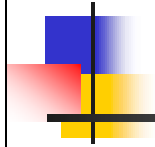


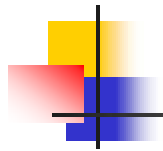
# **Probabilistic Aspects of Fatigue**

## **Case Studies**



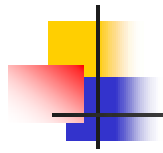
**Professor Darrell F. Socie**  
Department of Mechanical and  
Industrial Engineering

© 2003-2005 Darrell Socie, All Rights Reserved



# Probabilistic Aspects of Fatigue

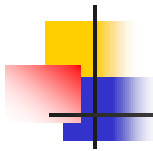
- Introduction
- Basic Probability and Statistics
- Statistical Techniques
- Analysis Methods
- Characterizing Variability
- **Case Studies**
- [FatigueCalculator.com](http://FatigueCalculator.com)
- [GlyphWorks](http://GlyphWorks.com)



## Case Studies

---

- DARWIN
  - Southwest Research
- Bicycle
- Loading Histories

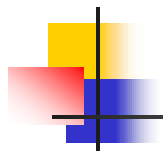


# A Software Framework for Probabilistic Fatigue Life Assessment

ASTM Symposium on  
Probabilistic Aspects of Life Prediction  
Miami Beach, Florida  
November 6-7, 2002

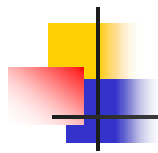
R. C. McClung, M. P. Enright, H. R. Millwater\*,  
G. R. Leverant, and S. J. Hudak, Jr.  
Southwest Research

Slides 6 – 27 used with permission of of Craig McClung



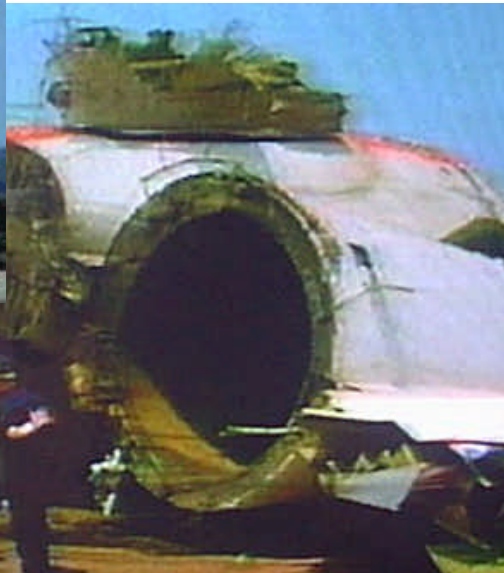
## Motivation

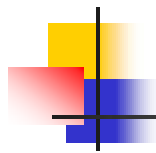




## UAL Flight 232

July 19, 1989





# Turbine Disk Failure

## Anomalies in titanium engine disks

Hard Alpha

- Very rare

- Can cause failure

- Not addressed by safe life methods

Enhanced life management process

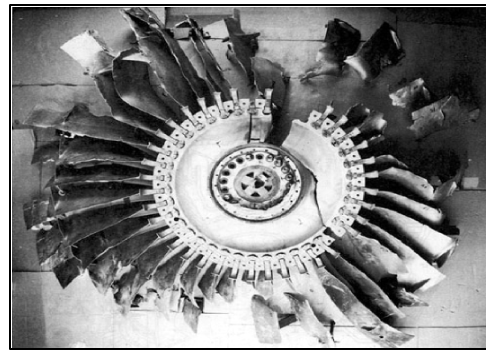
- Requested by FAA

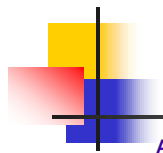
- Developed by engine industry

- Probabilistic damage tolerance methods

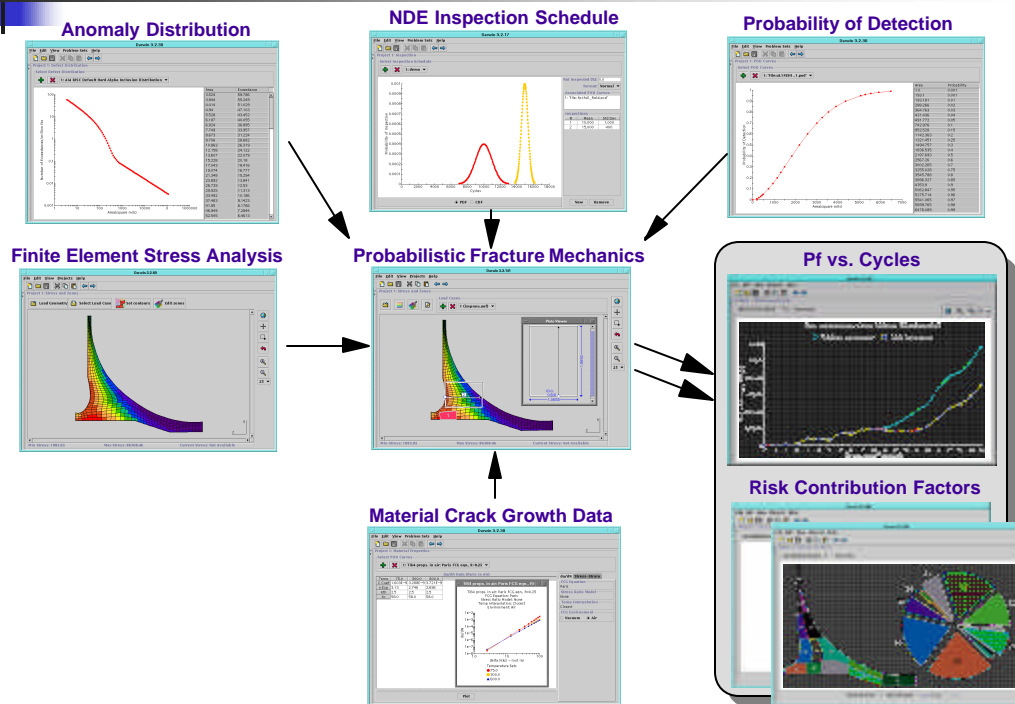
- Supplement to safe life approach

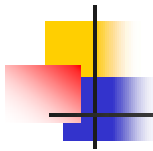
SwRI and engine industry developed DARWIN with FAA funding





# Probabilistic Damage Tolerance





## Zone-Based Risk Assessment

Define zones based on similar stress,  
inspection, anomaly distribution, lifetime

Total probability of fracture for zone:

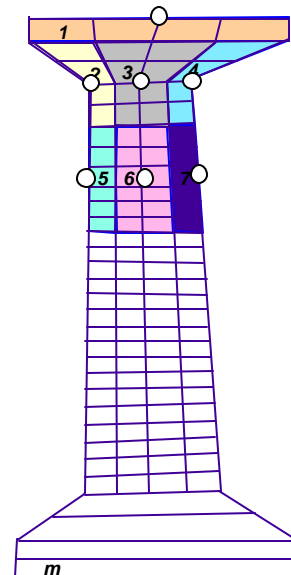
(probability of having a defect)  $\times$  (POF given  
a defect)

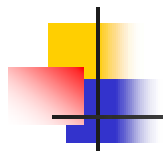
Defect probability determined by anomaly  
distribution, zone volume

POF assuming a defect computed with  
Monte Carlo sampling or advanced  
methods

POF for disk obtained by summing zone  
probabilities

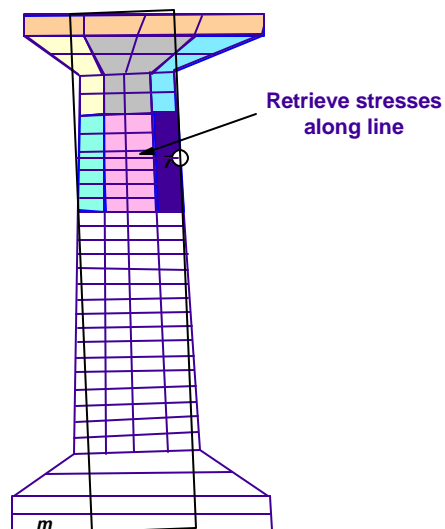
As individual zones become smaller (number of  
zones increases), risk converges down to  
“exact” answer



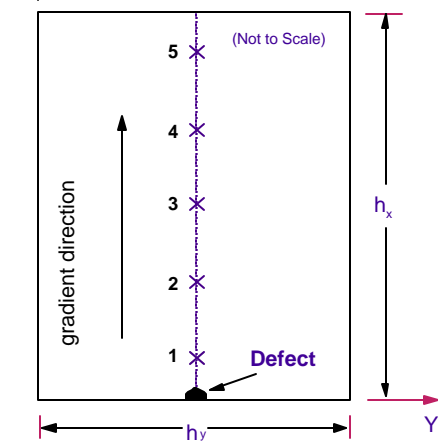


## Fracture Mechanics Model of Zone

**Finite Element Model**

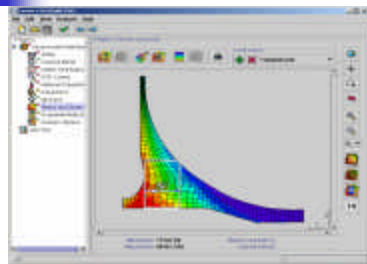


**Fracture Mechanics Model**

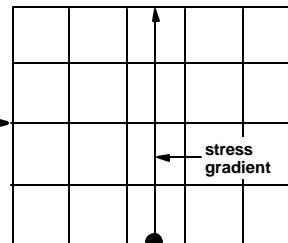




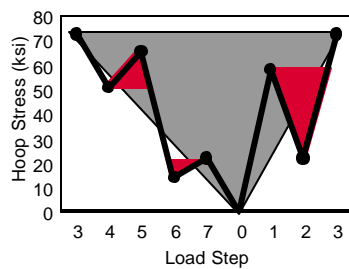
# Stress Processing



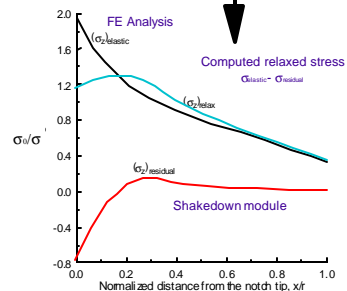
FE Stresses and plate definition



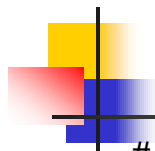
Stress gradient extraction



Rainflow stress pairing

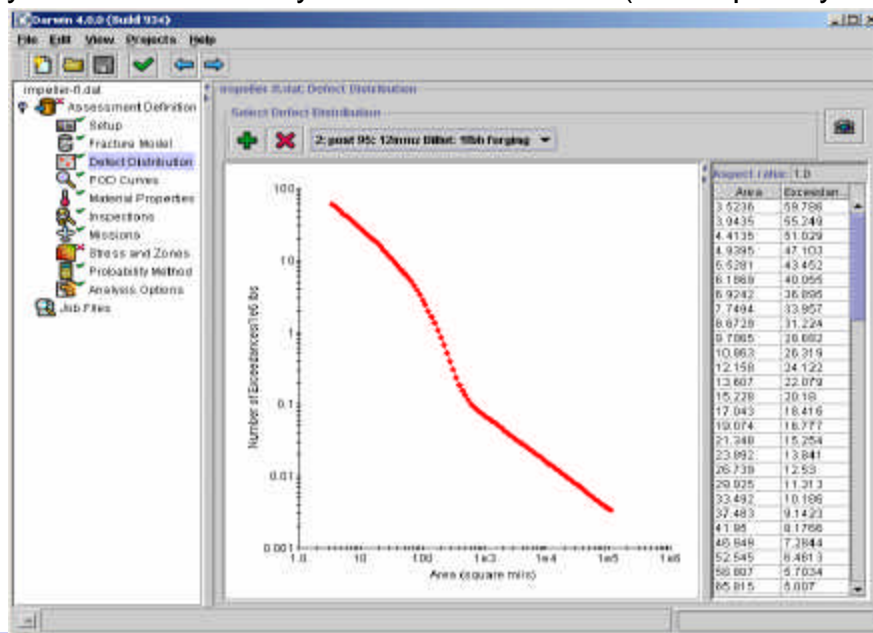


Residual stress analysis



## Anomaly Distribution

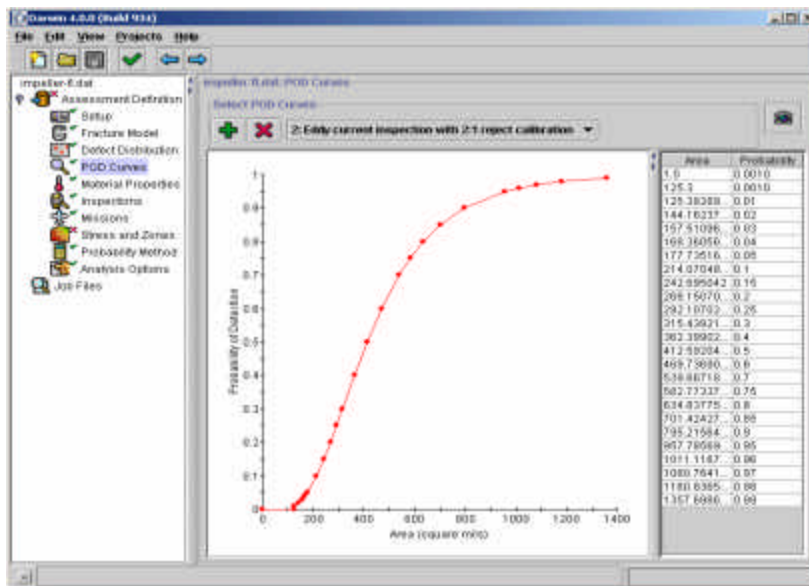
# of anomalies per volume of material as function of defect size  
Library of default anomaly distributions for HA (developed by RI SC)

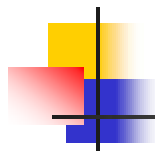




# Probability of Detection Curves

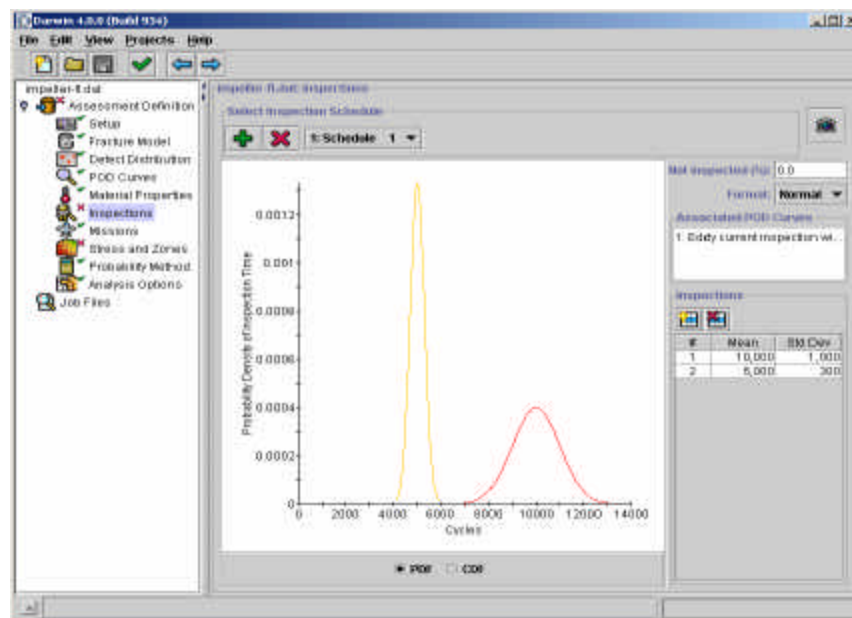
- Define probability of NDE flaw detection as function of flaw size
- Can specify different PODs for different zones, schedules
- Built-in POD library or user-defined POD



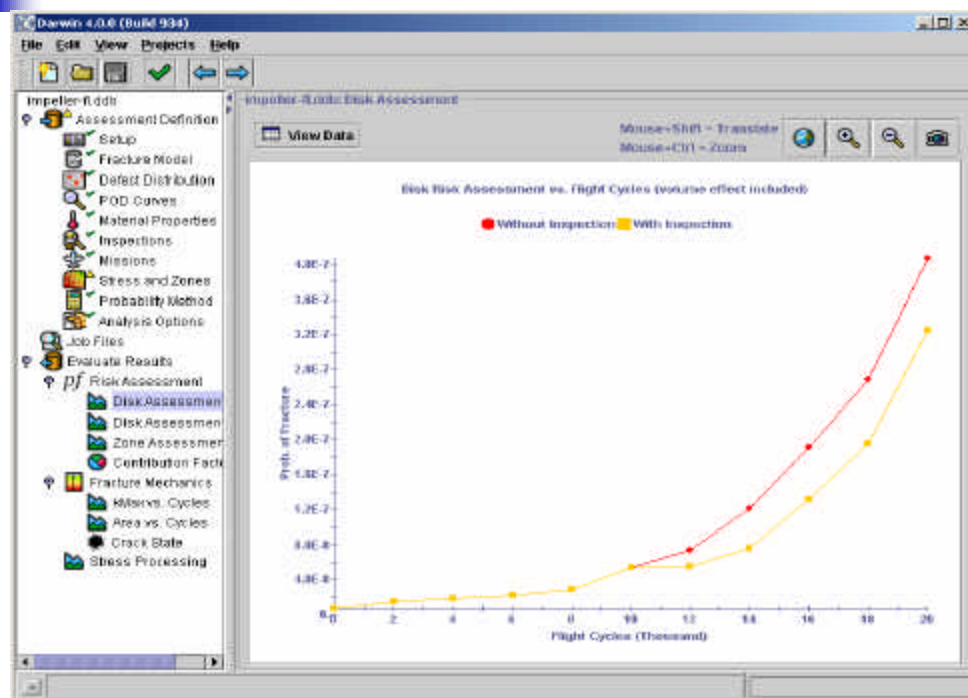


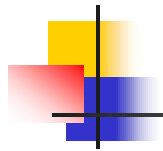
# Random Inspection Time

“Opportunity Inspections” during on-condition maintenance  
 Inspection time modeled with Normal distribution or CDF table



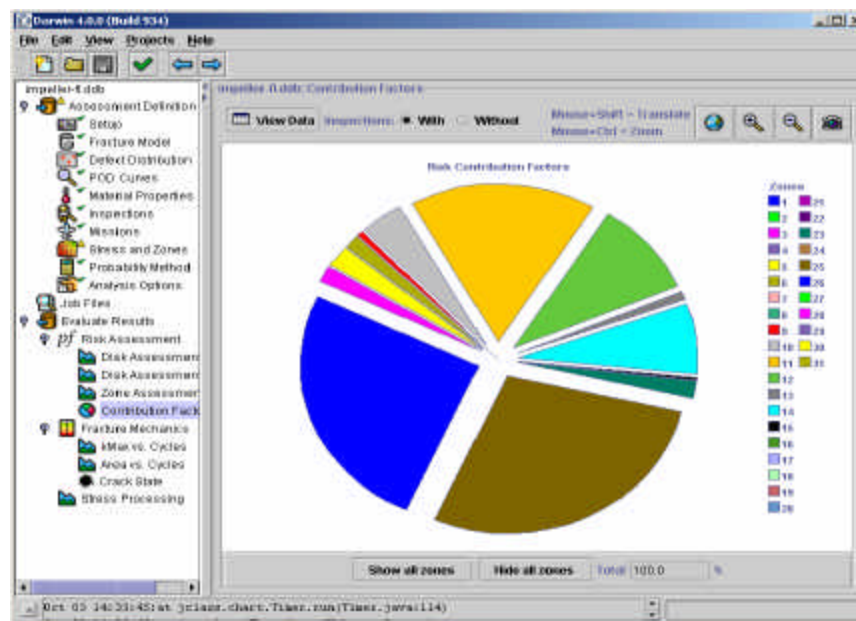
## Output: Risk vs. Flight Cycles

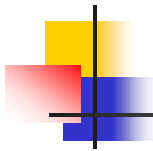




## Output: Risk Contribution Factors

Identify regions of component with highest risk

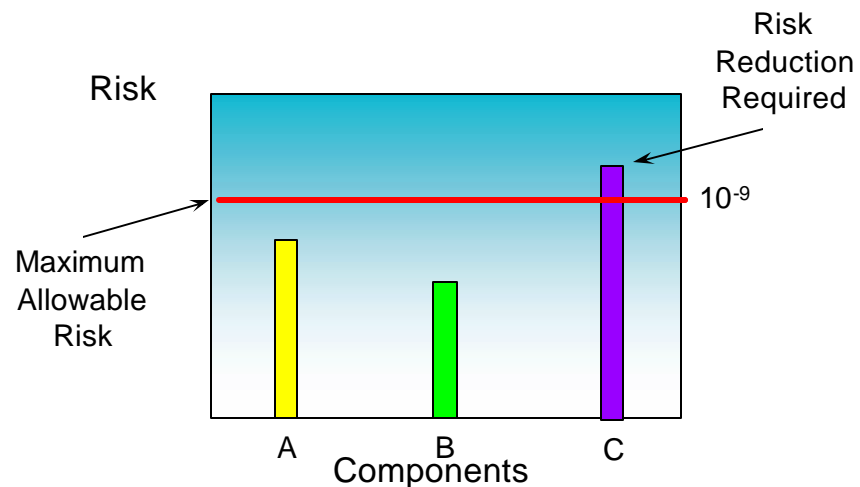




## Implementation in Industry

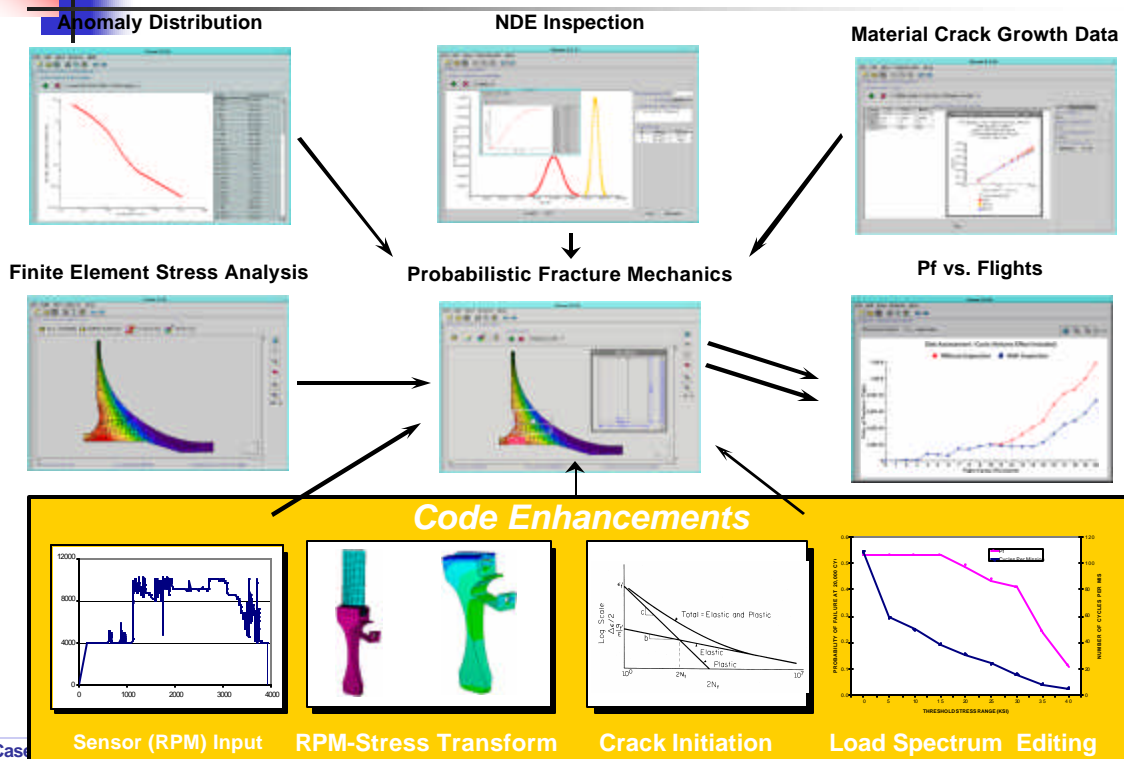
FAA Advisory Circular 33.14 requests risk assessment be performed for all new titanium rotor designs

Designs must pass design target risk for rotors



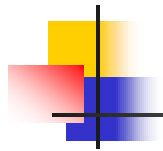


# DARWIN for Prognosis Studies



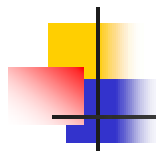
6 Case

62



## Three Sources of Variability

- Anomaly size (initial crack size)
- FCG properties (life scatter)
- Mission histories (stress scatter)



## Hard Alpha Defects in Titanium

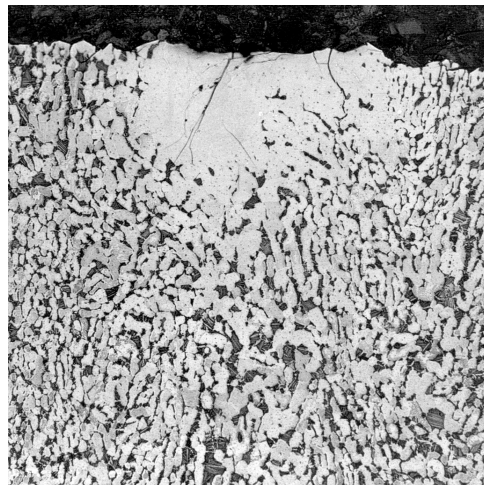
Initial DARWIN focus on  
Hard Alpha

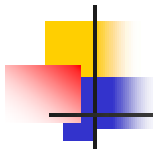
- Small brittle zone in  
microstructure

- Alpha phase stabilized by N  
accidentally introduced  
during melting

- Cracks initiate quickly

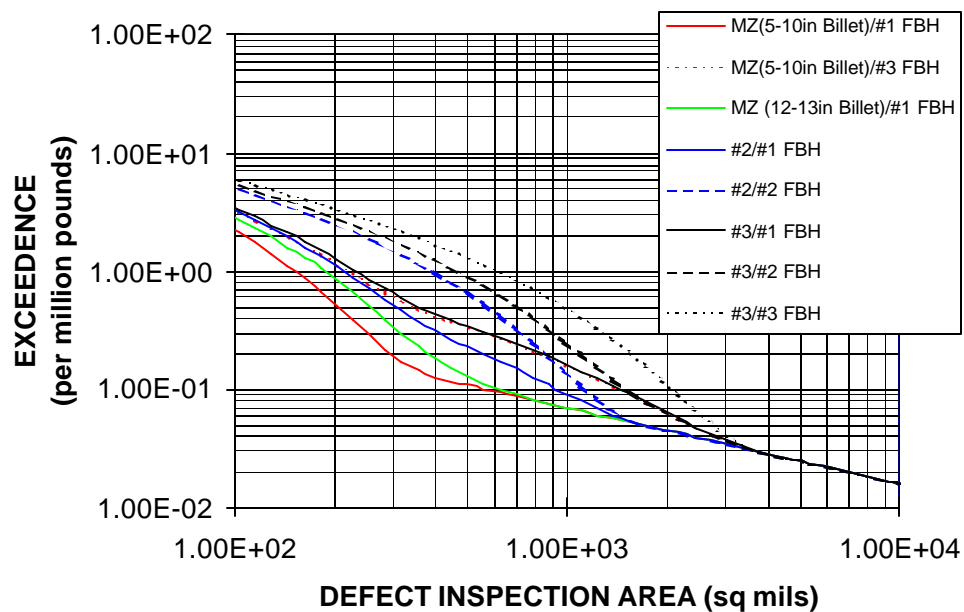
Extensive industry effort  
to develop HA  
distribution

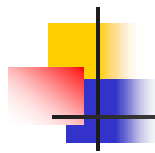




## Resulting Anomaly Distributions

### Post 1995 Triple Melt/Cold Hearth + Vacuum Arc Remelt



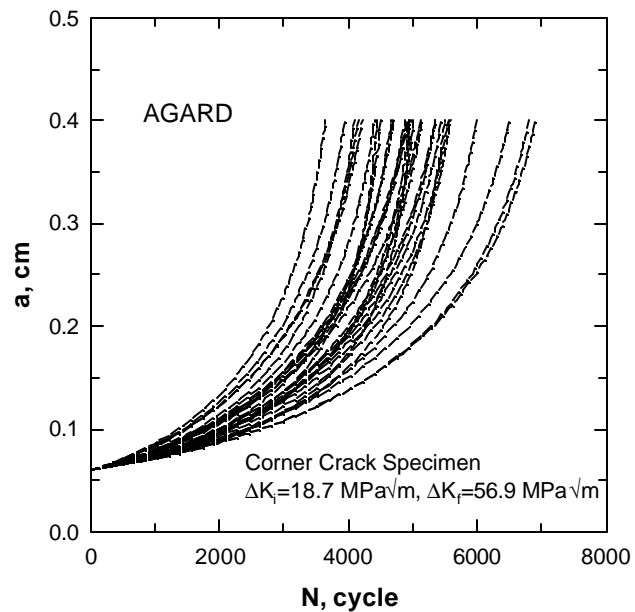


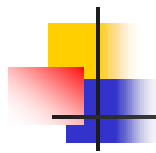
## FCG Simulations for AGARD Data

Use individual fits to generate set of  $a$  vs.  $N$  curves for identical conditions

Characterize resulting scatter in total propagation life

Lognormal distribution appropriate in most cases





# Engine Usage Variability

Stress/Speed:

$$\Delta\sigma \propto (\text{RPM})^2$$

Total Cyclic Life (LCF):

$$N_f = N_i + N_p$$

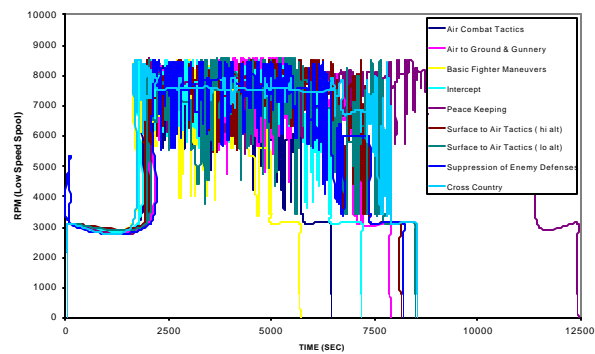
$$N_i \propto \Delta\sigma^{3-5}$$

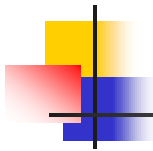
$$N_p \propto \Delta\sigma^{3-4}$$

Life/Speed:

$$N_f \propto (\text{RPM})^6$$

Component life is very sensitive  
to actual usage

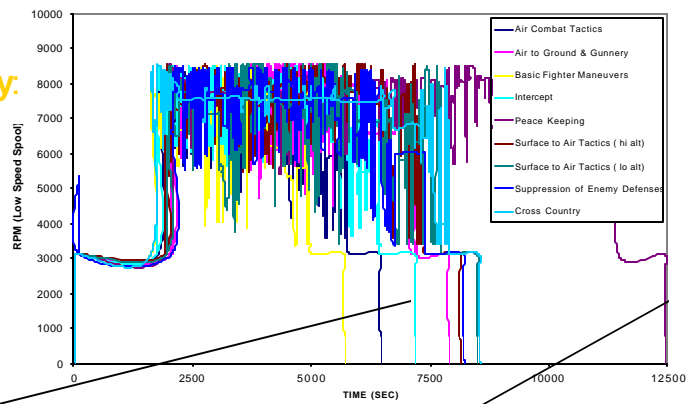




# Usage Variability

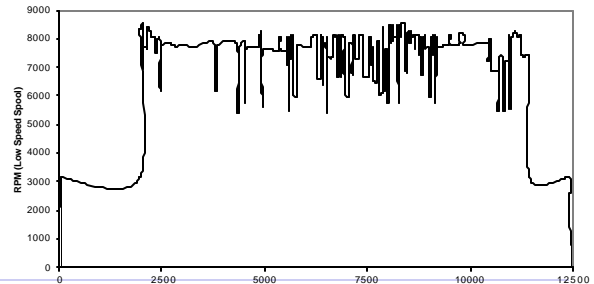
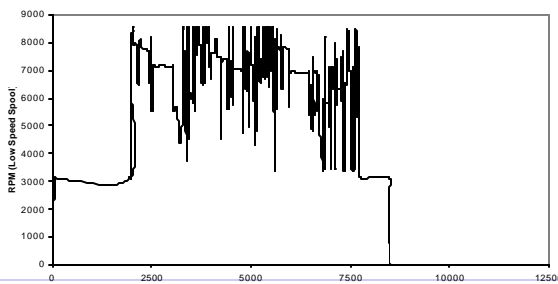
## Components of Usage Variability:

- Mission type
- Mission-to-mission variability
- Mission mixing variability

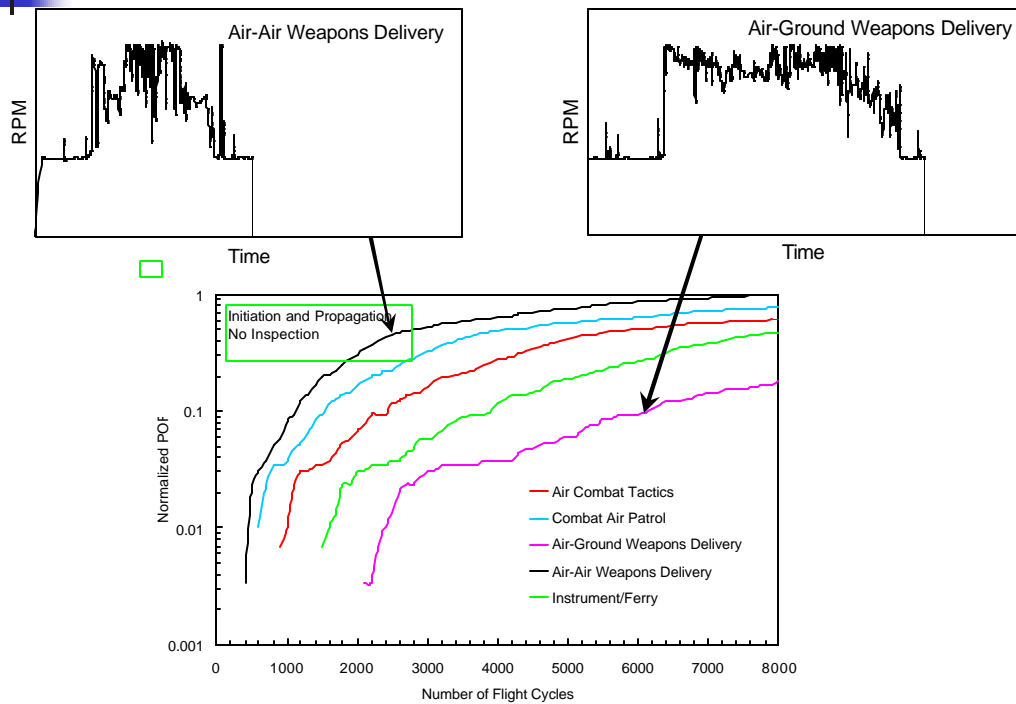


Surface to Air Tactics (lo alt)

Peace Keeping



## Variability in Mission Type



Web Site: [www.darwin.swri.org](http://www.darwin.swri.org)



**DARWIN**  
Design Assessment of Reliability  
With INSpection

Winner RAD 100 Award

- The DARWIN computer program predicts the probability of fracture of aircraft turbine rotor disks.
- DARWIN risk assessment considers finite element stress analysis, defect growth analysis, and nondestructive inspection simulation.
- DARWIN identifies the most likely failure regions and risk reduction associated with single or multiple inspections.

DARWIN is sponsored by the  
FEDERAL AVIATION ADMINISTRATION

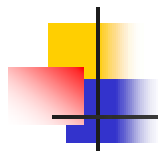
Steering Committee Members Page

USER Pages  
Username & Password required

View a video describing the main features of Darwin. RealPlayer 8 Basic is required to view the interactive video segments included in this tutorial. If you don't have the free RealPlayer 8 Basic, click [here to download](#).

[DARWIN Overview \(16 MB\)](#) [DARWIN GUI \(30 MB\)](#)

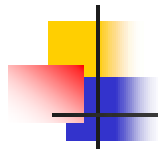
Download a printable Darwin brochure in Adobe PDF format. If you don't have the free Adobe Acrobat reader, click [here to download](#).



# Bicycle

Assess risk in a new design





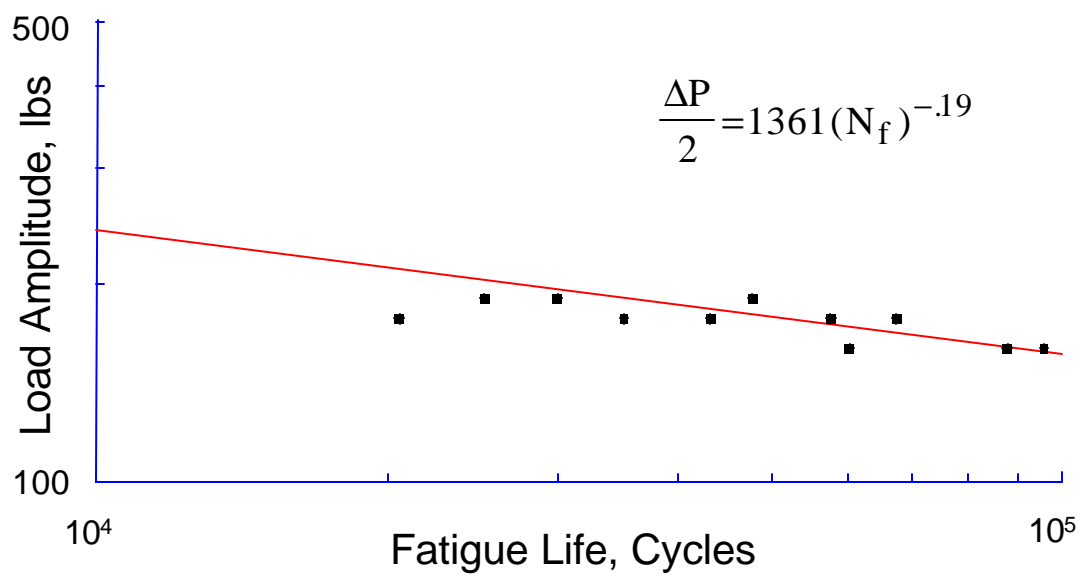
## Variability / Uncertainty

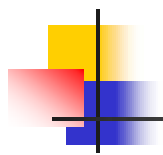
---

- Fatigue strength of fork
- Load history variability
- Load history uncertainty
- Analysis uncertainty

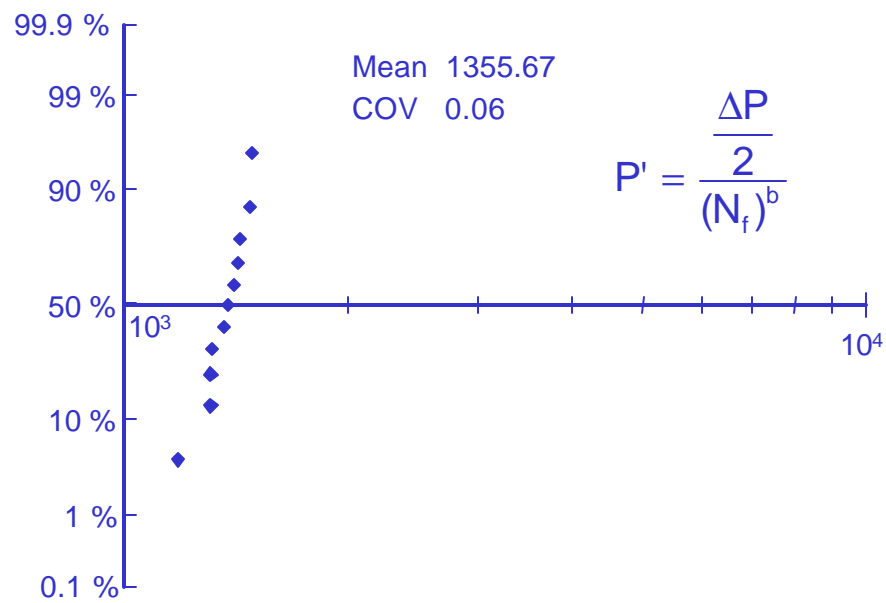


## Load-Life Data





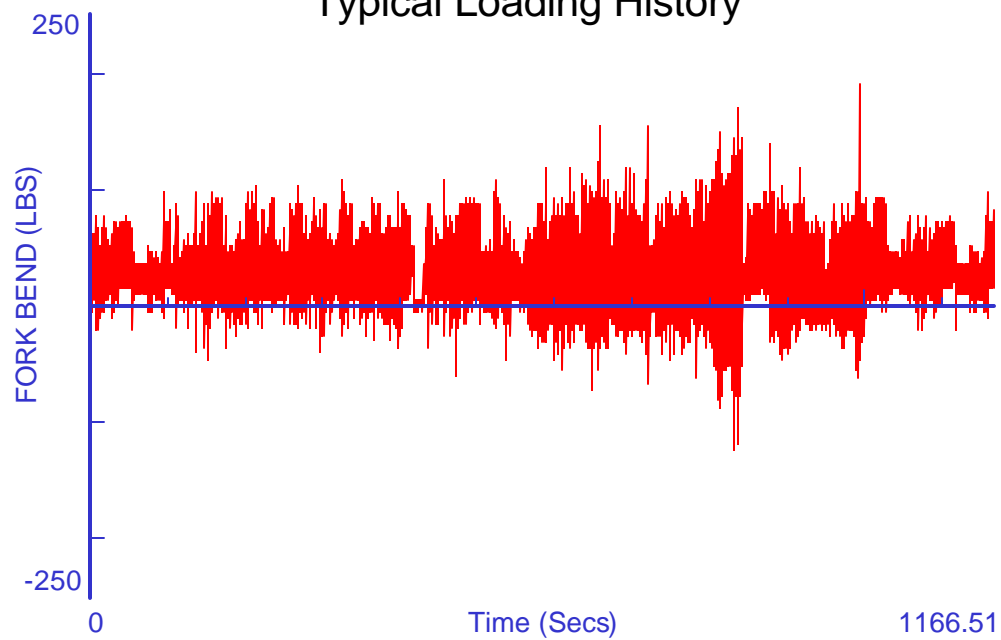
## Variability

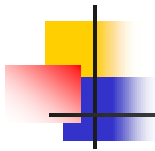




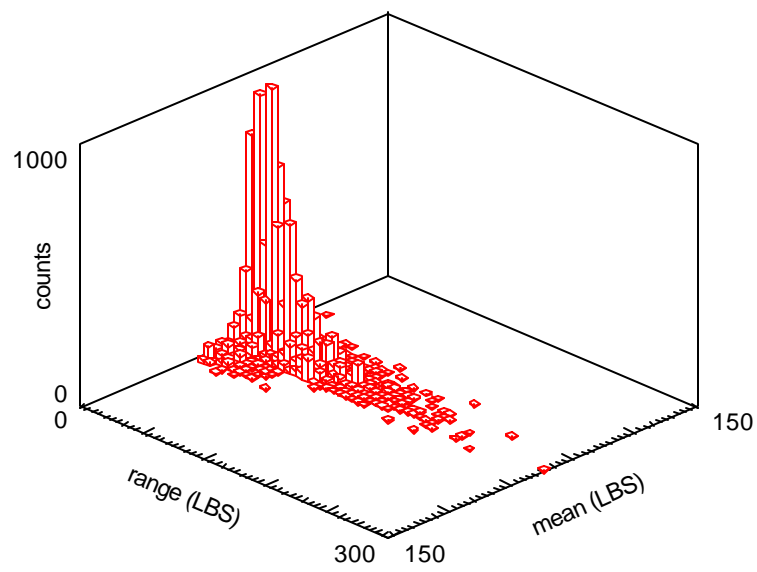
## Loading History

Typical Loading History



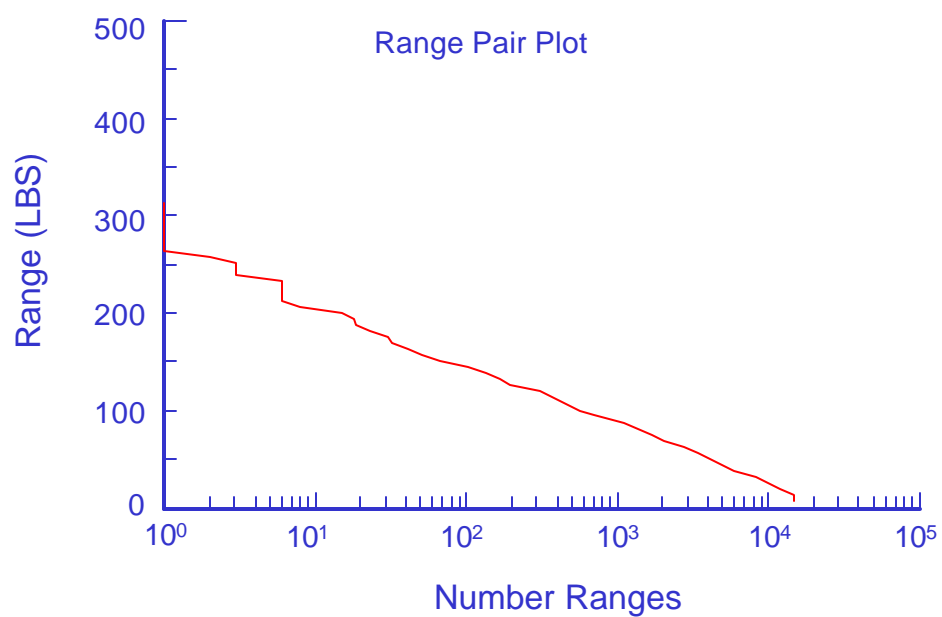


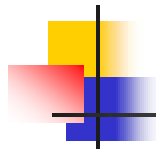
## Rainflow





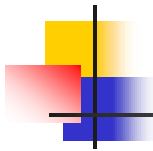
## Exceedance Diagram





## Random Variables

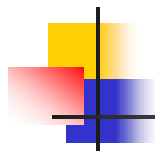
- Strength LN( 1356 , 0.06 )
- Loading History LN( 1 , 0.3 )
  - Estimated from other data
- Loading History Uncertainty in Mean
  - Could be “off” by a factor of 2 LN( 1.0 , 0.25 )
- Analysis
  - Estimated from other data LN( 1.0 , 1.0 )



## Combined Variability for Loads

$$\text{COV}_C = \sqrt{\prod_{i=1}^n (1 + C_{X_i}^2) - 1}$$

$$\text{COV} = \sqrt{(1 + 0.3^2)^2 (1 + 0.25^2)^2 - 1} = 0.58$$



# Analysis



**Load Life Analysis (P/N)**

Load Life | Constant Amplitude Analysis

Parameters

Scale Factor: 1.0 L 0.58

Damage Sum: 1.0 L 1.0

Analysis Options

☐ Cycle Hist.

☐ Damage Hist

☐ Materials Data

☐ Exceedance

☒ Statistical Analysis

Component Properties

Description: None

Curve Intercept: 1361 L 0.06

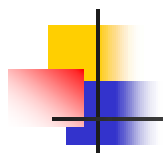
Curve Slope: -0.19

Load

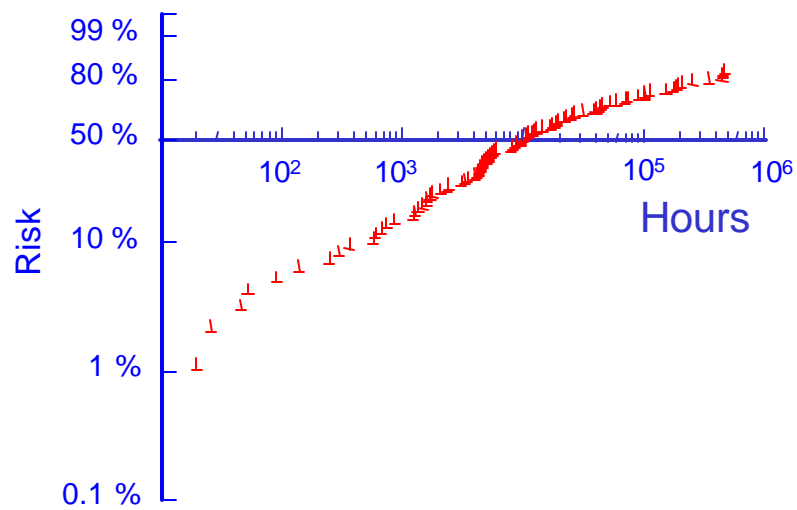
Save

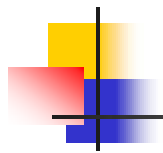
Plot PN Curve

OK Cancel Apply Help



## Results





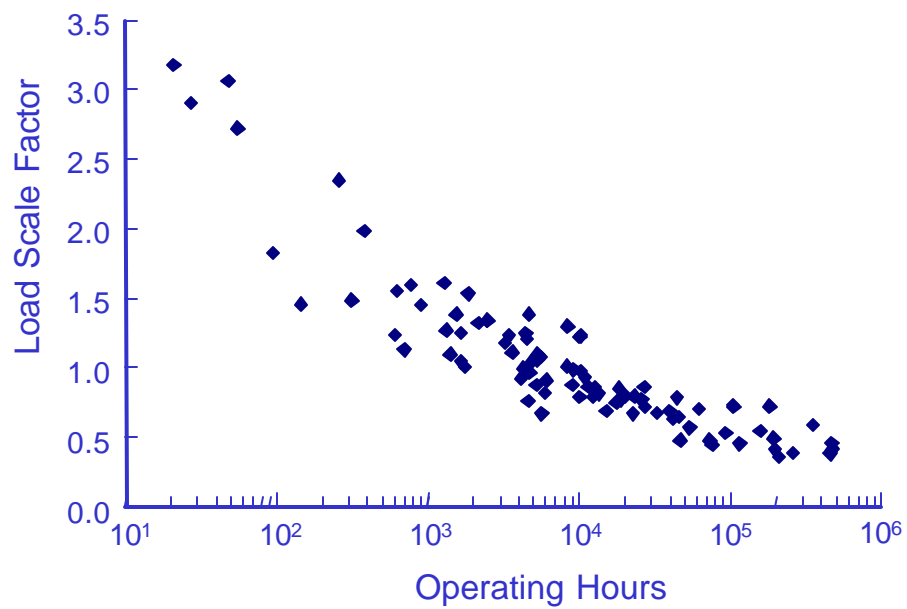
# Results

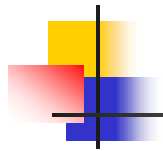
Monte Carlo Simulation Results Table

	mean	COV	$r^2$	$S_{\mu i}$	$\alpha_i$
Blocks	7.778e+04	6.132			
intercept	1.360e+03	0.061	0.015	5.4	0.10
slope	-1.900e-01	0.000	0.000	-13.9	
damage	9.856e-01	0.980	0.016	0.99	0.31
scale	9.981e-01	0.588	0.928	-5.4	0.94



## Correlation Coefficient

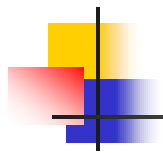




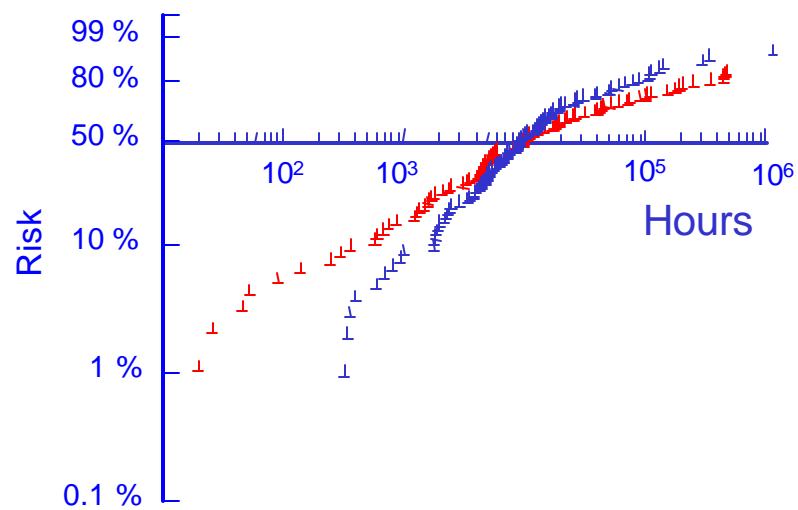
## Course of Action

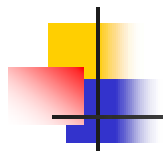
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- Make it stronger
- Run tests to reduce analysis uncertainty
- Field tests to reduce loading uncertainty

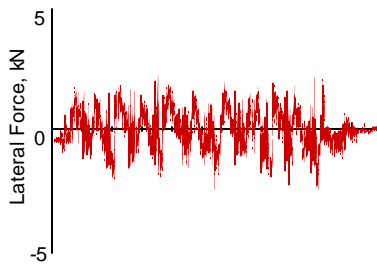


## Eliminate Mean Uncertainty

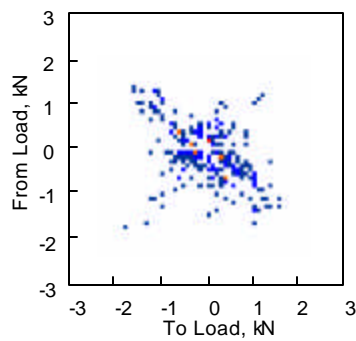




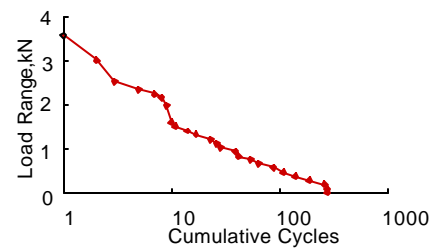
## Service Loading Spectra



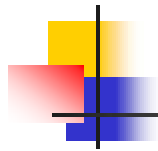
Time history



Rainflow  
Histogram



Exceedance  
Diagram



## Problem Statement

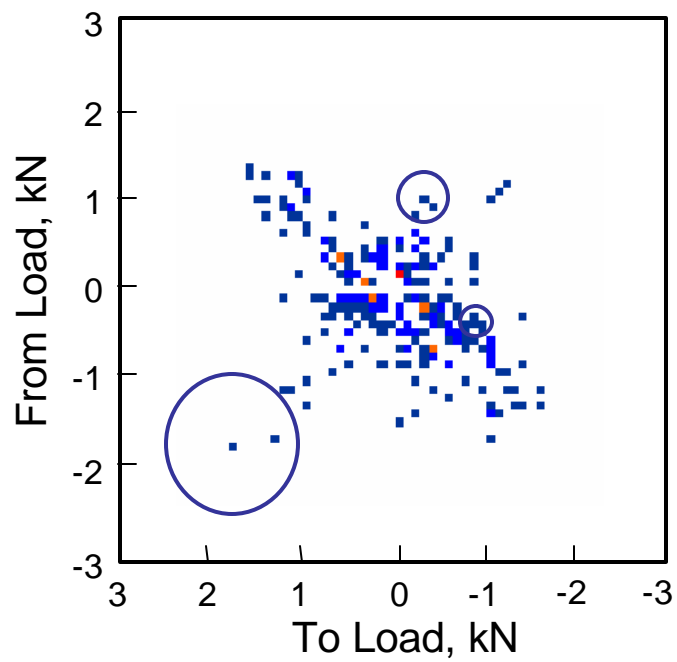
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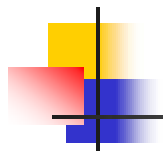
Given a rainflow histogram for a single user,  
extrapolate to longer times

Given rainflow histograms for multiple users,  
extrapolate to more users

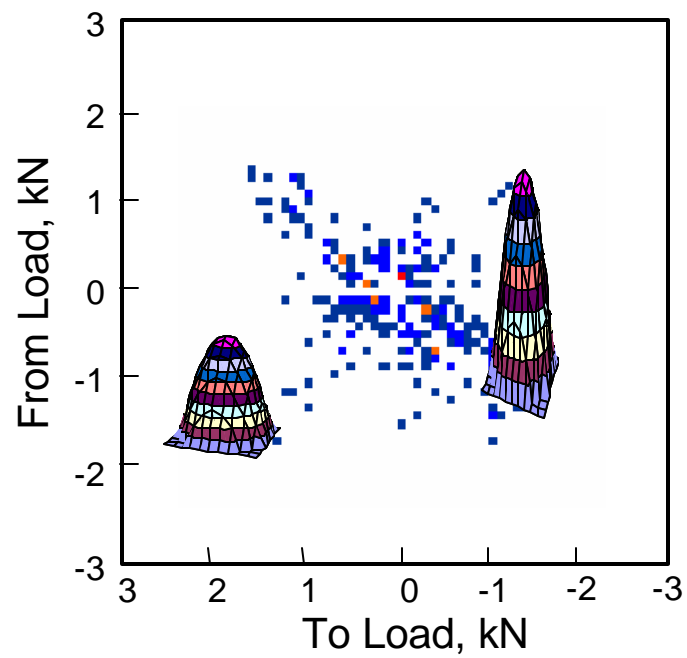


## Probability Density



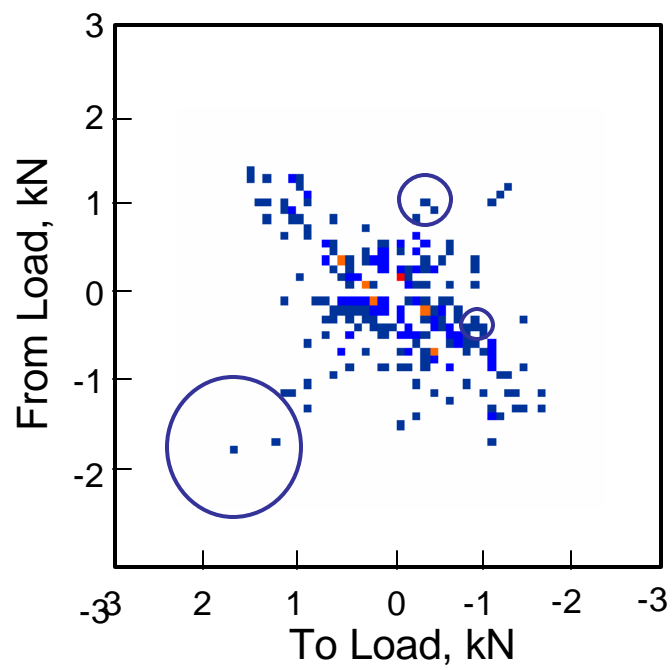


## Kernel Smoothing



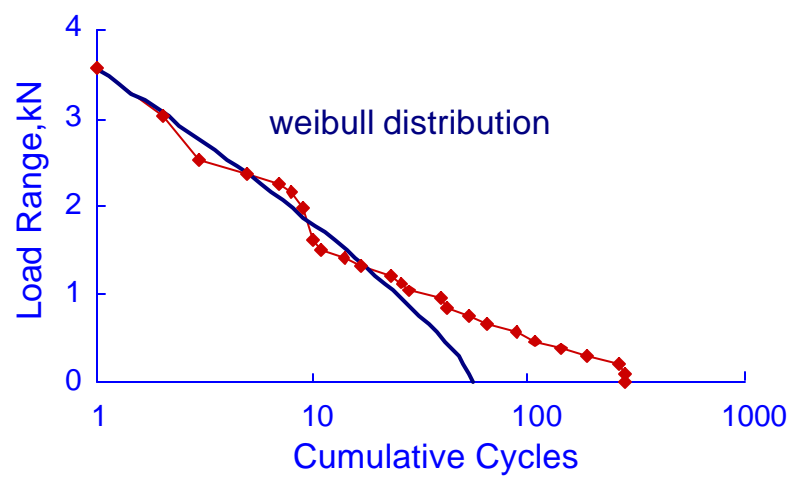


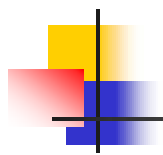
## Sparse Data



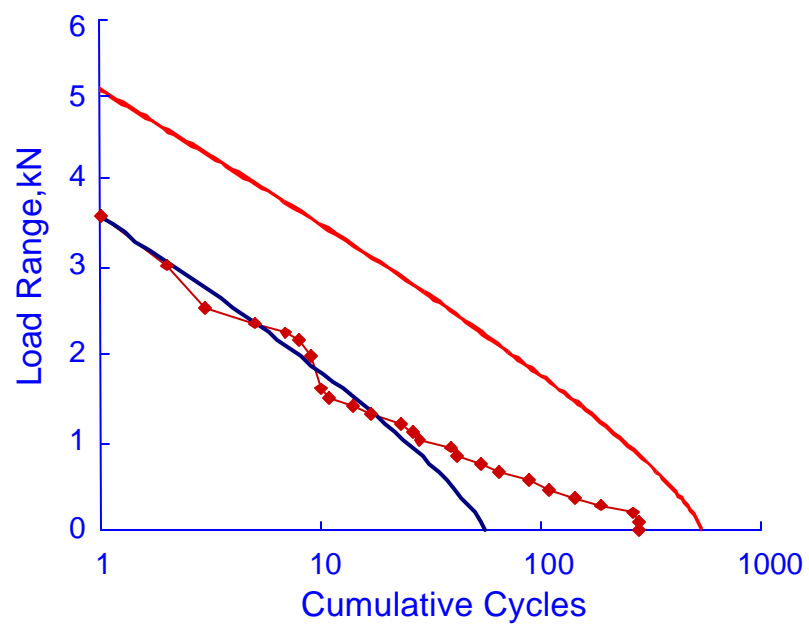


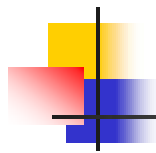
## Exceedance Plot of 1 Lap



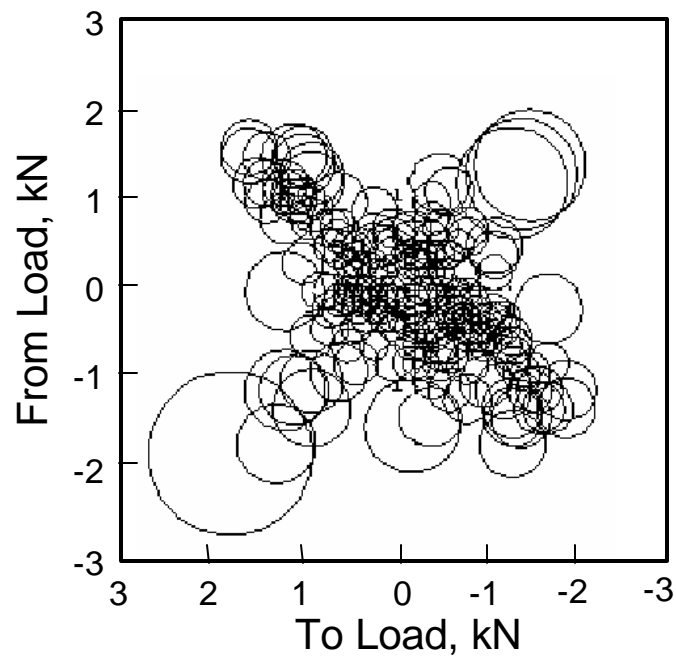


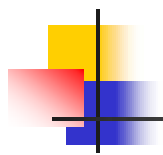
## 10X Extrapolation





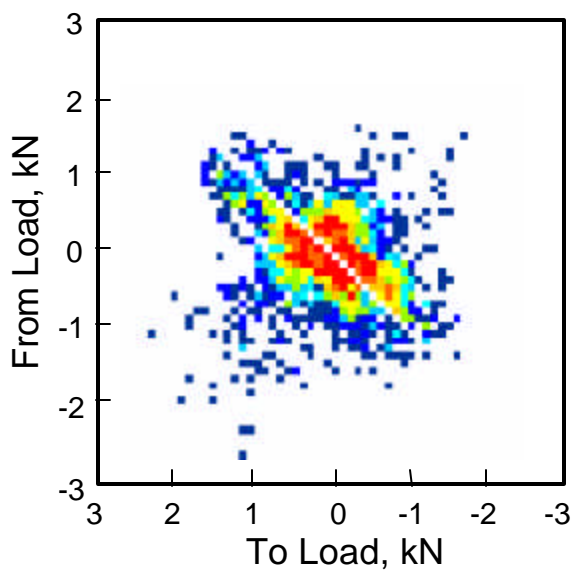
## Probability Density



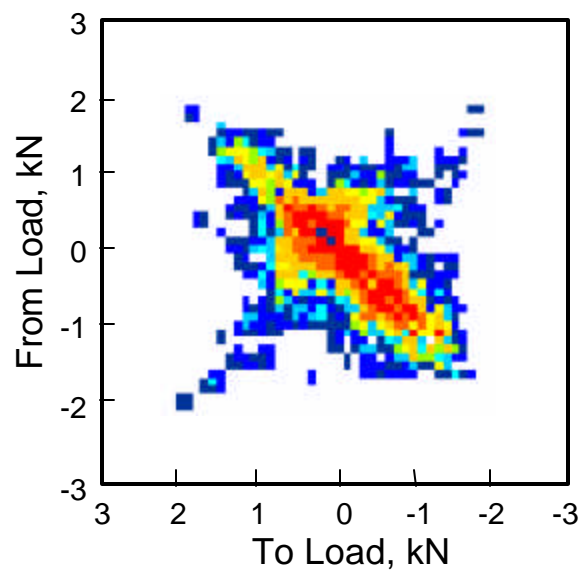


## Results

Simulation

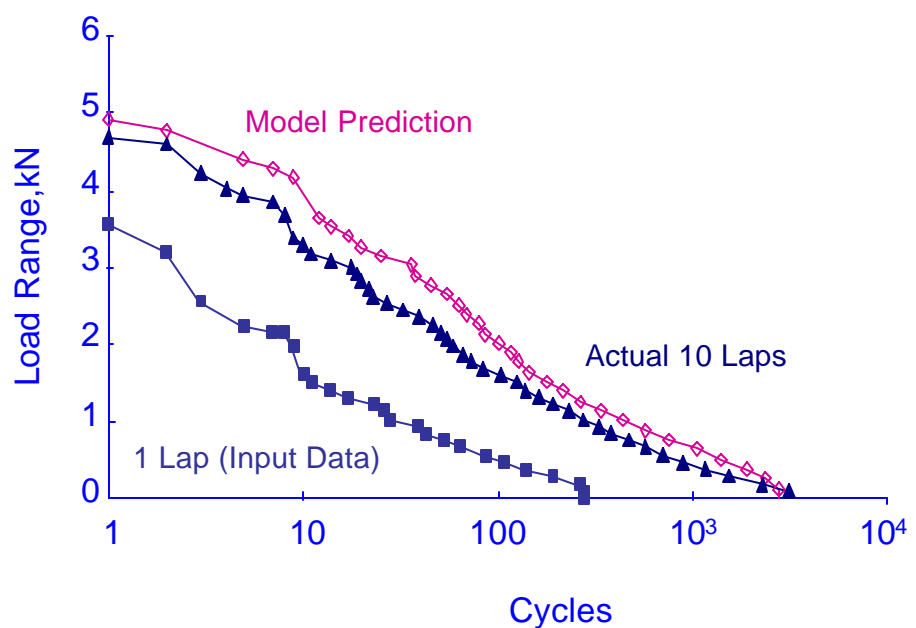


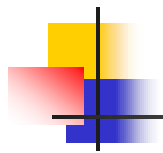
Test Data





## Exceedance Diagram





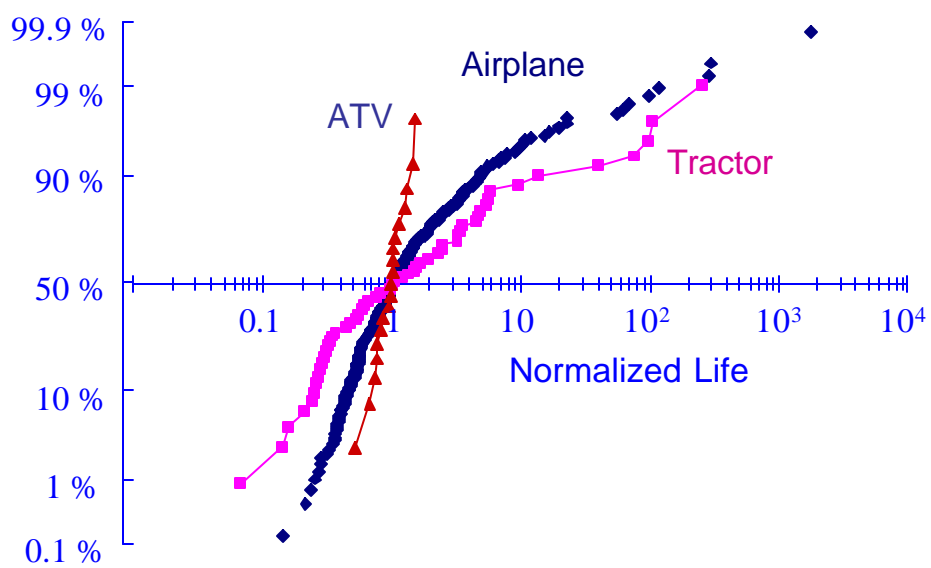
## Problem Statement

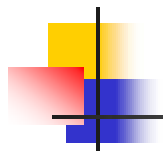
---

- Given a rainflow histogram for a single user, extrapolate to longer times
- Given rainflow histograms for multiple users, extrapolate to more users



## Extrapolated Data Sets





## Issues

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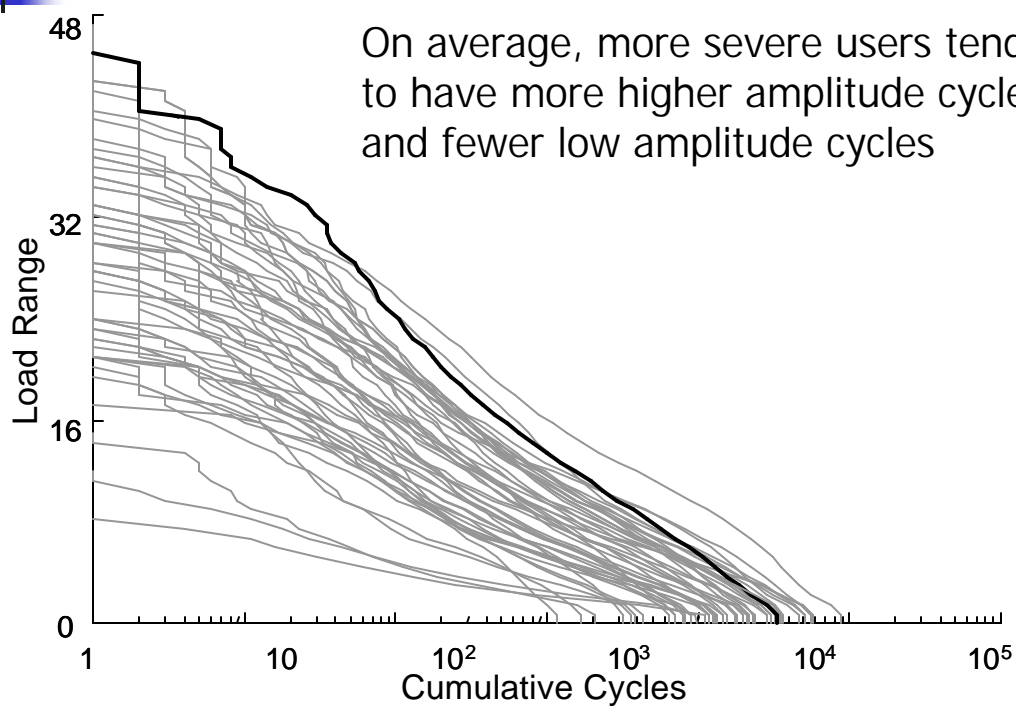
In the first problem the number of cycles is known but the variability is unknown and must be estimated

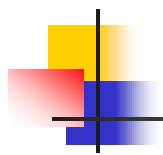
In the second problem the variability is known but the number and location of cycles is unknown and must be estimated



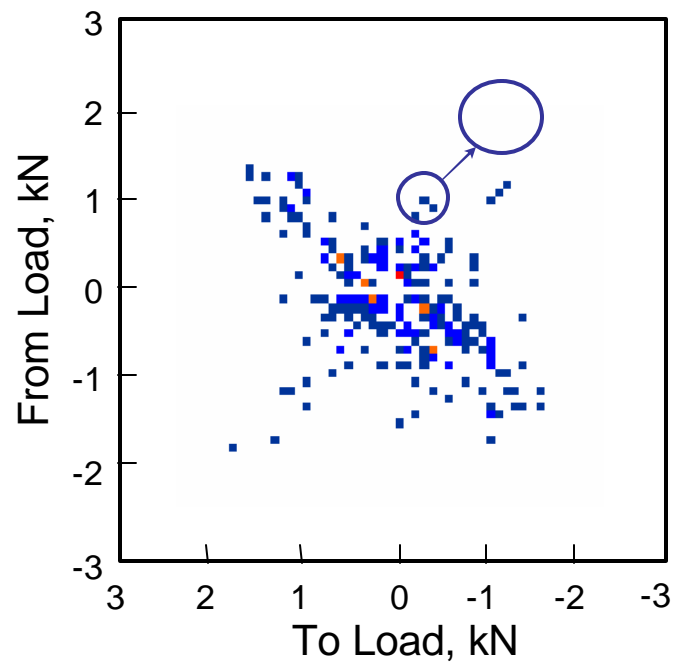
## Assumption

On average, more severe users tend to have more higher amplitude cycles and fewer low amplitude cycles



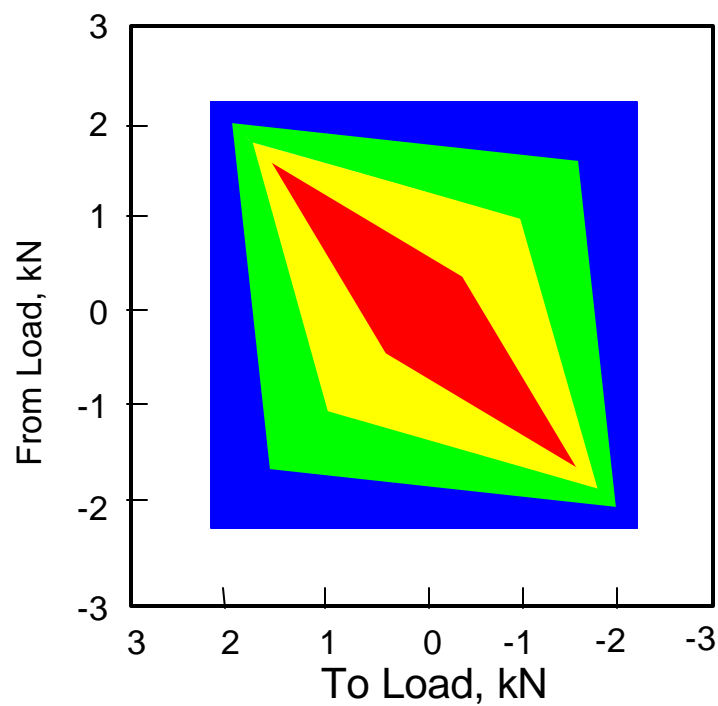


## Translation

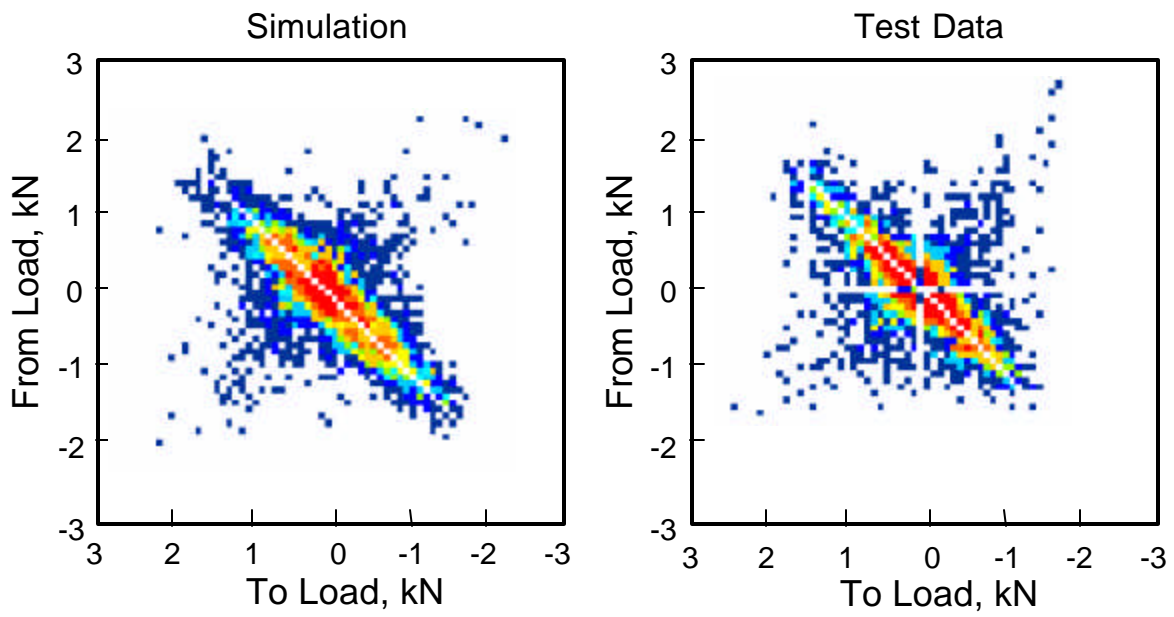


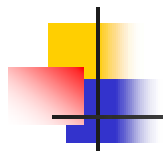


## Damage Regions

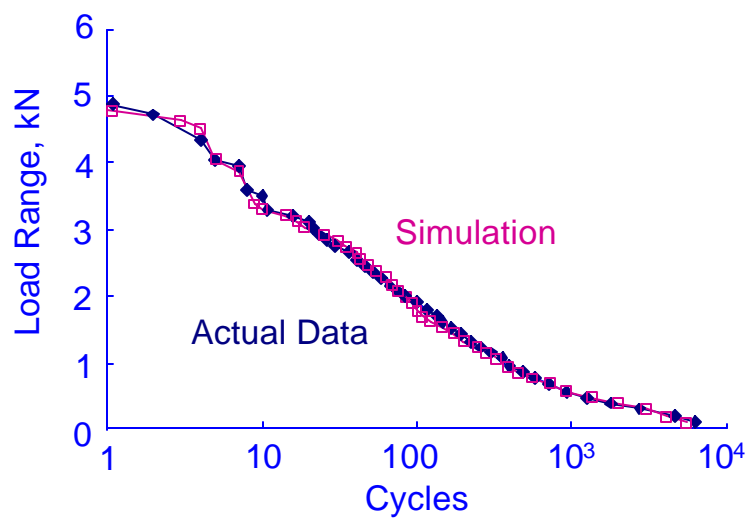


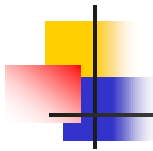
## ATV Data - Most Damaging in 19



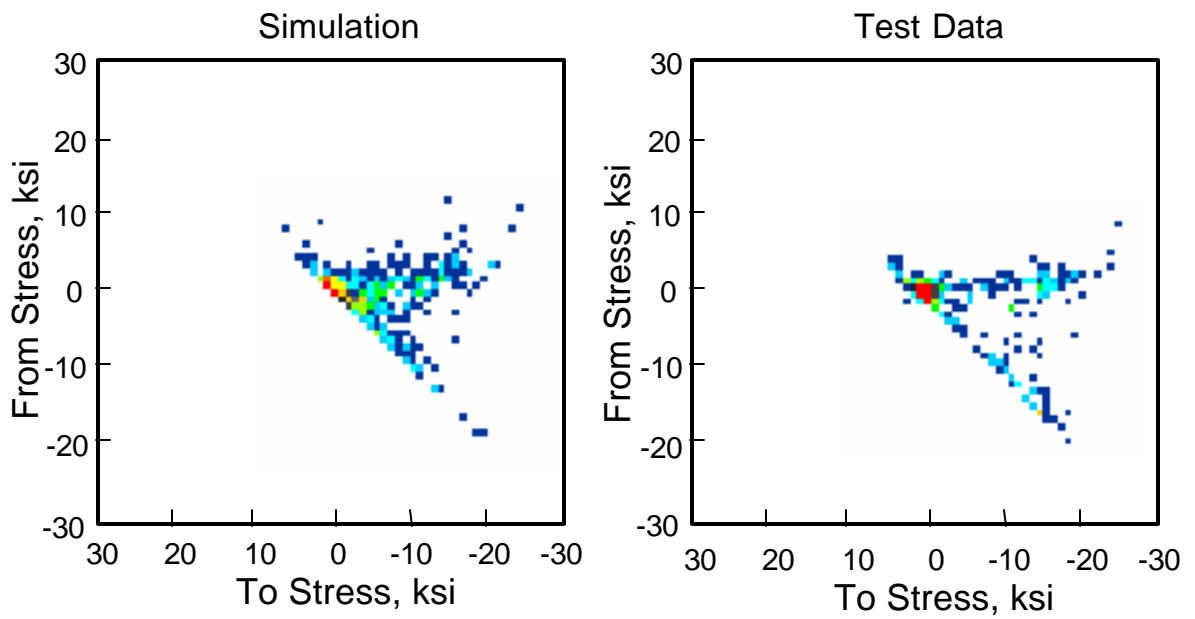


## ATV Exceedance



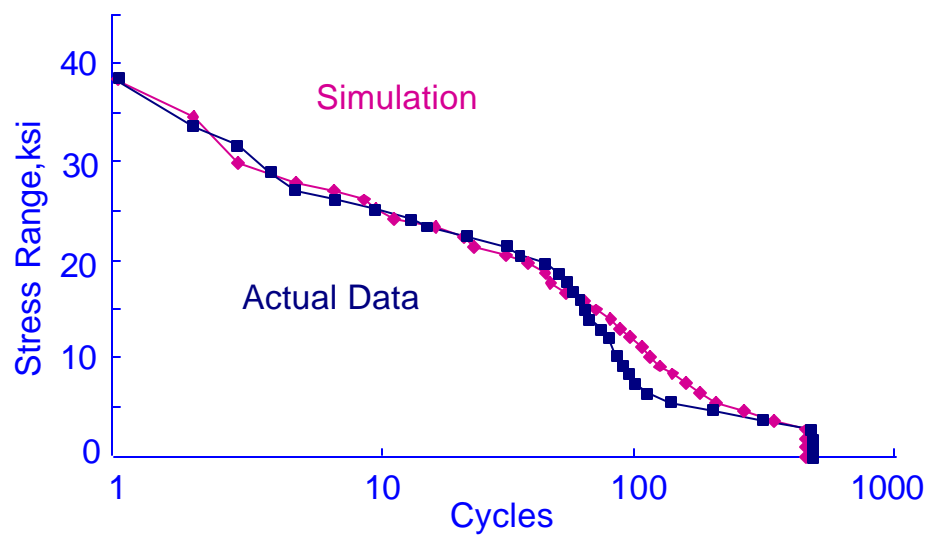


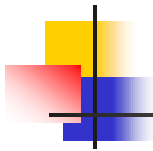
## Airplane Data - Most Damaging in 334



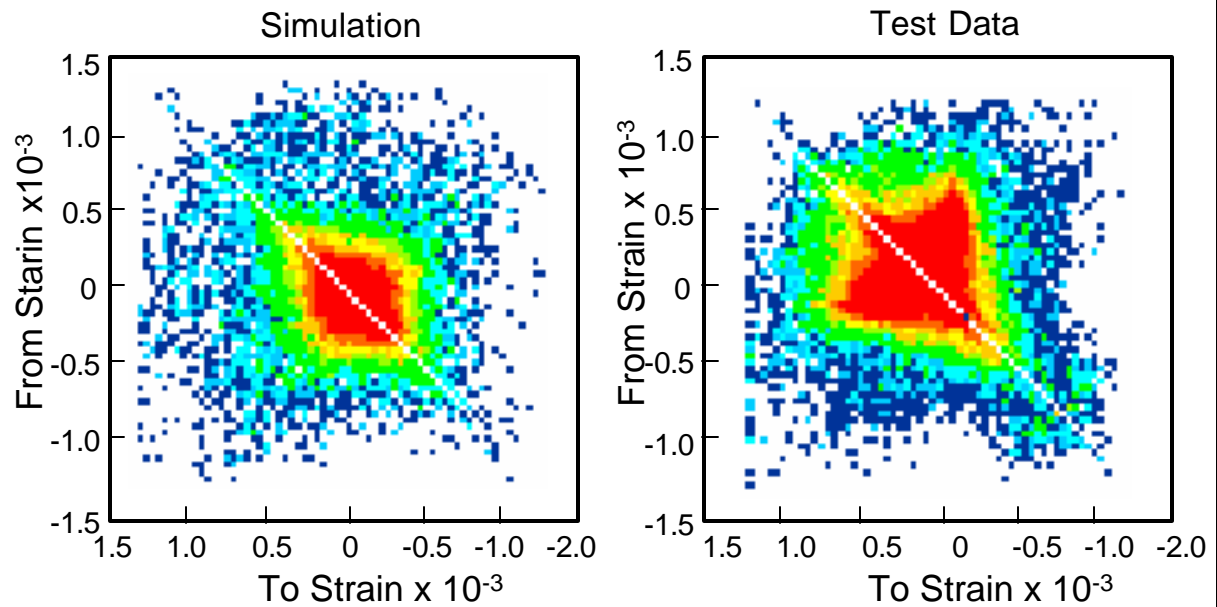


## Airplane Exceedance



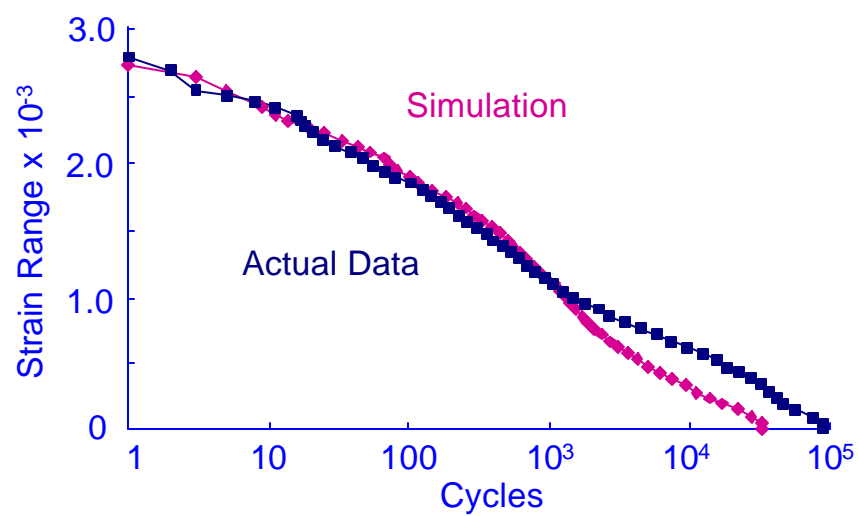


## Tractor Data - Most Damaging in 54





## Tractor Exceedance



# Probabilistic Aspects of Fatigue

