded joints to study weldment thickness effects. They measured the crack propagation life from crack size of  $1\sim2$  mm to final failure. The  $S_y$  and  $S_u$  of the base plate are close to the values listed in Table 2. Model predictions for the  $N_{P2}$  of T-joints were made for the specimens with base plate thickness of 22 mm. The initial crack length used for  $N_{P2}$  predictions was 1.5 mm. Figure 6 shows the predicted and experimental results.

In comparing the predicted S-N curves with the observed data, it is evident that Model A predictions are conservative for all three plate thicknesses. Model B in which the crack-shape development is considered still predicts smaller  $N_{P2}$  than the observed results. However, it should be noted that using this model and stress-intensity factors calculated using the finite element technique, Bell et al. [1] obtained predictions that agreed well with the test data: see Fig. 7. Most experimental data are bounded by the predicted S-N curves of Model C and D except the cases of t = 22 mm.

## **Estimates of the Total Fatigue Life:**

Four crack propagation models and the IP model were used to estimate  $N_{\rm T}$  of the T-joints discussed in the previous section: see Table 4.

Because of the bilinear modified Paris law assumed in model D', an additional set of material constants<sup>2</sup> for the modified Paris equation are listed in **Table 2** to account for the Stage I crack propagation. The required material properties for estimating  $N_{\rm I}$  using the I-P model are also listed in **Table 2**.

Figures 8 - 10 show the model predictions and the observed data obtained from Bell et al. and Yagi et al. Again, Model A' predicts much lower fatigue strengths than the

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<sup>&</sup>lt;sup>1</sup>It should be noted that C and C' listed in Table 8 should be dependent ( $C'=C/U^m$ ). Since these values were obtained from different studies, the listed numbers are not related. This difference causes the predicted lives using the crack-closure concepts to be higher than those obtained by LEFM, which is contradictory to the expected results that the LEFM overestimates the crack propagation life.

<sup>&</sup>lt;sup>2</sup>These data are scarce in literature. The values used were obtained from the test data of Verreman et al. [53] for ASTM A36 steel weldments.

test data. Model C' to E predicts more favorable results except that they are slightly higher at  $10^6 \sim 10^7$  cycles. The results of Model D' deviate from those of Model C' in the long-life regime. This deviation is due to the bilinear modified Paris law employed in Model D'. These deviations increase with decreasing base plate thickness. Model E predicts a same trend as Model D', that is, the slope of the predicted S-N curves becomes less in long-life regime. However, if one doesn't assume a bilinear modified Paris law, Model C' predicts no S-N curve slope change in the long-life regime. This change in the slope of S-N curves is very important for obtaining accurate predictions at lives larger than  $10^7$  cycles.

Table 4 MODELS CONSIDERED IN TOTAL FATIGUE LIFE PREDICTIONS

Model	Model Category	Description
A'	LEFM	Plane-front crack considered. No crack shape development. Initial crack length assumed to be $a_i = 0.1$ mm.
В'	LEFM	Crack Shape Development considered using the Bell and Vosikovsky forcing function. Initial crack length assumed to be $a_i = 0.1$ mm.
C'	CCN	Crack-closure model, crack-shape development, $U(a) = 1$ and $\sigma_{res} \neq 0$ assumed. Initial crack length assumed to be $a_i = 0.1$ mm.
D'	CCN	Crack-closure model, crack-shape development, $\sigma_{res} = 0$ assumed. Initial crack length assumed to be $a_i = 0.1$ mm.
E	IP	IP model, with crack closure, crack-shape development considered for $N_{\rm P2}$ and $a_{\rm i}=0.25$ mm assumed.

#### **Discussion and Summary**

To verify models and thus to promote their acceptance and usage, one must compare the predictions each model makes with experimental data. Good models predict the fatigue life and fatigue strength, the presence or absence of a fatigue crack, and the crack length under any condition. Bad models don't.

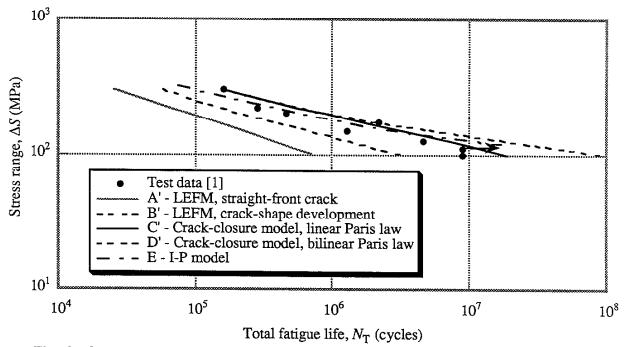


Fig. 8 Comparison of the model predictions with the tested total fatigue lives [1] for a plate thickness of t = 16 mm.

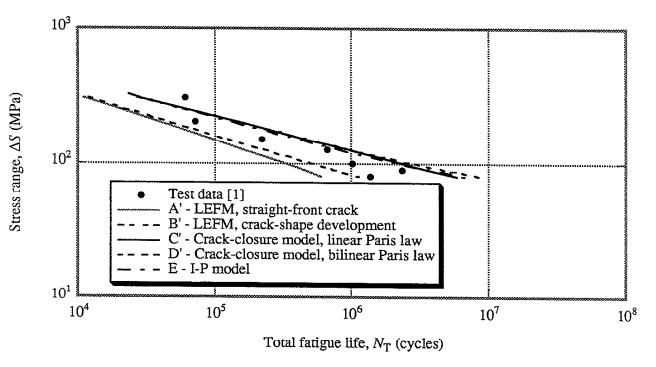


Fig. 9 Comparison of the model predictions with the observed total fatigue [1] lives for a plate thickness t = 78 mm.

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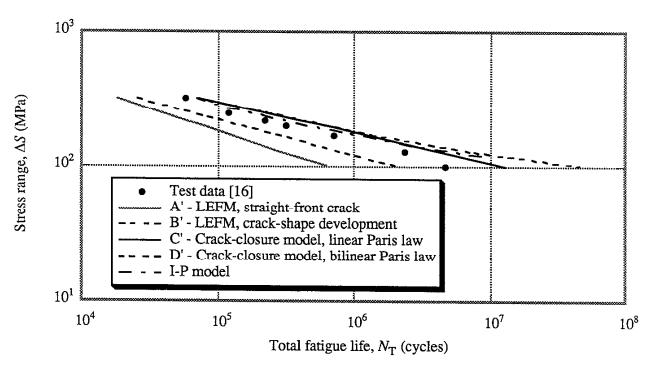


Fig. 10 Comparison of the model predictions with the observed total fatigue lives [14] for a plate thickness t = 22 mm.

Once one has confidence in a model, it can be used to study complex and important issues in weldment fatigue such as: the influence of weldment size, the role of residual stresses, the effect of mean stresses, the effect of various loading conditions, the response to variable load histories, and the sources of uncertainty in weldment fatigue resistance. Having proven models for the fatigue strength of weldments would permit the computer-aided fatigue design of weldments. With deterministic models, one could estimate both the average fatigue strength or fatigue life as well as the expected uncertainty in these values. Thus, models for the fatigue behavior of weldments can be a powerful design tool for the engineer.

The results of this study suggest that both the CCN and IP give good predictions; whereas, LEFM models which neglect crack initiation and the behavior of short cracks underestimate the total fatigue life particularly at long lives. LEFM models are substantially improved by considering crack shape development. Both the CCN and IP model seem to capture the long-life behavior of weldments: the former does so because of the use of a bi-linear da/dN versus  $\Delta K_{eff}$  relationship; and the latter does so because of the value of the fatigue strength exponent (b). Both of these attributes of the two models reflect the important contribution of the growth of small (short) cracks to the total fatigue life of weldments in the long-life regime.

While the CCN model is less empirical, more sophisticated and presumably more accurate than the IP model; both the give the same predictions. The IP model is rather simple to use; whereas, the CCN is a very complex, computationally intensive model. However the complexity of the CCN model should not be a deterrent to its future use as sophisticated computer simulations of engineering phenomena become more and more commonplace.

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