FCP Short Course

Ductile and Brittle Fracture

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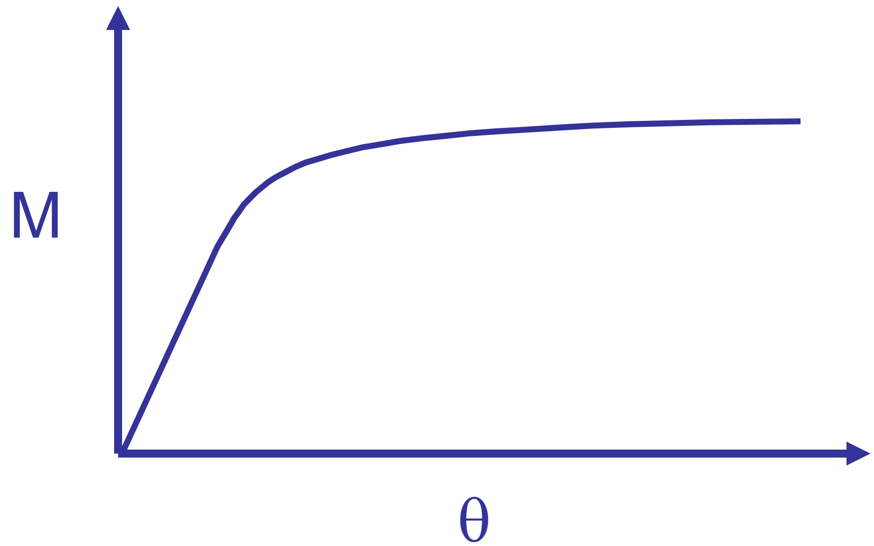
Mechanical Science and Engineering

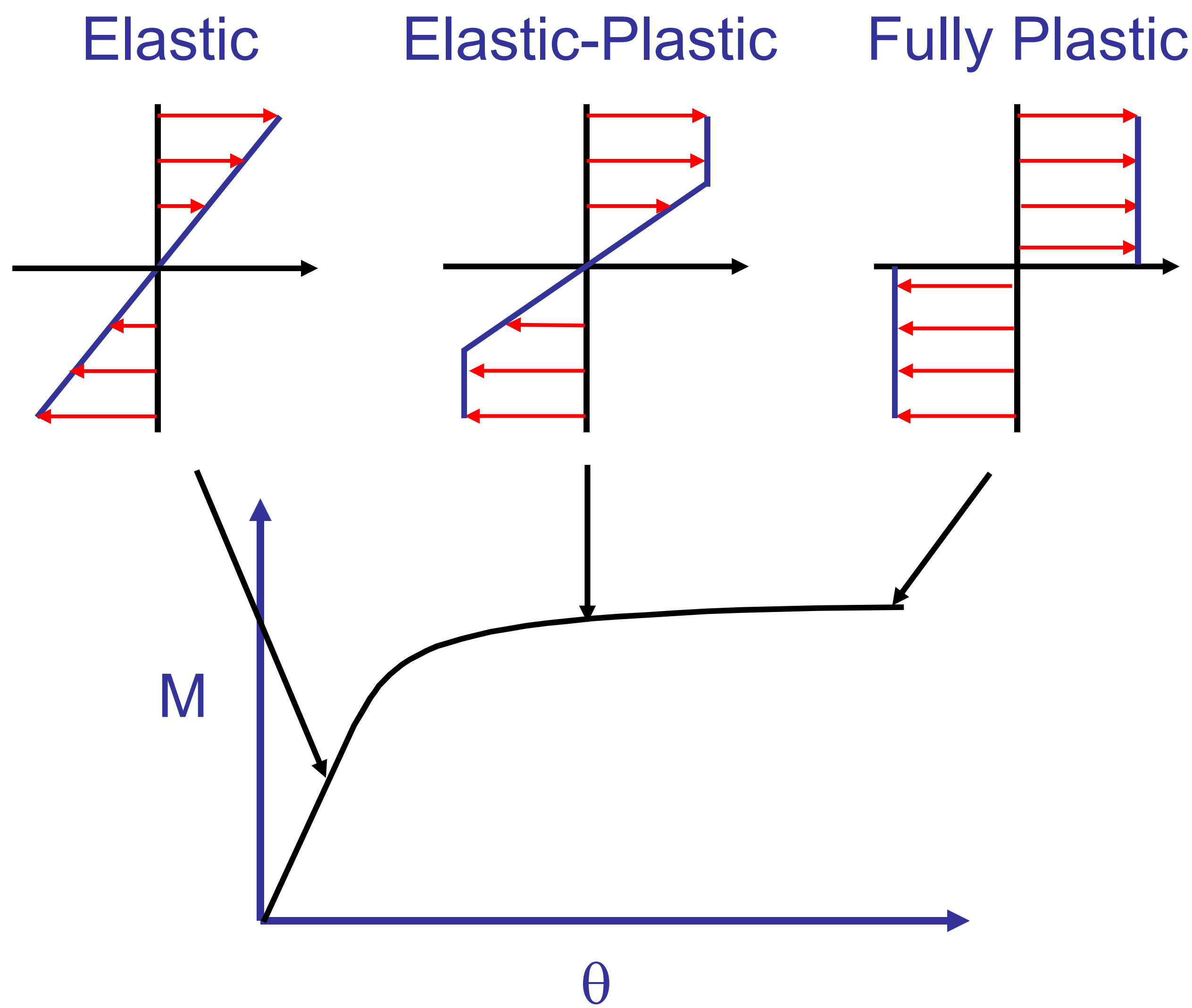
Agenda

- **Limit theorems**
- Plane Stress – Plane Strain
- Notched plates
- Notched bars
- Brittle Faracture

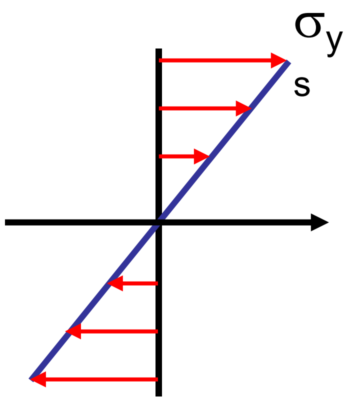
Beam in Bending



Stress Distribution

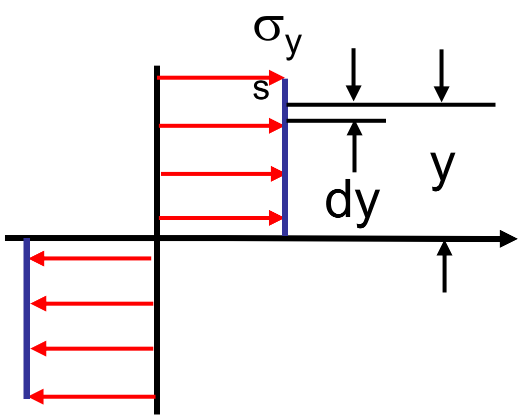


Plastic Moment



Elastic:

$$M_e = \frac{\sigma_{ys} I}{c} = \frac{\sigma_{ys} b h^2}{6}$$

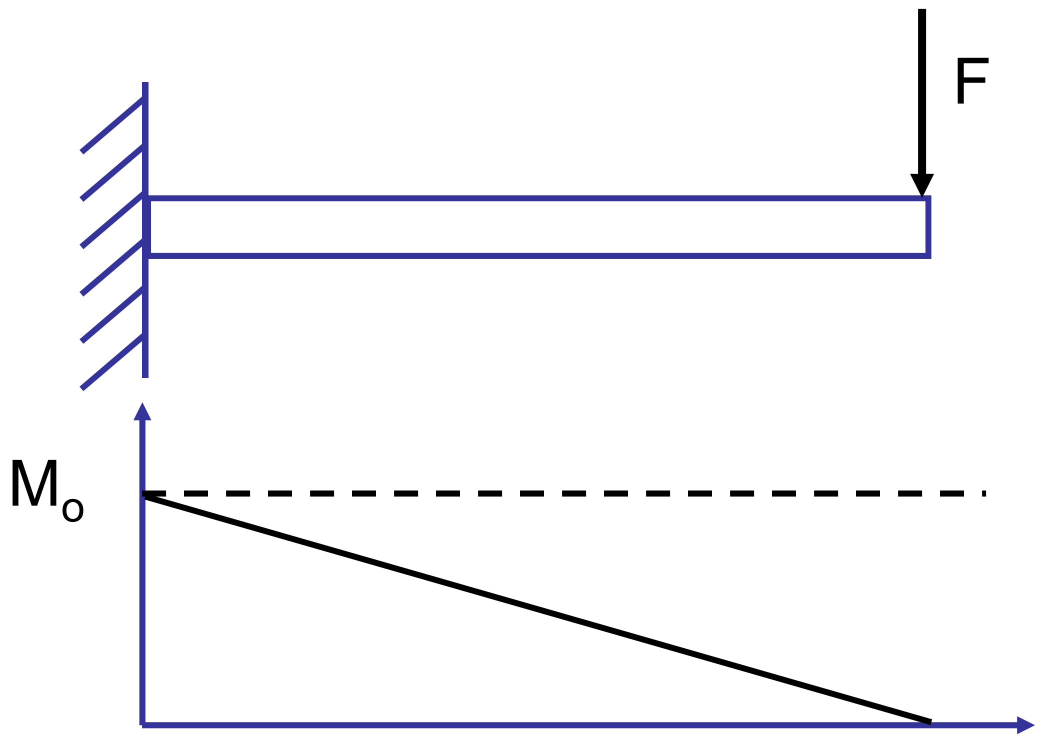


Fully Plastic:

$$M_o = 2\int_0^{\frac{h}{2}} \sigma_{ys}\, b\, y\, dy = \frac{\sigma_{ys} b h^2}{4}$$

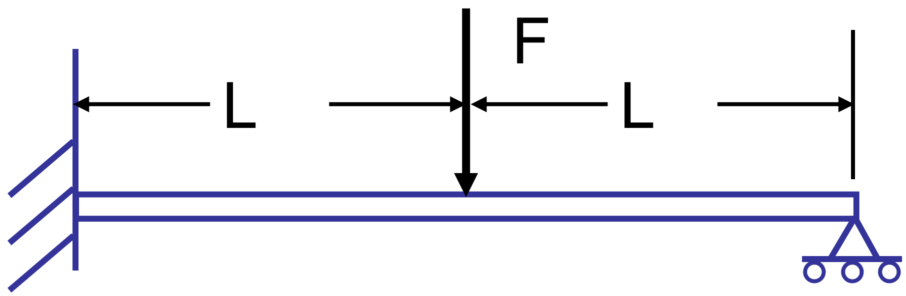
$$\frac{M_o}{M_e} = 1.5$$

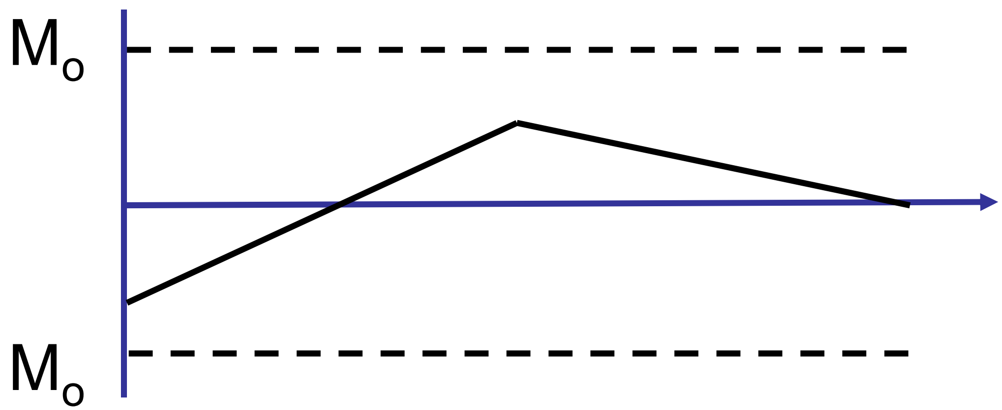
Fully Plastic Moment



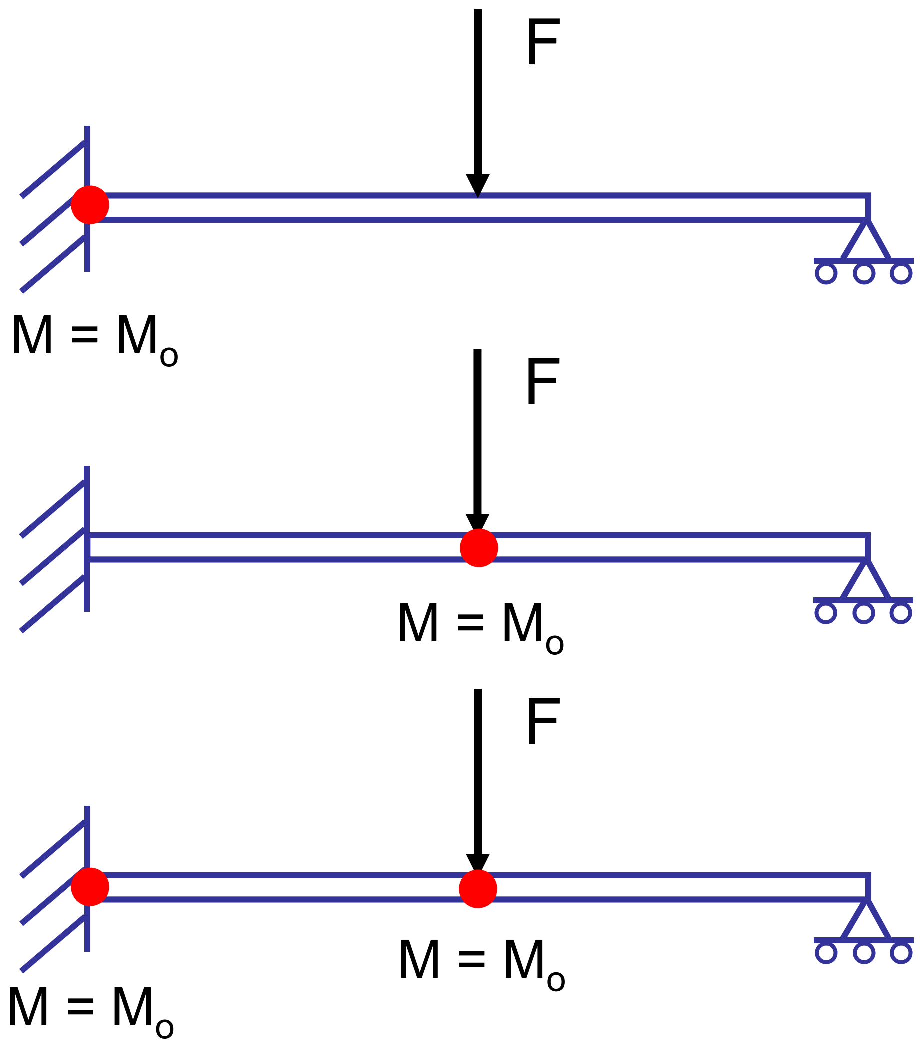
Beam can be loaded until M_o is reached after which a plastic hinge is formed and the beam is free to rotate.

Beam With a Support





Collapse



right end support prevents collapse

left end support prevents collapse

collapse

Lower Bound Theorem

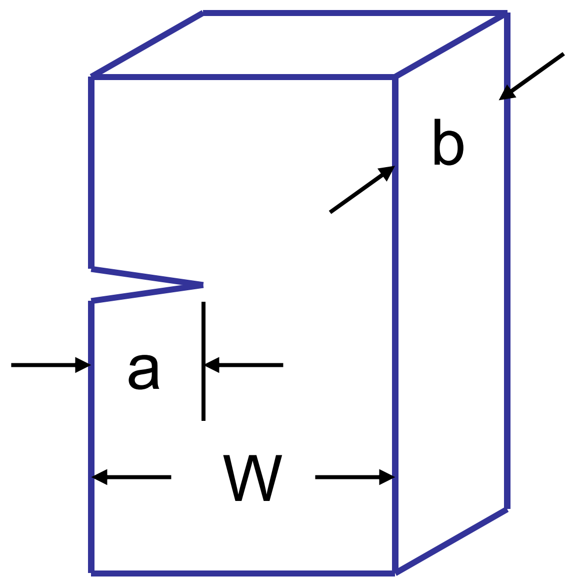
- If an equilibrium distribution of stress can be found which balances the applied load and is everywhere below or at yield, the structure will not collapse or just be at the point of collapse.
- Guaranteed load capacity where system will not have large deformations

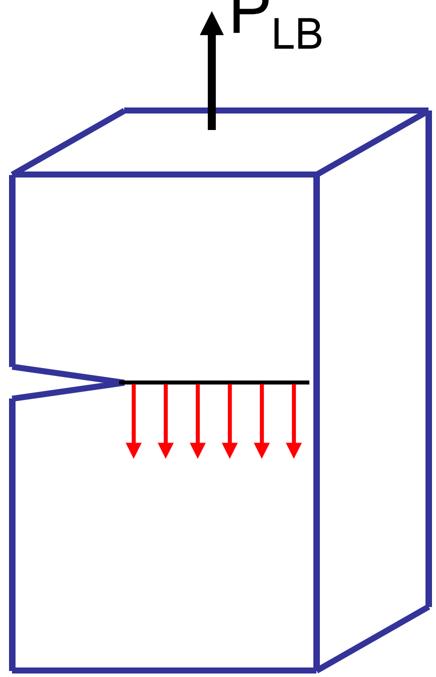
Upper Bound Theorem

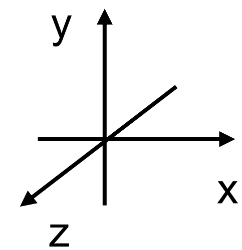
- The structure must collapse if there is any compatible pattern of plastic deformation for which the rate at which external work is equal to or greater than the rate of internal dissipation.
- Guaranteed load capacity where the system will have large deformations.

Application to a Cracked Plate

Assume a stress distribution that satisfied equilibrium







$$\sigma_y = \sigma_{ys}$$

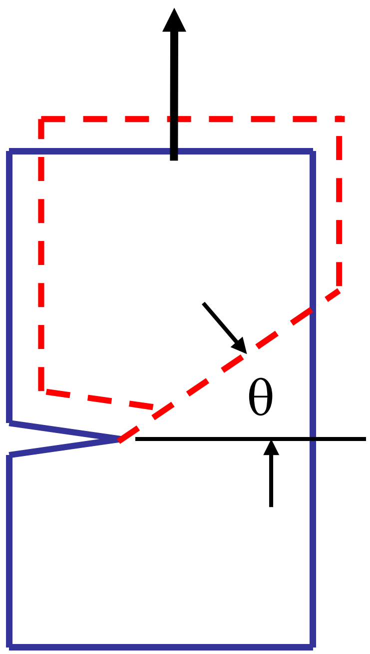
$$\sigma_x = 0$$

$$\sigma_z = 0$$

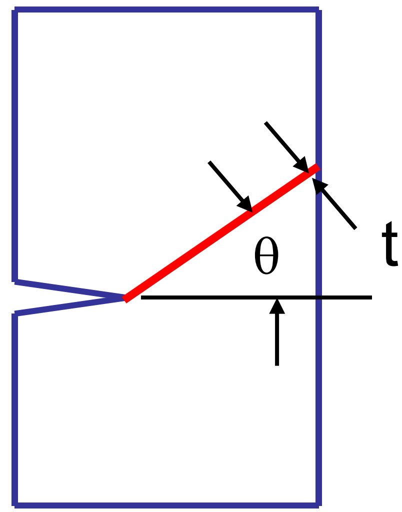
$$\tau_{xy} = 0$$

$$P_{LB} = \sigma_{ys} \, (W - a) \, b$$

Upper Bound Solution

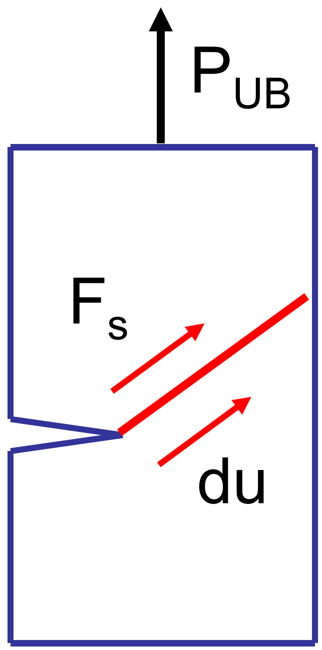


Rigid blocks allowed to slide



Shear band of thickness t

Upper Bound Solution (continued)



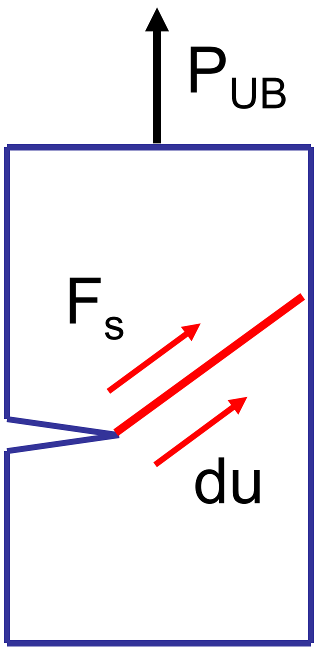
Slip plane area:

$$A_s = \frac{(W-a)b}{\cos\theta}$$

Force on shear plane:

$$F_s = \frac{\sigma_{ys}}{\sqrt{3}} \frac{(W-a)b}{\cos\theta}$$

Upper Bound Solution (continued)



Internal work:

$$dU = F_s \, du$$

$$dU = \frac{\sigma_{ys}}{\sqrt{3}} \frac{(W-a)b}{\cos\theta} du$$

External work:

$$dW = P_{UB} \, du \sin\theta$$

Upper Bound Solution (continued)

Upper Bound Theorem: $dU = dW$

$$\frac{\sigma_{ys}}{\sqrt{3}} \frac{(W-a)b}{\cos\theta} du = P_{UB}\, du \sin\theta$$

$$P_{UB} = \frac{\sigma_{ys}}{\sqrt{3}} \frac{(W-a)b}{\sin\theta\cos\theta} = \frac{2\sigma_{ys}}{\sqrt{3}} \frac{(W-a)b}{\sin 2\theta}$$

Lowest upper bound for $\sin 2\theta = 1$ or $\theta = 45°$

$$P_{UB} = 1.15\, \sigma_{ys} (W - a) b$$

$$P_{LB} = \sigma_{ys} (W - a) b$$

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- Limit theorems
- **Plane Stress – Plane Strain**
- Notched plates
- Notched bars
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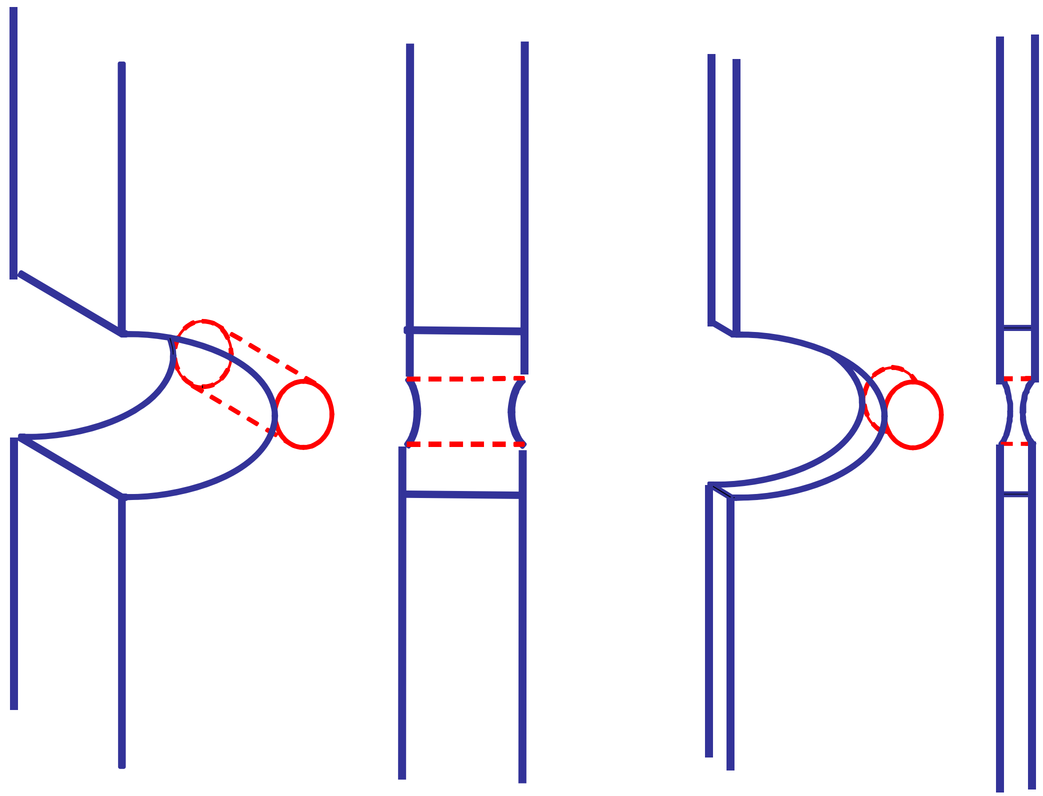
Plane Stress – Plane Strain

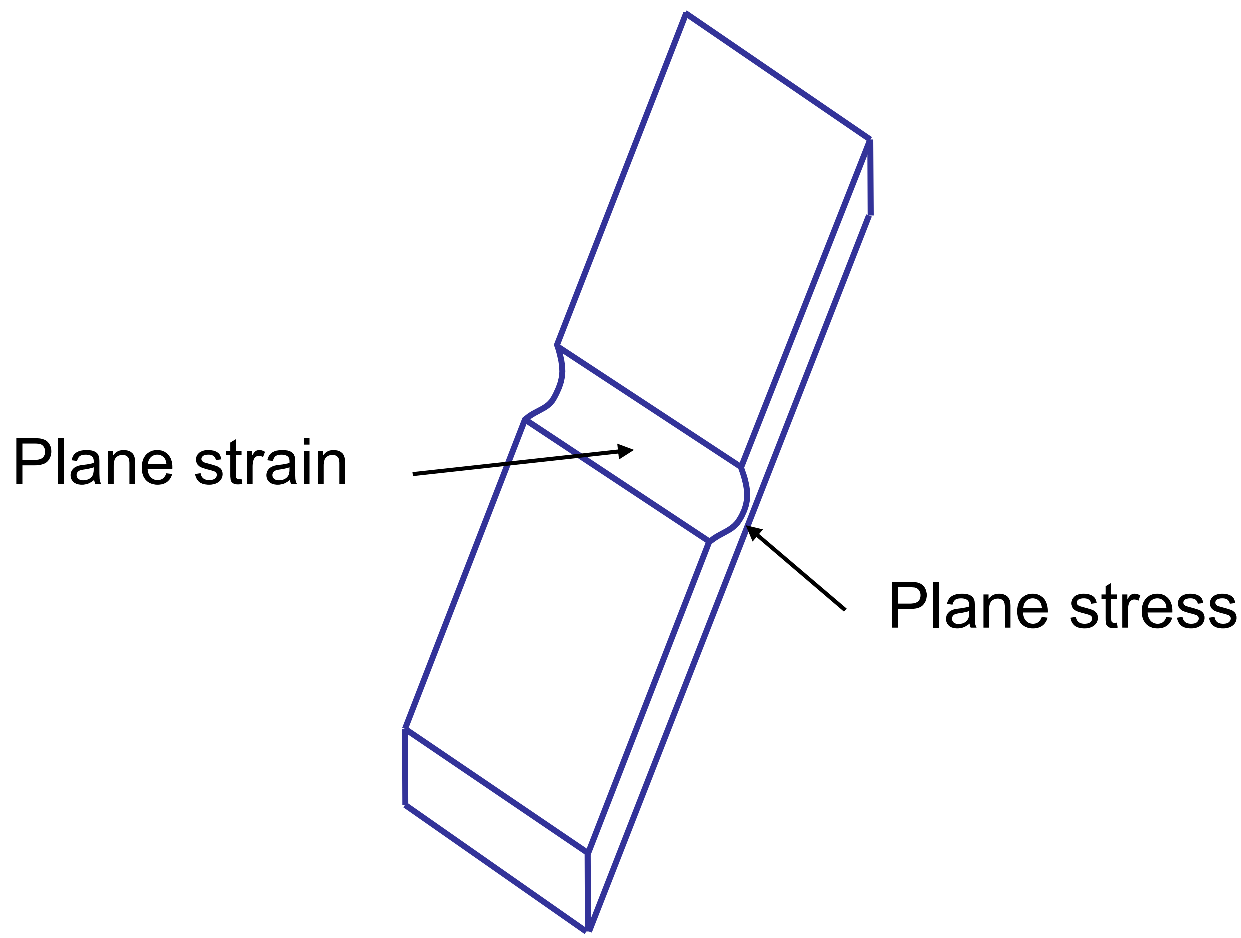


thick plate

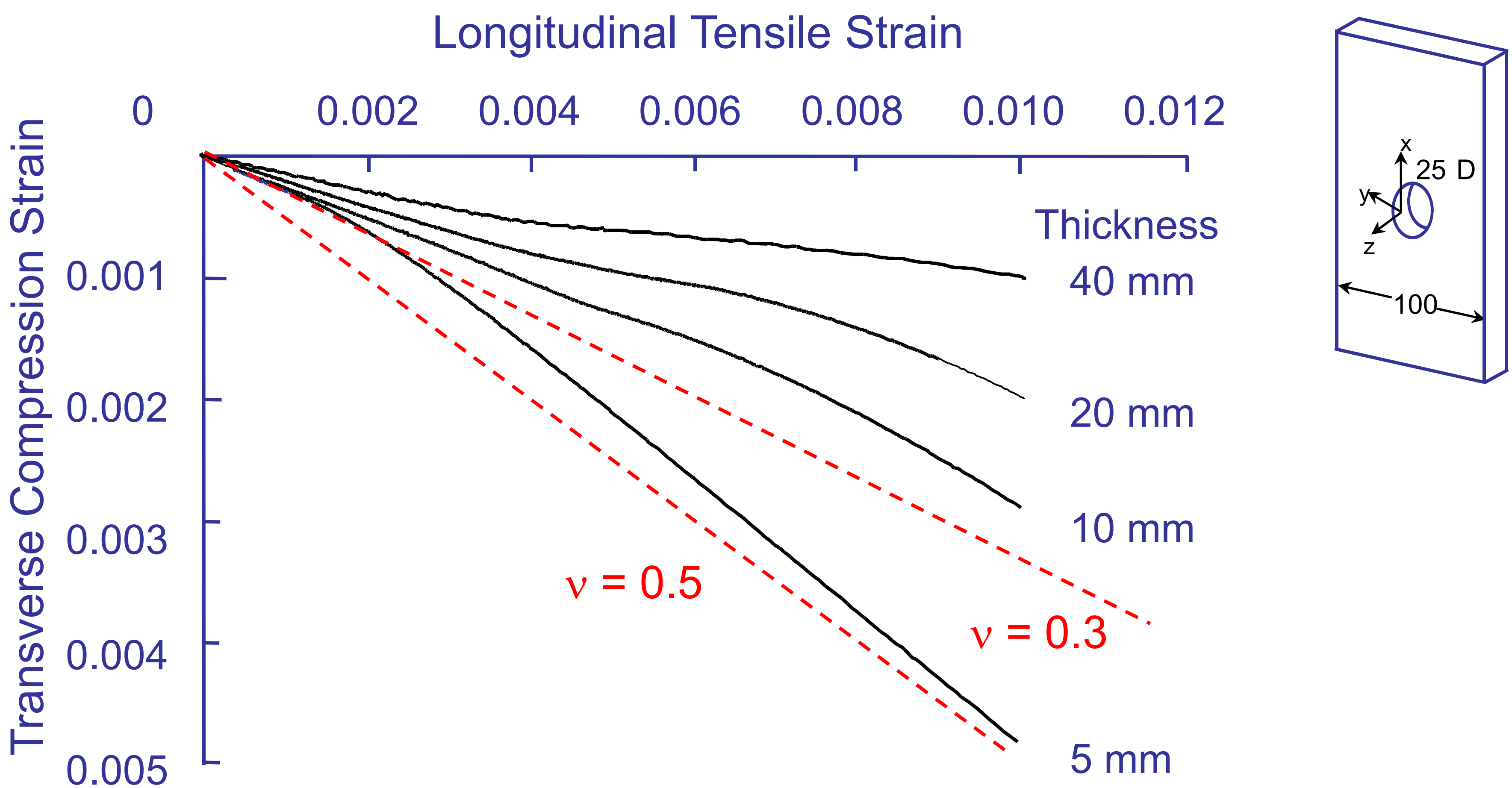
thin plate

plane strain $\sigma_z \neq 0$ $\varepsilon_z = 0$ plane stress $\sigma_z = 0$ $\varepsilon_z \neq 0$

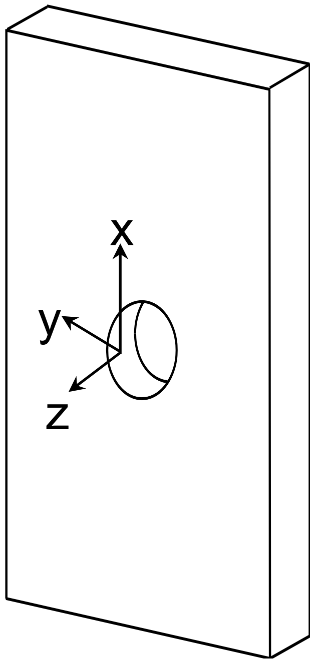
Thick or Thin ?



Transverse Strains



Notch Stresses



t	ε_x	ε_z	σ_x	σ_z
7	0.01	-0.005	63.5	0
15	0.01	-0.003	70.6	14.1
30	0.01	-0.002	73.0	21.8
50	0.01	-0.001	75.1	29.3

Fracture Surfaces

Fracture Surfaces

Yielding

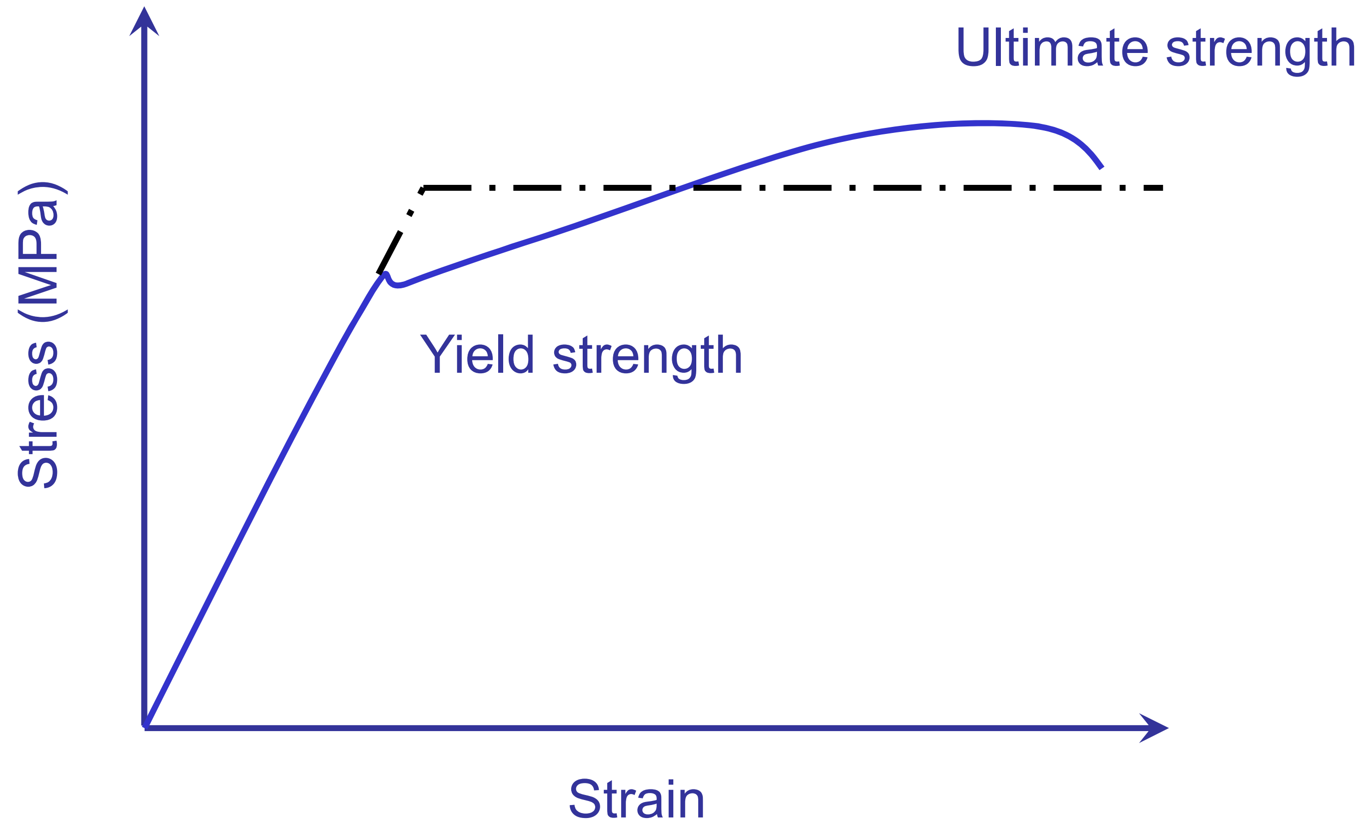
plane stress $\sigma_z = 0$ $\varepsilon_z \neq 0$

$$\sigma_x^{\,2} - \sigma_x\sigma_y + \sigma_y^{\,2} + 3\tau^2 = 3k^2$$

plane strain $\sigma_z \neq 0$ $\varepsilon_z = 0$

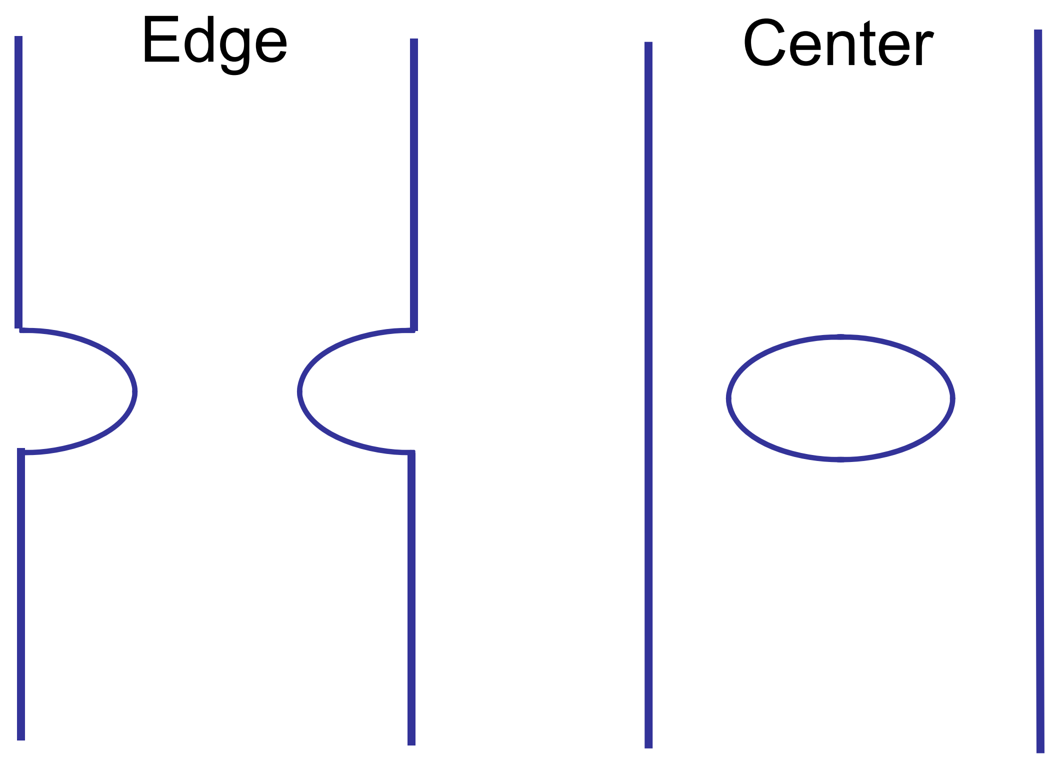
$$\frac{1}{4}\left(\sigma_x - \sigma_y\right)^2 + \tau^2 = k^2$$

Flow Stress



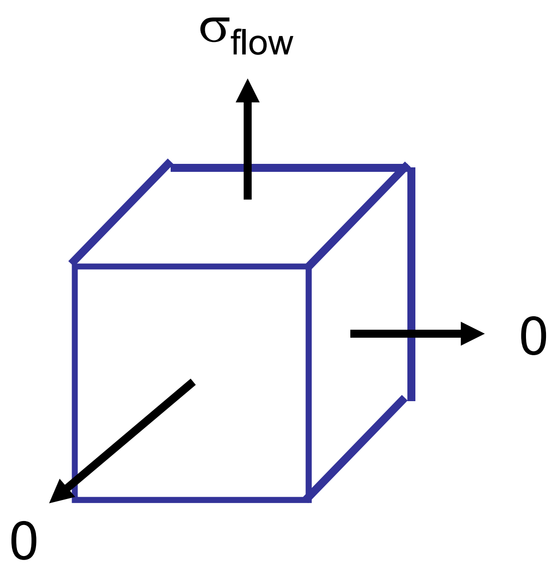
Flow Stress: $\sigma_{flow} = \dfrac{\sigma_{ys} + \sigma_{uts}}{2}$ $\quad k = \dfrac{\sigma_{flow}}{\sqrt{3}}$

2 Notches



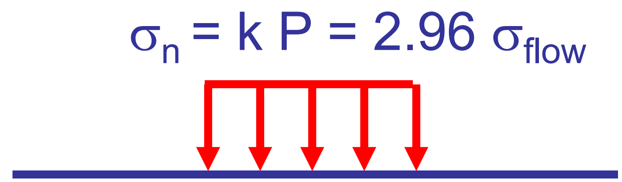
Same net section, same K_T

Plane Stress

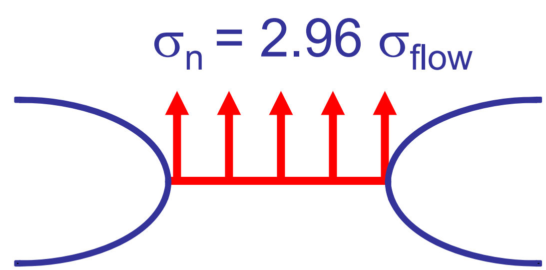


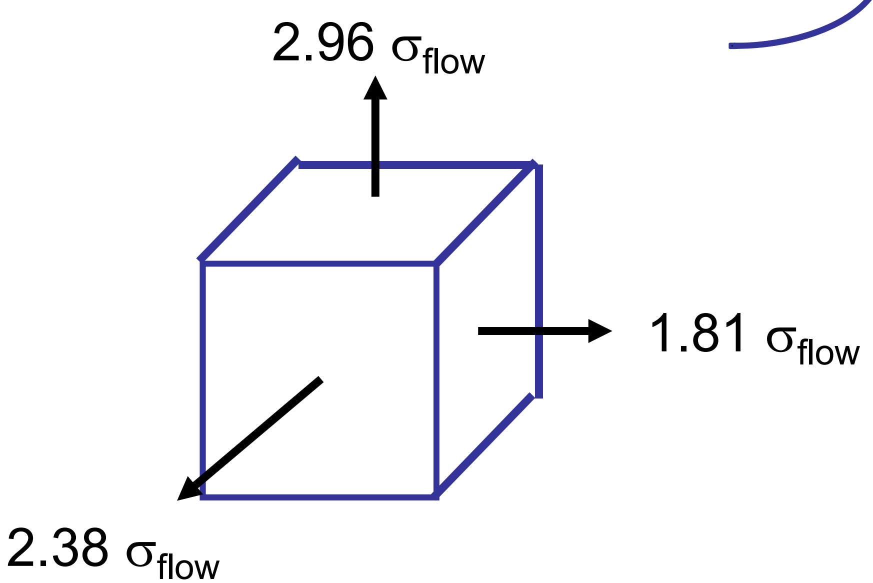
Edge Notch – Plane Strain

From the punch problem:



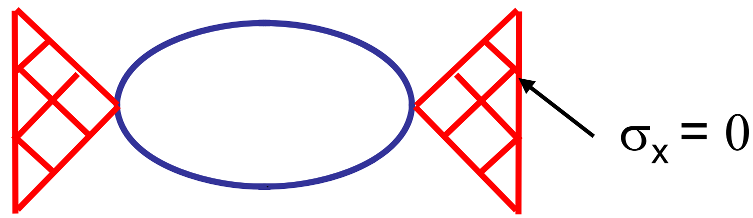
By analogy:

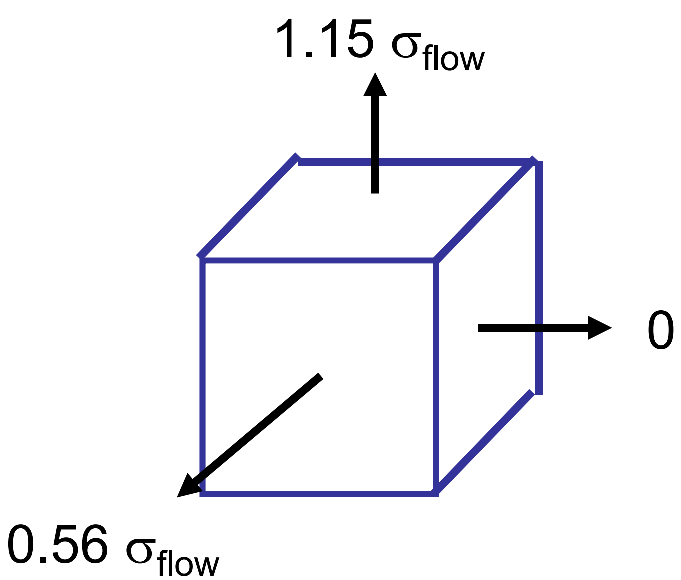




Center Notch – Plane Strain

Slip Line Field





Agenda

- Limit theorems
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- **Notched plates**
- Notched bars
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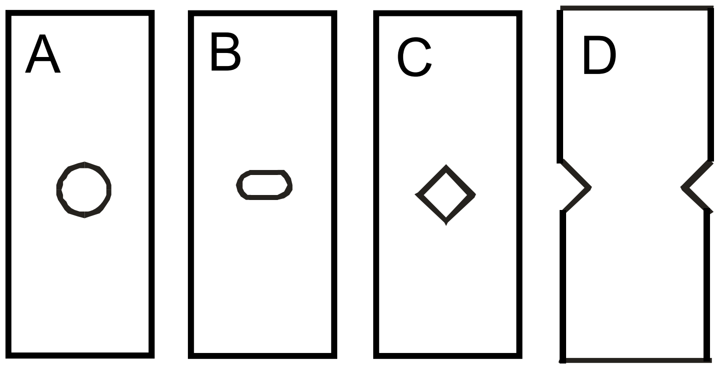
What does all this tell us?



Lower Bound Theorem: $P_A = P_B = P_C = P_D$

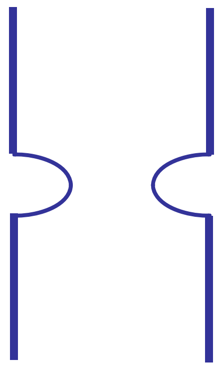
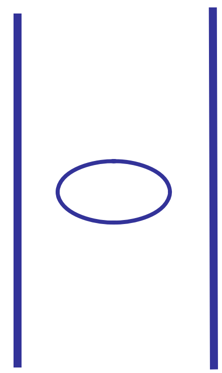
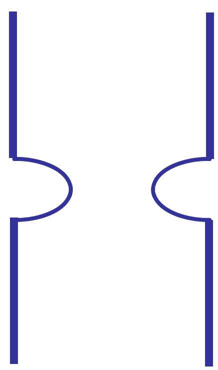
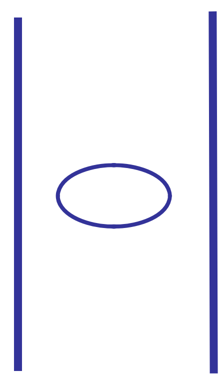
Upper Bound Theorem: $P_A = P_B = P_C < P_D$

Slip Line Theory: $P_A = P_B = P_C < P_D$

No effect of stress concentration on static strength !

Some stress concentrations can increase the strength !

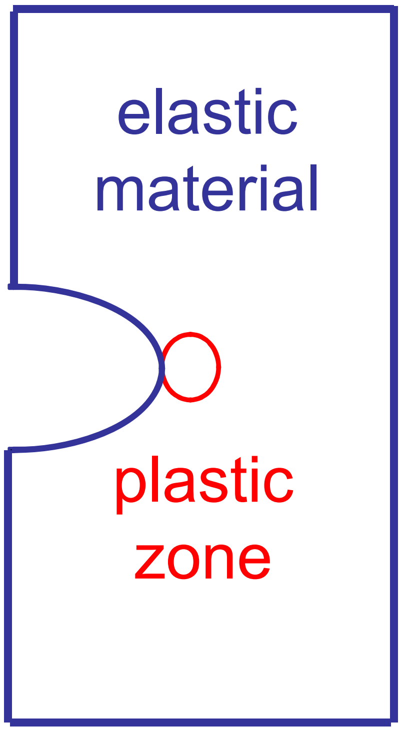
Failure Loads

	Edge		Center	
Plane Stress		$\frac{P}{A_{net}} = 1.15\,\sigma_{flow}$		$\frac{P}{A_{net}} = \sigma_{flow}$
Plane Strain		$\frac{P}{A_{net}} = 2.96\,\sigma_{flow}$		$\frac{P}{A_{net}} = 1.15\,\sigma_{flow}$

Conclusion

Net section area, state of stress and material strength control the failure load in a structure.

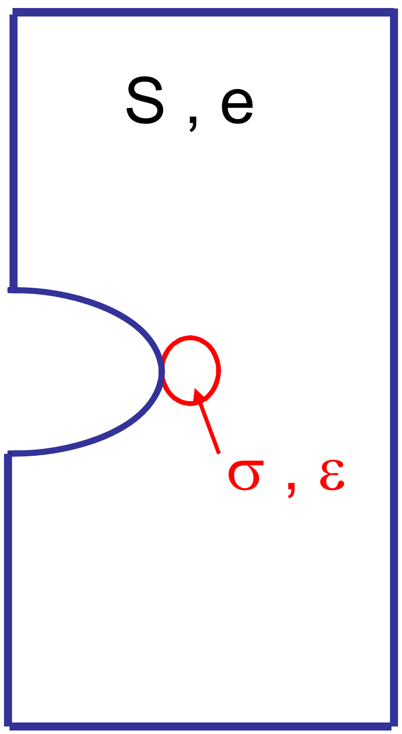
Stress or Strain Control?



Elastic material surrounding the plastic zone forces the displacements to be compatible, I.e. no gaps form in the structure.

Boundary conditions acting on the plastic zone boundary are displacements. Strains are the first derivative of displacement

Define K_σ and K_ε



Define: nominal stress, S

nominal strain, e

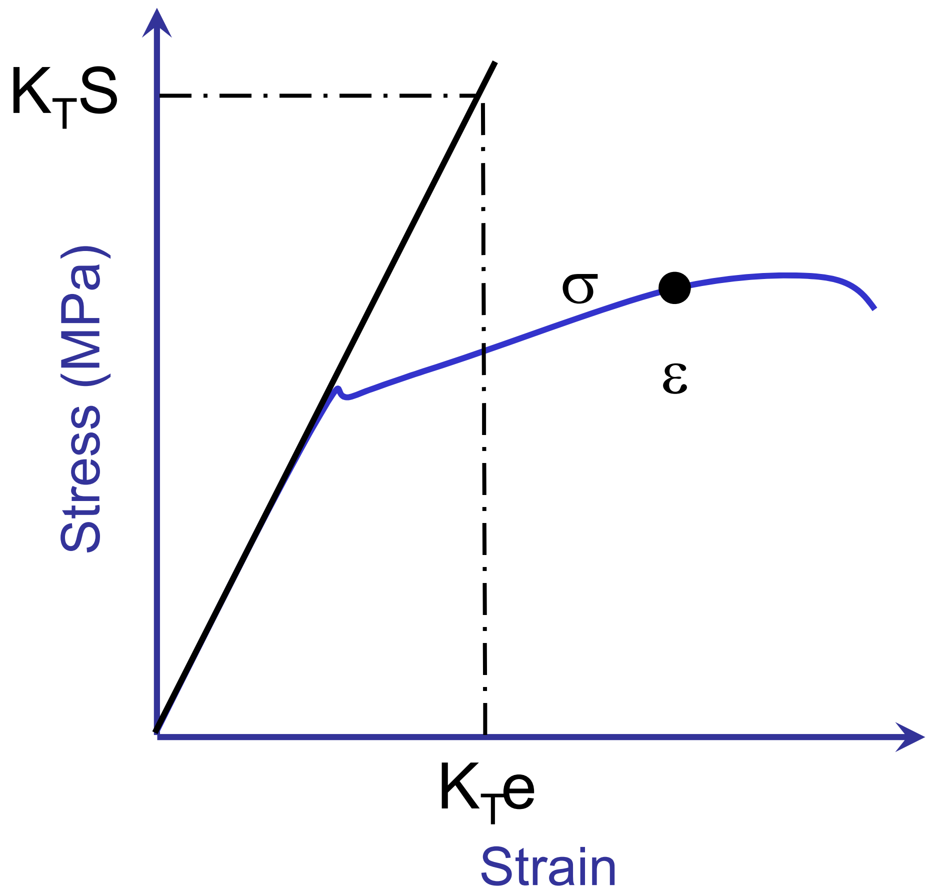
notch stress, σ

notch strain, ε

Stress concentration $K_\sigma = \dfrac{\sigma}{S}$

Strain concentration $K_\varepsilon = \dfrac{\varepsilon}{e}$

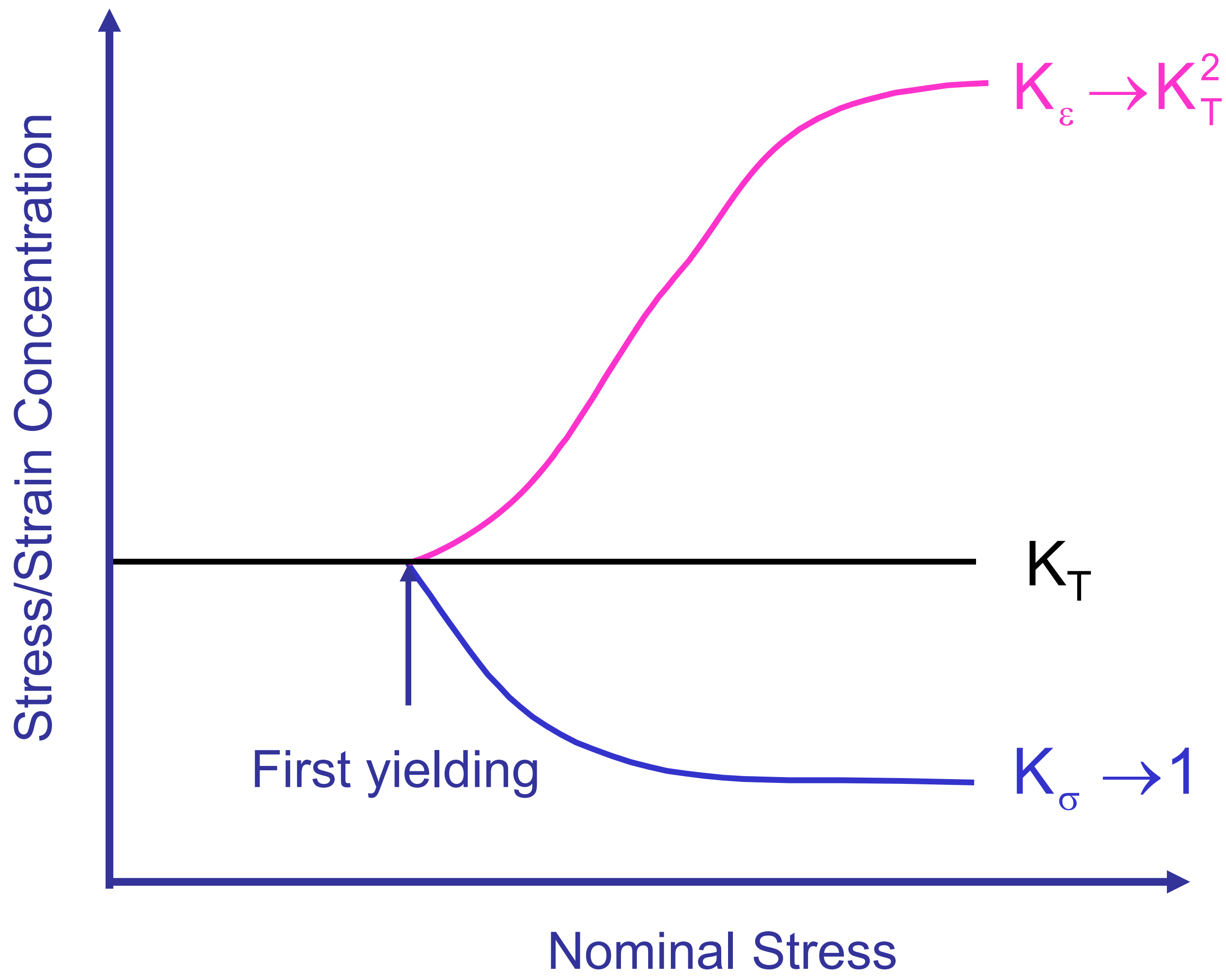
K_σ and K_ε



$$K_\sigma = \frac{\sigma}{S}$$

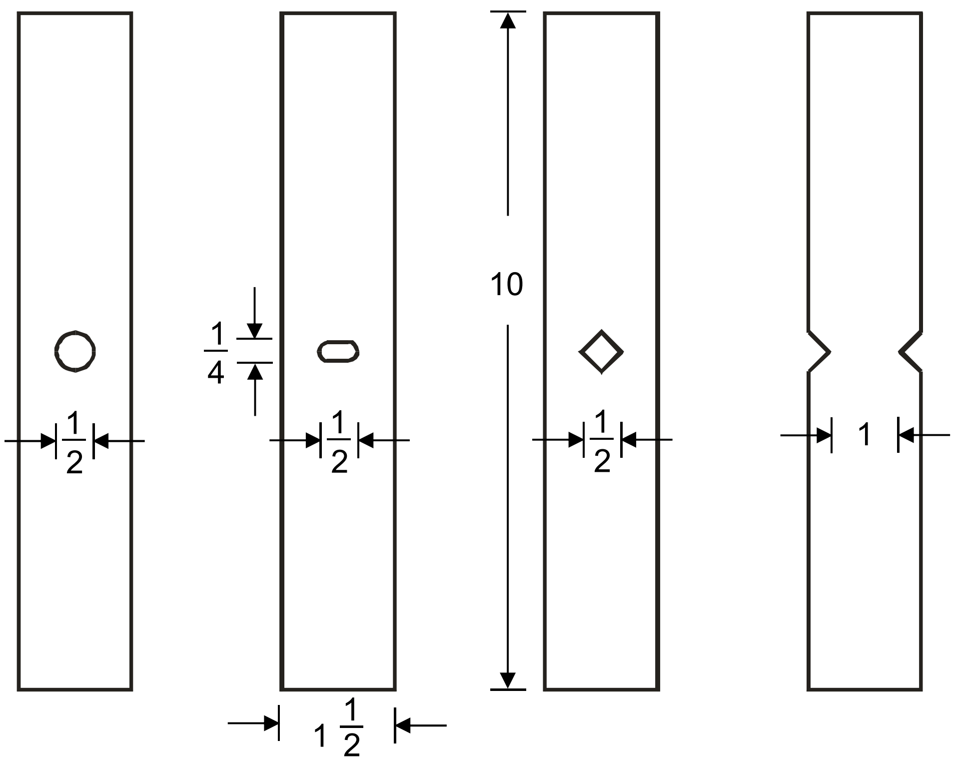
$$K_\varepsilon = \frac{\varepsilon}{e}$$

Stress and Strain Concentration



Failure of a Notched Plate

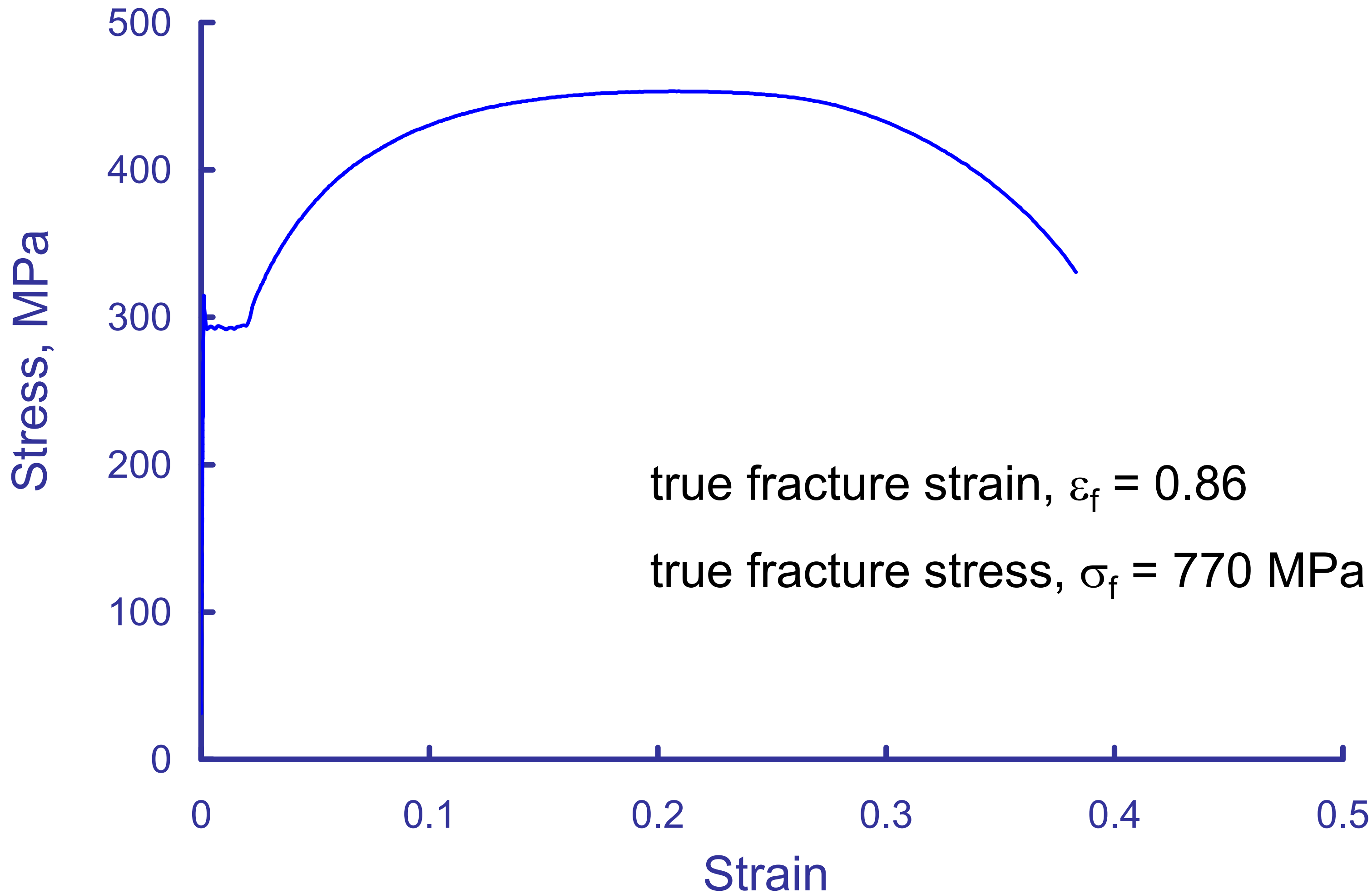
Real Data



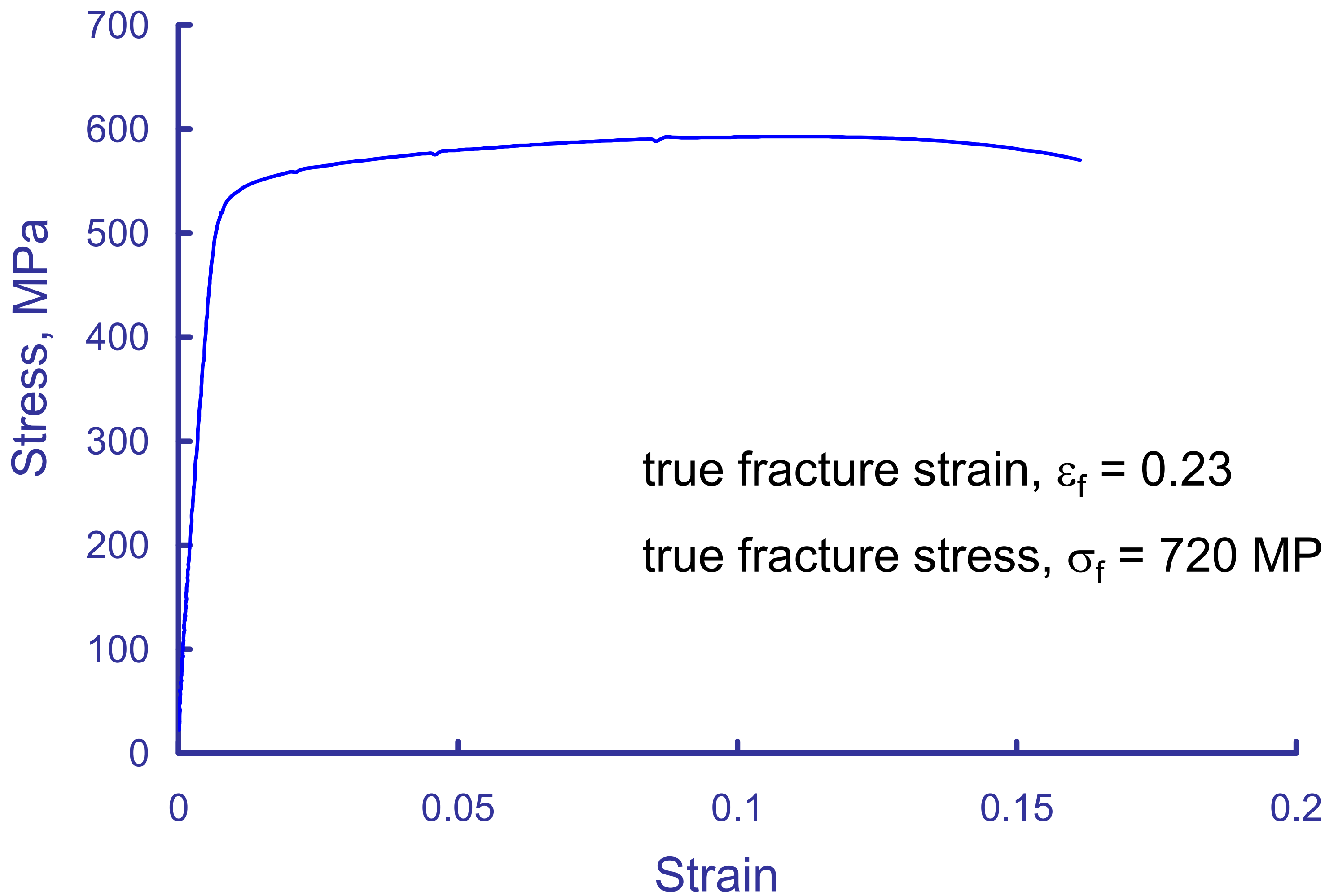
Materials:
1018 Hot Rolled Steel
7075-T6 Aluminum

1/4 thick

1018 Stress-Strain Curve



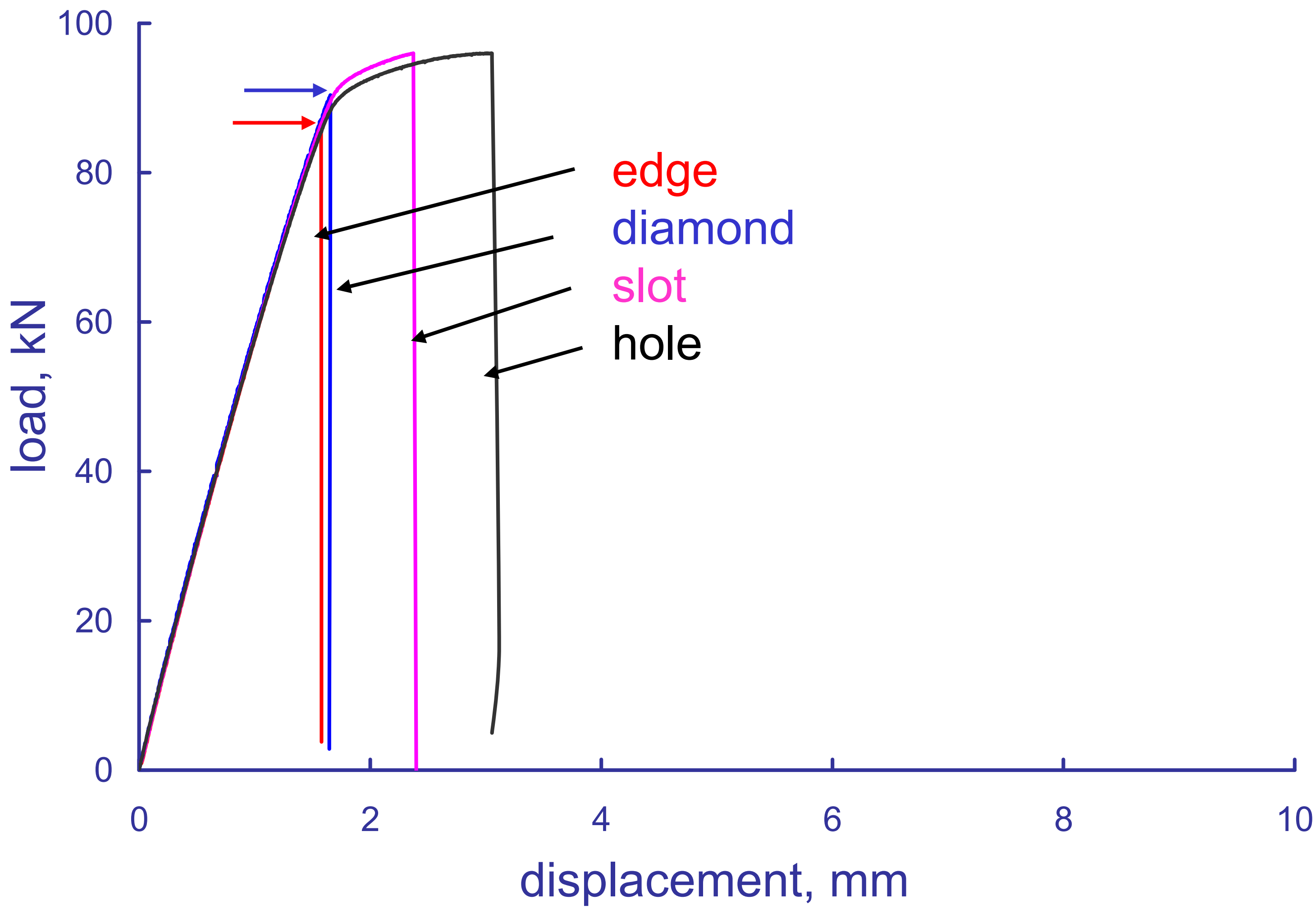
7075-T6 Stress-Strain Curve



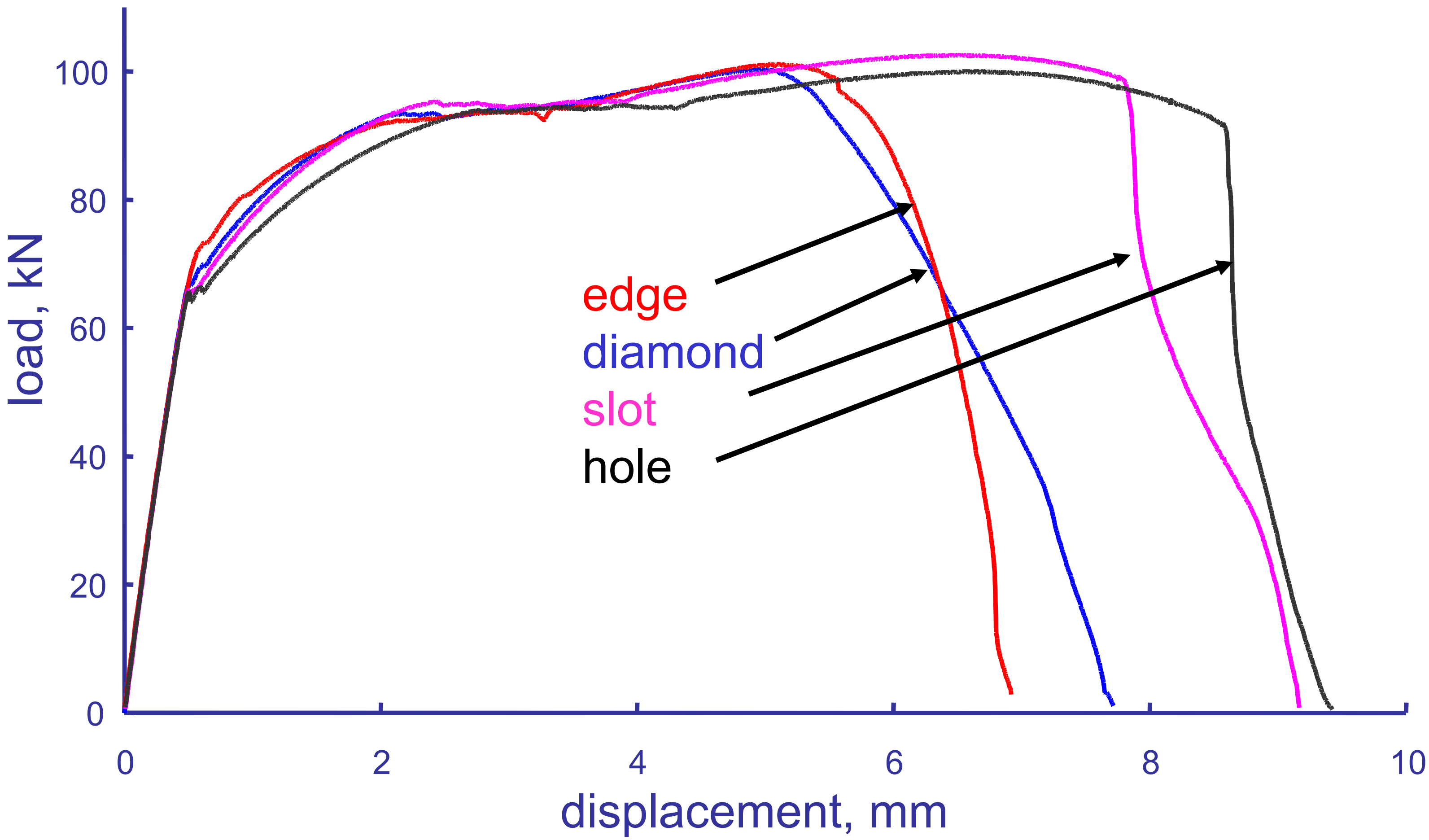
true fracture strain, $\varepsilon_f = 0.23$

true fracture stress, $\sigma_f = 720$ MPa

7075-T6 Test Data



1018 Steel Test Data



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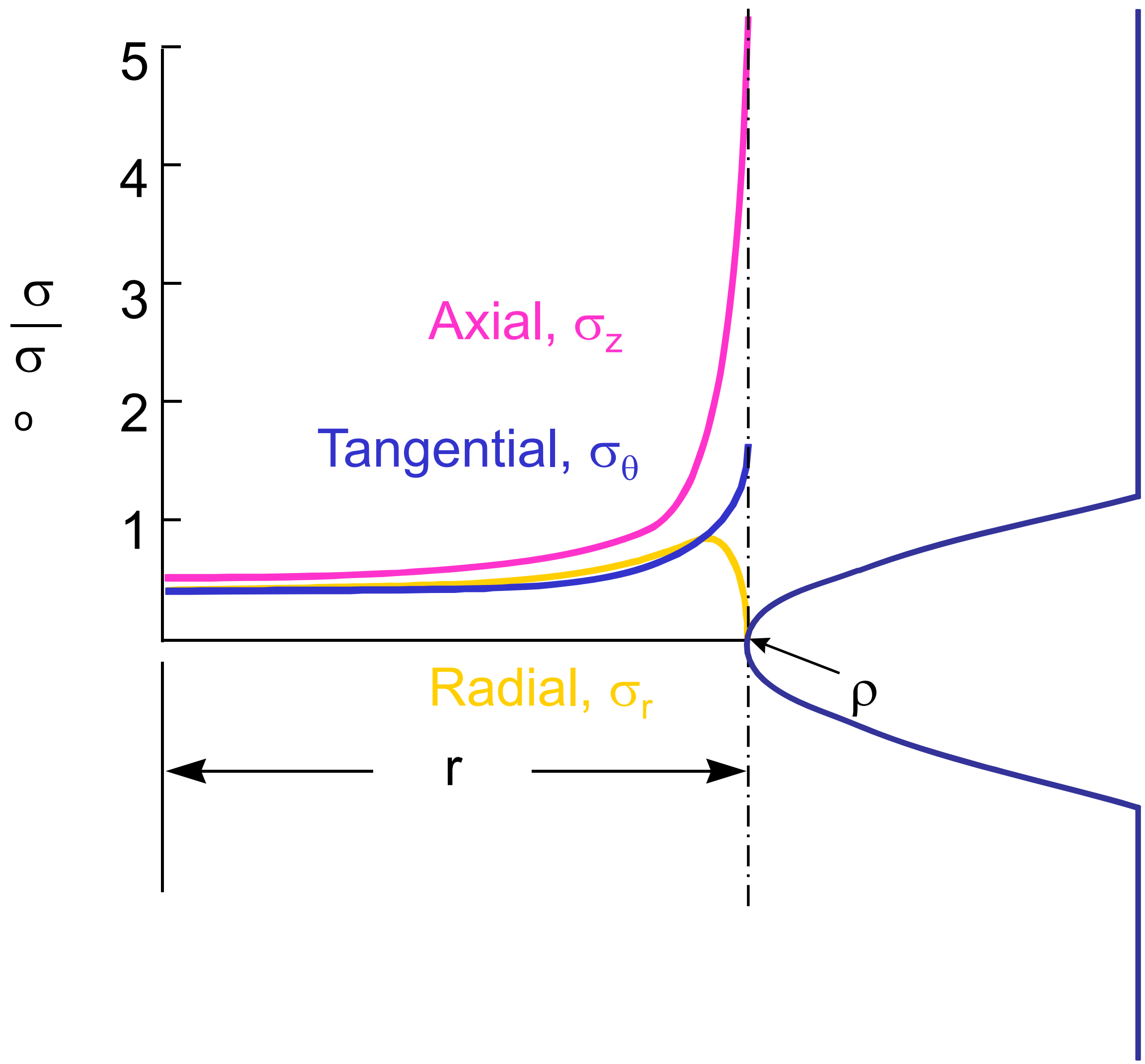
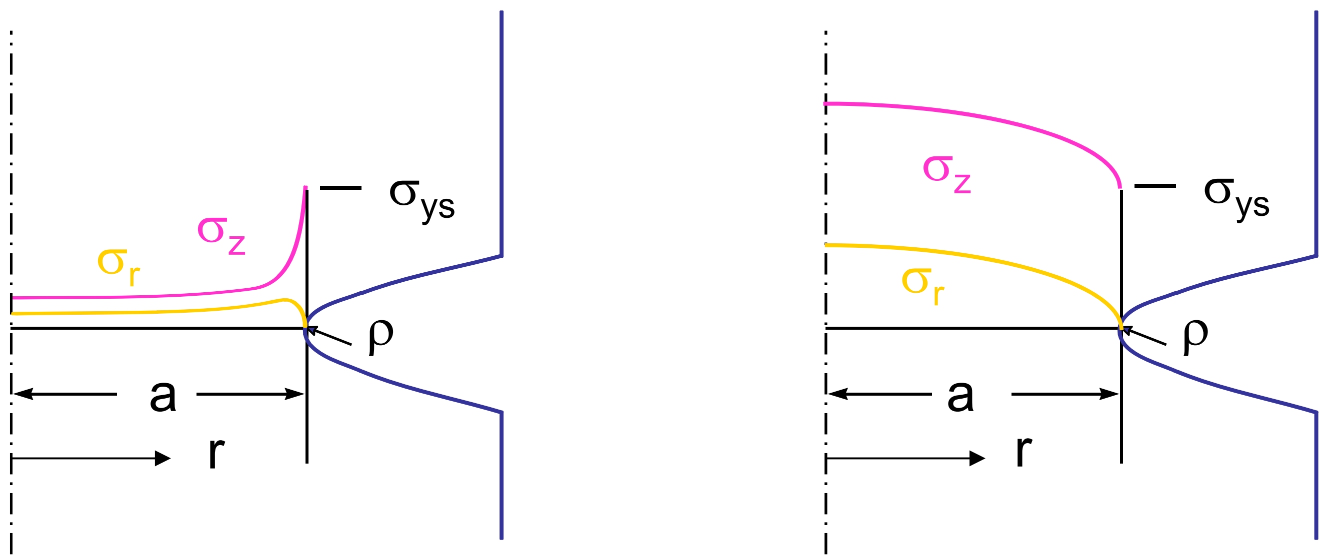
Stress Concentration



Bridgeman Analysis (1943)



Elastic stress distribution

Plastic stress distribution

$$\tau = \frac{\sigma_z - \sigma_r}{2} = \text{constant}$$

Stresses

$$\sigma_z = \sigma_o \left[1 + \ln\left(\frac{a^2 + 2a\rho - r^2}{2a\rho} \right) \right]$$

$$P_z = \int_0^a 2\pi r \, \sigma_z \, dr$$

$$P_{max} = \pi a^2 \sigma_{flow} \left(1 + \frac{2\rho}{a} \right) \ln\left(1 + \frac{a}{2\rho} \right)$$

$$P_{max} = A_{net} \, \sigma_{flow} \, CF$$

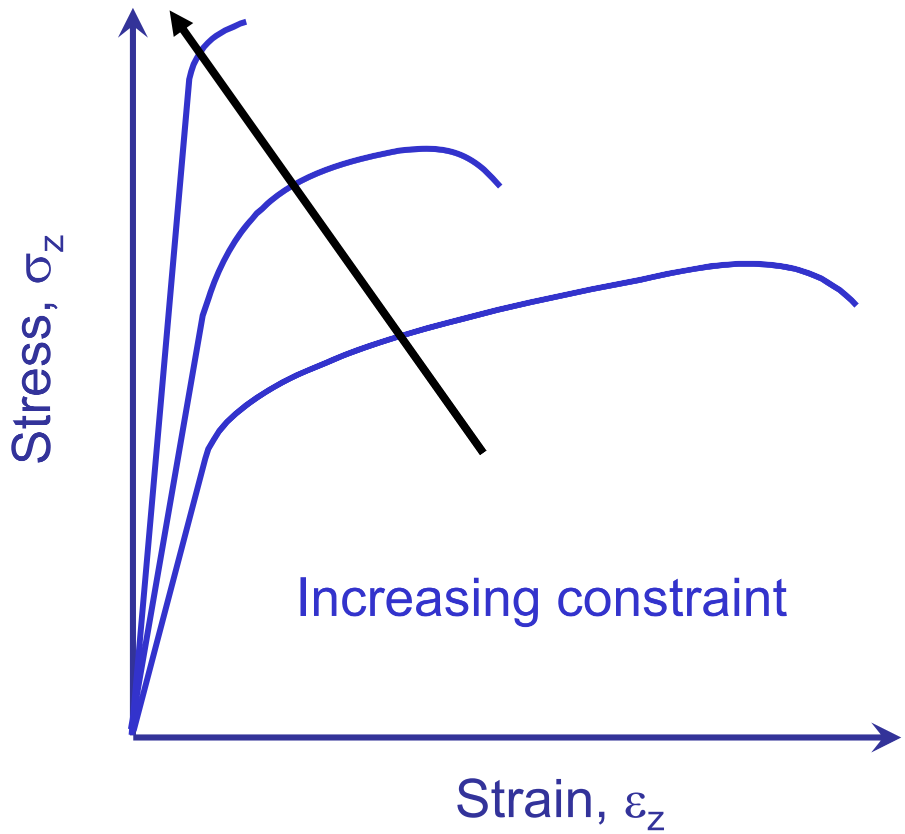
CF constraint factor

Constraint Factors

a/ρ	CF
0	1
1	1.21
2	1.38
4	1.64
8	1.73
20	2.63
∞	2.96

$$P_{max} = A_{net}\, \sigma_{flow}\, CF$$

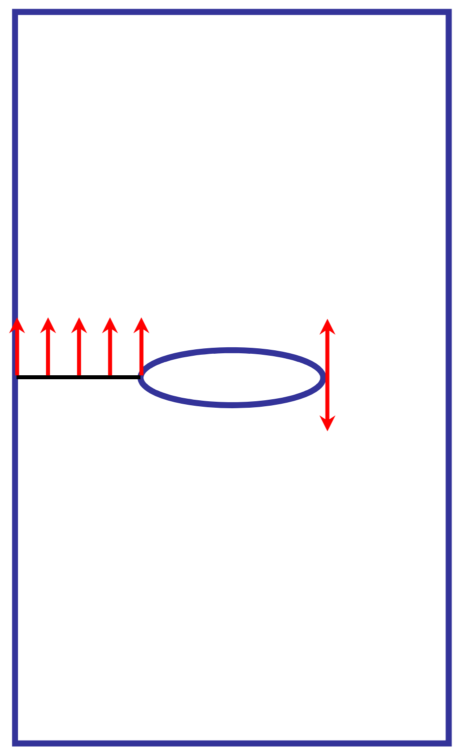
Effect of Constraint



Higher strength and lower ductility

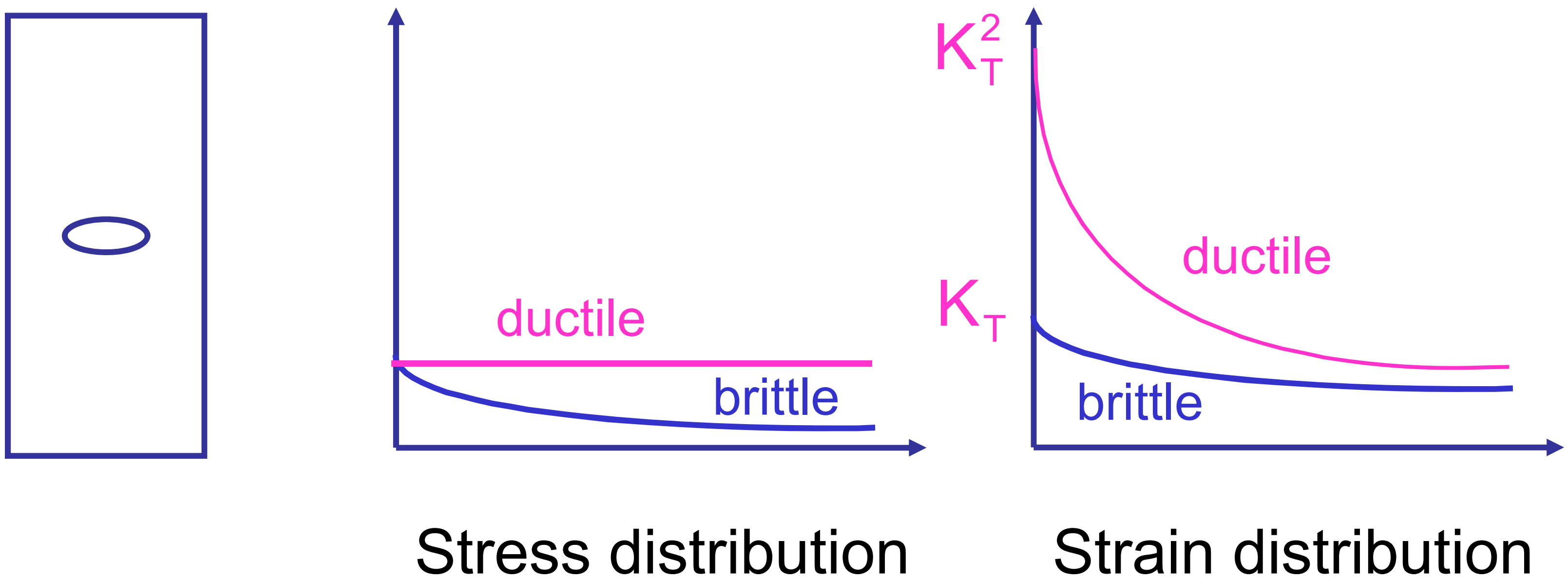
Failures from Stress Concentrations

Net section stresses must be below the flow stress



Notch strains must be below the fracture strain

Typical Stress Concentration

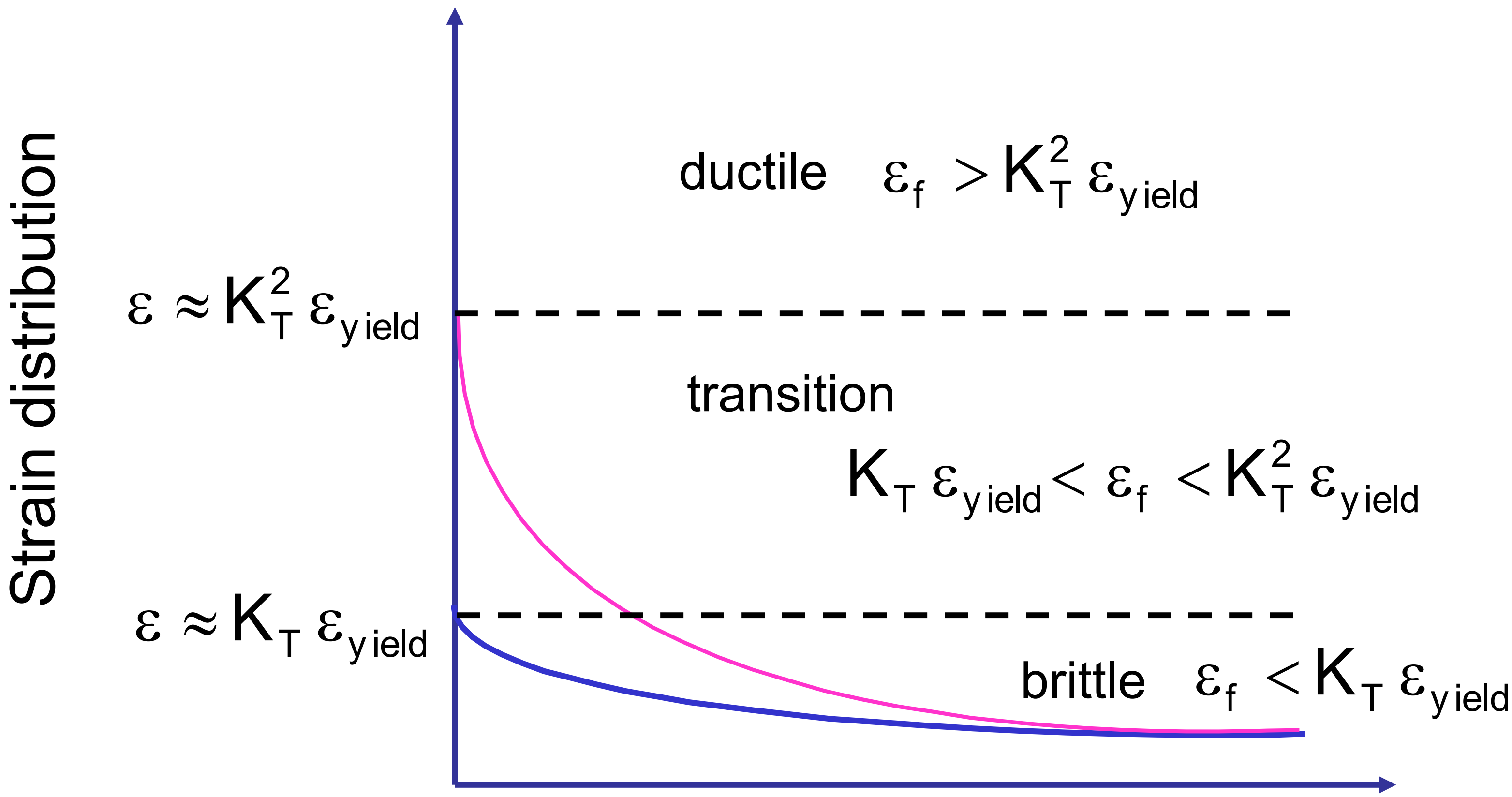


A ductile material has the capacity for very large strains and it reaches the strength limit first

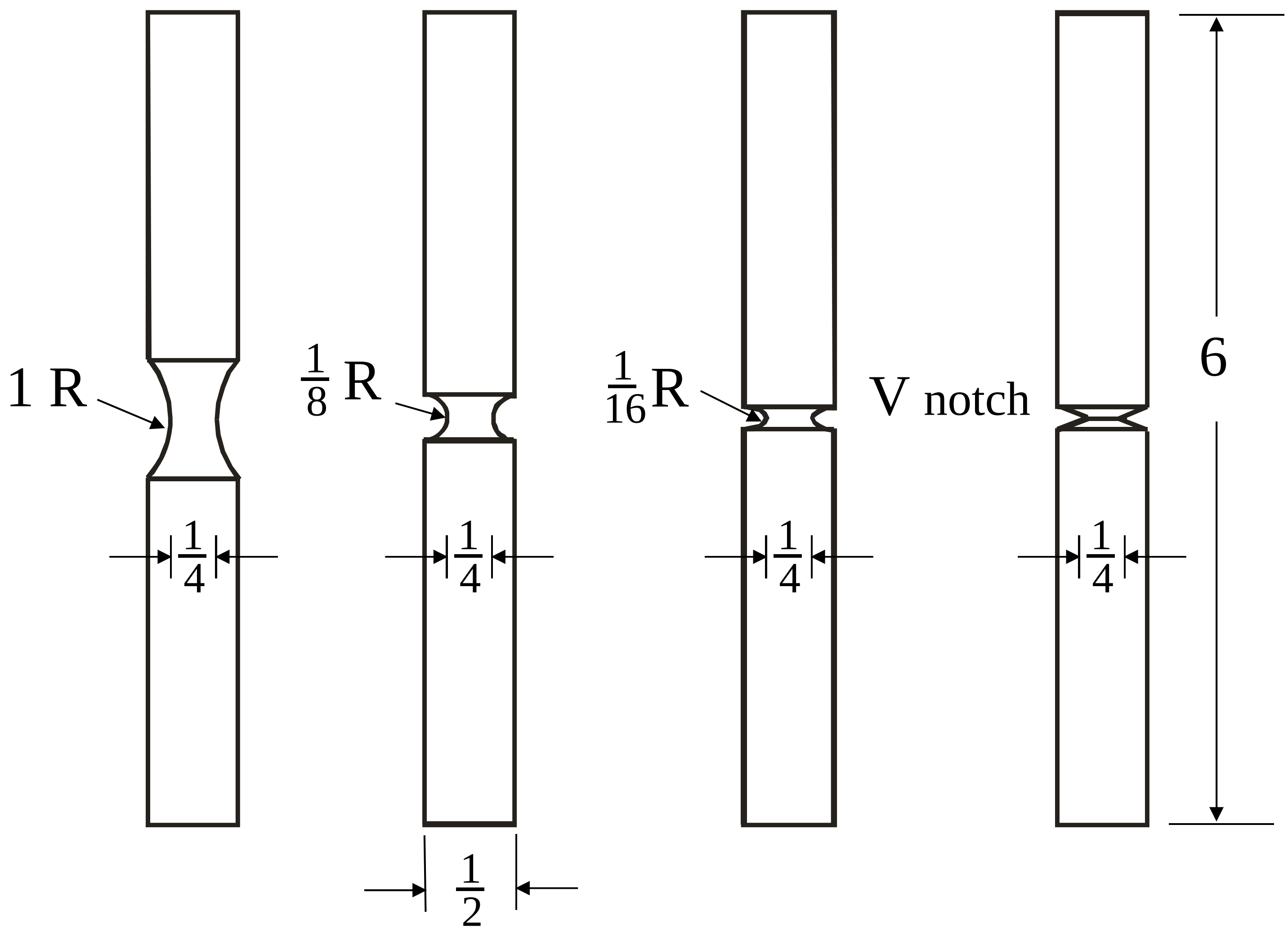
A brittle material reaches the strain limit first

Strain Distribution

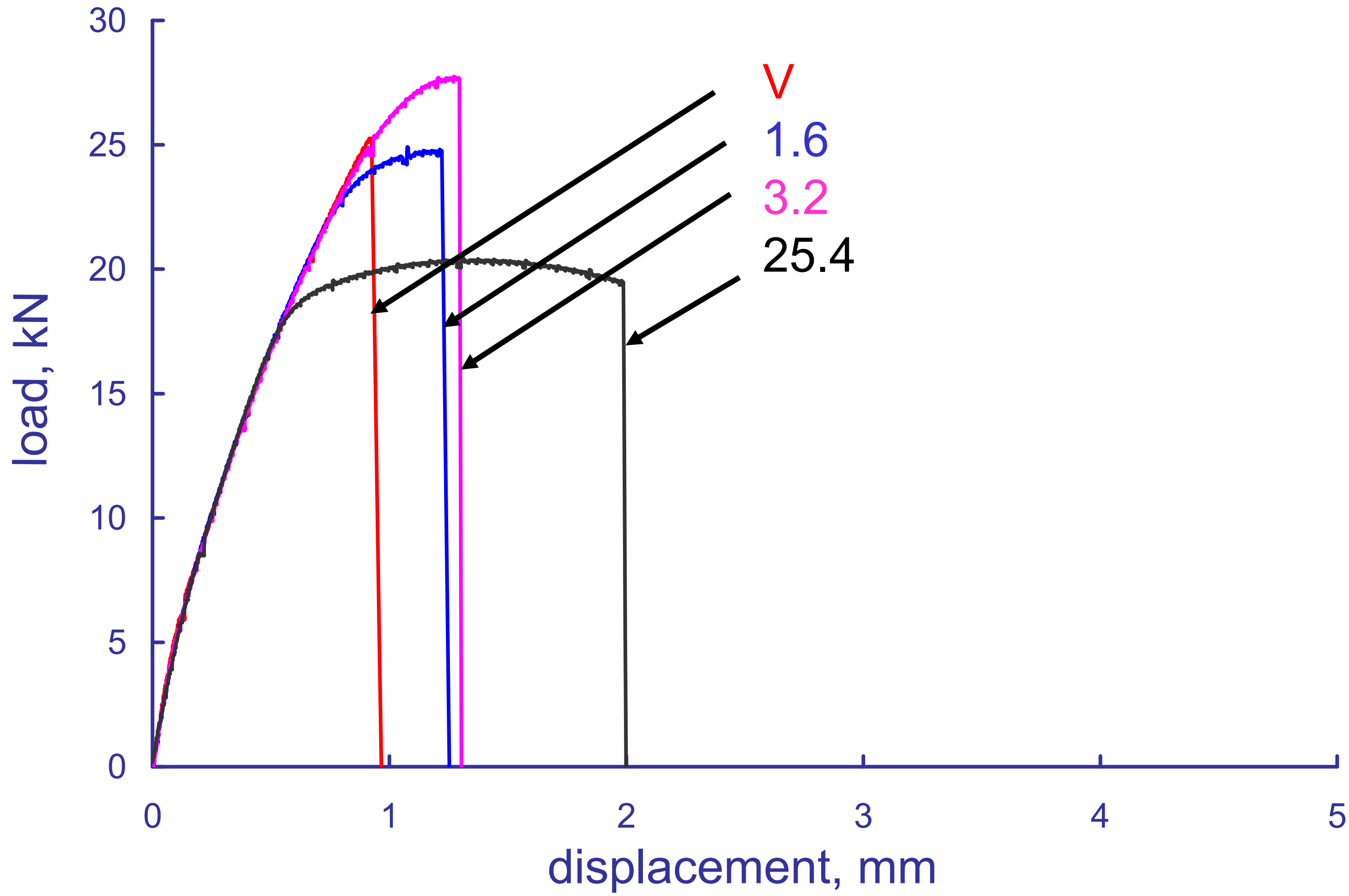
Nominal strain will reach the yield strain when the strength limit is reached



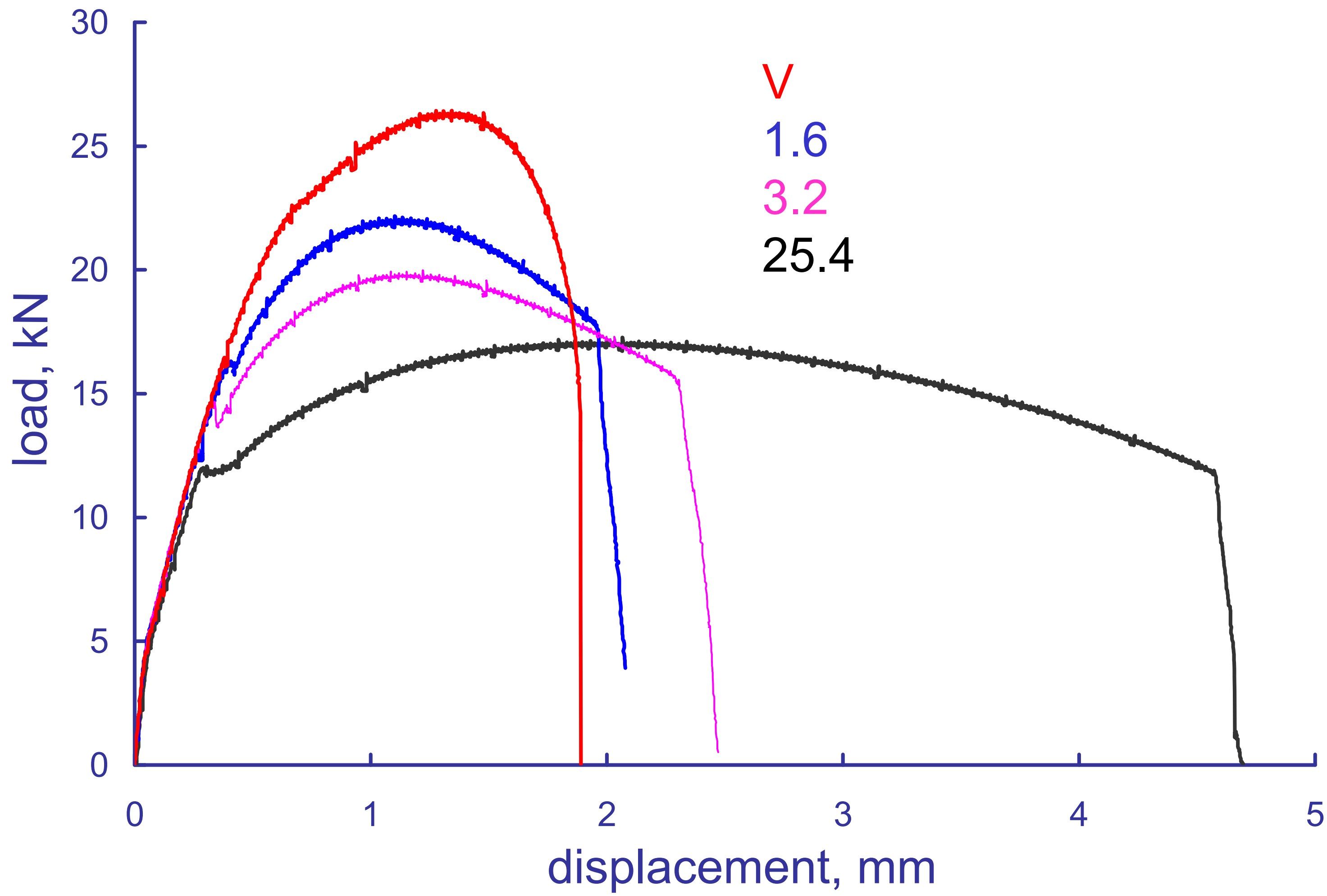
Notched Bars



7075-T6 Test Data



1018 Steel Test Data



Conclusion

Net section area, state of stress and material strength control the failure load in a structure only in ductile materials. In brittle materials, cracks will form before the maximum load capacity of the structure is reached.

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- Limit theorems
- Plane Stress – Plane Strain
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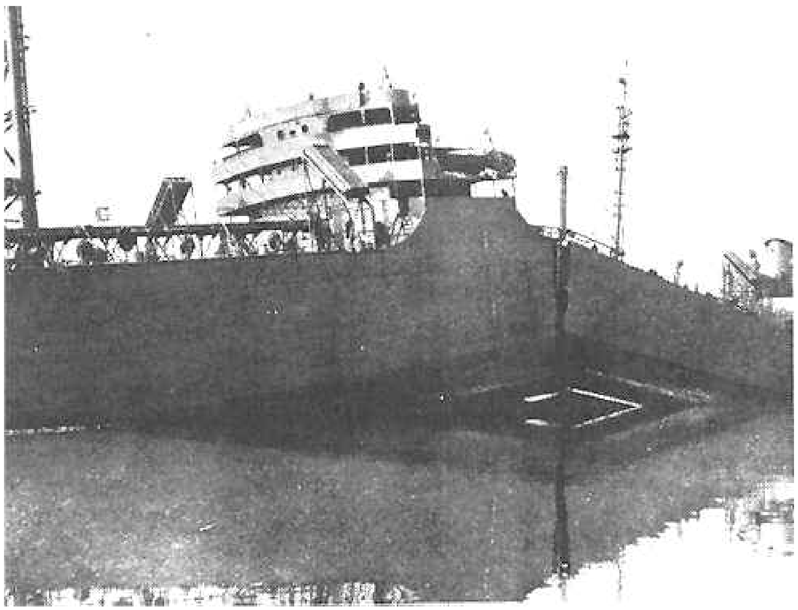
- Notched bars
- **Brittle Fracture**

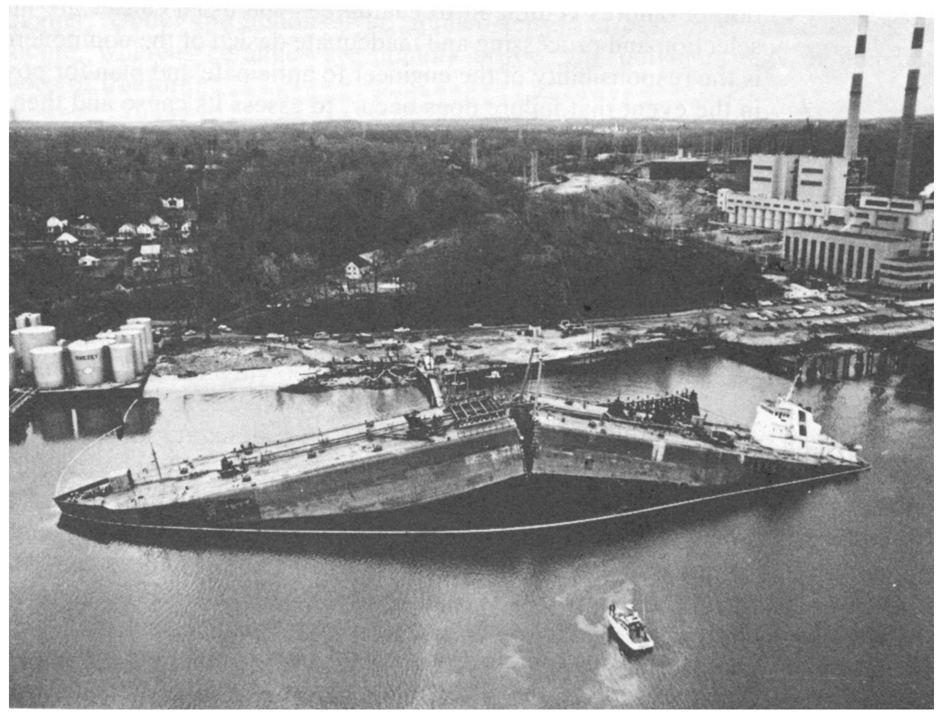
Brittle Fracture

Failure of structures by the rapid growth of cracks (or crack-like defects) until loss of structural integrity.

Fracture

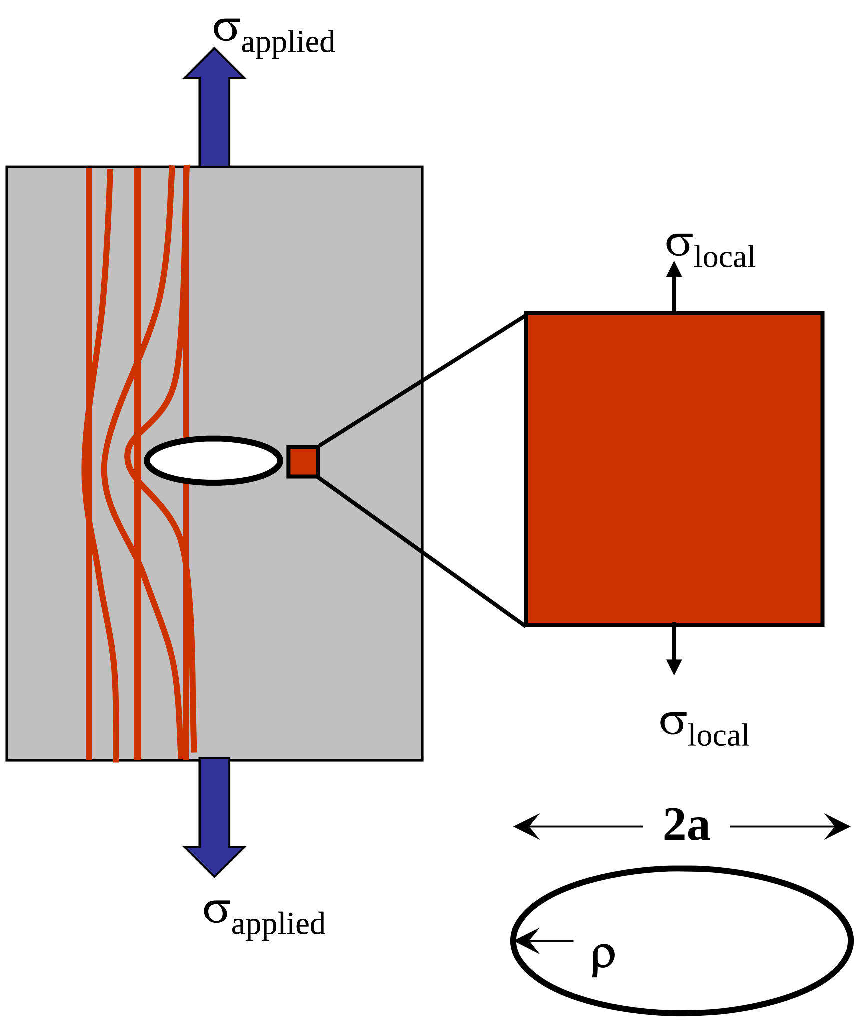


1943



1972

Stress Concentration



- Inglis Equation

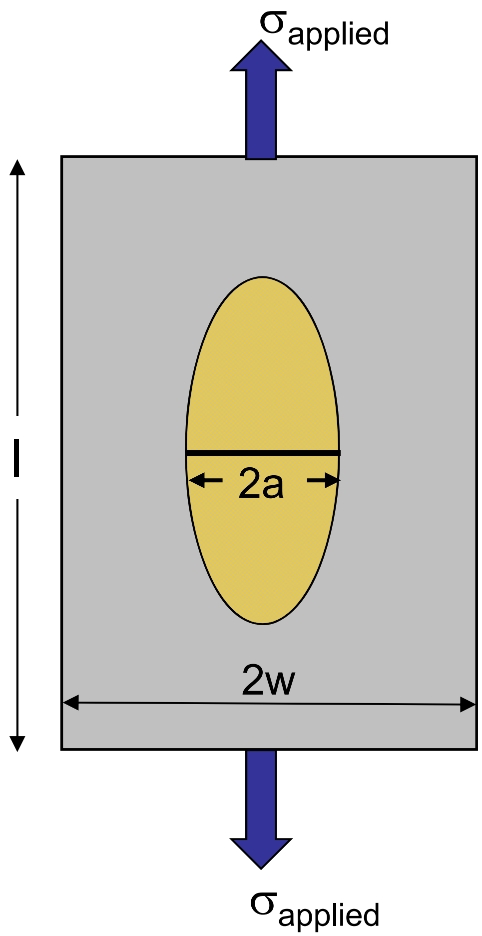
$$\sigma_{local} = \left(1 + 2\sqrt{\frac{a}{\rho}}\right)\sigma_{applied}$$

$$= K_t \sigma_{applied}$$

- Crack dimensions
  - $a = 10^{-3}$
  - $\rho = 10^{-9}$
- Does this make sense?

$$K_t = 2000$$

Griffith Approach



thickness, t

- Uncracked plate elastic energy

$$U_e = \frac{1}{2}\sigma\varepsilon\left(2w*l*t\right) = \frac{\sigma^2}{2E}\left(2w*l*t\right)$$

- Introduce a crack which
  - Reduces elastic energy by:

$$U_e^{2a} = \frac{\sigma}{2E}\left(2\pi a^2 * t\right)$$

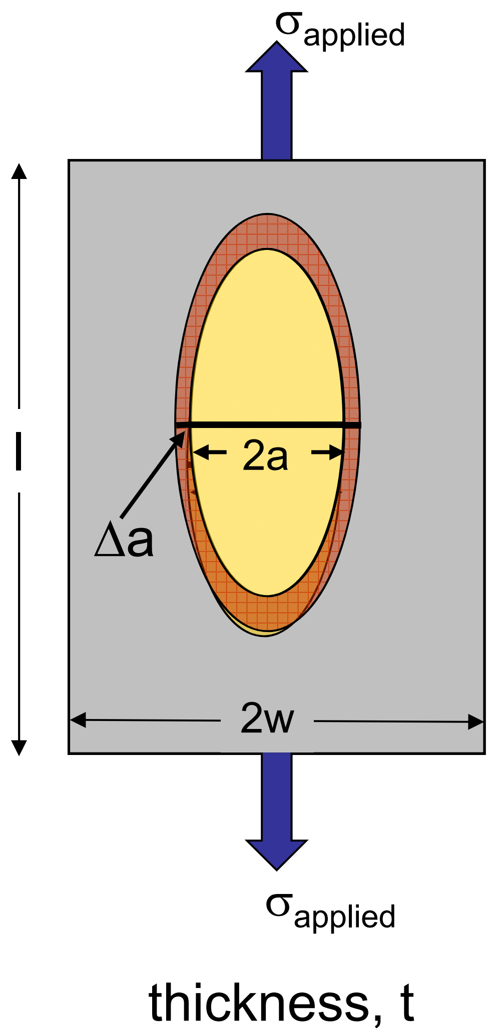
  - Increases total surface energy by:

$$\Gamma = 2\gamma_s \bullet 2at$$

- For a crack to grow, the energy provided by new surfaces must equal that lost by elastic relief

$$\Delta U_e^{2a} = \Delta\Gamma$$

Griffith Approach



- Minimum criterion for stable crack growth:
- Strain energy goes into surface energy

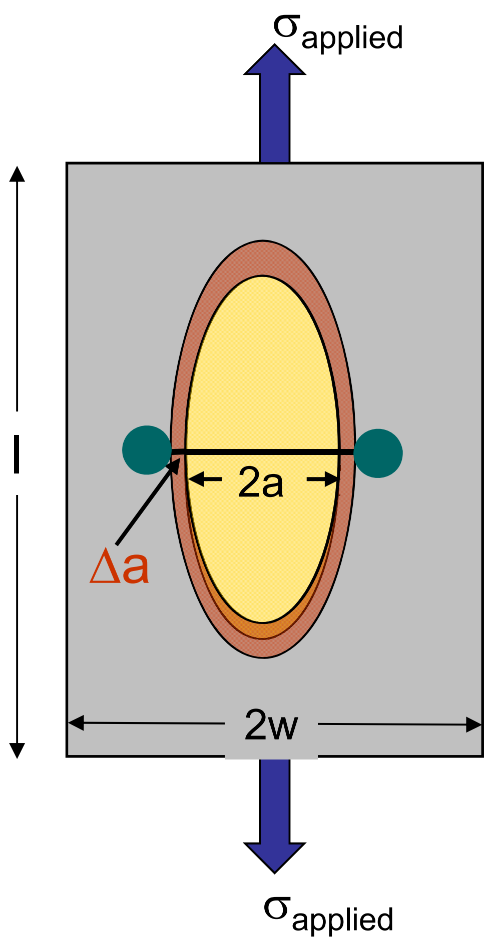
$$\frac{\partial U}{\partial a} = \frac{\partial \Gamma}{\partial a}$$

$$\frac{\partial}{\partial a}\left(\frac{\sigma^2}{2E}\pi\, 2a^2 t\right) = \frac{\partial}{\partial a}(\gamma_s 4at)$$

$$\sigma^2 \pi a = 2E\gamma_s$$

$$\sigma = \sqrt{\frac{2E\gamma_s}{\pi a}}$$

Plastic Energy Term



thickness, t

- Previous derivation is for purely elastic material
- Most metals and polymers experience some plastic deformation
- Strain energy (U) goes into surface energy (Γ) & plastic energy (P)

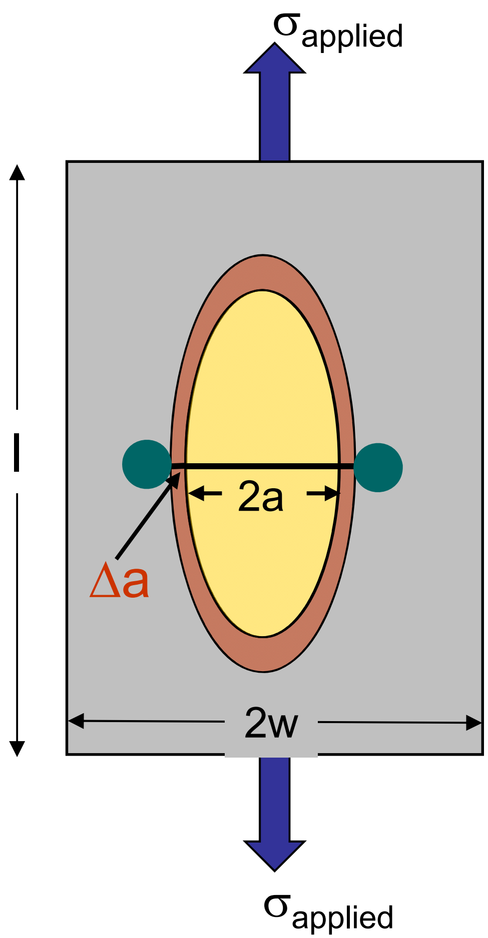
$$\frac{\partial U}{\partial a} = \frac{\partial \Gamma}{\partial a} + \frac{\partial P}{\partial a}$$

- Orowan introduced plastic deformation energy, γ_p

$$\sigma = \sqrt{\frac{2E(\gamma_s + \gamma_p)}{\pi a}} \qquad \gamma_p >> \gamma_s$$

- Materials which exhibit plastic deformation absorb much more energy, removing it from the crack tip

Energy Release Rate, G



thickness, t

- Irwin chose to define a term, G, that characterizes the energy per unit crack area required to extend the crack:

$$G = -\frac{1}{t}\frac{\partial U}{\partial a}$$

Change in crack area

$$G = \frac{\pi a \sigma^2}{E}$$

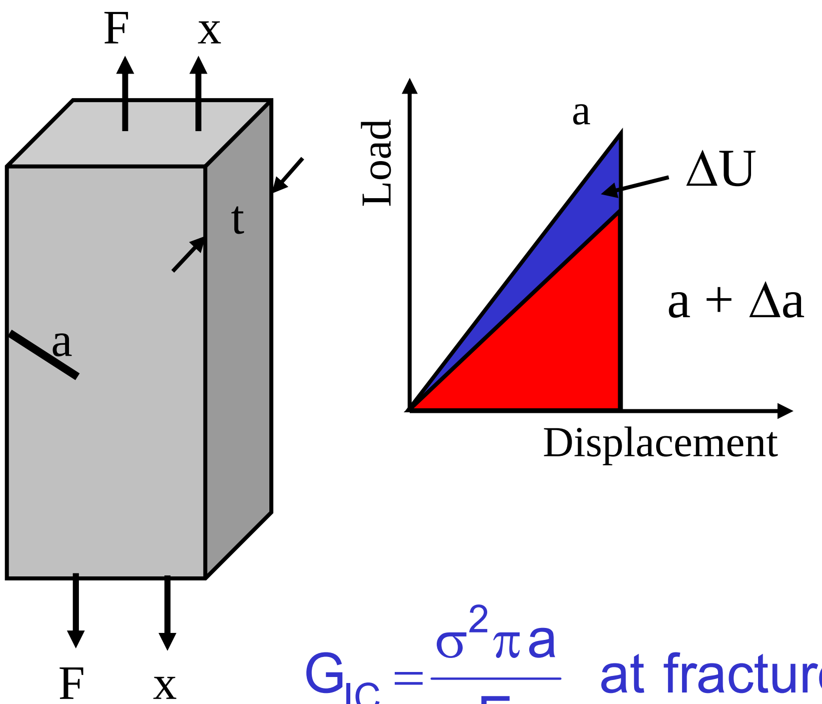
$$\sigma = \sqrt{\frac{EG}{\pi a}}$$

- Comparing to the previous expression, we see that:

$$G = 2(\gamma_s + \gamma_p)$$

- Works well when plastic zone is small ("fraction of crack dimensions")

Experimentally Measuring G

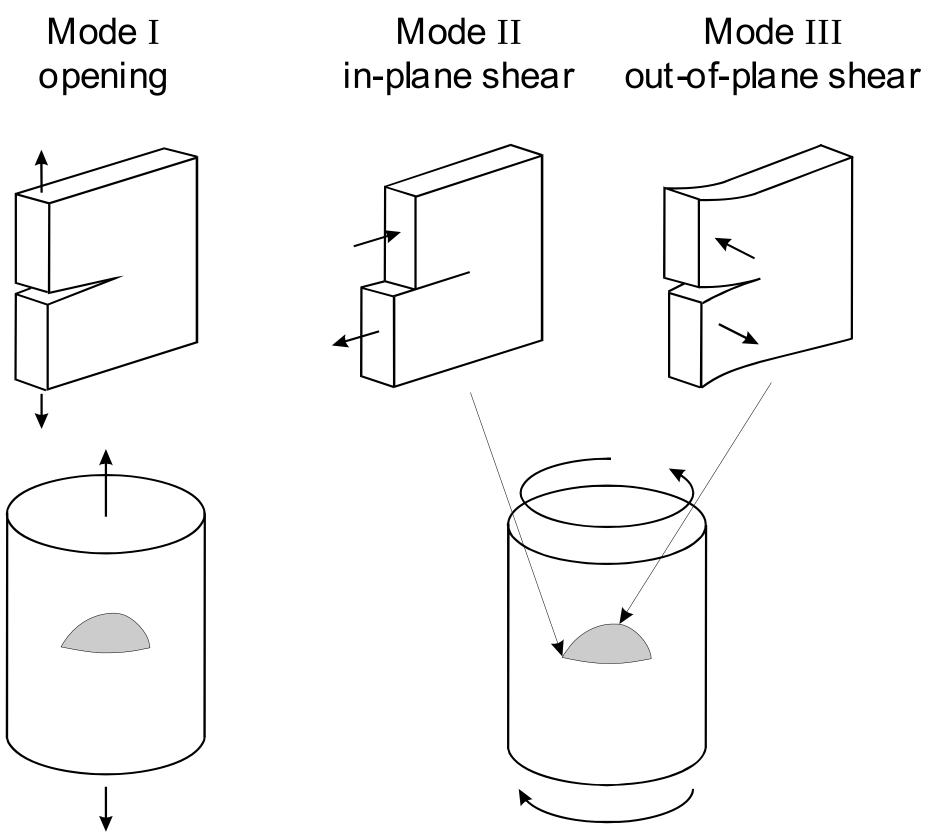


$$G_{IC} = \frac{\sigma^2 \pi a}{E} \quad \text{at fracture}$$

- Load cracked sample in elastic range
- Fix grips at given displacement
- Allow crack to grow length Δa
- Unload sample
- Compare energy under the curves
- G is the energy per unit crack area needed to extend a crack
- Experimentally measure combinations of stress and crack size at fracture to determine G_{IC}

Stress Based Crack Analysis

- Westergaard, Irwin analyzed fracture of cracked components using elastic-based stress theory
  - Three modes for crack loading



From: Socie

Stress Intensity Factor, K

$$K = \sigma \sqrt{\pi a}\, F\left(\frac{a}{b}\right)$$

Stress intensity factor

specimen geometry

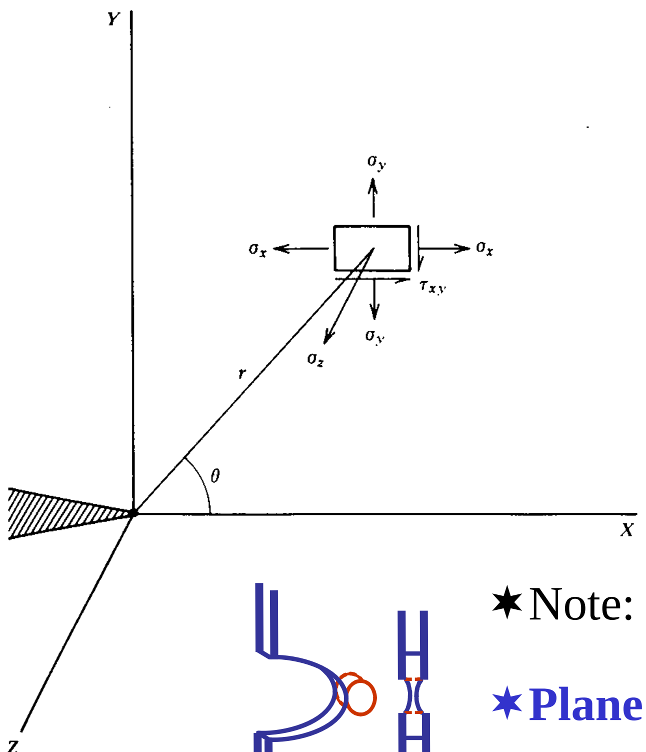
flaw size

operating stresses

- Critical parameter is now based on:
  - Stress
  - Flaw Size
- Specimen geometry included using “correction factor”
  - crack shape
  - specimen size and shape
  - type of loading (i.e. tensile, bending, etc)

Stress Intensity Factor, K

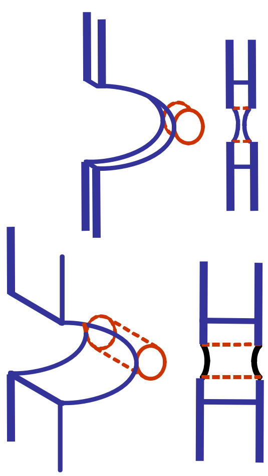
Stress Fields (near crack tip)



$$\sigma_x = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\sigma_y = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

$$\tau_{xy} = \frac{K}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$



✶Note: as r → 0, stress fields → ∞

✶**Plane stress** - too thin to support stress through thickness

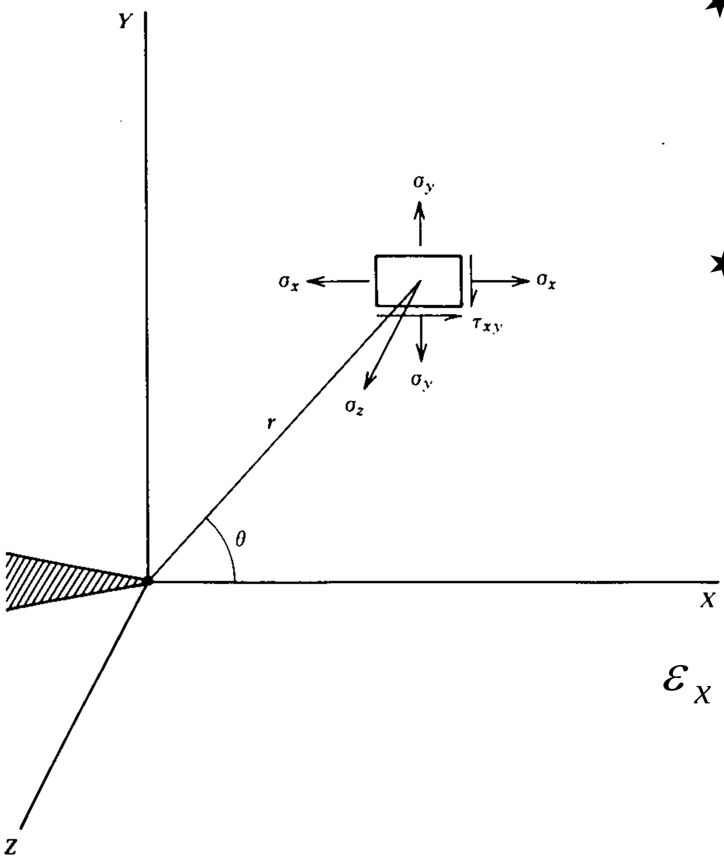
$$\sigma_z = 0 \quad \varepsilon_z \neq 0$$

✶**Plane strain** - so thick that constrains strain through thickness

$$\sigma_z = \nu\left(\sigma_x + \sigma_y\right) \quad \varepsilon_z = 0$$

Stress Intensity Factor, K

Displacement Fields



- Displacement in x-direction

$$u = \frac{K}{2G}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}\left[\frac{3-\nu}{1+\nu} - 1 + 2\sin^2\frac{\theta}{2}\right]$$

- Displacement in y-direction

$$v = \frac{K}{2G}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}\left[\frac{3-\nu}{1+\nu} - 1 + 2\cos^2\frac{\theta}{2}\right]$$

$$\varepsilon_x = \frac{\partial u}{\partial x} \qquad \varepsilon_y = \frac{\partial v}{\partial x} \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}$$

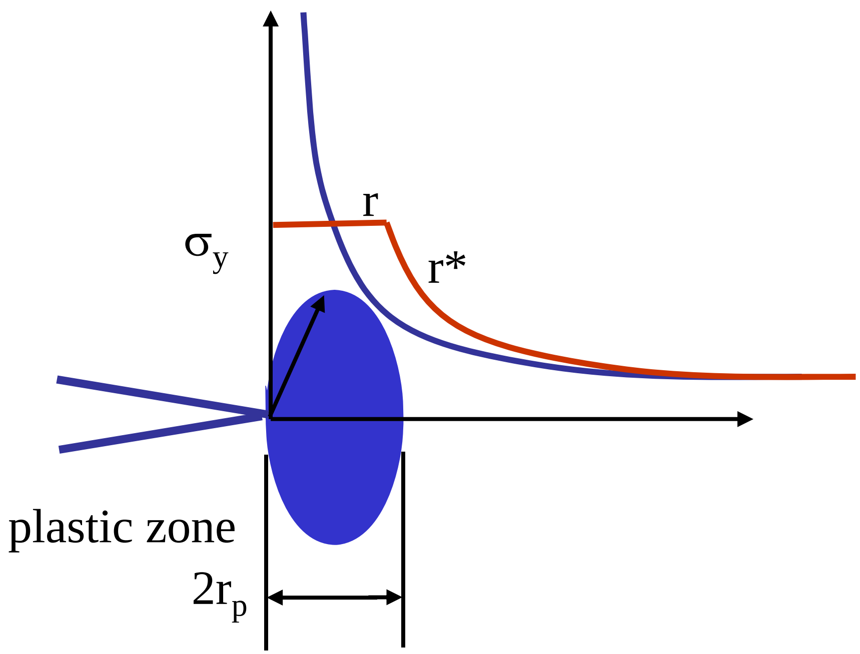
K and G

$$G = \frac{K^2}{E}$$ plane stress

$$G = \frac{\left(1 - \nu^2\right)K^2}{E}$$ plane strain

Linear Elastic Fracture Mechanics

- How can we use an elastic concept (K) when there is plastic deformation?
  - Small scale yielding
  - "Blunts" the crack tip



- At elastic-plastic boundary,

$$\sigma_y = \frac{K}{\sqrt{2\pi r^*}} \cos\frac{\theta}{2}\left[1 + \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right]$$

- Note: zone of yielding will vary with θ
- Consider stress at $\theta = 0$

$$\sigma_y = \frac{K}{\sqrt{2\pi r^*}}$$

Design Philosophy

$$K_{IC} = \sigma \sqrt{\pi a}\, f\left(\frac{a}{w}\right)$$

fracture toughness

flaw shape

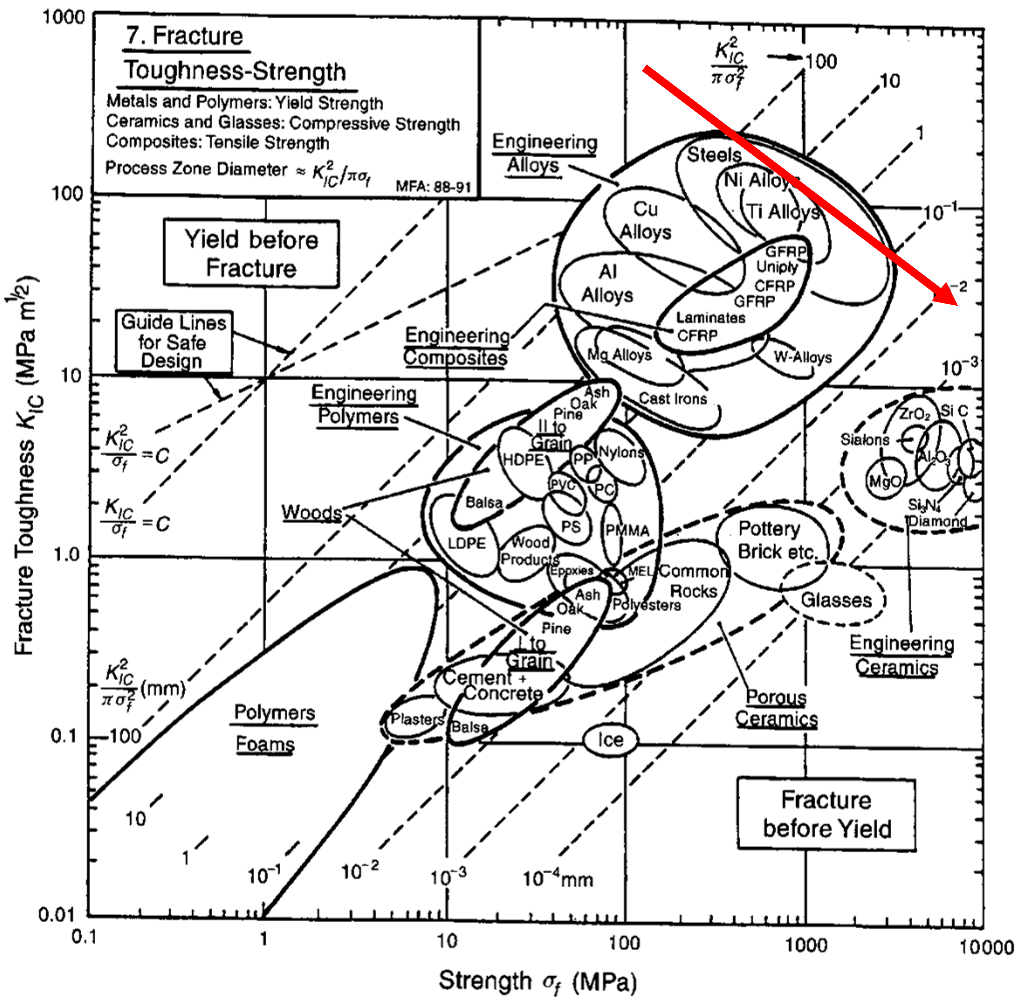
flaw size

operating stresses

Critical Crack Sizes

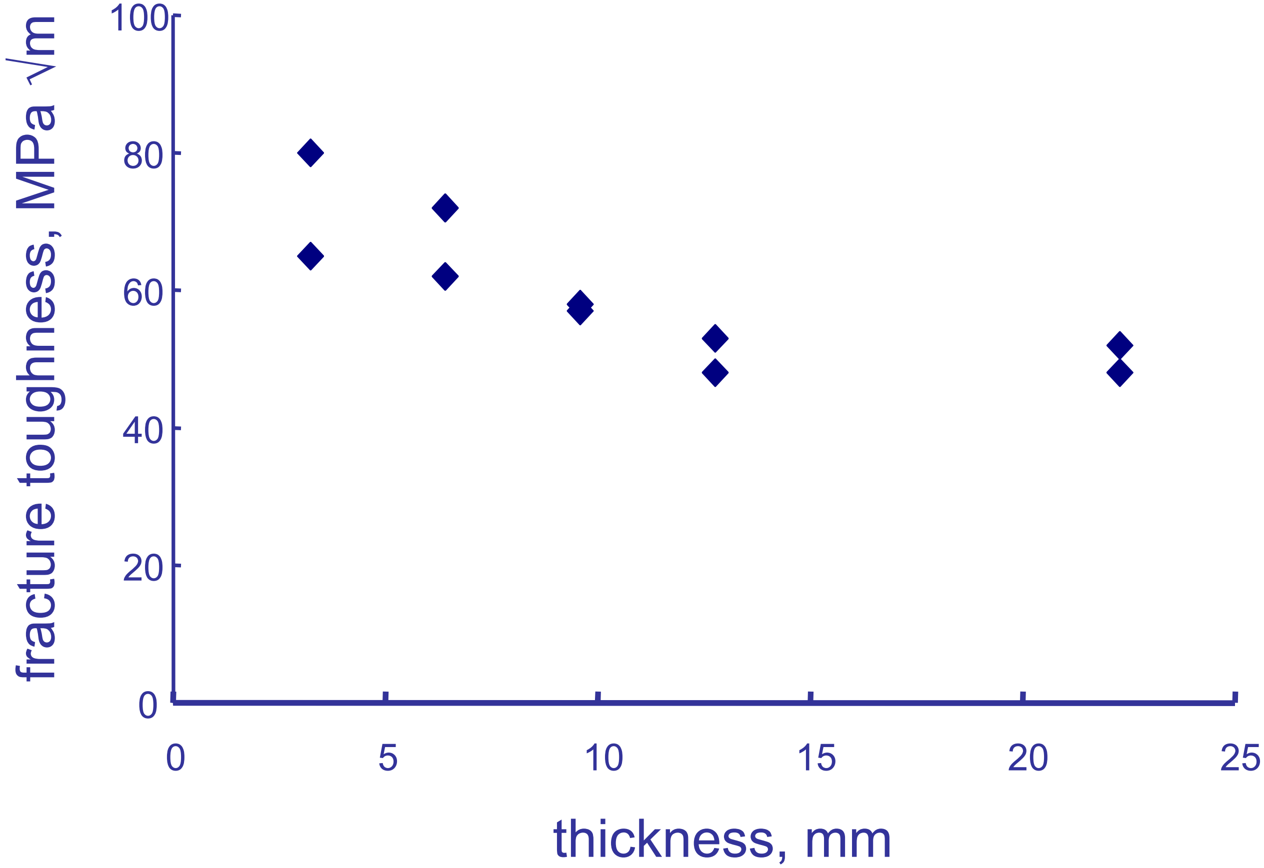
$$a_{critical} = \frac{1}{\pi}\left(\frac{K_{IC}}{\sigma_f}\right)^2$$

Fracture Toughness

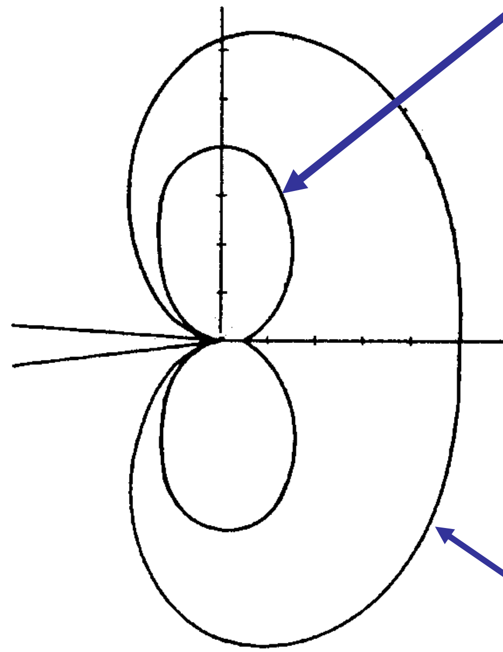


From M F Ashby, Materials Selection in Mechanical Design, 1999, pg 431

Thickness Effects



Plastic Zone Size



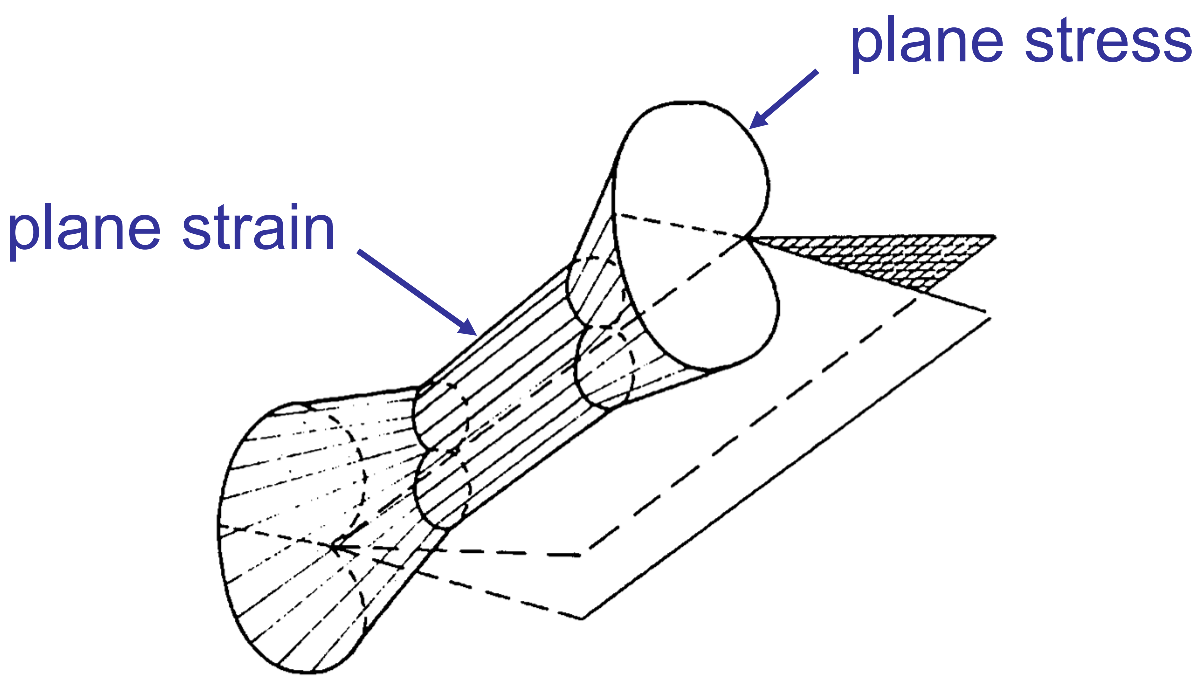
plane strain

$$r_p = \frac{1}{6\pi}\left(\frac{K}{\sigma_{ys}}\right)^2$$

plane stress

$$r_p = \frac{1}{2\pi}\left(\frac{K}{\sigma_{ys}}\right)^2$$

3D Plastic Zone



Fracture Surfaces

Size Requirements

$$t, W - a, a \geq 2.5\left(\frac{K}{\sigma_{ys}}\right)^2 \approx 50 r_p$$

	σ_{ys}	K_{Ic}	t, mm
2024- T3	345	44	40.7
7075 -T6	495	25	6.4
Ti-6Al-4V	910	105	33.3
Ti-6Al-4V	1035	55	7.1
4340	860	99	33.1
4340	1510	60	3.9
17-7 PH	1435	77	7.2
52100	2070	14	0.1