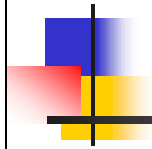


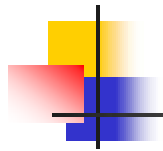
Probabilistic Aspects of Fatigue

www.FatigueCalculator.com



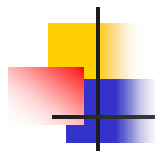
Professor Darrell F. Socie
Department of Mechanical and
Industrial Engineering

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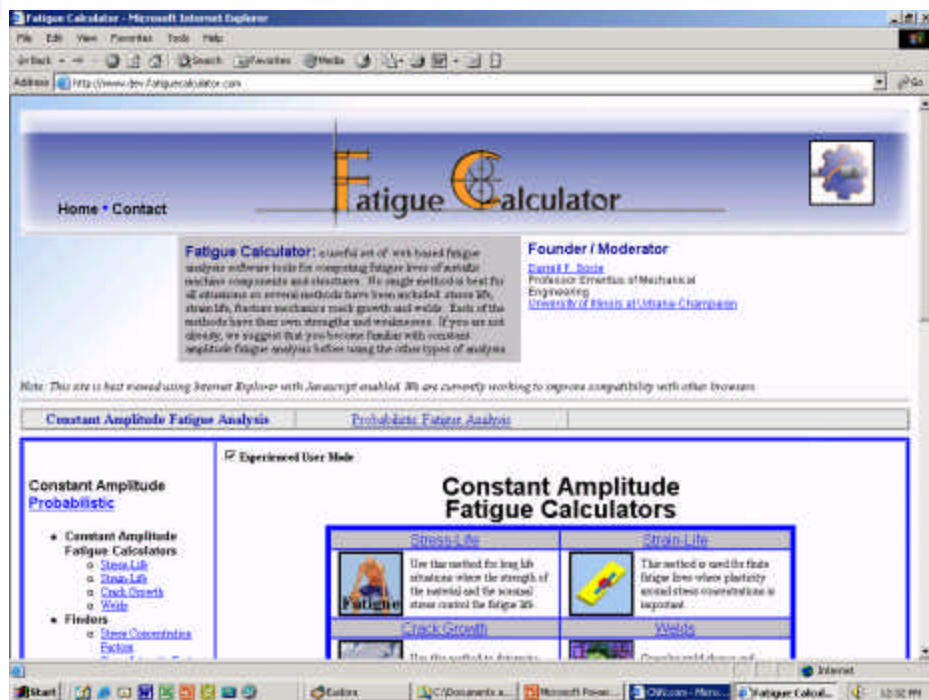


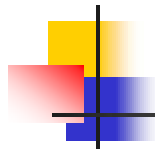
Probabilistic Aspects of Fatigue

- Introduction
- Basic Probability and Statistics
- Statistical Techniques
- Analysis Methods
- Characterizing Variability
- Case Studies
- **FatigueCalculator.com**
- GlyphWorks

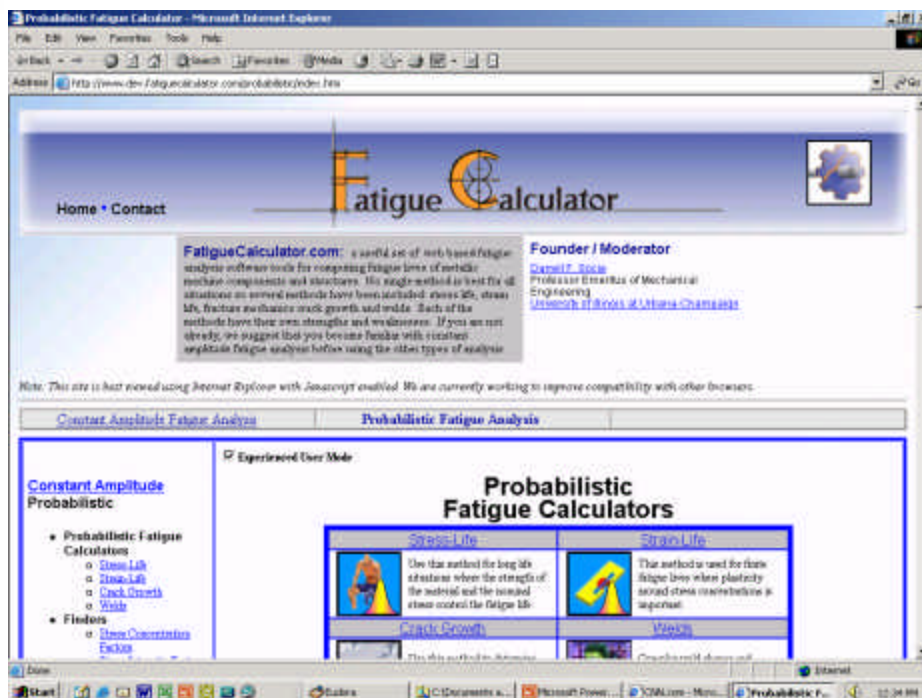


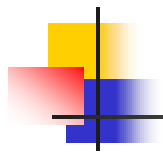
www.FatigueCalculator.com





Probabilistic Fatigue Analysis



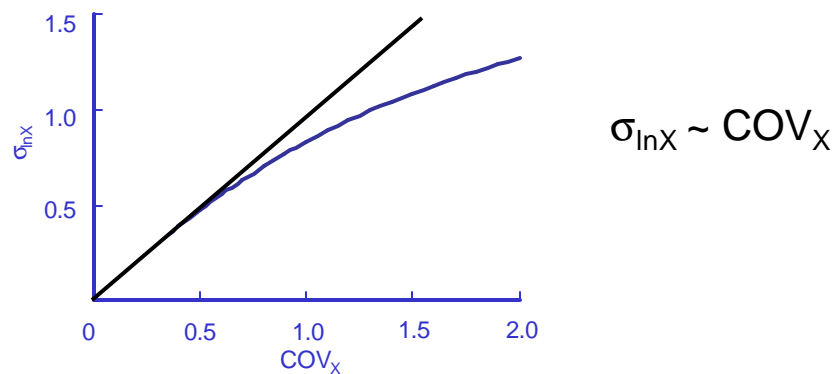


Modeling Variability

Central Limit Theorem:

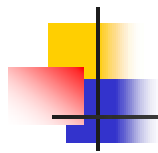
Products: $Z = X_1 \cdot X_2 \cdot X_3 \cdot X_4 \cdot \dots \cdot X_n$

$Z \rightarrow \text{LogNormal}$ as n increases



$$\sigma_{\ln X} \sim COV_X$$

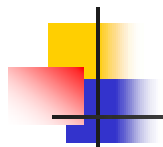
COV_X is a good measure of variability



COV and LogNormal Distributions

COV _x	Standard Deviation, Inx		
	1 68.3%	2 95.4%	3 99.7%
0.05	1.05	1.11	1.16
0.1	1.10	1.23	1.33
0.25	1.28	1.66	2.04
0.5	1.60	2.64	3.92
1	2.30	5.53	11.1

99.7% of the data is within a factor of ± 1.33 of the mean for a COV = 0.1



Strain Life Analysis

Strain Life Analysis - FatigueCalculator.com - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Address http://www.darrellsocie.com/fatiguecalculator/straining.htm

Home • Contact

Fatigue Calculator

Probabilistic Strain-Life Analysis

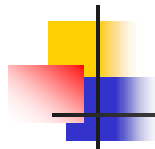
Enter as much data as you know. If it is not enough, you will be asked for more. Fields in red represent absolutely required data to begin calculations. Other data may become necessary as calculation proceeds. This is the probabilistic method, so select a distribution and coefficient of variation for each variable. You may choose None if you do not wish include this variable in the probabilistic analysis.

[Description of Distribution Types](#)

Loading

[Location of user](#)

		Value	Units	Distribution Type	Coefficient of Variation
Maximum	σ_{max} or ϵ_{max}	1000	macro-strain	Normal	0.2
Minimum	σ_{min} or ϵ_{min}	-1000	macro-strain	Normal	0.2
Range	$\Delta \sigma$ or $\Delta \epsilon$		macro-strain	Normal	0.2
Mean	σ_m or ϵ_m		macro-strain	Normal	0.2



Material Properties

Material Life Analysis - FatigueCalculator.com - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Address http://www.fatiguecalculator.com/submitdata.php#mprop

Material

[bottom of page](#)

You may use the Material Picker to look up the proper values for your material but you must specify the distribution you wish to use manually.

[Material Properties Editor](#)

Type:

	Value	Units	Distribution Type	Coefficient of Variance	Correlation Coefficient
Fatigue Strength Coefficient	382	MPa	Log-Normal	0.1	
Fatigue Strength Exponent	-0.118		None		
Fatigue Scatter Coefficient	0.16		Log-Normal	0.2	
Fatigue Scatter Exponent	-0.412		None		

If this section is left blank, values will be auto-filled

	Value	Units	Distribution Type	Coefficient of Variance	Correlation Coefficient
Elastic Modulus	210000	MPa	None		
Fatigue Limit	0.01	MPa	None		
Cyclic Strength Coefficient	1441	MPa	None		
Cyclic Strain Hardening Exponent	0.293		None		

Start

http://www.fatiguecalculator.com/submitdata.php#mprop

Internet




Stress Concentration

Strain Life Analysis - FatigueCalculator.com - Microsoft Internet Explorer

Address: http://www.fatiguecalculator.com/contributors/bs-williams.php

Elastic Modulus $E = 216800$ MPa
 Fatigue Limit $\sigma_{PL} =$ MPa
 Cyclic Strength Coefficient $\sigma'_c = 1440$ MPa
 Cyclic Strain Hardening Exponent $n' = 1.283$


Stress Concentration



 Stress Concentration Factor Finder
 $K_t = 1$ Coefficient of Variation: Normal 0.15

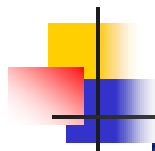
Either specify K_t directly or enter K_t and the radius. If you choose to use K_t the distribution specified for K_t will be used. There is no need to specify a distribution for either the radius or ultimate strength. The uncertainty in these measurements is insignificant when compared to the uncertainty in K_t .

Use K_t in analysis? ☐ No ☒ Yes

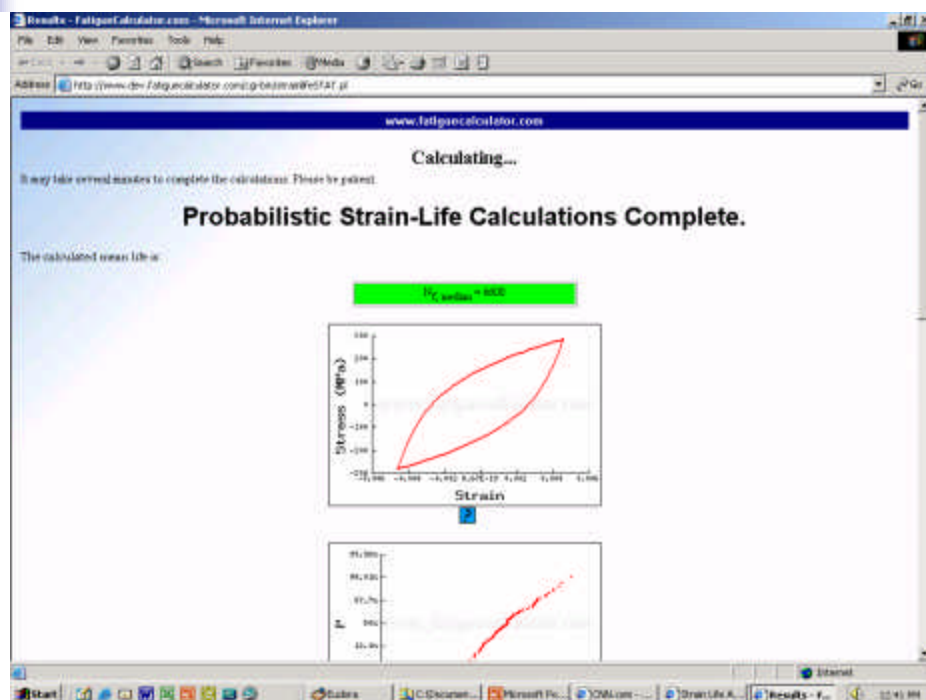
$R_{eff} =$ mm
 Material $S_u = 65$ ksi
 Ultimate Strength

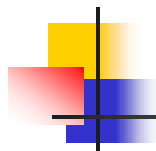

 Griffith (1893-1967)


 Eusebi (1922-1969)

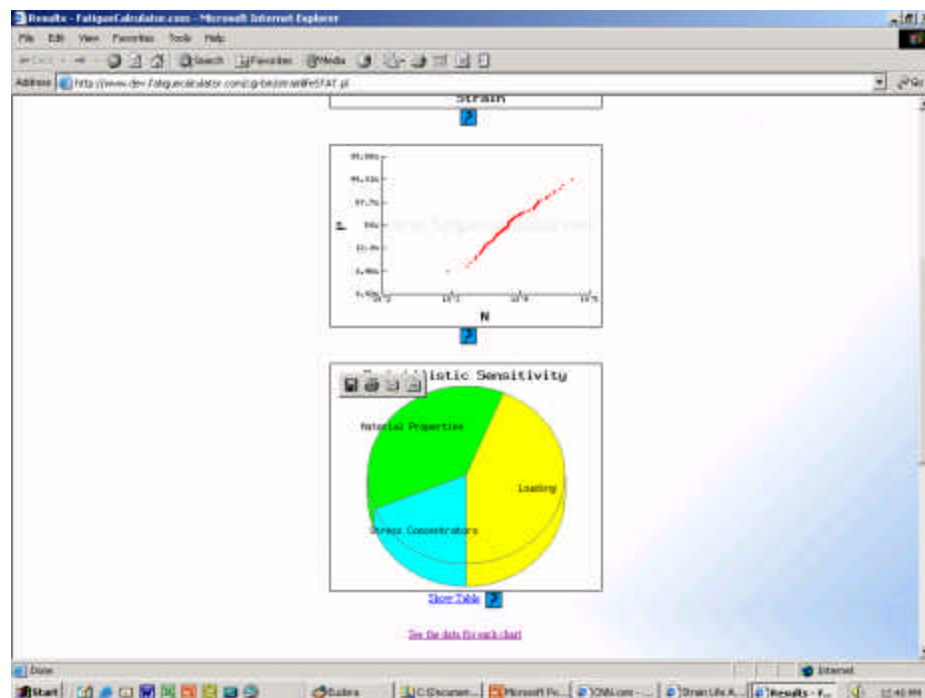


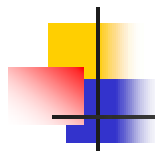
Results





Results (continued)

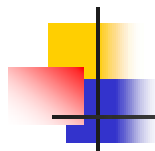




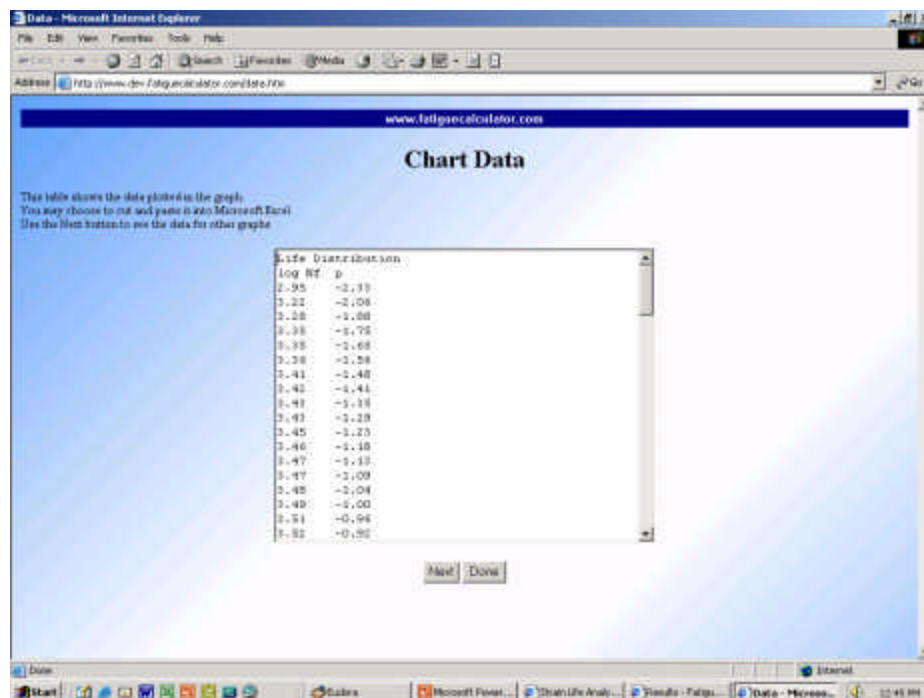
Results (continued)

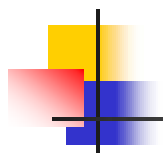
www.fatigacalculator.com

Mean Life	9938		6640	0.098
Variable	Deterministic Sensitivity	Probabilistic Sensitivity	Median	COV
Loading	-3.35	0.72		
$s_{\text{ten}}(N(0.005,0.2))$	-3.22	0.821	0.92×10^4	0.198
$s_{\text{ten}}(N(0.005,0.2))$	-4.3	0.304	0.00003	0.181
Material Properties	-6.99	0.821		
K'	0.666	0	1440	0
α'	-0.14	0	0.283	0
K	-1.57	0	2.07×10^2	0
α	-1.04	0	-8.118	0
ϵ	-0.02	0	-0.482	0
$\frac{1}{\sqrt{1+883,811}}$	0.944	0.332	383	0.0971
$B_1(0.016,0.2)$	2.17	0.407	0.16	0.262
Stress Concentration	-4.43	0.309		
$K(M(3,0.03))$	-4.43	0.339	3	0.6403



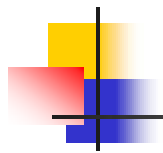
Results (continued)



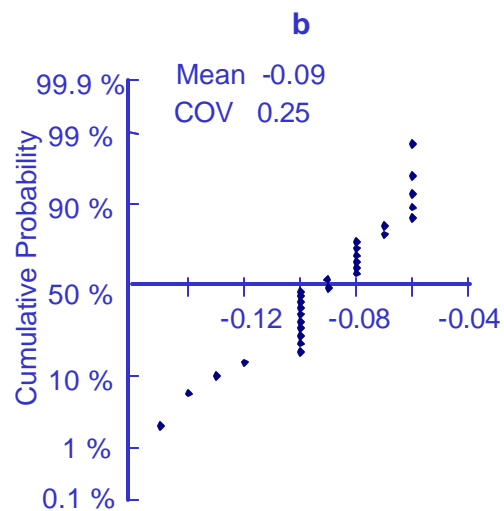
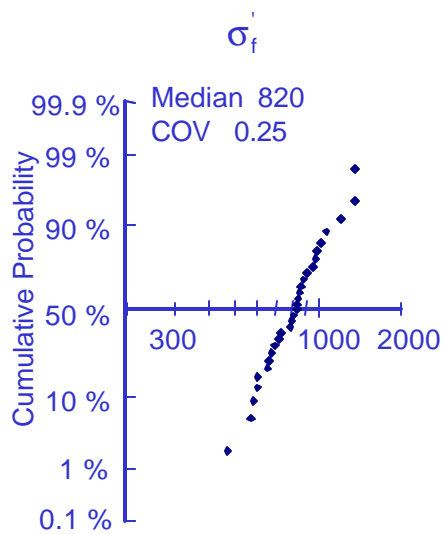


Ten Simulations

	Life	COV
	6470	0.959
	6930	0.898
	6710	0.688
	6640	0.908
	6580	0.869
	6470	0.959
	7010	0.723
	6690	0.908
	6170	0.791
	6560	0.971
Mean	6623	0.8674
COV	0.038	0.114

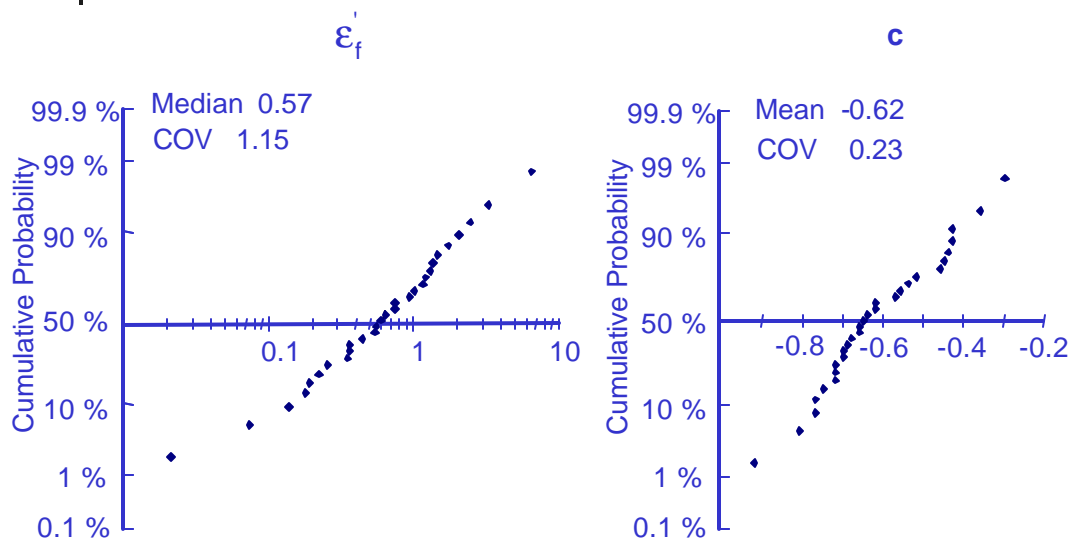


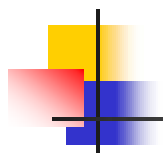
29 Individual Data Sets



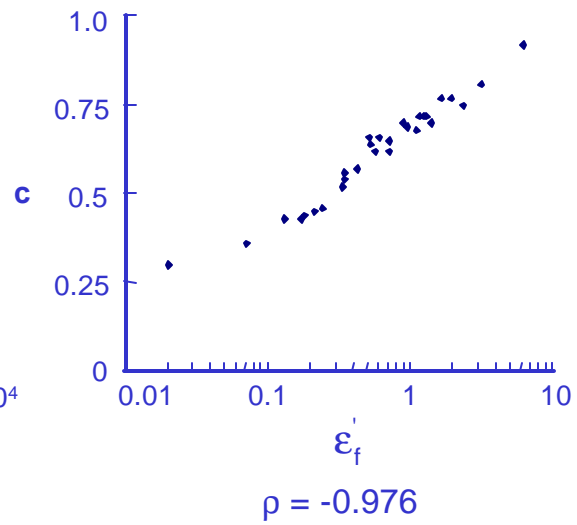
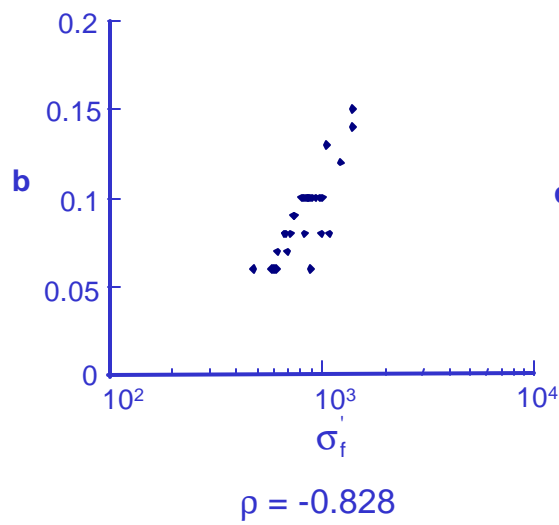


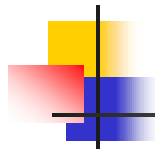
29 Individual Data Sets (continued)





Correlation





Correlated Variables

Strain Life Analysis - FatigueCalculator.com - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Address http://www.fatiguecalculator.com/submitdata/submit.asp

Material

This may use the Material Reader to look up the proper values for your material but you must specify the distribution you wish to use manually.

Material Property Reader

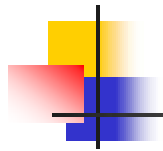
Type:

	Value	Units	Distribution Type	Coefficient of Variation	Correlation Coefficient
Fatigue Strength Coefficient	883	MPa	Log-Normal	25	
Fatigue Strength Exponent	-0.118		Normal	25	-0.81
Fatigue Ductility Coefficient	3.16		Log-Normal	1.15	
Fatigue Ductility Exponent	-0.412		Normal	23	-0.98

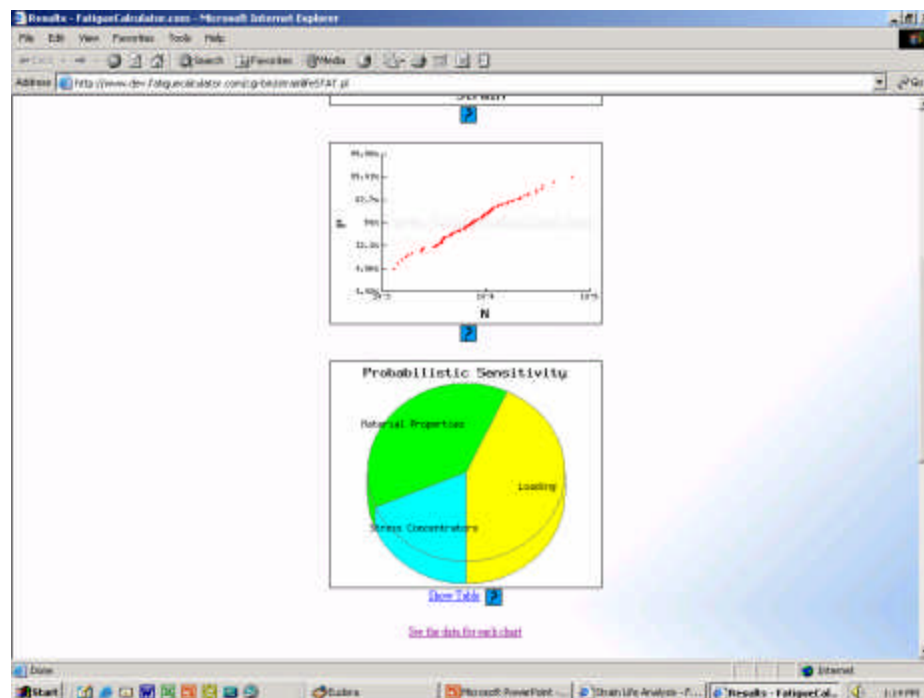
If this section is left blank, values will be estimated

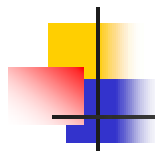
	Value	Units	Distribution Type	Coefficient of Variation	Correlation Coefficient
Elastic Modulus	E = 206000	MPa	None		
Fatigue Limit	<input type="text" value="S<sub>FL</sub> ="/>	MPa	None		
Cyclic Strength Coefficient	<input type="text" value="K' ="/>	MPa	None		
Cyclic Strain Hardening Exponent	<input type="text" value="n' ="/>		None		

Start Calculation



Results (continued)

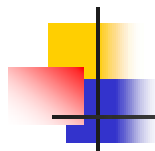




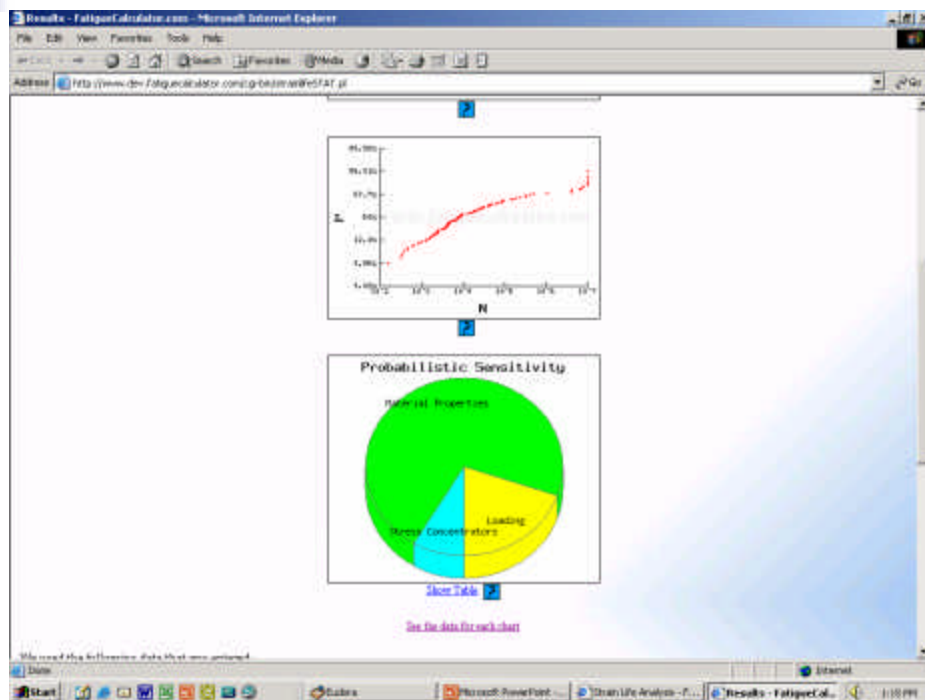
Results (continued)

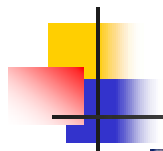
www.fatiguecalculator.com

Mean Life	1440		6640	0.013
Variable	Deterministic Sensitivity	Probabilistic Sensitivity	Median	COV
Loading	-3.33	0.71		
$\sigma_{\text{max}} N(0.901, 0.2)$	-2.23	0.603	9.34×10^{-4}	0.203
$\sigma_{\text{min}} N(-0.901, 0.2)$	-1.3	0.238	0.00331	0.203
Material Properties	-6.99	0.605		
K'	0.666	0	1440	0
n'	-0.14	0	0.203	0
E	-1.57	0	2.87×10^3	0
$k \text{ RE}(0.114, 25), \text{COV}=0.33$	-1.68	0.2	0.119	0.24
$\sigma_{\text{RE}}(-0.412, 25)$	-0.02	0.366	-0.41	0.9407
$S_u \text{ U}(881, 25)$	0.944	0.325	361	0.261
$E_y \text{ U}(0.16, 1.15)$	2.17	0.0344	0.16	0.0106
Stress Concentration	-4.43	0.305		
$E \text{ W}(3, 0.05)$	-4.43	0.380	3	0.0472



Uncorrelated Variables





Results (continued)

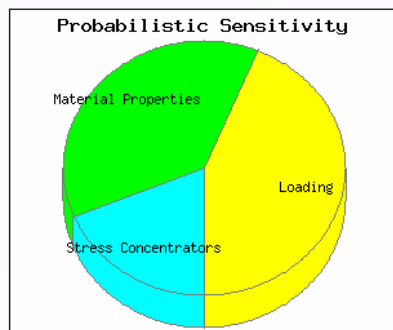
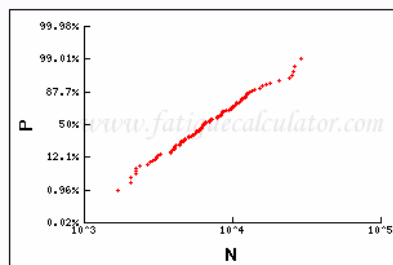
www.fatiguecalculator.com

Mean Life	1.00×10^6		6646	1.00
Variable	Deterministic Sensitivity	Probabilistic Sensitivity	Median	COV
Loading	-3.33	0.263		
σ_{max} (N(0.001, 0.2))	-3.22	0.227	8.94×10^4	0.202
σ_{min} (N(-0.001, 0.2))	-1.3	0.113	-0.0001	0.218
Material Properties	-6.90	0.330		
E'	0.666	0	1440	0
ν'	-0.14	0	0.283	0
E	-1.37	0	2.07×10^5	0
$\frac{1}{2} H$ (N(115, 20))	-1.04	0.132	-8.119	0.26
c (N(-0.412, 20))	-0.02	0.341	-6.401	0.23
$\frac{1}{2} L$ (N(893, 20))	0.944	0.12	863	0.262
R_L (N(0.16, 1.15))	2.17	0.0127	0.16	0.0106
Stress Concentration	-4.43	0.113		
K (N(2, 0.05))	-4.43	0.113	3	0.0472



Strain Amplitude

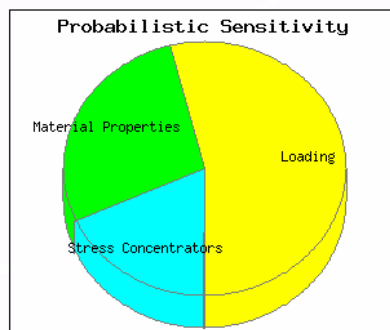
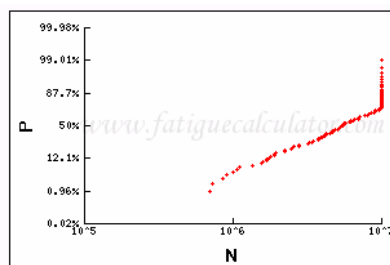
± 1000



[Show Table](#)

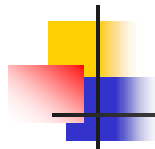


± 250



[Show Table](#)





Deterministic Analysis

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Address http://www.darrell.socie.com/fatiguecalc.htm

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Fatigue Calculator

Constant Amplitude Stress-Life Analysis

Enter as much data as you know. If it is not enough, you will be asked for more. Fields in red represent absolutely required data to begin calculations. Other data may become necessary as calculation proceeds.

Loading

You may choose to specify either the stress or the desired life / safety factor and compute the other. If you choose to calculate the stress, leave this section blank.

Maximum $\sigma_{max} = \sigma_{max}$ MPa

Minimum $\sigma_{min} = \sigma_{min}$ MPa

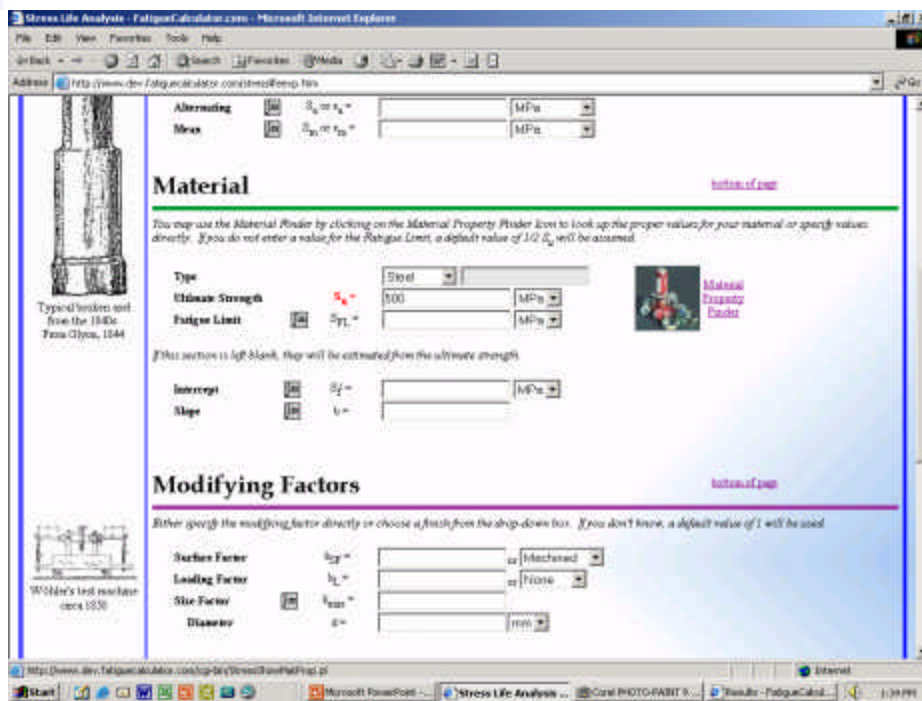
Or

Alternating $\sigma_a = \sigma_a$ MPa

Mean $\sigma_m = \sigma_m$ MPa

Material

Deterministic Analysis (continued)




Stress Life Analysis - FatigueCalculator.com

File Edit View Favorites Tools Help

Address http://www.fatiguecalculator.com/

Material

You express the Material Reader by clicking on the Material Property Reader icon to look up the proper values for your material or specify values directly. If you do not enter a value for the Fatigue Limit, a default value of 1/2 S_u will be assumed.

Type: 

Ultimate Strength: $S_u =$ MPa

Fatigue Limit: $S_{FL} =$ MPa

If this section is left blank, they will be estimated from the ultimate strength.

Intercept: $S_f =$ MPa

Slope: $b =$

Modifying Factors

Enter specify the modifying factor directly or choose a finish from the drop-down box. If you don't know, a default value of 1 will be used.

Surface Factor: $k_{SF} =$ or

Loading Factor: $k_{LF} =$ or

Size Factor: $k_{SZ} =$

Diameter: $d =$ mm

Alternating: $S_a = S_e =$ MPa

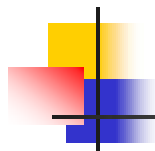
Mean: S_m or $r_m =$ MPa

Wöhler's test machine circa 1850

Typical tension test from the 1940s From Glynn, 1944

Internet

Start Microsoft PowerPoint Stress Life Analysis Corel PHOTO-Paint 8.0 Photoshop - FatigueCalc...



Deterministic Analysis (continued)

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Stress Concentration


button to page

K_t

Do K_t in analysis?

K_f

Radius


 Stress Concentration Factor


Safety Factor for Infinite Life

If you wish to calculate a safety factor, leave this section blank and click the Calculate Life / Safety Factor button below. If you wish to calculate the stresses, you must specify the desired safety factor and provide some additional information about the stresses.

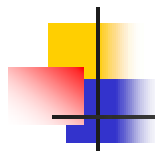
Safety Factor

Mean Stress Definition

 Data Acquisition icon

 Safety Stop Chart





Probabilistic Analysis

Stress-Life Analysis - FatigueCalculator.com - Microsoft Internet Explorer

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Address http://www.darrellsocie.com/fatiguecalculator.asp

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Fatigue Calculator

Constant Amplitude Stress-Life Analysis

Enter as much data as you know. If it is not enough, you will be asked for more. Fields in red represent absolutely required data to begin calculations. Other data may become necessary as calculation proceeds.

Loading

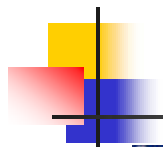
[bottom of page](#)

You may choose to specify either the stresses or the desired life / safety factor and compute the other. If you choose to calculate the stresses, leave this section blank.

Maximum	<input type="checkbox"/> $\sigma_{max} = \sigma_{tens}$	<input type="text" value="20"/>	MPa
Minimum	<input type="checkbox"/> $\sigma_{min} = \sigma_{comp}$	<input type="text" value="-20"/>	MPa
Or			
Alternating	<input type="checkbox"/> $\sigma_a = \sigma_s$	<input type="text"/>	MPa
Mean	<input type="checkbox"/> $\sigma_m = \sigma_b$	<input type="text"/>	MPa

Material

[bottom of page](#)



Probabilistic Analysis (continued)

Probabilistic Stress Life Analysis - FatigueCalculator.com - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Address http://www.darrellsocie.com/fatiguecalculator.com/probabilisticstresslifeanalysis.htm

Mean $\sigma_a = \sigma_m =$ MPa Normal 2.2

Material

You may use the Material Picker by clicking on the Material Property Picker icon to look up the proper values for your material or specify values directly. If you do not enter a value for the Fatigue Limit, a default value of $1/2 S_u$ will be assumed.

Material

	Value	Units	Distribution Type	Coefficient of Variation	Coefficient of Correlation
Ultimate Strength	$S_u =$ 550	MPa	Normal	0.1	
Fatigue Limit	$S_{FL} =$	MPa	None		
Elastic Modulus	$E =$	MPa	None		
Intercept	$\log S_f =$	MPa	Log-Normal		
Slope	$b =$		None		

If this surface is left blank, values will be estimated.

Modifying Factors

Enter specific modifying factor directly or choose a finish from the drop-down box. If you don't know, a default value of 1 will be used.



Probabilistic Analysis (continued)

Probabilistic Stress Life Analysis - FatigueCalculator.com - Microsoft Internet Explorer

File Edit View Favorites Tools Help

Address http://www.dar.fatiguecalculator.com/probabilisticstresslife.htm


Modifying Factors

Either specify the modifying factor directly or choose it from the drop down box. If you don't know, a default value of 1 will be used.

Surface Factor	$k_{sp} =$	Value or Factor	Distribution Type	Coefficient of Variation
Leading Factor	$k_L =$	or MacFadden	Normal	0.1
Size Factor	$k_{sz} =$	or None		
Diameter	$d =$			

Stress Concentration

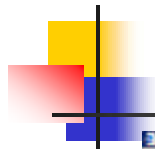
Either specify K_t directly or enter K_f and the radius.



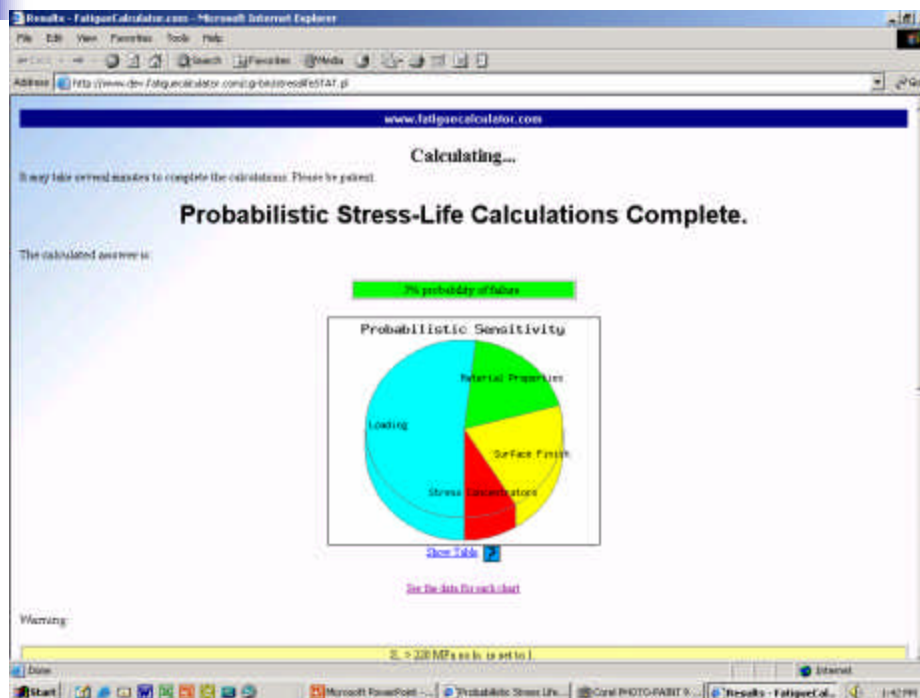
Stress Concentration Factor

Use K_f in analysis?	Value	Distribution Type	Coefficient of Variation
K_f	1	Normal	0.15
Radius			

Calculate Life Clear Form



Probabilistic Analysis Results



Probabilistic Aspects of Fatigue

