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CYCLE COUNTING FOR VARIABLE AMPLITUDE CRACK GROWTH

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Variable amplitude crack growth tests were conducted to determine the best cycle counting method. Rainflow counting provided better estimates of variable amplitude crack growth rate than range counting for two structural steels.

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INTRODUCTION

Constant amplitude test data may be employed to predict crack growth under variable amplitude loading, providing an appropriate cycle counting method is used to identify load ranges.

One of the unique features of cyclic crack growth is the formation of crack front arrest lines called striations. Early work by McMillian and Pelloux (1) showed that one striation forms during the loading portion of each loading cycle. Later work by Abelkis (2) on variable amplitude loading histories showed that striations are associated with the increasing load between successive load reversals. Figure 1 shows the loading history and corresponding fractograph of the fracture surface from reference 2. Striations can be identified with each reversal of the loading history.

One conclusion that may be drawn from this work is that the crack grows during each load reversal. Therefore, range counting should be appropriate for identifying cycles in a variable amplitude loading history.

The crack growth rate for the variable amplitude loading history could then be calculated from constant amplitude test data corrected for stress ratio and sequence effects.

Striation formation is not the mechanism of crack growth but the result of deformation around the crack tip. Several investigators (3-5) have proposed models for fatigue crack growth in terms of the damage accumulated in the plastic zone ahead of the crack tip. That is, plastic strains ahead of the crack tip control the fatigue crack growth rate.

This model is schematically shown in Fig. 2. It is now well established that rainflow counting gives the best life estimates for strain cycle fatigue tests under variable amplitude loading (6-7).

In rainflow counting cycles are identified as closed hysteresis loops. Details of this method may be found in reference 7. Therefore, rainflow counting would be appropriate for crack growth if the mechanism is damage accumulation in the plastic zone ahead of the crack.

For the simple load history shown in Fig. 1, range counting identifies one cycle with amplitude A-A', six cycles with amplitude B-B', etc. Rainflow counting identifies one overall cycle with amplitude A-B' and smaller cycles.

To determine the most appropriate cycle counting method, simple loading histories were constructed. The histories were selected to show the difference between the two cycle counting methods.

SPECIMEN AND MATERIALS

Two structural steels, Man-Ten and RQC-100, were selected for this study. They were part of a cooperative testing program conducted by the Society of Automotive Engineers Fatigue Design and Evaluation Committee. Details of the experimental program may be found in reference 8. The testing program established constant amplitude crack growth rates for the two materials at various stress levels using the WOL specimen shown in Fig. 3. The specimens employed in this investigation were part of the original testing program. Constant amplitude growth rates are shown in Figs. 4 and 5.

The higher strength RQC-100 (σ_y = 825 MPa) the crack growth rate is more sensitive to stress ratio than the lower strength Man-Ten (σ_y = 325 MPa).

ANALYSIS

The crack growth rate for a variable amplitude block of loading, $\Delta a/\Delta B$, may be expressed in the following form.

$$\frac{\Delta a}{\Delta B} = \sum_{1}^{N} \Delta a \qquad (1)$$

where N is the number of cycles in the loading history and Δa is the incremental extension for each load cycle. Crack growth per cycle is calculated from the constant amplitude crack growth rate.

$$\Delta a \simeq \frac{da}{dN} = C\ \Delta K^{m} \qquad (2)$$

where

- da/dN crack growth rate
- ΔK cyclic stress intensity
- C, m material constants

Stress ratio effects may be included by employing a relationship of the following form

$$\Delta a \simeq \frac{da}{dN} = \frac{C\ \Delta K^{m}}{(1-R)^{k}} \qquad (3)$$

where

- R stress ratio
- k = 1.0 for RQC-100
- 0 for Man-Ten

The solid lines in Fig. 5 are from Eq. 3. The general form of the cyclic intensity factor is

$$\Delta K = \Delta P\ f\left(\frac{a}{W}\right) \qquad (4)$$

where ΔP applied load range for each cycle

$f(a/W)$ geometry term dependent on crack size, shape and dimensions of the specimen

Equations 1, 3 and 4 may be combined into the following relationship.

$$\frac{\Delta a}{\Delta B} = C\ f\left(\frac{a}{W}\right)^m \sum_1^N \left(\frac{\Delta P}{(1-R)^k}\right)^m \qquad (5)$$

The first two terms are constants depending on crack length and material. The crack growth rate for a variable amplitude loading history depends on the manner in which load ranges are summed. When the maximum loads are equal, the relative growth rate between constant amplitude and variable amplitude loading may be expressed as follows:

$$\frac{\Delta a/\Delta B}{da/dN} = \frac{\sum_1^n \left(\frac{\Delta P}{(1-R)^k}\right)^m}{P^m} \qquad (6)$$

The relative growth rate is independent of crack length and stress intensity factor. This equation may then be used to determine the relative growth rates for various loading histories if they are normalized in terms of K_{max}.

Threshold stress intensity factor must be considered when analyzing this type of data. The threshold stress intensity factor for those materials is estimated to be 5 $MPa\sqrt{M}$. Stress ratio effects may be included by employing the following relationship (9)

$$\Delta K_{TH} = \Delta K_0 \sqrt{1-R} \qquad (7)$$

where ΔK_0 threshold stress intensity factor, R = 0

ΔK_{TH} threshold stress intensity factor at other stress ratios

No crack growth is calculated for cycles below ΔK_{TH}.

LOADING HISTORIES

Five simple variable amplitude loading histories were employed in this study. The maximum load in each history was held constant while the number and amplitude of small cycles were varied. The loading histories are shown in Fig. 4. Range counting identifies N_2 cycles with a range of R_2. Rain-flow counting identifies one cycle with a range of R_1 and N_2-1 cycles with a range of R_3. One repetition of the loading history is defined as a block.

RESULTS

Equation 6 was used to calculate the relative growth rates for the loading histories and materials employed in this investigation. Analytical results are summarized in Table I. Threshold stress intensity effects were not considered in these calculations. Stress ratio effects were only included for the RQC-100 calculations.

Table II shows analytical results including threshold effects when the maximum stress intensity is 20 $MPa\sqrt{M}$. Cycles that were below the threshold were eliminated from the loading history before cycle counting. Equation 6 was then used to calculate the relative growth rate. Only histories E and F have cycles below the threshold. There is very little difference between the analytical results in Tables I and II.

In the Man-Ten material where the growth rate is not dependent on stress ratio, the smaller high stress ratio cycles do not cause crack growth. The increase in predicted growth for load history F is due to a larger initial cycle that results from removing small amplitude cycles below the threshold stress intensity.

Experimental data for the Man-Ten tests are shown in Fig. 7. Test results for RQC-100 specimens are shown in Fig. 8. Crack growth rate per block is plotted on the vertical axis. The maximum load in all loading histories was constant. The maximum stress intensity in the loading history was used to normalize and plot the growth rate data.

DISCUSSION

Figures 5 and 6 show that the addition of small cycles did not change the crack growth rate for either material. Predictions based on rainflow counting predicted the observed results much better than range counting.

Consideration must be given to those loading histories where rainflow counting is not appropriate. Rainflow counting matches the largest and smallest load to form the overall load range. In the example shown in Fig. 9 events A-B and part of C-D would be paired and counted as a large cycle. When there are enough small cycles between B and C to cause the crack to grow, out of the plastic zone due to loading A-B, these events should not be paired together because the crack has advanced into new undamaged material (see Fig. 2). Rainflow counting would not be appropriate for these kinds of loading histories.

CONCLUSIONS

Rainflow counting is a better cycle counting method than range counting for estimating crack growth under variable amplitude loading histories employed in this investigation.

ACKNOWLEDGMENT

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Man-Ten

Load History	Relative Growth Rate	
	Range Counting	Rainflow Counting
A	1.00	1.00
B	.50	1.02
C	.28	1.02
D	.11	1.01
E	.02	1.00
F	.02	1.01

RQC-100

Load History	Relative Growth Rate			
	Range	Range with Stress Ratio	Rainflow	Rainflow with Stress Ratio
A	1.00	1.00	1.00	1.00
B	.53	.76	1.03	1.09
C	.31	.54	1.02	1.09
D	.14	.32	1.01	1.06
E	.03	.13	1.00	1.01
F	.03	.20	1.01	1.02

TABLE I ANALYTICAL RESULTS

Man-Ten

Load History	Relative Growth Rate	
	Range Counting	Rainflow Counting
A	1.00	1.00
B	.50	1.02
C	.28	1.02
D	.12	1.01
E	.16	1.01
F	.63	1.00

RQC-100

Load History	Relative Growth Rate			
	Range	Range with Stress Ratio	Rainflow	Rainflow with Stress Ratio
A	1.00	1.00	1.00	1.00
B	.53	.76	1.03	1.09
C	.31	.54	1.02	1.09
D	.14	.32	1.01	1.06
E	.19	.29	1.00	1.01
F	.65	.78	1.01	1.02

TABLE II ANALYTICAL RESULTS INCLUDING THRESHOLD STRESS INTENSITY EFFECTS

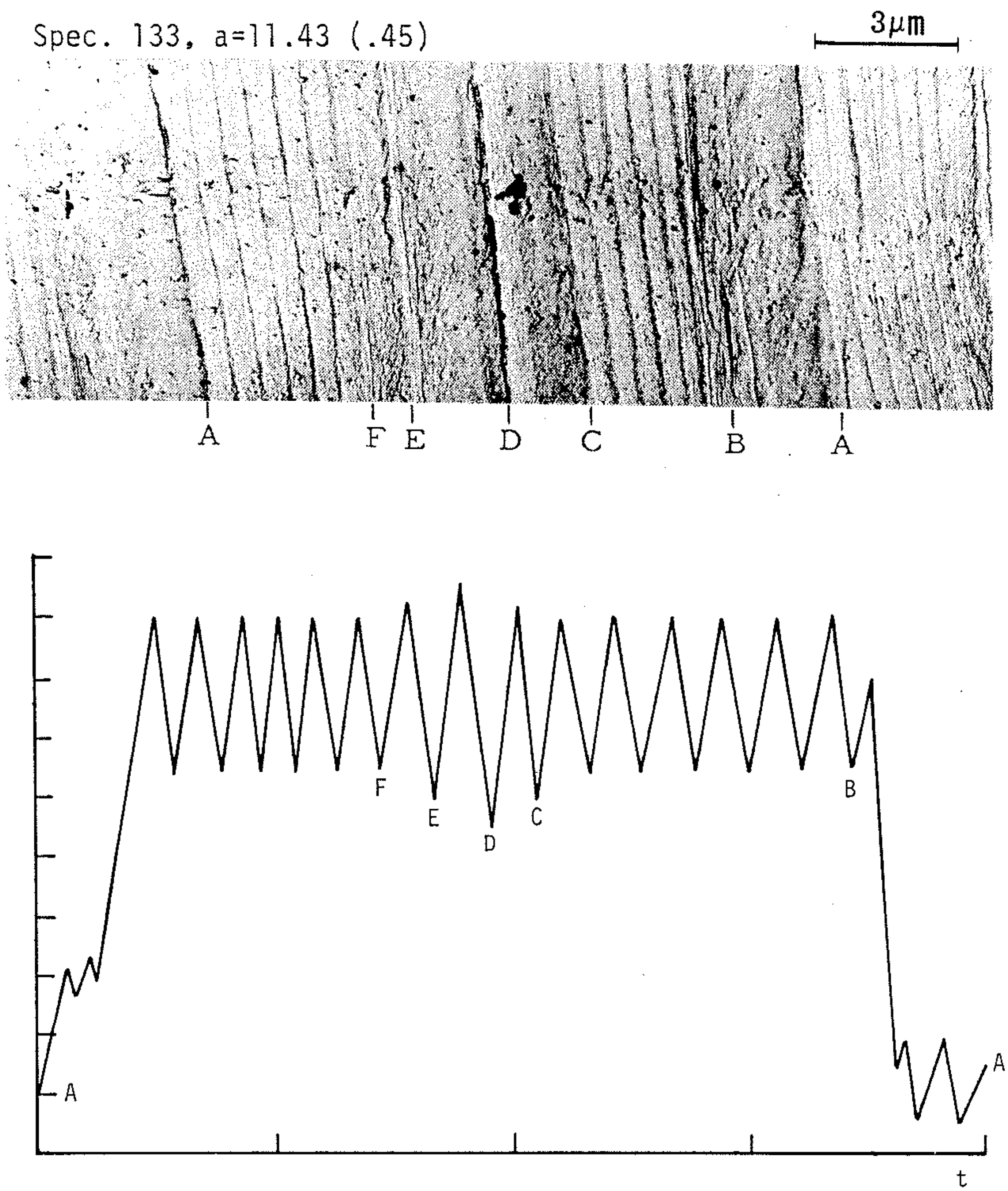


FIG. 1 VARIABLE AMPLITUDE LOADING FATIGUE STRIATIONS

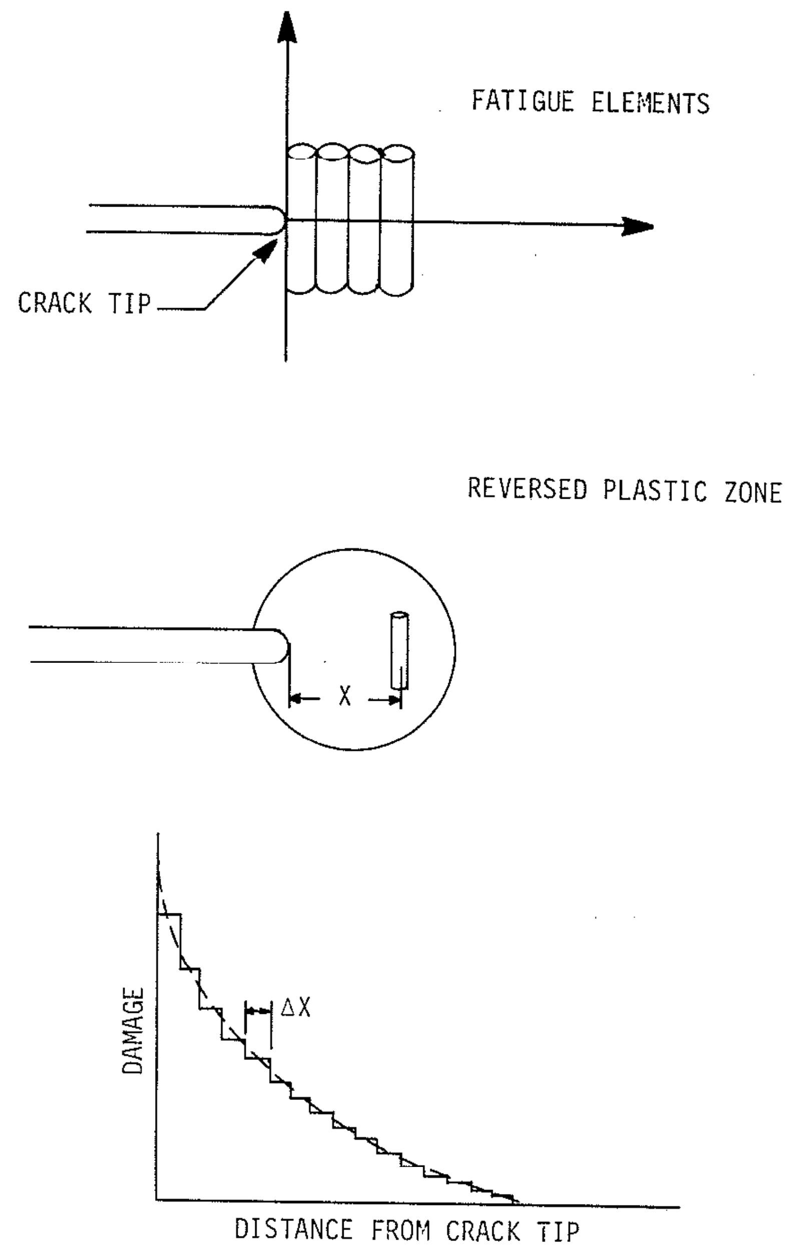


FIG. 2 SCHEMATIC ILLUSTRATION OF FATIGUE CRACK GROWTH PROCESS

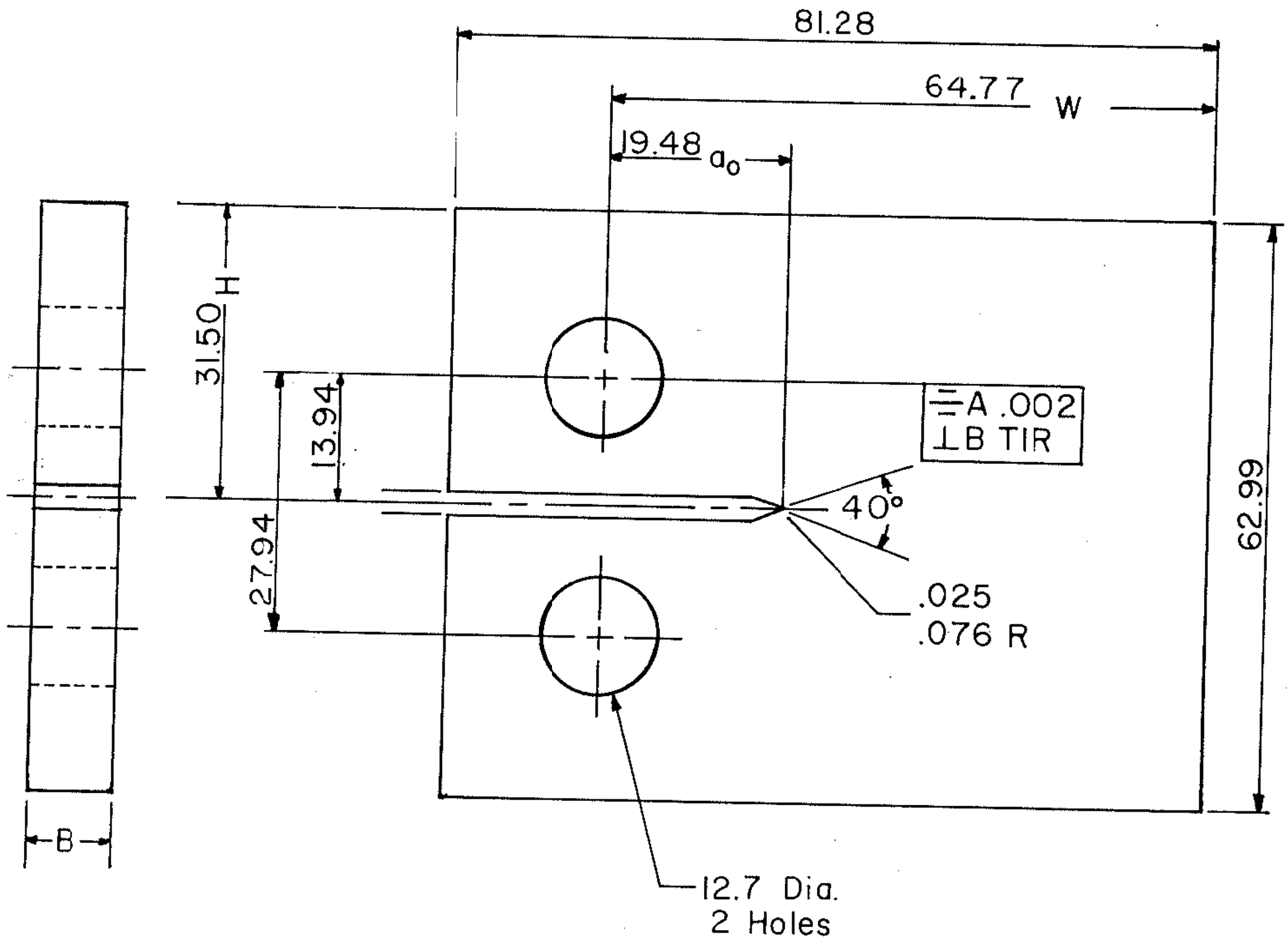


FIG. 3 FATIGUE CRACK GROWTH RATE SPECIMEN (H/W = 0.486)

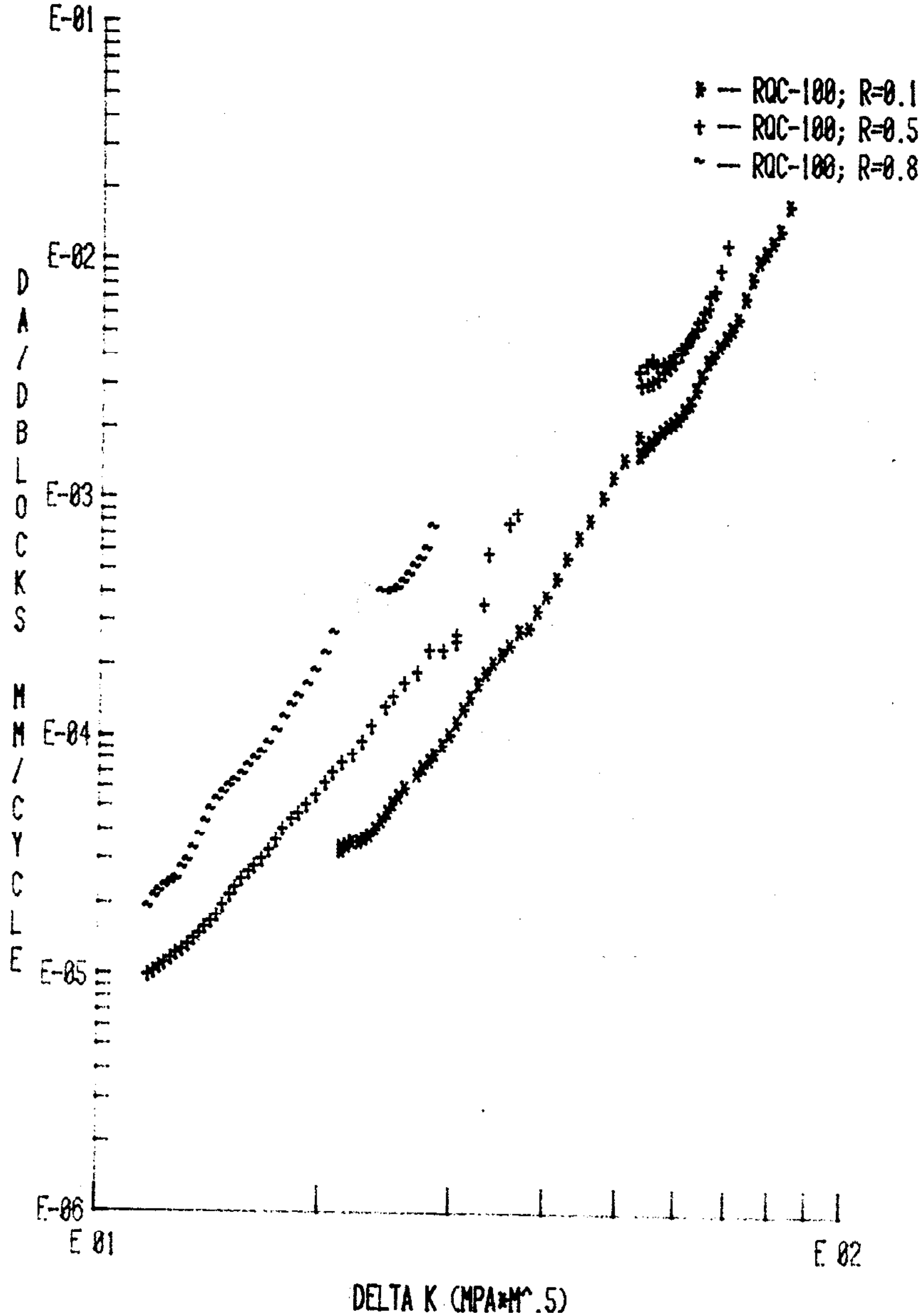


FIG. 4 CONSTANT AMPLITUDE CRACK GROWTH DATA FOR MAN-TEN

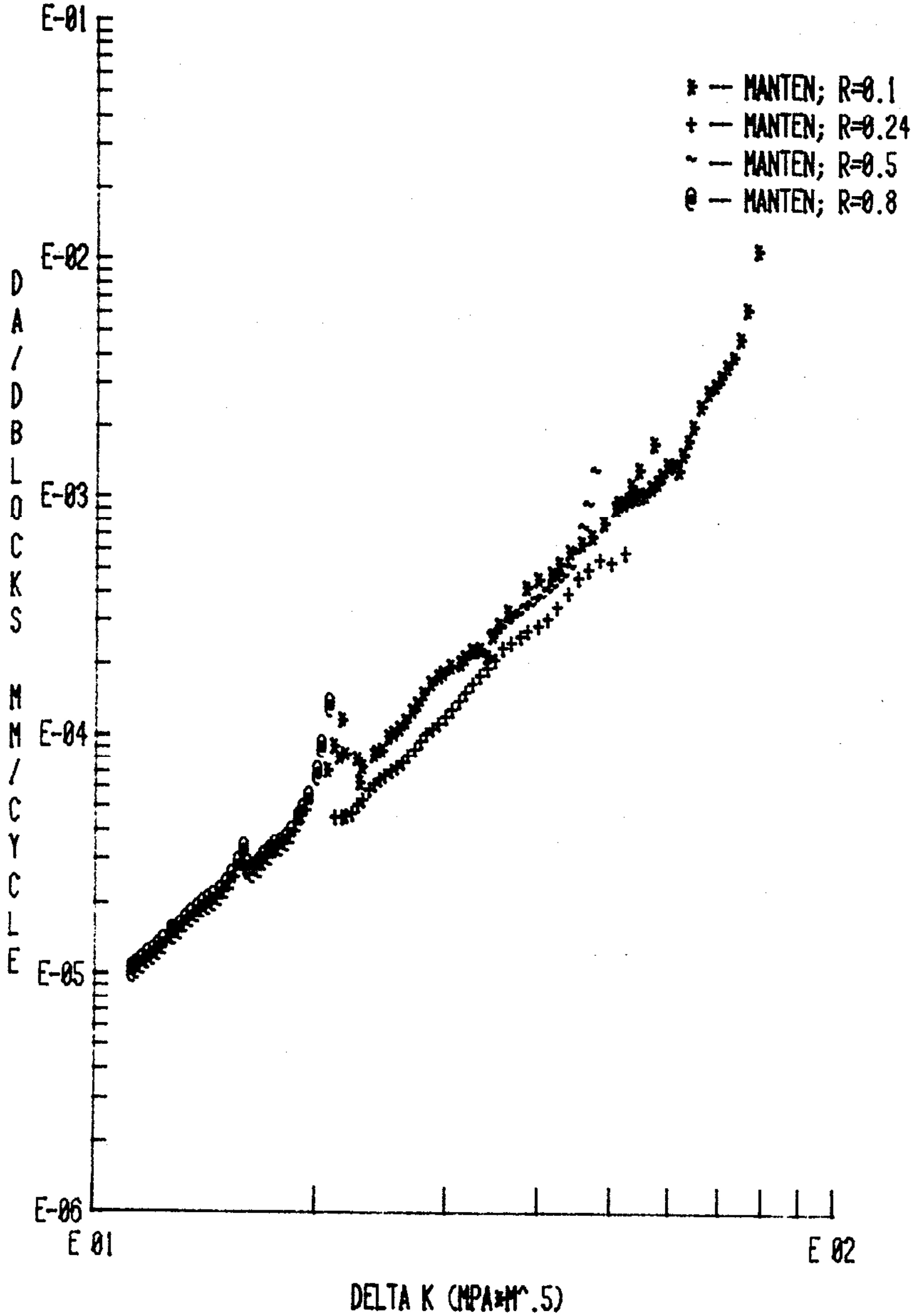
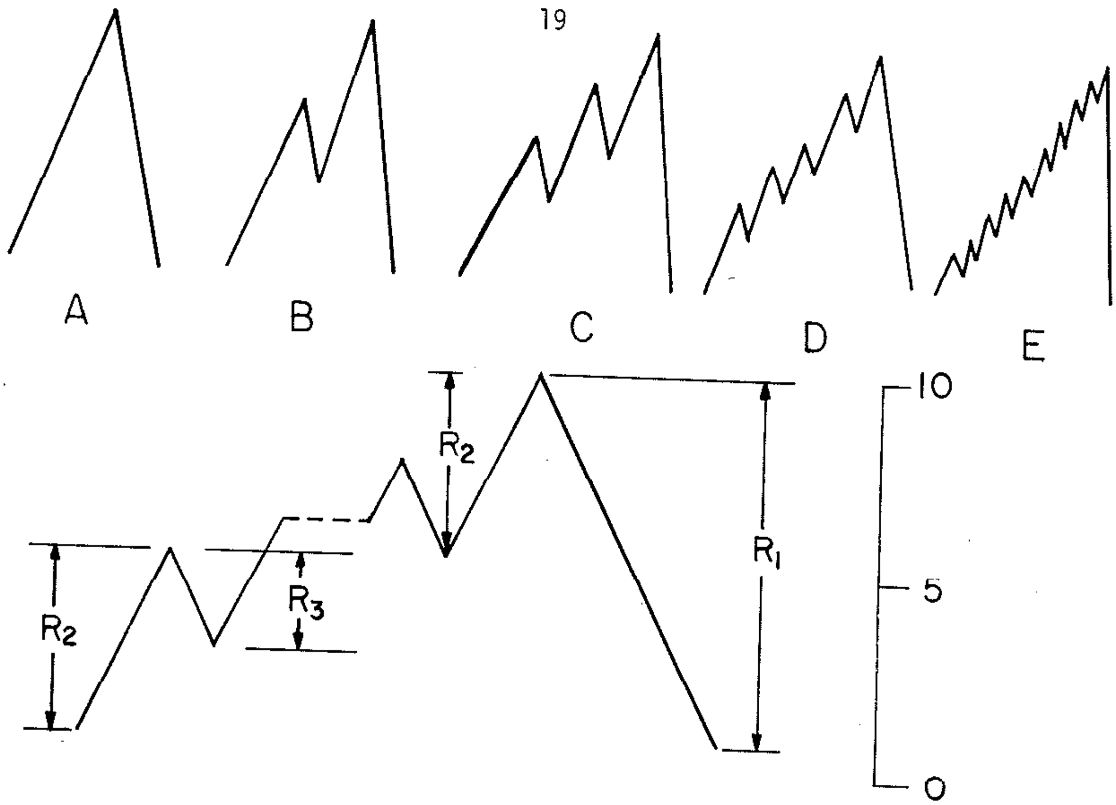


FIG. 5 CONSTANT AMPLITUDE CRACK GROWTH DATA FOR RQC-100



Load History	R_1	N_1	R_2	N_2	R_3
A	9	1	0	0	0
B	9	1	6	2	3
C	9	1	4.5	3	2.25
D	9	1	3	5	1.5
E	9	1	1.5	10	.75
F	9	1	0.9	50	.75

FIG. 6 VARIABLE AMPLITUDE LOAD HISTORIES

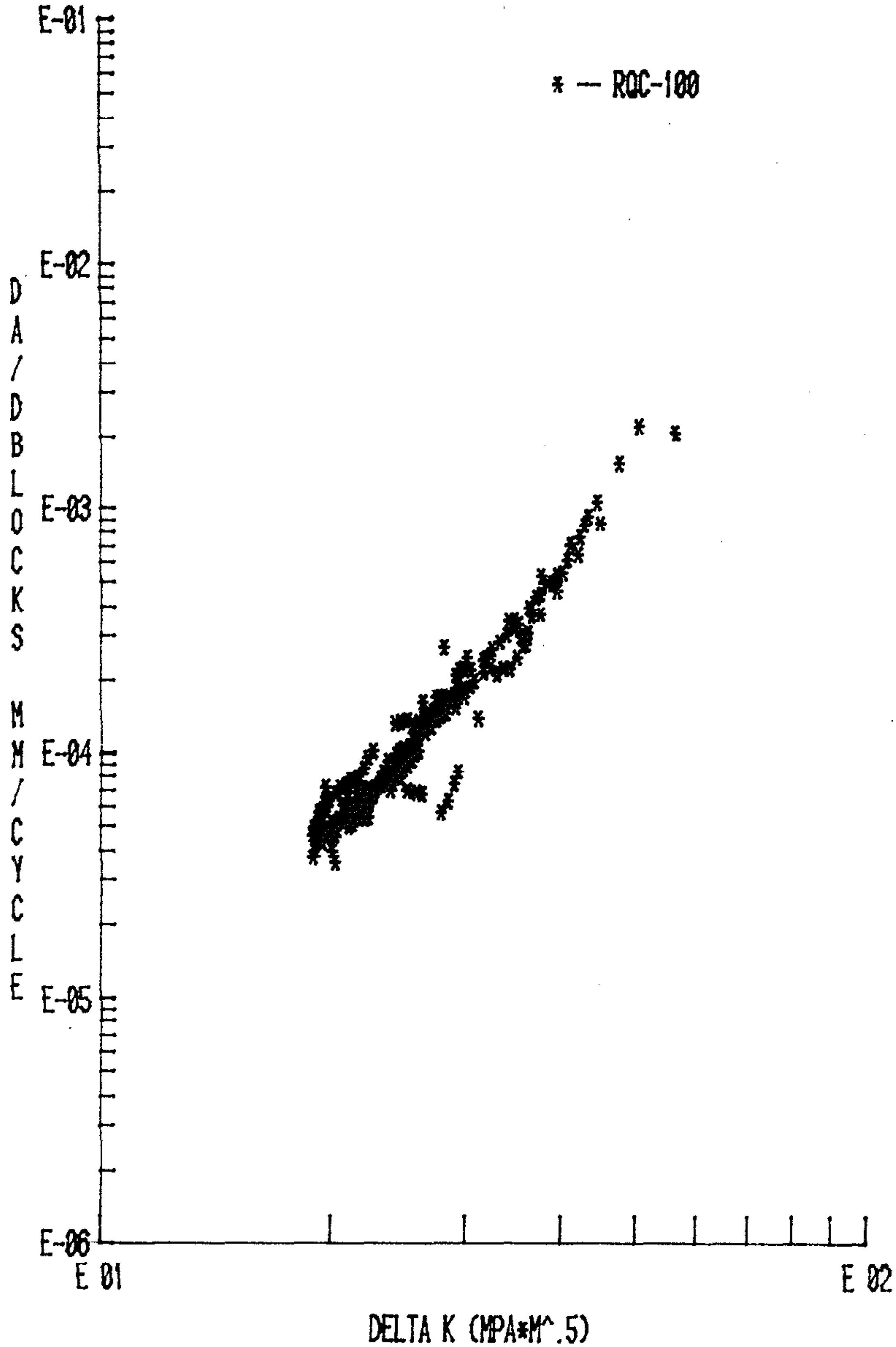


FIG. 7 VARIABLE AMPLITUDE CRACK GROWTH DATA FOR MAN-TEN

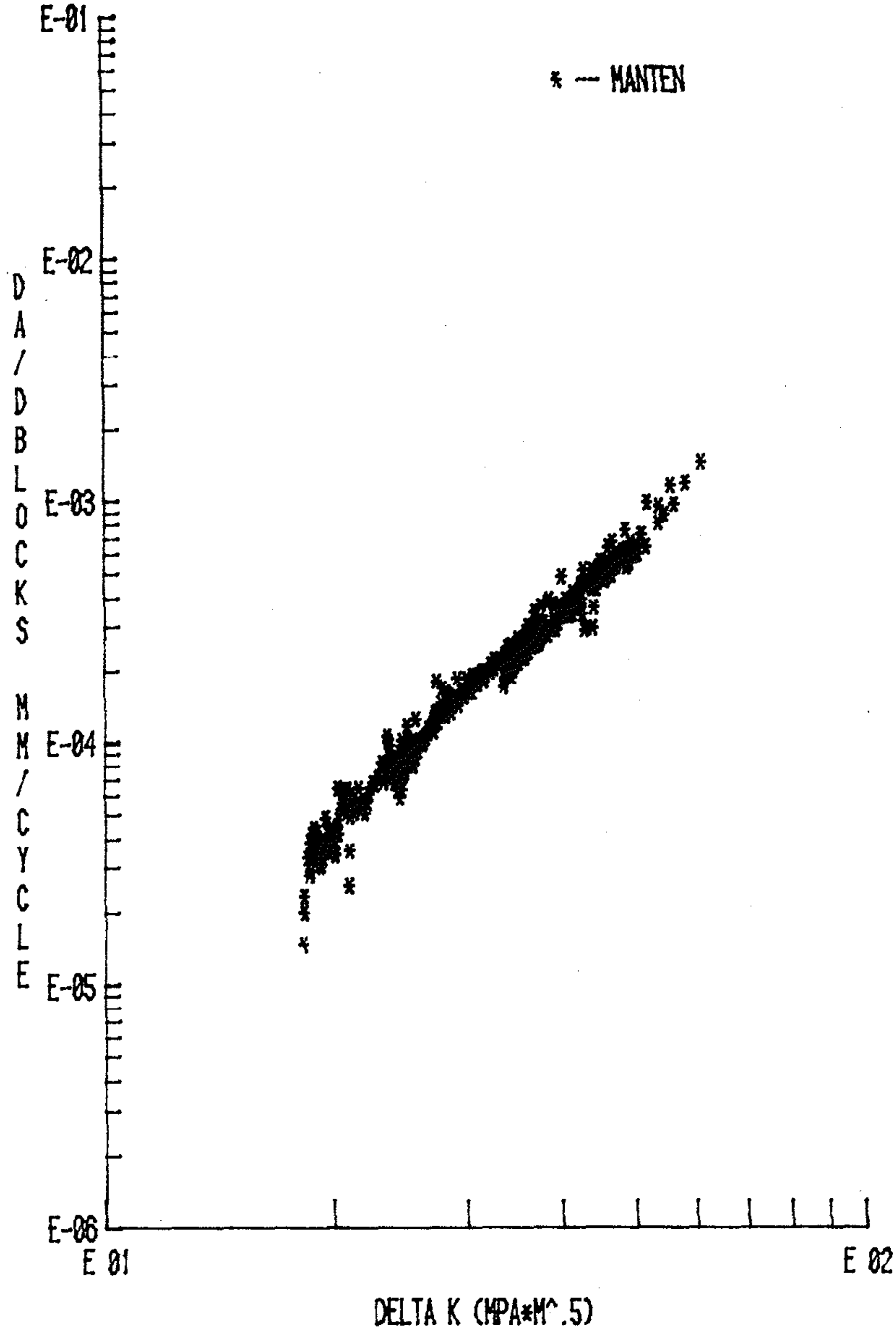


FIG. 8 VARIABLE AMPLITUDE CRACK GROWTH DATA FOR RQC-100

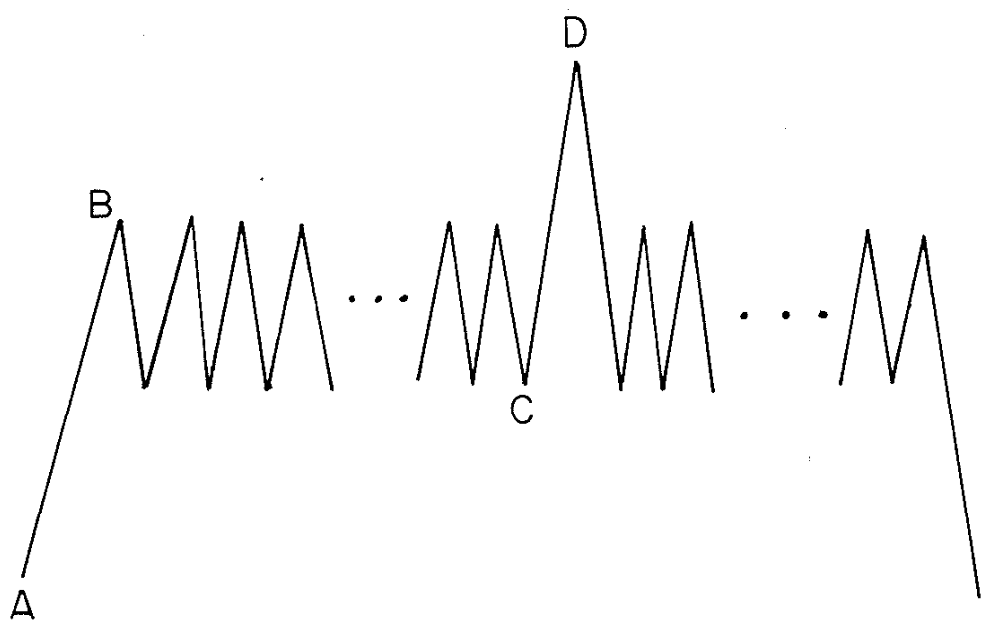


FIG. 9 SAMPLE PROBLEM

PARTIAL LIST OF FRACTURE CONTROL PROGRAM REPORTS

No.

14 "Effects of Graphite Morphology, Matrix Hardness and Structure on the Fatigue Resistance of Gray Cast Iron," M. R. Mitchell, December, 1974.

15 "Review of the Mechanical Properties of Cast Steels with Emphasis on Fatigue Behavior and the Influence of Microdiscontinuities," M. R. Mitchell, June, 1975.

16 "An Analysis of Fretting Damage on Fatigue Strength," Hsing-Chung Yeh, June, 1975.

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24 "Review of Contemporary Approaches to Fatigue Damage Analysis," D. F. Socie and JoDean Morrow, December, 1976.

25 "Material Characterization of SAE 950 XK; Monotonic and Cyclic Stress-Strain Behavior and Strain-Life Response," M. R. Mitchell, February, 1977.

26 "Fundamentals of Modern Fatigue Analysis," D. F. Socie, M. R. Mitchell and E. M. Caulfield, April, 1977.

27 "Material Characterization of a 1% Nickel Steel; Monotonic and Cyclic Stress-Strain Behavior and Strain-Life Response," M. R. Mitchell and R. B. Zweigoron, September, 1977.

28 "Evaluation of Fracture Mechanics Parameters for A27 Cast Steel," E. M. Caulfield, December, 1977.

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30 "Characterization of Ductile Fracture in Wrought Steels and Weldments," E. P. Cox, F. V. Lawrence and G. M. Sinclair, June, 1978.

31 "Effect of Spectrum Editing on Fatigue Crack Initiation and Propagation in a Notched Member," Darrell F. Socie and Paul F. Artwohl, December, 1978.

32 "A Model for Joining the Fatigue Crack Initiation and Propagation Analyses," W.-C. Chen and F. V. Lawrence, November, 1979.

33 "Finite Element Simulation of the Microstructure of Gray Cast Iron," Edward S. Russell, December, 1979.

34 "Cyclic Deformation and Fatigue Behavior of Carburized Steel," John M. Waraniak, May, 1980.

35 "Fatigue Crack Initiation and Growth in Gray Cast Iron," James W. Fash, October, 1980.

36 "Predicting the Fatigue Resistance of Welds," F. V. Lawrence, Jr., N. J. Ho and P. K. Mazumdar, October, 1980.