

## A REVIEW OF FRETTING FATIGUE

by

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### ABSTRACT

The mechanisms of fretting fatigue are briefly reviewed. Seven important variables which influence fretting fatigue strength are discussed and various methods of minimizing fretting damage are presented. Some directions for future work are suggested.

A Report of the  
FRACTURE CONTROL PROGRAM

College of Engineering, University of Illinois  
Urbana, Illinois 61801  
May, 1973

## ACKNOWLEDGMENT

The authors are indebted to Professors T. J. Dolan and H. T. Corten for many helpful suggestions.

Thanks are also due to Mrs. R. A. Mathine, who typed the manuscript, and to Mr. H. T. James, who prepared the figures for this report.

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## I. INTRODUCTION

Fretting is a form of surface damage which occurs when two surfaces in contact undergo repeated sliding or rubbing motion. In metals, high spots or "asperities" on the surfaces in contact may undergo cold welding and be sheared off. In air, these small bits are readily oxidized, in the case of steel or aluminum, and debris in the form of brown or blackish powder appears. This is known as fretting corrosion. Accompanying this action, small surface cracks may be produced which can grow, under fatigue loading, and substantially reduce the normal fatigue strength of the material. Production of an oxide debris does not appear to be a necessary part of the fretting fatigue damage process, since fretting experiments conducted in an inert atmosphere or vacuum continue to show serious loss of fatigue strength with no debris production.

In service, fretting fatigue failures are found in assemblies, such as clamped or bolted joints and press fits of wheels or gears on rotating shafts. In small scale laboratory fatigue testing it becomes more of a nuisance than a safety hazard, since it can lead to unexpected failures in the grip region rather than the test section of the specimen.

Fretting fatigue is a serious industrial problem which deserves particular attention, since reductions in fatigue strength to only 20 percent of the original value may occur under some service conditions. It is the purpose of this report to make a brief "state of the art" review of fretting fatigue literature, which is particularly applicable to the Fracture Control Program.

Considering the fact that fretting fatigue is a rather common industrial problem, it is surprising that only a few reports have been published on this subject in the United States in the last decade. By contrast, the Japanese have devoted considerable effort to fretting research in recent years, concentrating

mainly on medium and low carbon steels. One factor contributing to a lack of enthusiasm for research on this topic may be the lengthy list of variables which have been reported to have a significant effect on fretting fatigue strength. With the advent of new research tools, such as the scanning electron microscope, and improved understanding of factors, such as surface residual stress and fracture mechanics aspects of crack propagation, it is believed that the list of critical variables for fretting fatigue can be reduced to a manageable level in the near future.

## II. FACTORS INFLUENCING FRETTING FATIGUE STRENGTH

The initial references in the bibliography (1-22) deal with fretting corrosion and its mechanism. One of the first systematic investigations of fretting fatigue was carried out by Peterson and Wahl (23), who studied the fatigue of shafts at fitted members. Since that time, investigators have reported a great many factors which affect fretting fatigue damage. Rimbe (43), for example, has compiled a listing of 42 variables from a review of the literature. This extensive listing, however, appears to contain many redundant factors or items of questionable importance. The list of variables which have been demonstrated to be important by a majority of investigators is much shorter and can be summarized as follows.

### 1. Relative Amplitude of Displacement

Fenner and Field (36) found that the fretting fatigue strength of aluminum alloy, based on failure of the specimen, decreases with increasing relative slip amplitude. When the amplitude was greater than 15 microns ( $1 \text{ micron} = 10^{-4} \text{ cm}$ ), they found no further reduction in fretting fatigue strength as is illustrated in Fig. 1. Nishioka and Hirakawa (59) reported essentially the same results based on the failure

of medium carbon steel specimens. However, the fatigue stress at which surface cracks are initiated appears to go through a minimum at a relative displacement of 15 microns. The crack initiation stress appears to increase for larger displacement, but this observation may be due to the fact that wear processes dominate at large slip. Fretting cracks, once formed, would be quickly worn away. The fact that the curves based on failure of the specimen reach a minimum at 15 microns of relative slip and remain low thereafter, as the slip amplitude is increased, supports this viewpoint.

## 2. Clamping Pressure

Corten (28) studied fretting fatigue strength 24ST aluminum and found that the clamping pressure influenced fatigue strength only at very low values of a few thousand psi. Beyond that level, increased clamping pressure resulted in little further reduction of fatigue strength. Essentially the same results were obtained later by Liu, Corten, and Sinclair (32) on a titanium alloy and by Nishioka and Hirakawa (59) on medium carbon steels. This data is summarized in Fig. 2. The hypothesis advanced by Liu et al. (32) suggests that fretting fatigue strength should be insensitive to contact pressure, and this argument is briefly reviewed in Appendix B.

## 3. Materials of the Mating Parts

Liu et al. (32) investigated the fretting fatigue strength of a titanium alloy as influenced by a wide variety of different gripping materials. A portion of their results, which illustrate general trends, are given in Fig. 3. While there are some substantial differences based on the mating materials alone shown in this figure, the overall trend appears clear. If the gripping material is much harder than the specimen, fretting fatigue strength is severely lowered. If the gripping material is much softer than the specimen, fretting fatigue strength is practically unaffected. Points one and

two for magnesium and aluminum (commercially pure) reduce the fretting fatigue strength to 95 percent of the ordinary fatigue strength value. If the grip material is equal in hardness to the specimen material, major fretting damage occurs and further increase in the hardness of the gripping material causes a relatively small further decrease in fretting fatigue strength. The greatest reduction in fretting fatigue strength was observed when a hardened steel grip was employed at the highest gripping pressure of 40,000 psi, as shown in Point 11. The results imply that under some service conditions, fretting fatigue might be avoided or alleviated by separating the rubbing parts by means of soft metal sacrificial shims or inserts of commercially pure aluminum, magnesium, etc. The fretting wear, of course, would then be transferred to the soft sacrificial insert and these would have to be replaced periodically.

Nishioka and Hirakawa studied fretting fatigue of medium carbon steel using different heat treatments to vary the hardness of the specimen material from 170 to 630 Vickers hardness number. The hardness of the grip or "fretting shoe" was held constant at 310 Vickers hardness number. They reported specimen hardness had little effect on fretting fatigue strength based on failure; however, the material displayed slightly greater resistance to fretting crack initiation as hardness was increased. These results are summarized in Fig. 4.

#### 4. State of Stress and Residual Stress

The beneficial effects of a surface residual compressive stress in alleviating fretting fatigue are qualitatively well documented. The precise contribution of the surface residual stress is somewhat obscured by the fact that all methods commonly employed to produce the beneficial surface stress also result in increased surface hardness which is also beneficial. Shot peening, surface rolling, carburizing, nitriding, and surface induction hardening have all been reported to produce substantial increases in fretting fatigue strength. To date, no author has clearly

documented the separate contributions of increased surface hardness and compressive surface residual stress.

In a related study Collins and Marco (38) used a two-step fretting fatigue test to study the effects of static pretension and static precompression of the specimen. In both cases the first step was fretting up to  $10^5$  cycles under the prestress condition. The second step was a simple fatigue test of the fretted sample. They found that the reduction in fatigue strength was much greater for the case of the static precompressive stress than that of the static tensile prestress, as illustrated in Fig. 5. Their explanation of the phenomena is illustrated in Fig. 6. In the case of static precompression, the fretting action produces a local high tensile stress in the surface when the prestress is removed. In the case of the static tensile prestress, fretting action results in a local residual compressive stress in the specimen when the prestress is removed. Their results indicate that for this condition there was practically no reduction in fretting fatigue strength below that of the ordinary fatigue strength of the material.

Nishioka and Hirakawa (49) studied the effect of mean stress on fretting fatigue. They reported that the alternating stress amplitude at which no fatigue cracks are initiated is not affected by mean stress. The fatigue limit, however, based on propagation of minute cracks, decreases with an increase in tensile mean stress. Compressive mean stress retards propagation of the cracks and improves the fatigue limit. They decided that the influence of fretting on stress state was limited to only a very thin surface layer of the specimen. After a small amount of crack growth from the surface, there existed no influence of fretting on the stress state which controlled the propagation of fatigue cracks.

Waterhouse and Taylor (54) studied the origin of fretting cracks by means of the scanning electron microscope and decided that the cracks originated in the boundary between slip and nonslip areas in the contact region. They state that this should

be a region of high stress concentration. When a crack is formed in this region, it relieves the stress concentration and the boundary moves inward, resulting in the initiation of another crack which propagates more rapidly than the first.

## 5. Environment

Oding and Ivanova (31) have found that the fretting fatigue damage in air is greater than that in hydrogen, as is shown in Fig. 7. Waterhouse et al. (58) reported that if the material has a normally protective oxide film on the surface, corrosion fatigue is less damaging than fretting fatigue. However, if the material is not corrosion resistant, corrosion fatigue is more damaging than fretting fatigue. The combination of corrosion plus fretting fatigue produces no more damage than corrosion fatigue alone, as is illustrated in Fig. 8.

Nishioka and Hirakawa (59) studied fretting fatigue of medium carbon steel in air and in an argon atmosphere and found little difference between the inert dry argon results and those of the oxidizing air atmosphere. They concluded that this was due to the fact that the mechanical damage process plays the major role in reducing the fatigue limit through fretting. The oxide debris is incidental to the fretting fatigue process and not a requirement for lowering fatigue strength. This conclusion is in agreement with the findings of Liu et al. (32).

## 6. Cyclic Frequency

Endo et al. (51) studied the effect of cyclic frequency on fretting fatigue strength of a 0.34 percent carbon steel. They found that the fretting fatigue damage increases with lower cyclic frequencies in both bending and twisting tests, as shown in Figs. 9 and 10. They reported that the fretting fatigue damage reaches a saturation value very early in the total fatigue life and the subsequent crack propagation period constitutes the larger portion of life. The lower the frequency, the shorter the saturation period for fretting fatigue crack formation.

## 7. Contact Friction

Since the formation of fretting fatigue cracks appears to be associated with the cold welding at asperities and since this cold welding phenomena or tendency should be reflected in the coefficient of friction, some recent workers have begun to study coefficient of friction under dynamic conditions as a function of number of cycles. Nishioka and Hirakawa (48) found the coefficient of friction increased steadily and the relative slip decreased with an increasing number of cycles in the very early stages of testing. A saturation value of coefficient of friction was achieved at something less than 1000 cycles. The material used in their investigation was a steel.

Milestone (52) used a specially designed apparatus to study the coefficient of friction in fretting joints and had essentially the same results, as illustrated in Fig. 11. The materials employed in his study were a 7075 T6 aluminum alloy and a titanium 6 aluminum 4 vanadium alloy. It appears that for the purpose of fretting fatigue analysis, which usually concerns itself with lifetimes in excess of  $10^5$  cycles, we may consider that the contact friction remains at a saturation value, since fretting debris is accumulated between the contact surfaces and the frictional force stabilizes at some maximum value.

## III. MECHANISMS OF FRETTING FATIGUE

As fretting fatigue cracks are initiated by fretting, an understanding of the mechanisms of fretting is useful in the investigation of fretting fatigue phenomena. Since the emphasis of this report is to be on fretting fatigue, the mechanisms of fretting can be stated briefly. It is usually postulated that the asperities or high points of the contacting metal surfaces become cold welded under the sliding action or relative displacement. The stronger metal then tears bits from the asperities of

the weaker metal. These bits are subsequently oxidized and turn up in the form of an oxide debris, which contributes further to abrasive wear. The mechanisms of fretting thus include both a chemical factor and a mechanical factor. Since fretting damage can occur in a vacuum or in a non-oxidizing atmosphere, such as dry argon, we may consider that the mechanical factor plays the major role in the fretting damage process. A brief discussion of the mechanisms of fretting is provided in Appendix A.

It is well known that the damage of fretting may cause the fatigue failure of parts at very low applied stress levels. Many investigators have studied factors which influence fretting fatigue strength, but only a few have proposed a mechanism for fretting fatigue (27, 32, 47, 48, 49, 53, 54, 60).

#### 1. Horger (27)

Horger studied the effect of fretting corrosion on the fatigue strength of press-fitted axle shafts. As the shaft rotated the alternating contraction and elongation of the axial fibers produced a minute sliding action on the end portion of the hub. Horger suggested this periodic sliding under pressure led to a "molecular plucking" of the surface metal and ultimately to fatigue crack initiation. Once fatigue cracks are initiated in the fretting region at relatively low stresses, then occasional higher stresses will propagate such incipient cracks to failure. Horger also concludes that if the environment (sliding contact member) producing fretting or corrosion is removed, once the crack has initiated after a large number of cycles of low stress values, then such cracks will show little if any propagation at the same stress required to start the crack. Finally, Horger concludes that the geometry or shape of the member and introduction of favorable residual stresses offer the best means for improvement of fretting fatigue strength.

## 2. Liu, Corten and Sinclair (32)

An investigation was made to determine the influence of a number of variables on the fretting fatigue strength of RC 130 B titanium alloy. The variables studied included different gripping materials, hardness of the gripping materials, gripping pressure, surface preparation of specimens, dry lubricants, metallic coatings, and special screen gripping shims.

An analysis of experimental results suggested that the primary mechanism responsible for fretting damage was the repeated frictional shear stress on the asperities or surface "high spots" which were in contact.

Based on prevention of fatigue crack initiation by this mechanism, a mathematical expression was proposed which relates the fretting fatigue strength to the fatigue limit of the specimen material, the hardness of the gripping material, and the coefficient of friction. The relationship is expressed as shown in Eq. 1. A more detailed derivation is provided in Appendix B.

$$\sigma_{alt} = \sqrt{4\tau_{alt}^2 - 1.04 \times 10^6 \frac{\mu^2 H^2}{0.151 + 4\mu^2}} \quad (\text{Eq. 1})$$

$\sigma_{alt}$  = allowable alternating bending stress in specimen, psi.

$\tau_{alt}$  = fatigue strength of specimen material in shear, psi.

$\mu$  = coefficient of friction

$H$  = diamond pyramid hardness number of grip material,  
Kg/mm<sup>2</sup>

For particular values of  $\mu$  and  $H$ , fretting fatigue cracks will be initiated if the applied repeated bending stress is greater than  $\sigma_{alt}$  given by Eq. 1.

### 3. Nishioka and Hirakawa (47, 48, 49)

In a study of a medium carbon and two low alloy steels using a cylindrical steel shoe on a plate, they reported that microcracks initiated but did not propagate to complete fracture after  $10^7$  cycles. Fatigue cracks were observed in the early stage of fretting. They note that the phenomena that cracks which had initiated in the early stage and did not propagate is similar to the behavior of fatigue cracks that have been observed in specimens with very sharp notches (non-propagating cracks).

They analyzed the stress state in the fretted region and defined a stress concentration factor  $K_t$  as follows in Eq. 2. Details of the derivation are provided in Appendix C.

$$K_t = \left| \frac{\sigma_R}{2\sigma_a} \right| = \left| 1 - 2\mu \left( \frac{\chi}{a} \right) \left( \frac{P_0}{\sigma_a} \right) \right|$$

$$= \left| 1 - 4\mu \left( \frac{\chi}{a} \right) \left( \frac{P}{\pi a \sigma_a} \right) \right| \quad (\text{Eq. 2})$$

$\sigma_R$  = range of alternating normal stress

$\sigma_a$  = alternating bending stress

$\mu$  = coefficient of friction

$\chi$  = distance from the line of maximum contact pressure  
( $\chi \leq a$ )

$a$  = half contact width

$P_0$  = maximum contact pressure (Hertzian)

$P$  = contact load

From the relationship of  $K_t$  on the fretted surface, they concluded that the initiation of fatigue cracks at low alternating stress under fretting conditions is due to the high stress concentration at the contact surface caused by the frictional force of fretting. The occurrence of cracks seemed to depend on the stress state but not

on the materials. The influence of fretting on the stress state is limited to only a very thin surface layer of the specimen. After a slight crack growth from the surface, there exists no influence of fretting on the stress state which controls the propagation of fatigue cracks.

#### 4. Waterhouse and Taylor (54)

In a study of a 0.7 percent carbon steel the authors concluded that the origin of fatigue cracks in fretting fatigue is the boundary between the slip and non-slip areas in the contact region. They arise at this boundary because of the high stress concentration. When a crack is formed it relieves the stress concentration at this point and the boundary between the slip and non-slip areas moves inwards resulting in the initiation of another crack which propagates more rapidly than the first crack because of the higher stress concentration. They suggest that crack formation can be prevented or retarded by insuring that slip occurs over the entire contact region.

#### 5. Hoepfner and Goss, (53, 60)

From a review of the literature and their experimental results Hoepfner and Goss concluded that fretting fatigue was a result of mechanical effects between the contacting surfaces and that chemical factors were a secondary consideration. They suggested that small cracks are initiated at points of high stress concentration near contacting asperities. They introduced fatigue cycle ratio and damage boundary concepts as shown in Figs. 12 and 13. The cycle ratio was defined as the ratio of the number of cycles to initiate a crack of specified size at a given stress to the number of cycles to failure at that stress,  $C_R = \frac{N_i}{N_f}$ . The effect of fretting is to reduce this cycle ratio. Fretting damage thus defined is permanent and once the fretting damage boundary (C or D in Fig. 13) is exceeded, removal of the source of fretting should have no effect on the remaining fatigue life.

#### 6. Collins and Tovey (62)

Two postulated mechanisms of fretting fatigue were examined experimentally. First, an abrasive particle-pit digging mechanism and second, an asperity contact-microcrack initiation mechanism were studied. From experimental results it was concluded that the asperity contact-microcrack initiation mechanism was dominant. Furthermore, it was found that the direction of fretting motion relative to the direction of subsequent fatigue loading had a marked influence on fatigue strength. Fretting in the same direction as subsequent fatigue loading produced the greatest loss of strength.

### IV SUMMARY AND SUGGESTED METHODS OF IMPROVING FRETTING FATIGUE STRENGTH

A preponderance of evidence indicates that fretting fatigue is primarily a mechanical rather than a chemical problem. The process involves two stages, (1) a crack initiation stage wherein small cracks are formed at regions of stress concentration near cold weld junctions and (2) a propagation stage wherein the minute cracks may be propagated to failure by the fluctuating nominal stress.

The prime variables controlling these processes are (a) the surface hardness or yield strength of the materials in contact, (b) the contact pressure and (c) the coefficient of friction between the two. Increasing surface hardness or yield strength, decreasing contact pressure and reducing coefficient of friction all tend to reduce the "initiation" crack size of stage 1 and consequently improve fretting fatigue strength. Surface compressive residual stress fields can stop or retard propagation of these cracks in stage 2 and, thus, also increase fretting fatigue strength.

Case studies of successful "fixes" of fretting fatigue problems are neither numerous nor well documented, nevertheless, all appear to involve application of one or more of the aforementioned principles. Specific methods of improving strength vary widely in cost and effectiveness, each possesses its own special characteristics and drawbacks

and, in general, the method selected is determined by the service conditions. Suggested methods of improving fretting fatigue strength are as follows:

#### 1. Induce Surface Compressive Residual Stress

The methods commonly employed to induce surface compressive stress, all, also produce increased surface hardness and, thus, provide a twofold benefit.

a) Shot Peening is reasonably effective and relatively inexpensive. It produces a shallow, work hardened, compressive surface layer which, on small components, can recover up to 50% of the normal strength loss due to fretting.

Typical results (42) are shown in Fig. 14.

b) Surface rolling with hardened rollers produces results similar to shotpeening but is far more expensive. It is used when extra penetration and compressive layer depth are required, as in larger shafts and components. The effects of surface rolling can penetrate to 1/4 inch or more below the surface. Some typical results for surface rolling (40) are given in Fig. 15.

c) Nitriding or carburizing followed by quenching can produce a shallow, very hard, wear resistant surface layer which is compressively stressed. It should be quite effective in raising the fretting fatigue strength of small components but hard documentation is lacking.

d) Surface induction hardening will produce results similar to carburizing in a much deeper surface layer. It should be better for larger components.

While industrial application to fretting problems is common, quantitative data as to percent improvement to be expected is lacking.

2. Tensile prestrain accompanied by fretting produces a shallow compressively stressed surface layer in the fretted region. Laboratory results (38) show that the metal retains nearly 90% of the original fatigue strength but it must be noted that the layer is very shallow and would eventually be worn away.

3. Sacrificial, soft metal shims or inserts of commercially pure aluminum or magnesium between the normal contacting surfaces can leave the fatigue strength at 95 to 100% of its normal value. The drawback to this technique is that the soft metal is fretted away and must be replaced periodically.
4. Lubrication to reduce the coefficient of friction ranges widely in effectiveness depending on the character of the lubricant. Improvement in fretting fatigue strength due to treatment with oils or greases ranges from 0 to only a few percent. The high viscosity materials seem to do best but, in any event, improvement is only marginal. Some results for the lubrication of steel surfaces in contact, such as those summarized in Fig. 16, appear significant until the stress scale is examined more carefully. Here, the normal scatter to be anticipated in long life fatigue strength would occupy much of the range shown.

Dry lubricants, such as molybdenum disulphide, are better able to resist high contact pressure and improvements of up to 20 percent in fretting fatigue strength have been reported.

5. Special surface coatings remain a relatively unexplored area. Under relatively low contact pressures (1000 psi) teflon coating produces good results until the teflon is worn through and contact of the base metals occurs. Thin coatings of soft metals such as aluminum provide little improvement in fretting fatigue strength as they quickly wear away although Bowers (42) shows some improvement following spraying with an aluminum, 1% zinc alloy in combination with shotpeening in Fig. 14. Hard metal plating (chromium) and flame spraying with tungsten provide little improvement in fretting fatigue strength. While the wear rate is reduced by these coatings they apparently contain many surface fissures which soon lead to crack growth in the base metal. One exception to this experience with hard metal coatings is shown in Fig. 17 where a significant improvement in strength accompanied molybdenum coating.

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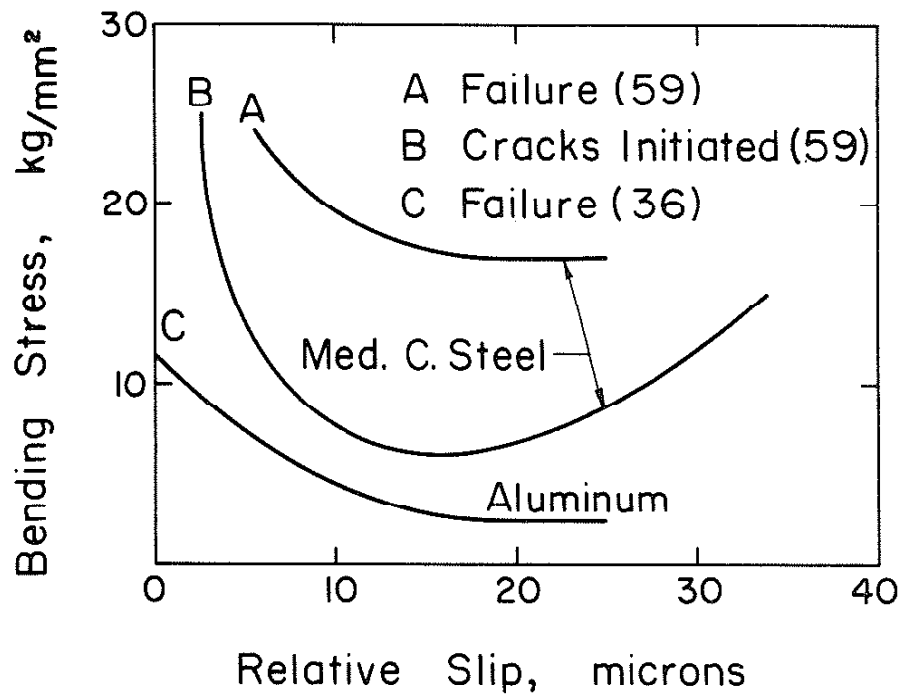


Fig. 1 Effect of Relative Slip on the Fretting Fatigue Strength(Ref. 36, 59)

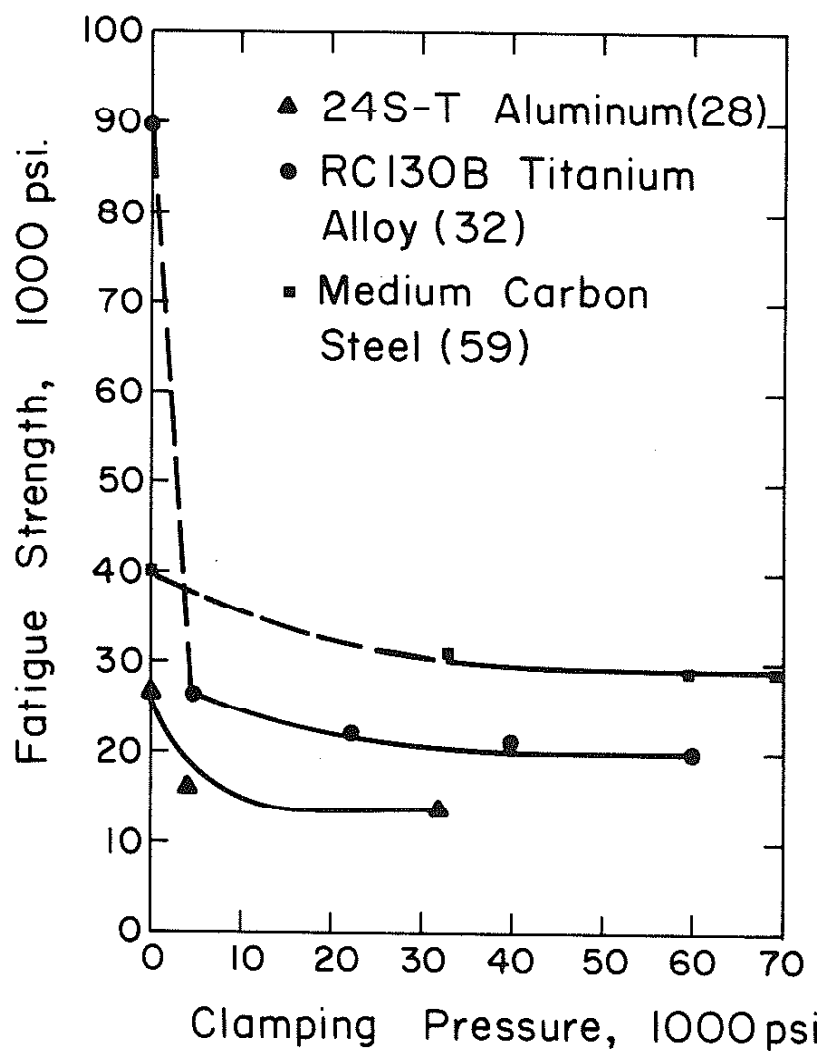


Fig. 2 Effect of the Clamping Pressure on the Fretting Fatigue Strength (Ref. 28, 32, 59)

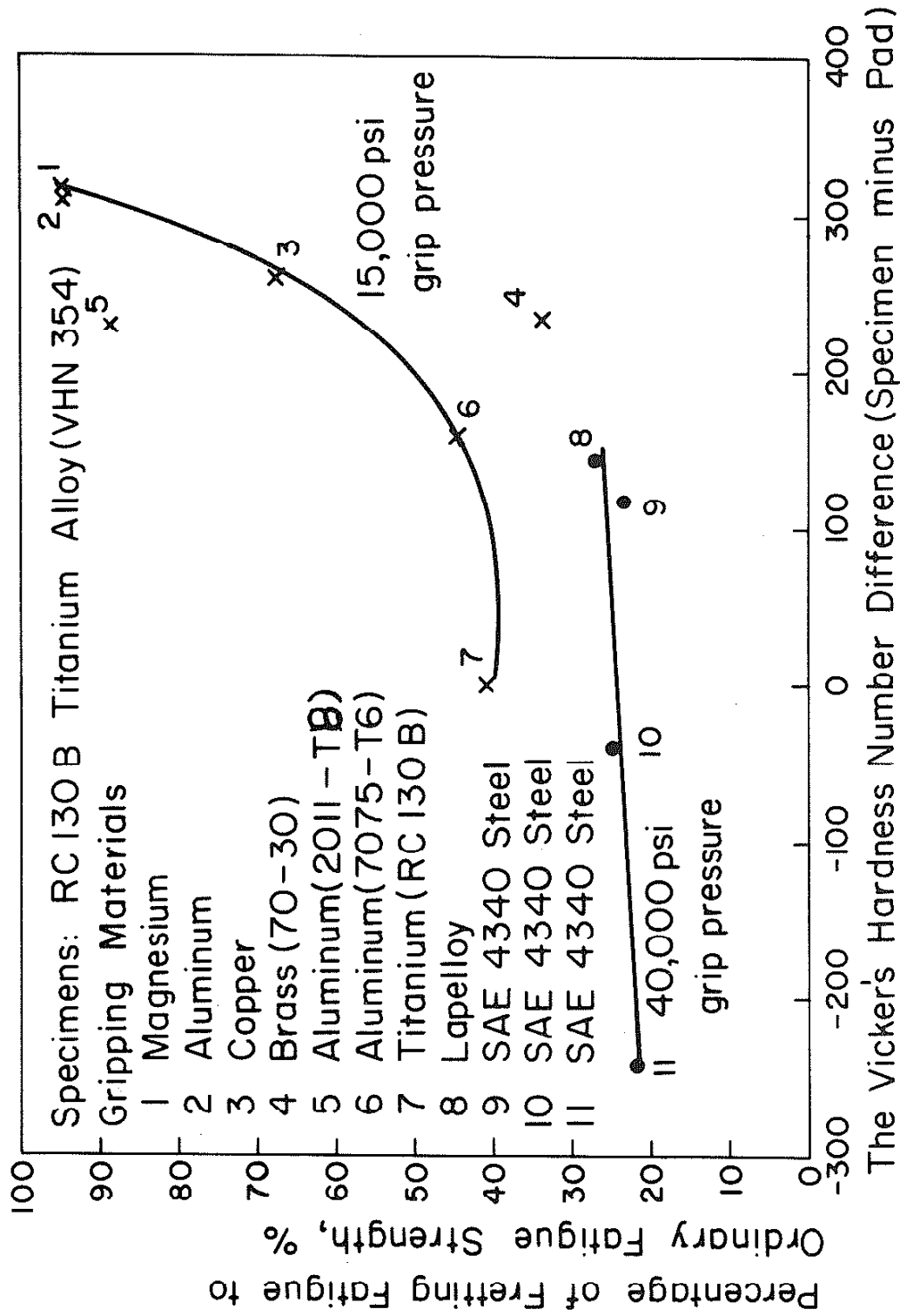


Fig. 3 The Relation Between the Fretting Fatigue Strength and the Hardness Number Difference Between Specimen and Pad (Ref. 32)

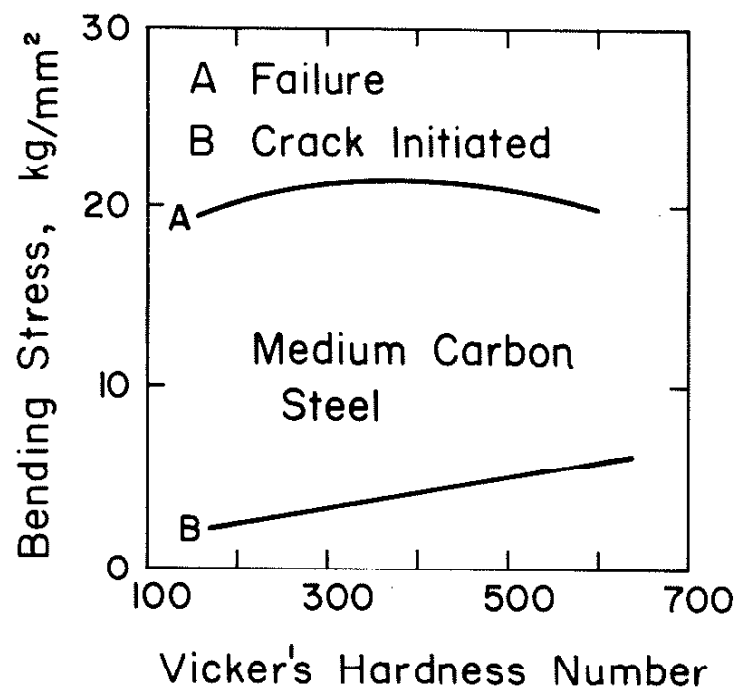


Fig. 4 Effect of the Hardness of Specimen on the Fretting Fatigue Strength (Ref. 57)

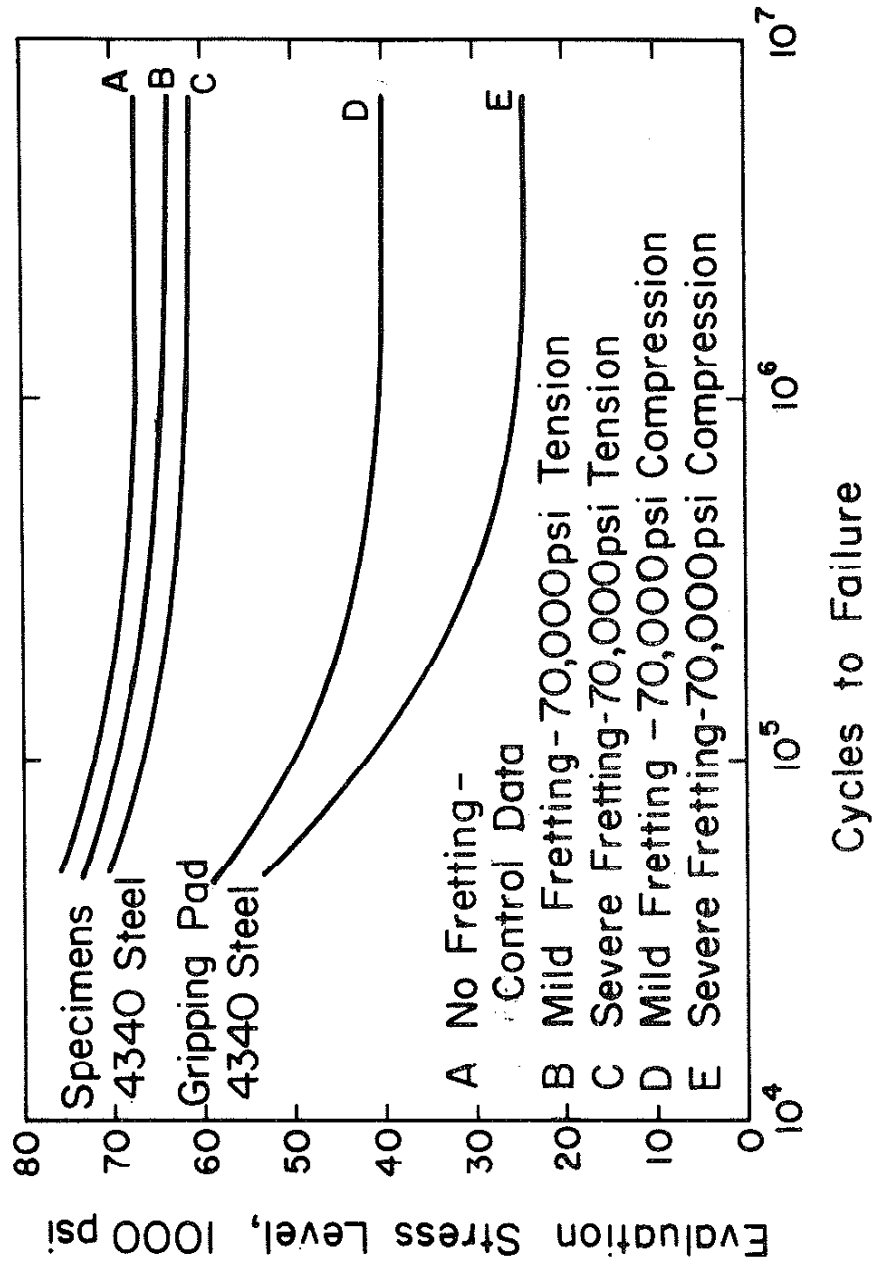


Fig. 5 Summary of Stress Versus Cycles to Failure Data  
for all Fretting-Fatigue Tests Conducted (Ref. 38)

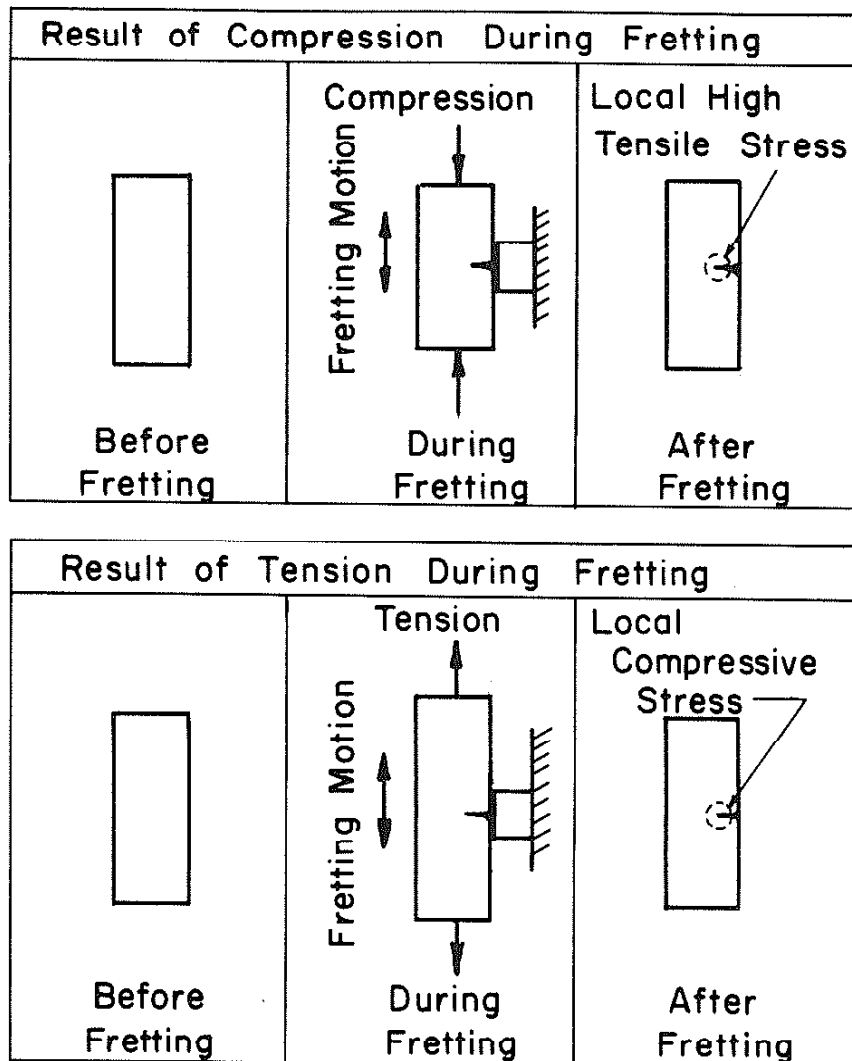


Fig. 6 Sketches Illustrating a Possible Explanation of Why Specimens Fretted Under Static Compressive Stresses Exhibit a Lower Fatigue Strength Than Specimens Fretted Under Static Tensile Stresses(Ref. 38)

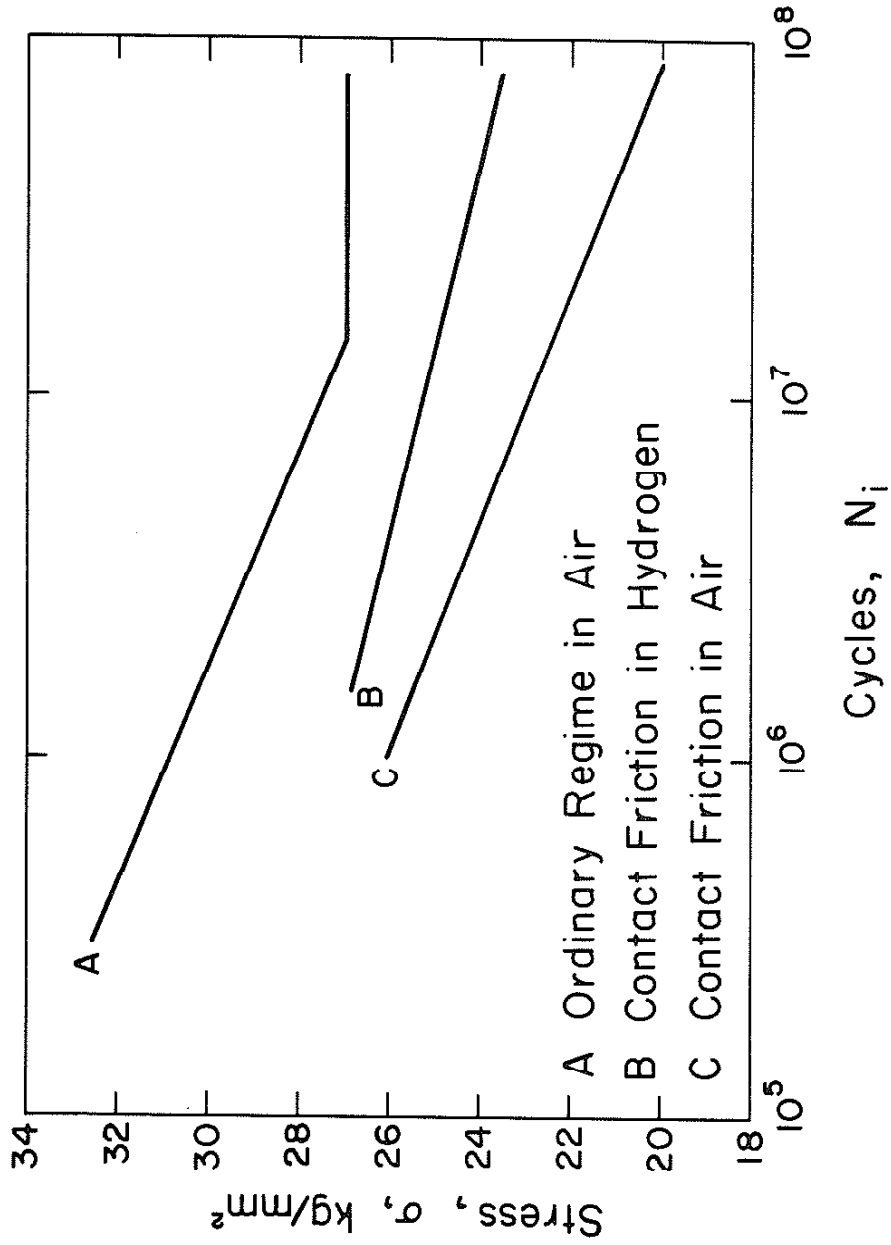


Fig. 7 Relation Between Stress  $\sigma$  and Number of Cycles to Failure  $N_i$  for XH3M Steel (Ref. 31)

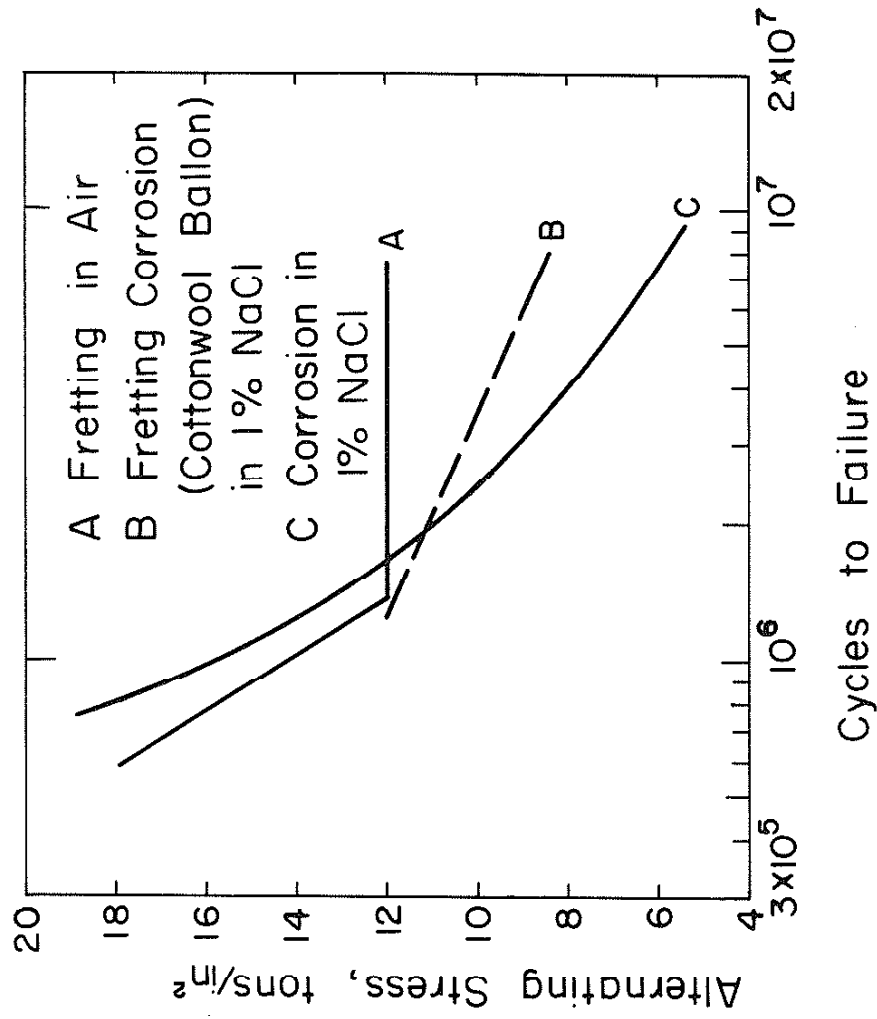


Fig. 8 S-N Curves for 0.7C Steel (Ref. 58)

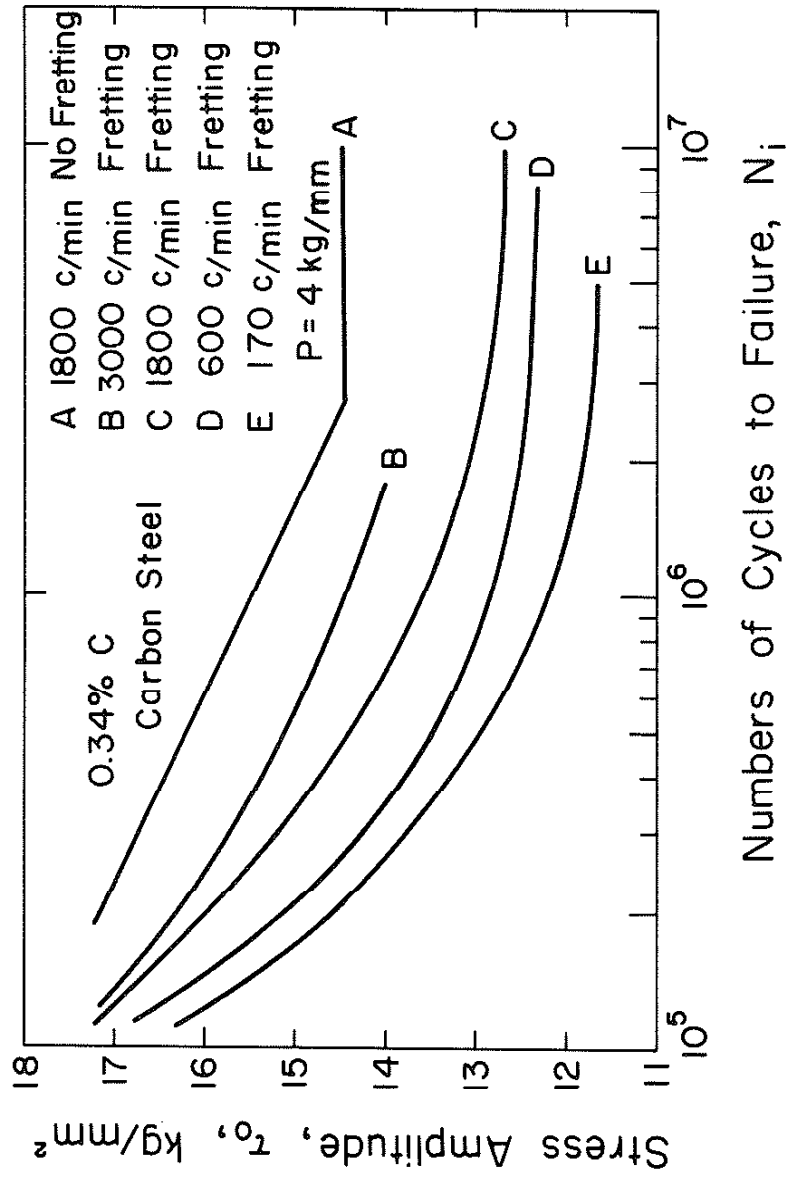


Fig. 9 S-N Diagram of Fretting Fatigue Tests and No Fretting Fatigue Tests under Completely Reversed Twisting at Various Frequencies (Ref. 51)

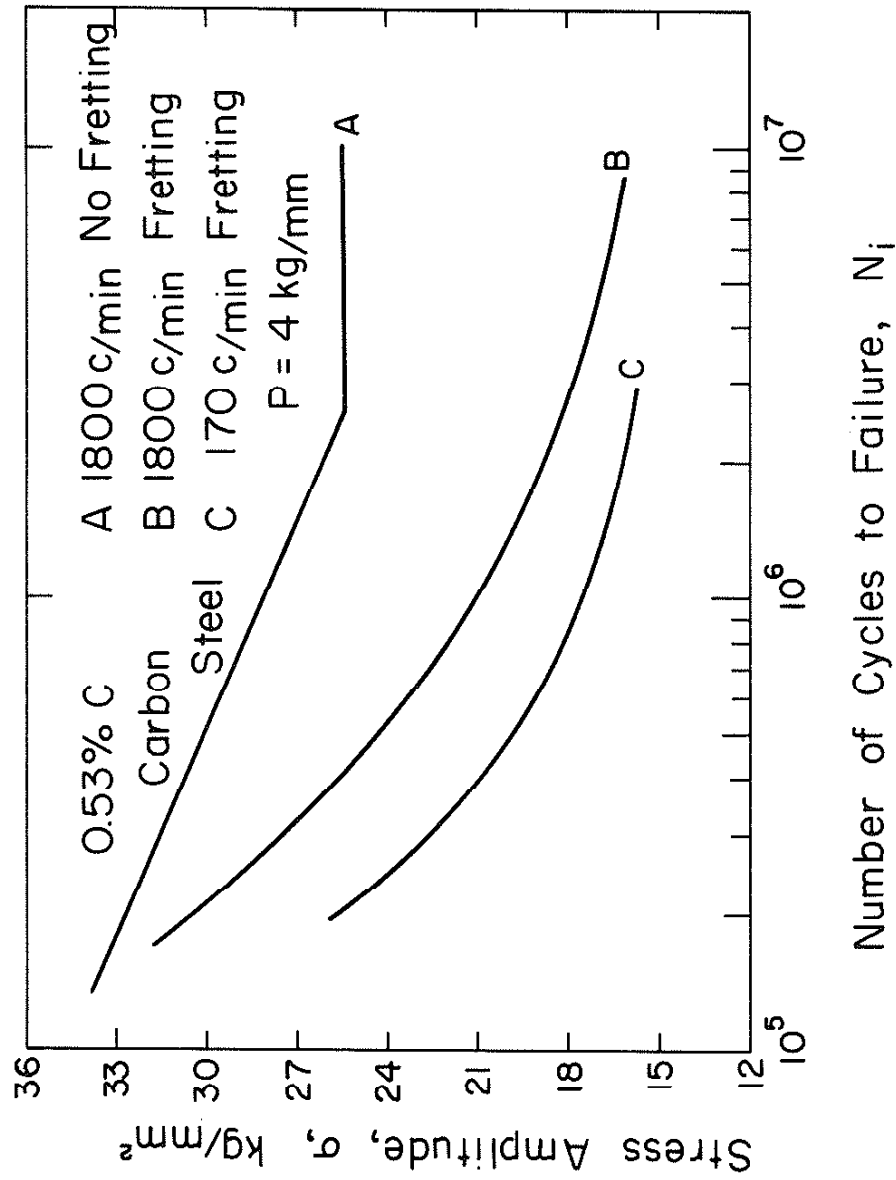


Fig. 10 S-N Diagram of Fretting Fatigue Tests and No Fretting Fatigue Tests under Completely Reversed Bending of Two Frequencies (Ref. 51)

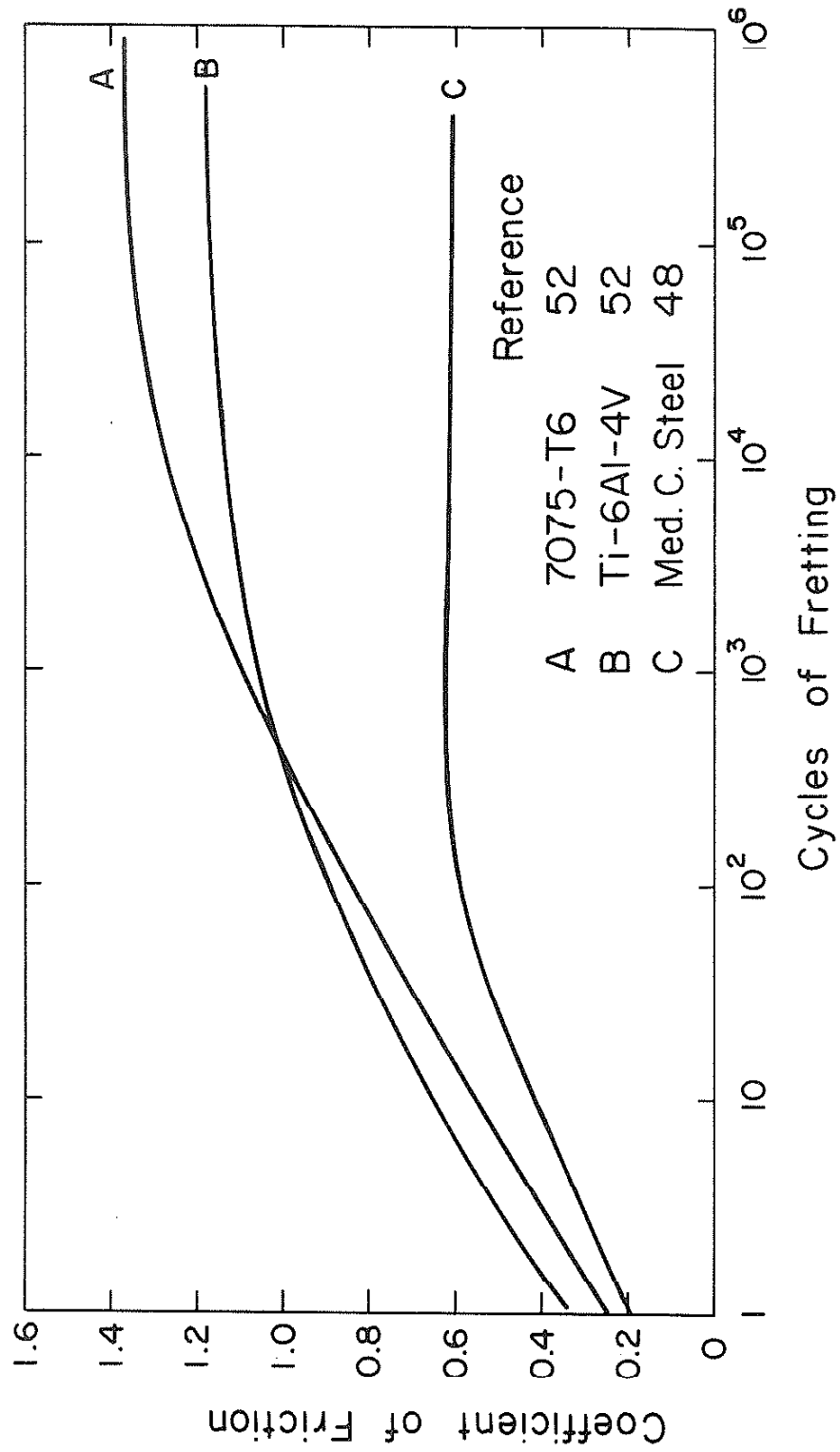
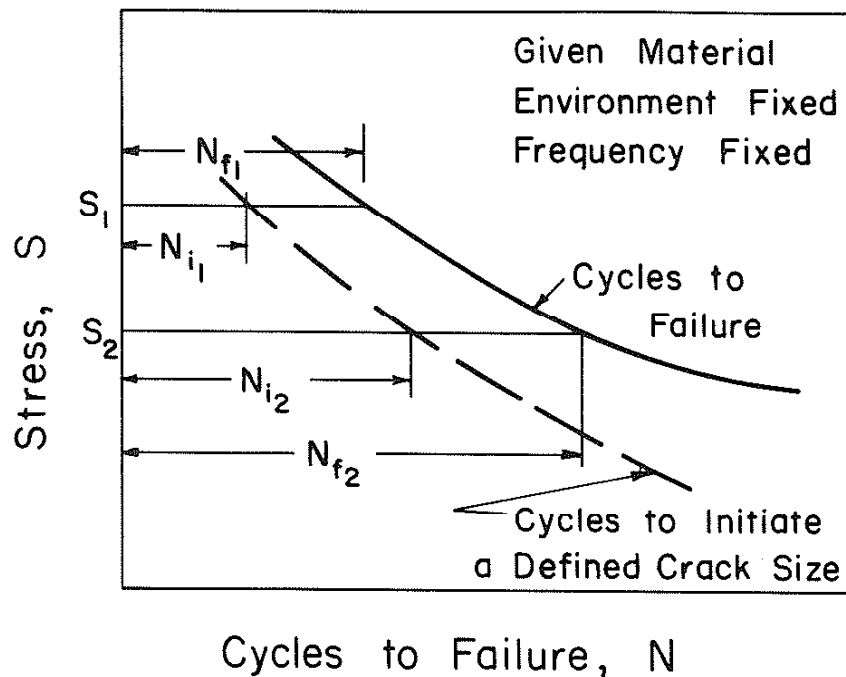


Fig. II Coefficient of Friction Versus the Number of Cycles of Fretting (Ref. 48, 52)



$$C_R = \frac{N_i}{N_f}$$

= Cycle ratio, the ratio of the number of cycles to initiate a crack at a given stress to the number of cycles to failure at the same stress.

Fig. 12 Schematic Representation of "Damage" Line for a Given Material, Surface Condition, and Environment Subjected to Cyclic Loading under Controlled Stress or Strain Conditions (Ref. 53)

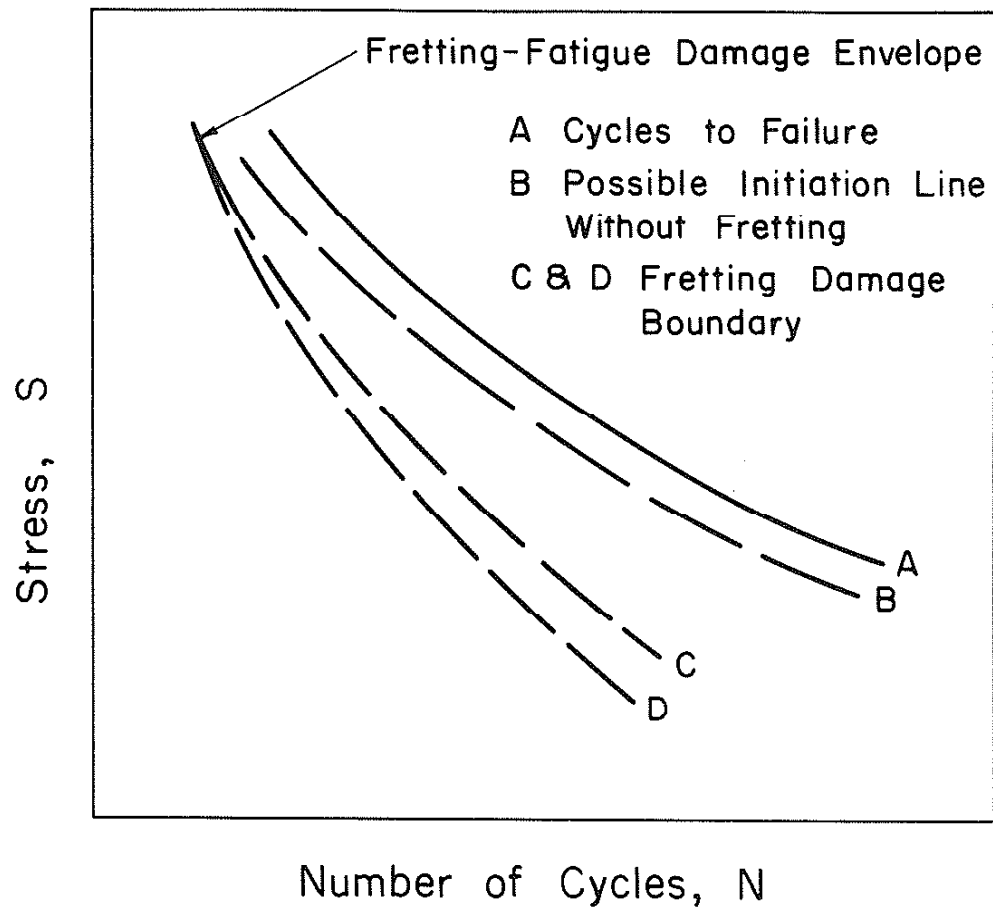
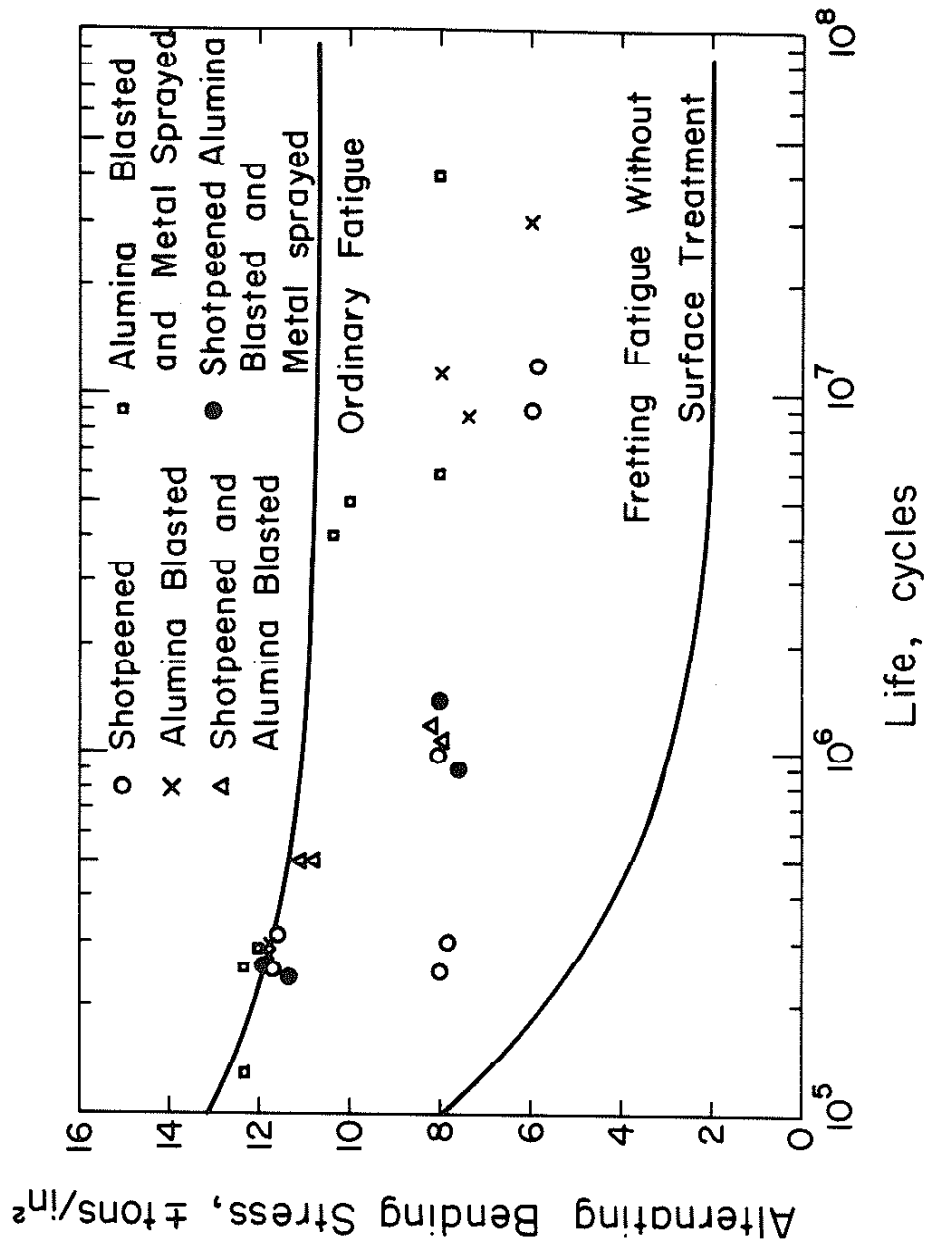


Fig. 13 The "Fretting-Fatigue Damage Envelope" Concept as it Relates to a Stress-Number of Cycles Curve (Ref. 53)



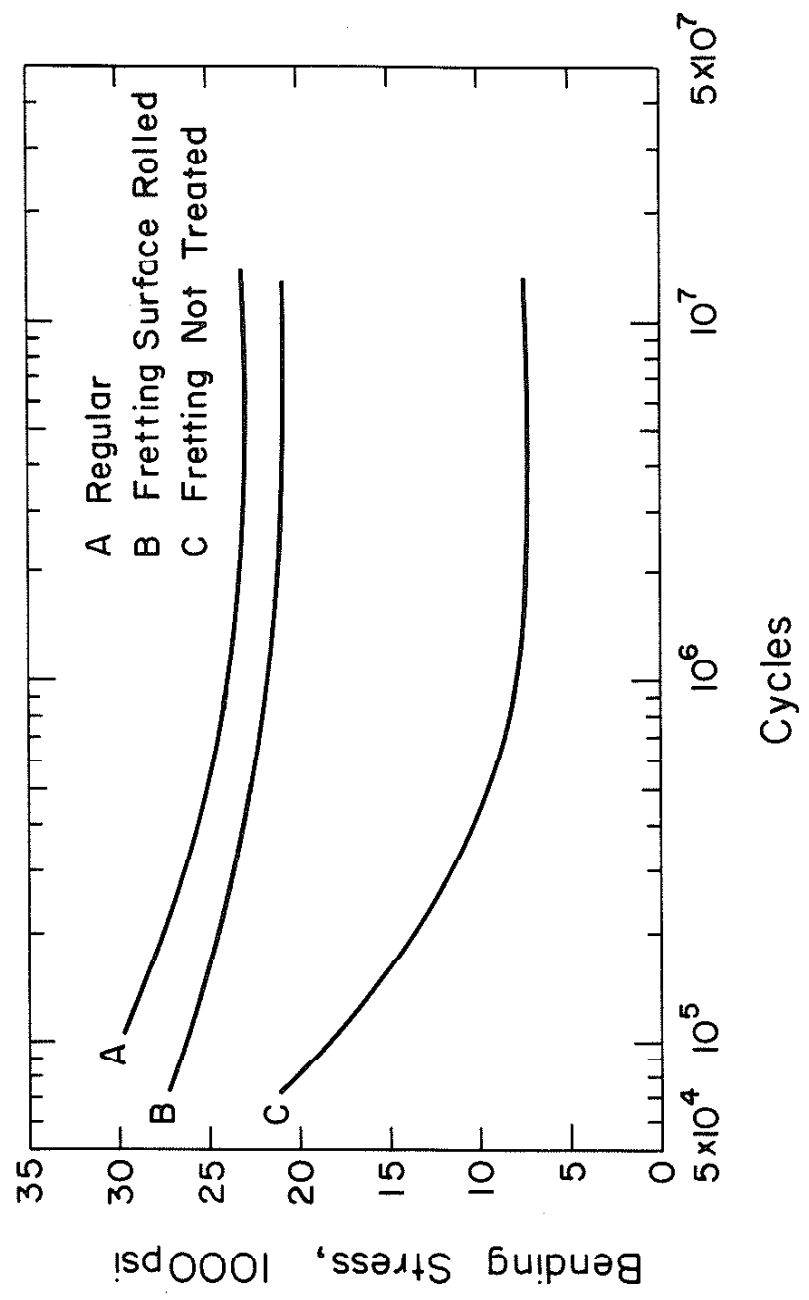


Fig. 15 Stress-Cycle Curves for Regular Fatigue and for Fretting Fatigue of Surface-Rolled and Untreated Specimens of Magnesium Alloys (Ref. 40)

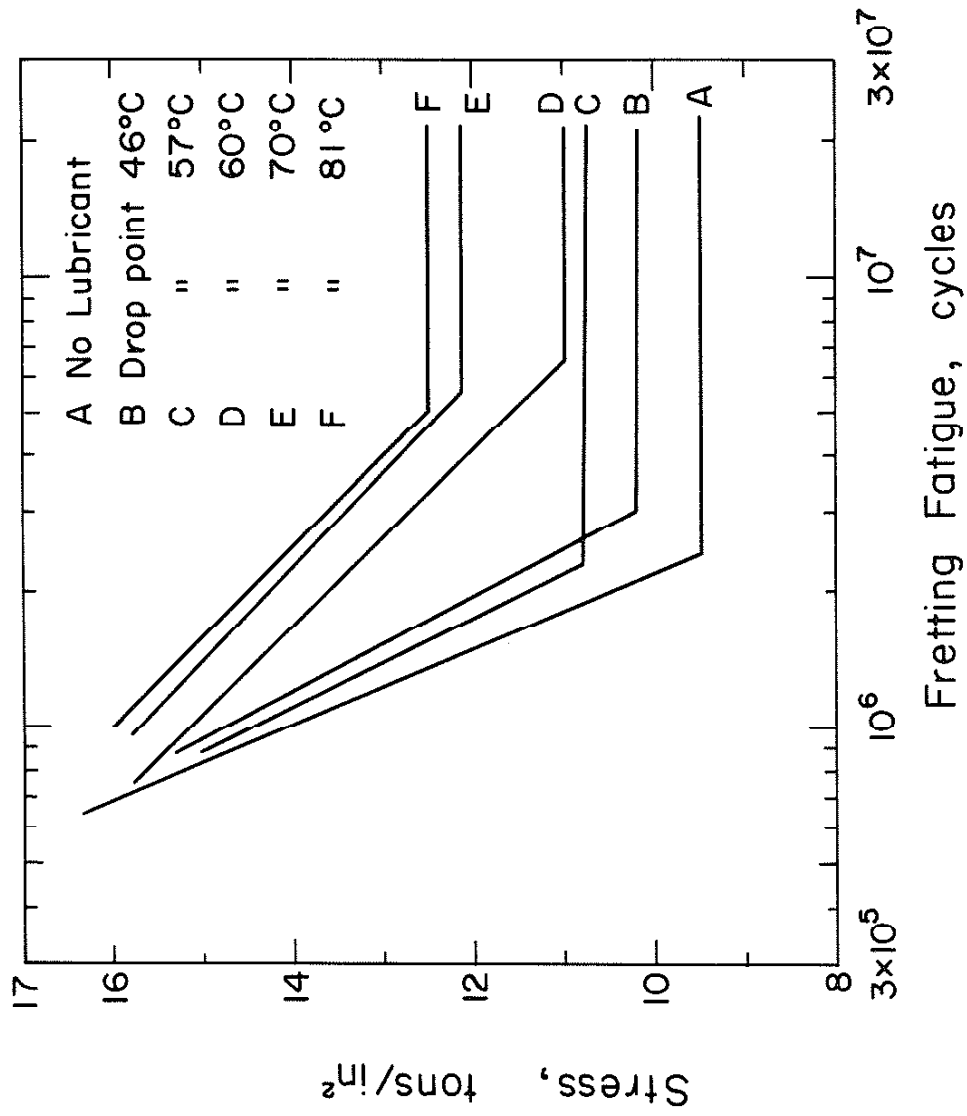


Fig. 16 Fretting-Fatigue Curves for the Rubbing Steel Lubricated With Commercial Lubricants (Reproduced from Ref. 55)

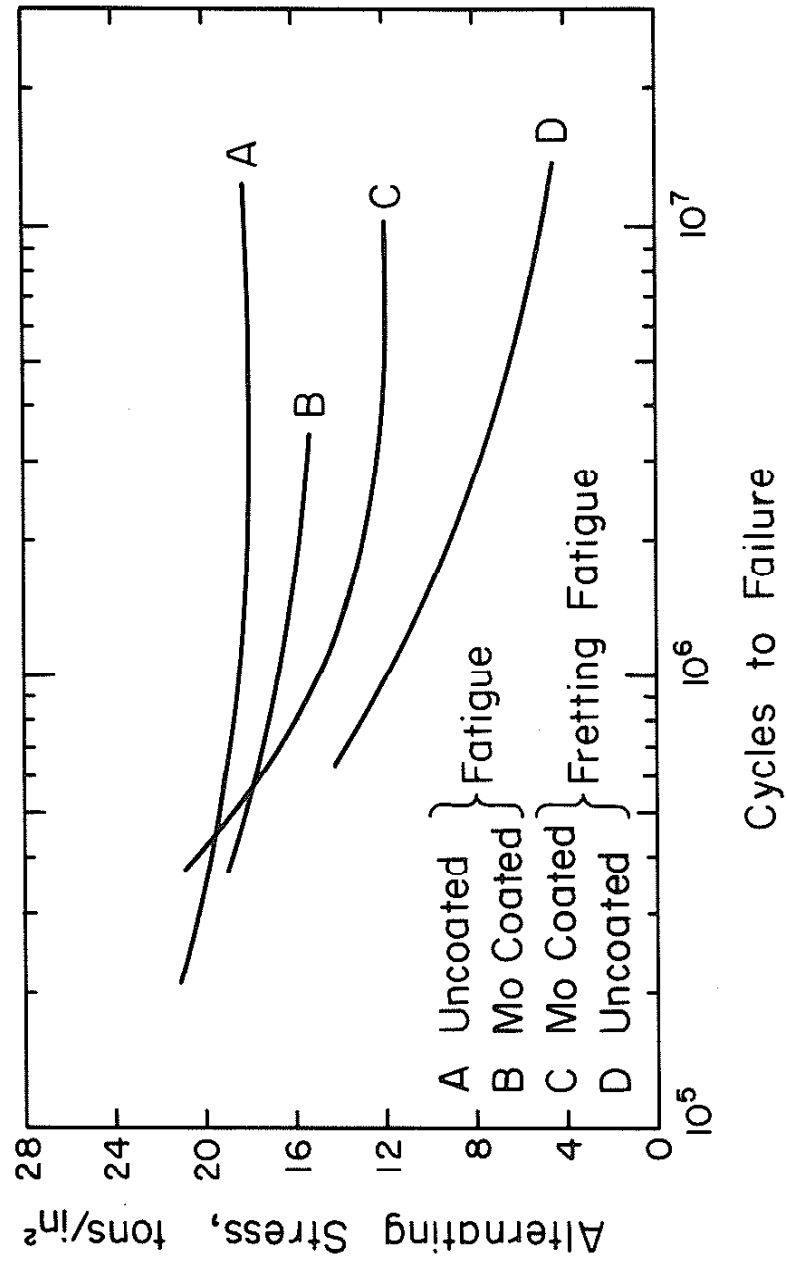


Fig. 17 Fatigue Curves for En 8 Steel with and without Mo Coating and with and without Fretting (Ref. 61)

## APPENDIX A

## Mechanisms of Fretting

Fretting was first mentioned in the literature about sixty years ago when Eden, Rose, and Cunningham (1) conducted fatigue tests to determine the endurance limit of metals. They noted corrosion of their test pieces and holders and suggested it was associated with the application of alternating stress. Since then, many investigators have studied fretting phenomena and have proposed various theories to explain fretting damage. Some excellent reviews such as those by Campbell (4), Allsop (17), Hurricks (19) and Sachs and Horger (40) have summarized the theories and important test results. The most detailed and complete work to date is the monograph by Waterhouse (63). For the present purpose it is sufficient to briefly note some of the research work on this topic.

1. In 1927 Tomlinson (2) determined fretting to be the cause of rusting of steel surfaces in relative motion at their common points of contact. He suggested that the damage was a result of molecular cohesion between the surfaces and noted that it could be observed even when the relative displacement was as small as  $6.5 \times 10^{-8}$  inches. Later Tomlinson, Thorpe and Gough (3) investigated the fretting corrosion of closely fitting surfaces and suggested that the corrosion was mechanical rather than chemical in character. They decided that some surface slip, alternating in direction, was a necessary condition to cause damage. A process of molecular attrition was proposed.
2. Uhlig (5, 11, 12) reviewed the existing facts and suggested that the mechanism of fretting corrosion included a chemical factor and a mechanical factor, with observed damage, in general, resulting from both. An asperity rubbing on a metal surface is considered to produce a track of clean metal which is immediately oxidized, or upon which gas is rapidly adsorbed. The next asperity wipes off the oxide or initiates reaction of metal with adsorbed

gas to form oxide and thus provide a chemical factor of fretting. Asperities also dig below the surface to cause a certain amount of wear by welding or shearing action in which metal particles are dislodged and this accounts for a mechanical factor as well. Uhlig suggested a quantitative expression describing weight loss through fretting corrosion as a function of load, number of cycles, slip distance, frequency and several material constants.

3. Wright (6, 10, 13, 30) studied ferrous surfaces fretting in air by making continuous electrical resistance measurements of the junction and showed that the metallic contact was quickly broken and the surfaces soon separated by a layer of ferric oxide. Oxidation processes were considered to play an important role in fretting corrosion. Oxidation was not an essential factor for fretting damage to occur but, in general, the extent of the damage was increased when it was present.
4. From the fact that fretting corrosion of steel was reduced in a non-oxidizing atmosphere, Waterhouse (13, 14) suggested that three possible processes were involved:
  - (1) mechanical abrasion by grinding or welding,
  - (2) abrasion by hard particles of oxide and (3) continual mechanical rupture of otherwise protective oxide films. He proposed that fretting damage in air was largely due to a chemical process and that the first two processes were not as important as the third.
5. A mechanism of fretting based on the mechanism of wear was suggested by Feng and Rightmire (7). According to this proposal, when contacting asperities carry a normal load great enough to cause plastic deformation, the interface of asperities will be roughened by the deformation as shown in Fig. A-1. This roughening of the interface produces a mechanical interlocking effect and the application of a tangential force would shear off the peak of a high spot

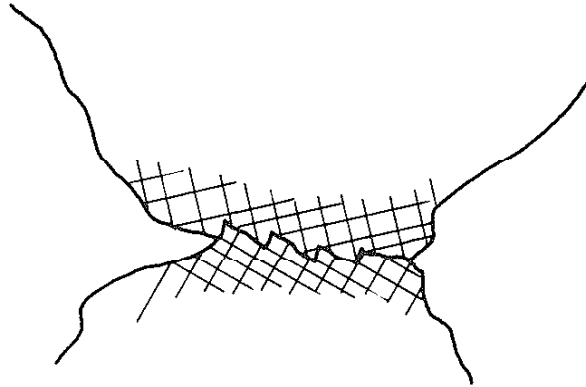


Fig. A-1 Mechanism of Wear-Roughening of the Interface of Contact High Spots Producing a Mechanical Interlocking Effect (Ref. 7)

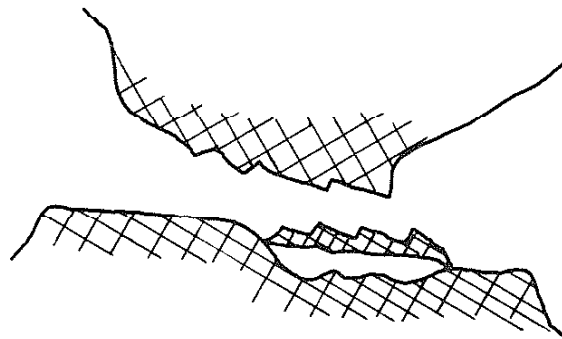


Fig. A-2 Mechanism of Wear-Formation of a Loose Wear Particle (Ref. 7)

which would then become a loose wear particle as shown in Fig. A-2. The process then continues through a period during which the accumulation of wear particles shift the wear action to abrasive action. Finally a stage is reached in which damage is almost entirely caused by abrasion.

6. Holliday and Hirst (15,19) conducted fretting corrosion experiments on mild steel and concluded that fretting corrosion involved a sequence of processes. The first stage is one of plastic flow of the surface contact points leading to the formation of intermetallic junctions. After welding, the junctions rupture leading to the production of loose metallic fragments and to scarring the opposing metal surfaces. The presence of loose oxidized debris which accumulates and tends to roll between the rubbing contacts would reduce the coefficient of friction. If the debris prevents metallic contact, the mild wear mechanism persists at loads far in excess of those in which continuous sliding brings a change to severe wear.
7. Through damage measurements and the use of the scanning electron microscope, Bill (21), concluded that fretting occurs as a sequence of three mechanisms. First, fretting damage is initiated by an adhesion junction growth - fracture process, much as in unidirectional sliding, in the first few hundred fretting cycles. Second, the cyclic stresses associated with the fretting motion lead to the development of a fatigue process which is clearly in evidence after a thousand cycles. Third, fretting in air produces significant amounts of loose oxidized debris.

From the foregoing brief reviews it can be seen that there is some difference of opinion concerning the role of oxidation during fretting and its effect on subsequent damage. The evidence suggests that oxidation and accelerated wear concepts may operate simultaneously.

## APPENDIX B

## Derivation of Equation (1)

When two metal surfaces are pressed into contact, only the high asperities of these two surfaces actually contact. These asperities yield and cold weld together as shown schematically in Fig. B (a), (b).

The stresses on an infinitesimal block of the gripping pads at the interface shows in Fig. B (c). The maximum shear stress on the block is

$$\tau_{\max} = p \sqrt{\left(\frac{1-k}{2}\right)^2 + \mu^2} \quad (\text{B-I})$$

where

$\tau_{\max}$  = maximum shear stress in the asperities of gripping pad, psi

$p$  = normal pressure on the asperities, psi

$k$  = the ratio of stress in x direction to that in y direction (in a two-dimensional solution of an ideal plastic metal loaded by a flat punch without friction  $k = \frac{\pi}{2 + \pi}$ )

$\mu$  = coefficient of friction.

For the fully plastic condition of asperities,  $\tau_{\max}$  is equal to the shearing yield strength of the gripping material, that is

$$\tau_{\max} = \frac{\sigma_Y}{2} \quad (\text{B-II})$$

where

$\sigma_Y$  = yield strength of gripping material.

From the plastic indentation of metals we have the following relationships

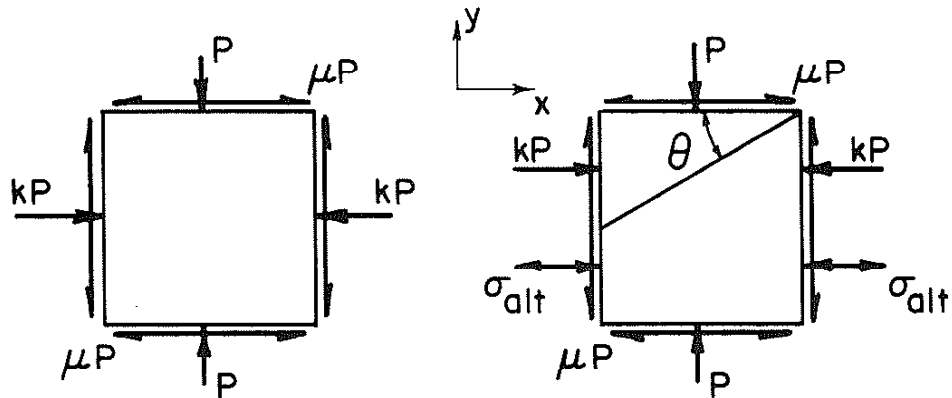
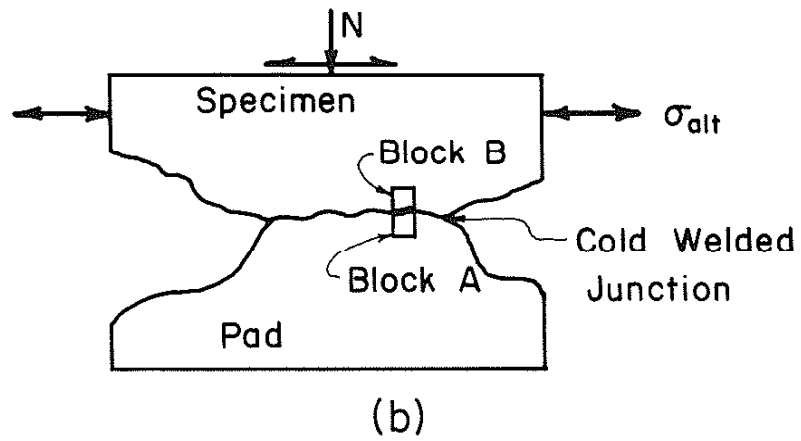
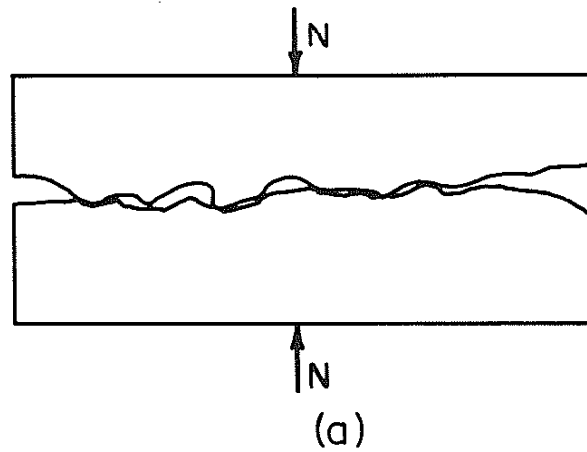
$$\begin{aligned} P_Y &= 3\sigma_Y \\ &= 1420 \times \frac{H}{0.927} \end{aligned} \quad (\text{B-III})$$

$P_Y$  = yield pressure of the weaker metal in the fully plastic state, psi

where

$\sigma_Y = \frac{H}{3 \times 0.927}$  for a 136 deg diamond pyramid indenter and 1420 is the conversion factor for kg per sq mm to psi

$H$  = diamond pyramid hardness number of gripping pads, kg per sq mm



(c) Block A- Pad      (d) Block B- Specimen

Fig. B Contacting Asperities and Associated Stress (Ref. 32)

Combining equations (B-I), (B-II) and (B-III), the relation between  $p$ ,  $H$  and  $\mu$  is

$$p^2 = 2.61 \times 10^5 \frac{H^2}{0.151 + 4\mu^2} \quad (\text{B-IV})$$

The stresses acting on an infinitesimal block of the specimen at the interface shows in Fig. B (d) on plane AC, when  $\sigma_{alt}$  is tensile, the shear stress  $\tau_1$  is equal to

$$\tau_1 = \frac{1}{2}(1 - k)p \sin 2\theta - \mu p \cos 2\theta + \frac{1}{2} \sigma_{alt} \sin 2\theta$$

when  $\sigma_{alt}$  is compressive, the shear stress on plane AC.  $\tau_2$  is equal to

$$\tau_2 = \frac{1}{2}(1 - k)p \sin 2\theta + \mu p \cos 2\theta - \frac{1}{2} \sigma_{alt} \sin 2\theta$$

The amplitude of the repeated shear stress  $\tau_{alt}$  is equal to

$$\begin{aligned} \tau_{alt} &= \frac{1}{2} (\tau_1 - \tau_2) \\ &= \frac{1}{2} \sigma_{alt} \sin 2\theta - \mu p \cos 2\theta \end{aligned}$$

The maxima of  $\tau_{alt}$  with respect to  $\theta$  are

$$\tau_{alt} = \frac{1}{2} \sqrt{4\mu^2 p^2 + \sigma_{alt}^2} \quad (\text{B-V})$$

Solving for  $\sigma_{alt}$

$$\sigma_{alt} = \sqrt{4\tau_{alt}^2 - 4\mu^2 p^2} \quad (\text{B-VI})$$

Substituting (B-IV) into (B-VI) gets

$$\sigma_{alt} = \sqrt{4\tau_{alt}^2 - 1.04 \times 10^6 \frac{\mu^2 H^2}{0.151 + 4\mu^2}} \quad (\text{Eq. 1})$$

Hence for a given gripping pad hardness, coefficient of friction, the ordinary fatigue strength (two times the shearing fatigue strength), the limiting values of  $\sigma_{alt}$  for preventing fatigue crack initiation can be calculated.

## APPENDIX C

## Derivation of Equation (2)

A steel cylinder is pressed onto a plate specimen at a contact load  $P$  and subjected to an alternating tangential load  $T$  as shown in Fig. C. The analysis was made under the following assumptions

- a) The coefficient of friction is constant and is designated as  $\mu$
- b) The force which acts on the contact surface is equal to  $\mu p$
- c) Contact pressure  $p(x)$  is assumed to be distributed in Hertzian fashion even in the presence of tangential load  $T$

$p(x)$  is expressed as

$$p(x) = p_0 \sqrt{1 - \left(\frac{x}{a}\right)^2} \quad (C-I)$$

$P_0$  = maximum contact pressure

$a$  = half contact width

The distribution of tangential force,  $q(x)$ , is assumed to be equal to  $\mu p(x)$

$$q(x) = \mu p_0 \sqrt{1 - \left(\frac{x}{a}\right)^2} \quad (C-II)$$

From Hertzian analysis

$$p_0 = \frac{2P}{\pi a} \quad (C-III)$$

$$2a = \sqrt{8PR(1 - \nu^2)/\pi E} \quad (C-IV)$$

$P$  = normal load per unit length in  $y$  direction

$\nu$  = poisson's ratio

$E$  = modulus of elasticity

$R$  = radius of cylinder

The stresses on the contact surface can be calculated from the following equations which have been obtained by Smith and Liu\*

\* J. O. Smith and C. K. Liu: Journal Applied Mechanics, Vol. 20, No. 2, 1953, p. 157

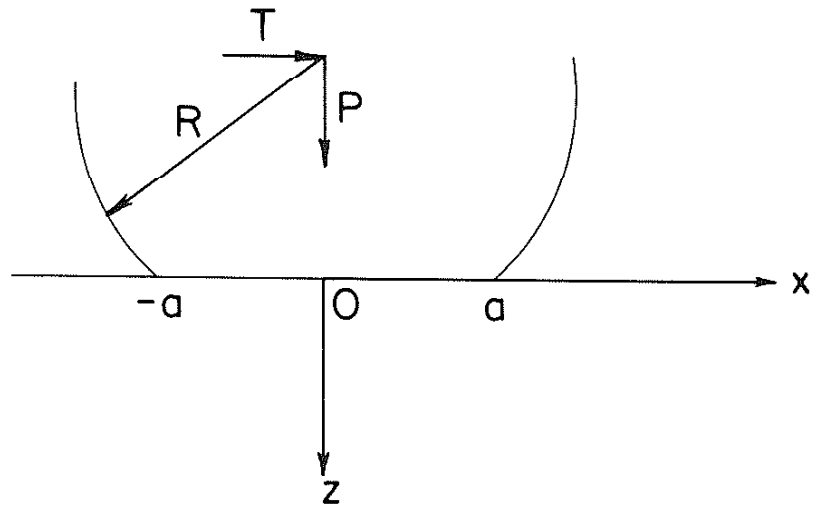


Fig. C An Illustrative Model for the Stress Analysis (Ref. 48)

$$\frac{\sigma_x}{p_o} = -2\mu \left( \frac{x}{a} \right) - \sqrt{1 - \left( \frac{x}{a} \right)^2} \quad (C-V)$$

$$\frac{\sigma_z}{p_o} = -\sqrt{1 - \left( \frac{x}{a} \right)^2} \quad (C-VI)$$

$$\frac{\tau_{xz}}{p_o} = -\mu \sqrt{1 - \left( \frac{x}{a} \right)^2} \quad (C-VII)$$

When the alternating bending stress  $\sigma_a$  is positive, the tangential force  $T$  acts also in the positive direction of the  $x$  axis and vice versa therefore:

$$\frac{\sigma_{\max}}{p_o} = \frac{\sigma_a}{p_o} - 2\mu \left( \frac{x}{a} \right) - \sqrt{1 - \left( \frac{x}{a} \right)^2} \quad (C-VIII)$$

$$\frac{\sigma_{\min}}{p_o} = -\frac{\sigma_a}{p_o} + 2\mu \left( \frac{x}{a} \right) - \sqrt{1 - \left( \frac{x}{a} \right)^2} \quad (C-IX)$$

The range of alternating normal stress  $\sigma_R$ , in one cycle of loading can be obtained as

$$\frac{\sigma_R}{p_o} = \frac{\sigma_{\max} - \sigma_{\min}}{p_o} = -4\mu \left( \frac{x}{a} \right) + 2 \left( \frac{\sigma_a}{p_o} \right) \quad (C-X)$$

The mean value of  $\sigma_R$  becomes

$$\frac{\sigma_m}{p_o} = -\sqrt{1 - \left( \frac{x}{a} \right)^2} \quad (C-XI)$$

If the fretting fatigue stress concentration factor is defined as  $K_t$  then

$K_t$  = the ratio of  $\sigma_R$  to the range of alternately nominal bending stress,  $2\sigma_a$ .

$$\begin{aligned} \therefore K_t &= \left| \frac{\sigma_R}{2\sigma_a} \right| \\ &= \left| 1 - 2\mu \left( \frac{x}{a} \right) \left( \frac{p_o}{\sigma_a} \right) \right| \\ &= \left| 1 - 4\mu \left( \frac{x}{a} \right) \left( \frac{P}{\pi a \sigma_a} \right) \right| \end{aligned} \quad (Eq. 2)$$