Multiaxial Fatigue

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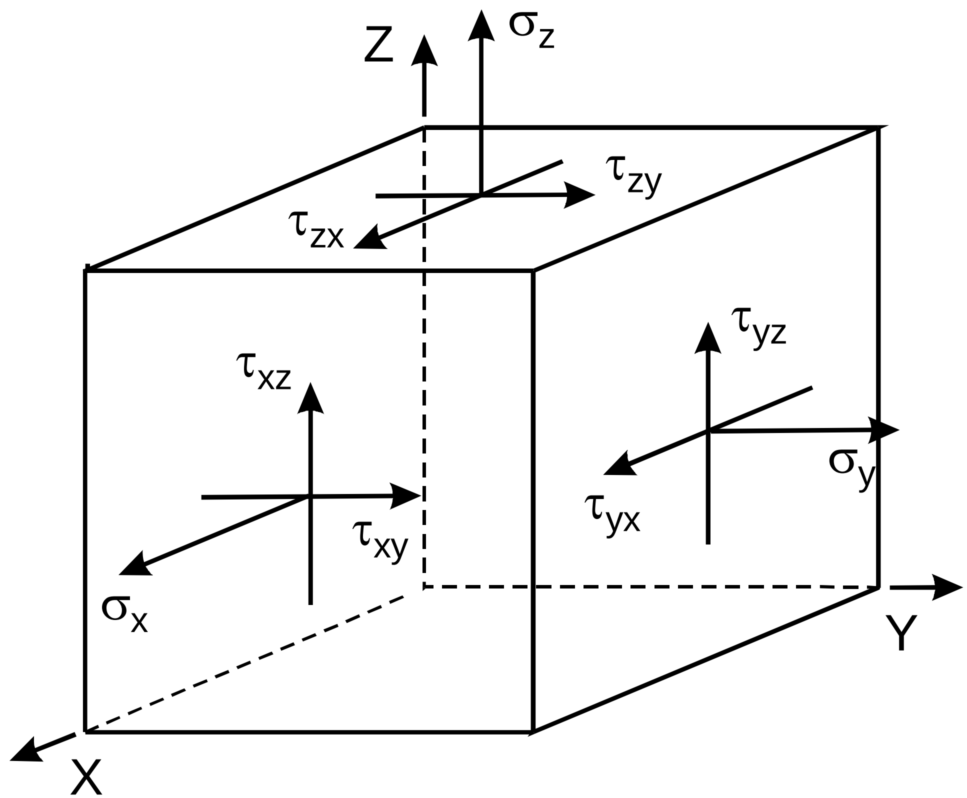
Outline

- State of Stress (Chapter 1)
- Fatigue Mechanisms (Chapter 3)
- Stress Based Models (Chapter 5)
- Strain Based Models (Chapter 6)
- Fracture Mechanics Models (Chapter 7)
- Nonproportional Loading (Chapter 8)
- Notches (Chapter 9)

State of Stress

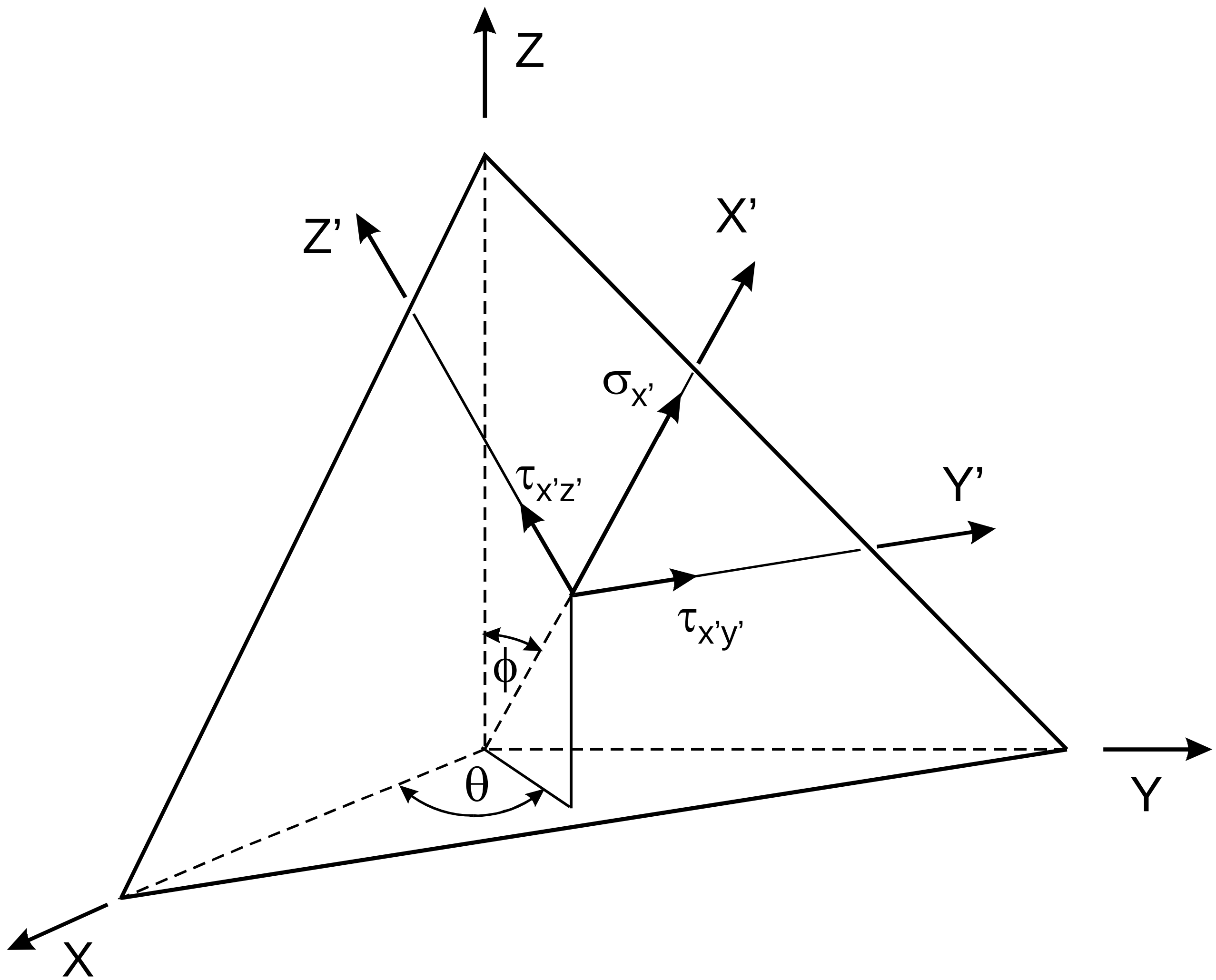
- Stress components
- Common states of stress
- Shear stresses

Stress Components

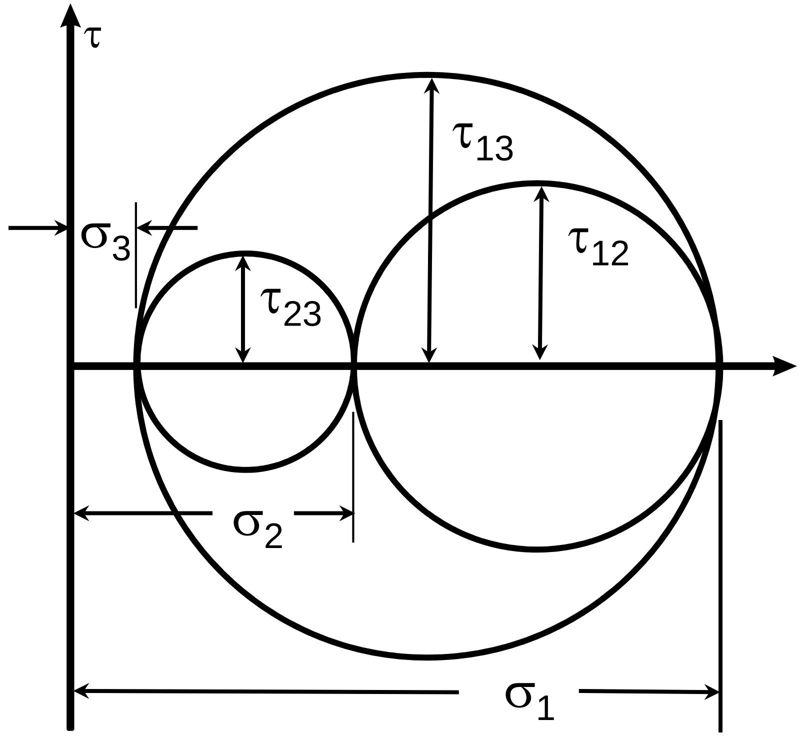


Six stresses and six strains

Stresses Acting on a Plane

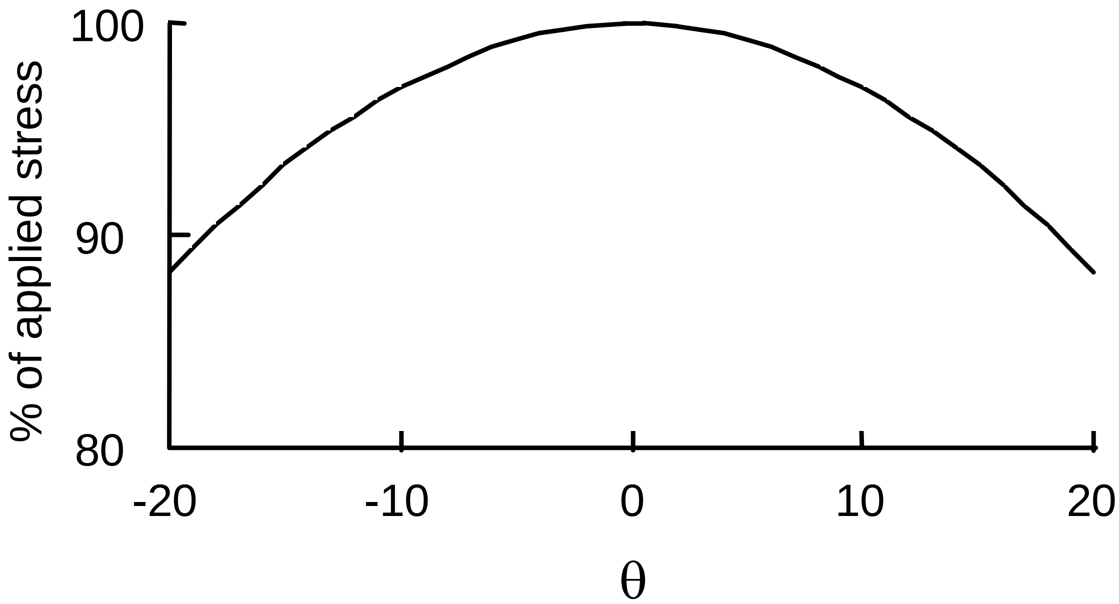


Principal Stresses



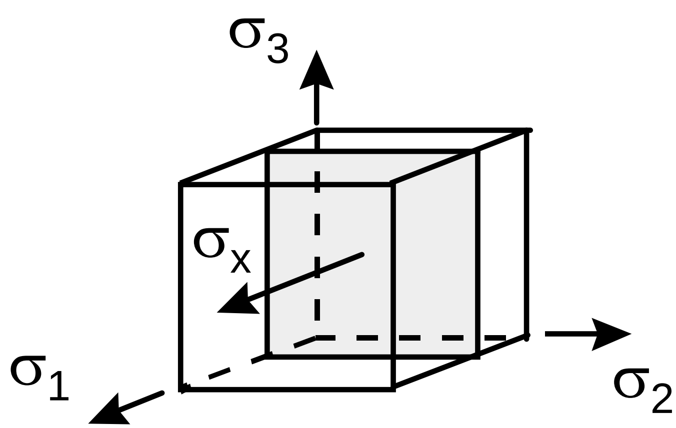
$$\sigma^3 - \sigma^2(\sigma_X + \sigma_Y + \sigma_Z) + \sigma(\sigma_X\sigma_Y + \sigma_Y\sigma_Z\sigma_X\sigma_Z - \tau^2_{XY} - \tau^2_{YZ} - \tau^2_{XZ})$$
$$- (\sigma_X\sigma_Y\sigma_Z + 2\tau_{XY}\tau_{YZ}\tau_{XZ} - \sigma_X\tau^2_{YZ} - \sigma_Y\tau^2_{ZX} - \sigma_Z\tau^2_{XY}) = 0$$

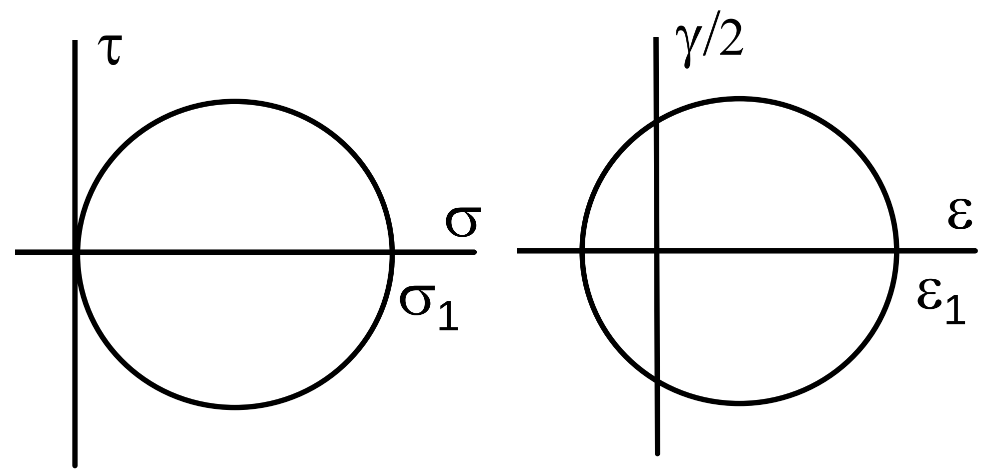
Stress and Strain Distributions



Stresses are nearly the same over a 10° range of angles

Tension

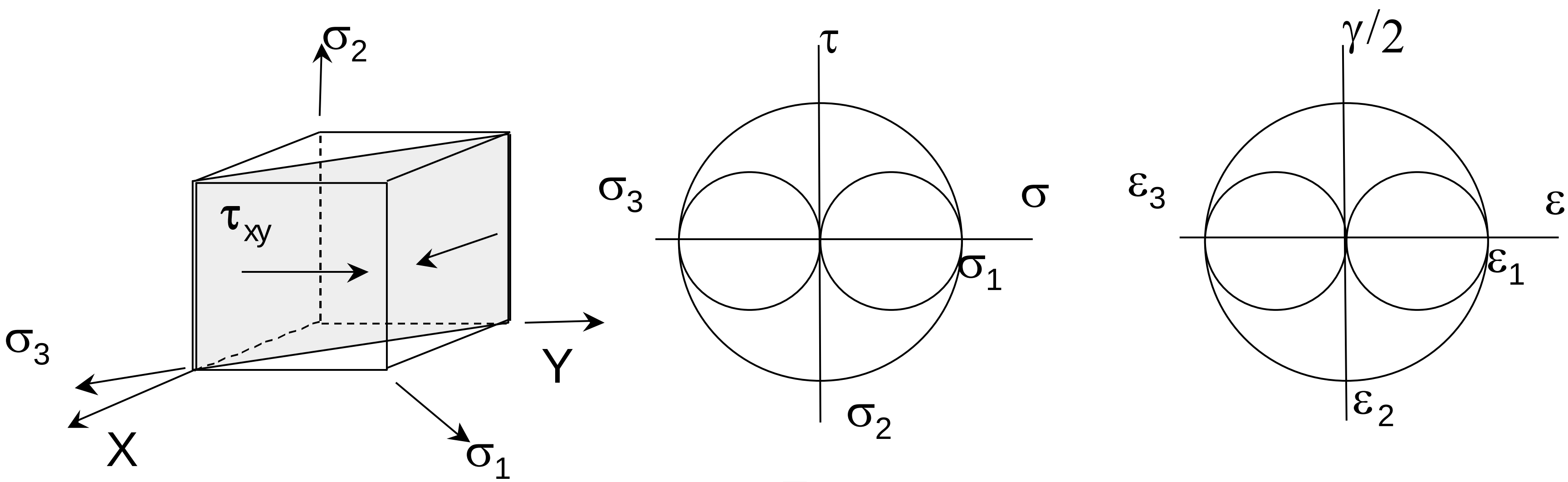




$\sigma_2 = \sigma_3 = 0$

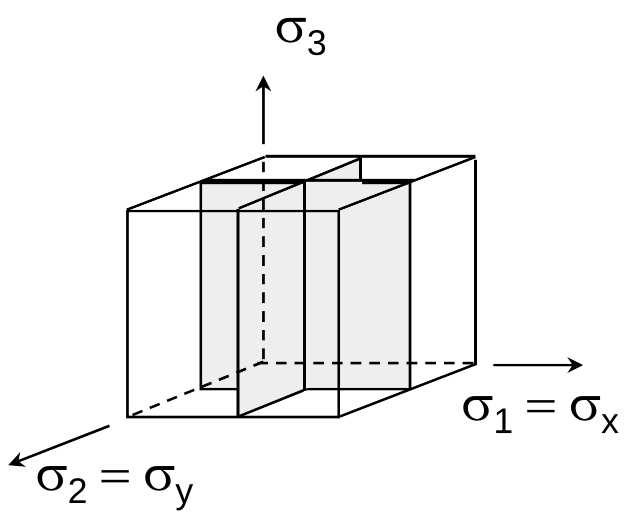
$\varepsilon_2 = \varepsilon_3 = -\nu\varepsilon_1$

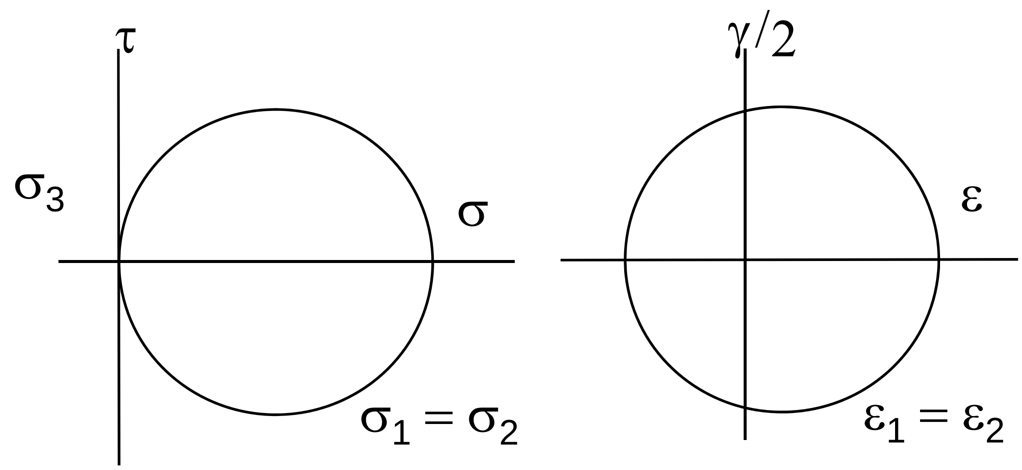
Torsion



$\sigma_1 = \tau_{xy}$

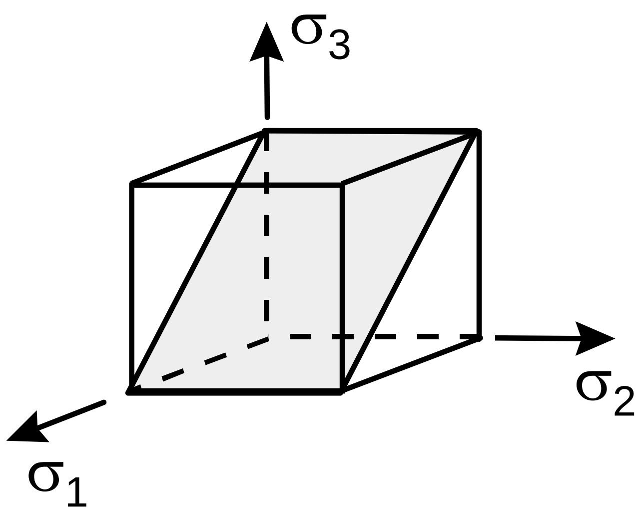
Biaxial Tension

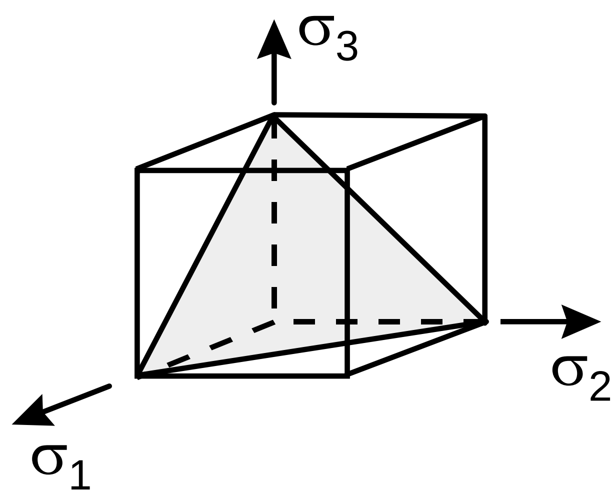




$$\varepsilon_3 = -\frac{2\nu}{1-\nu}\varepsilon$$

Shear Stresses





Maximum shear stress

Octahedral shear stress

$$\tau_{13} = \frac{|\sigma_1 - \sigma_3|}{2}$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2}$$

Mises: $\bar{\sigma} = \frac{3}{\sqrt{2}} \tau_{oct}$

$$\tau_{oct} = \frac{3}{2\sqrt{2}} \tau_{13} = 0.94\, \tau_{13}$$

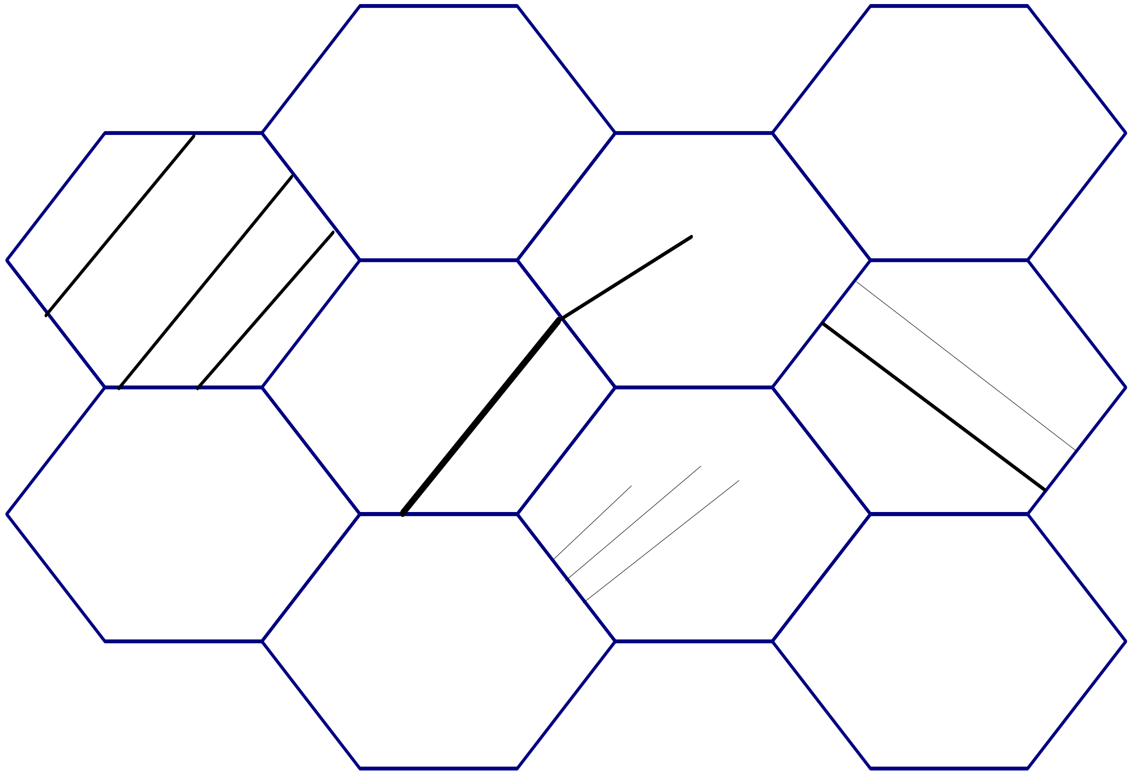
State of Stress Summary

- Stresses acting on a plane
- Principal stress
- Maximum shear stress
- Octahedral shear stress

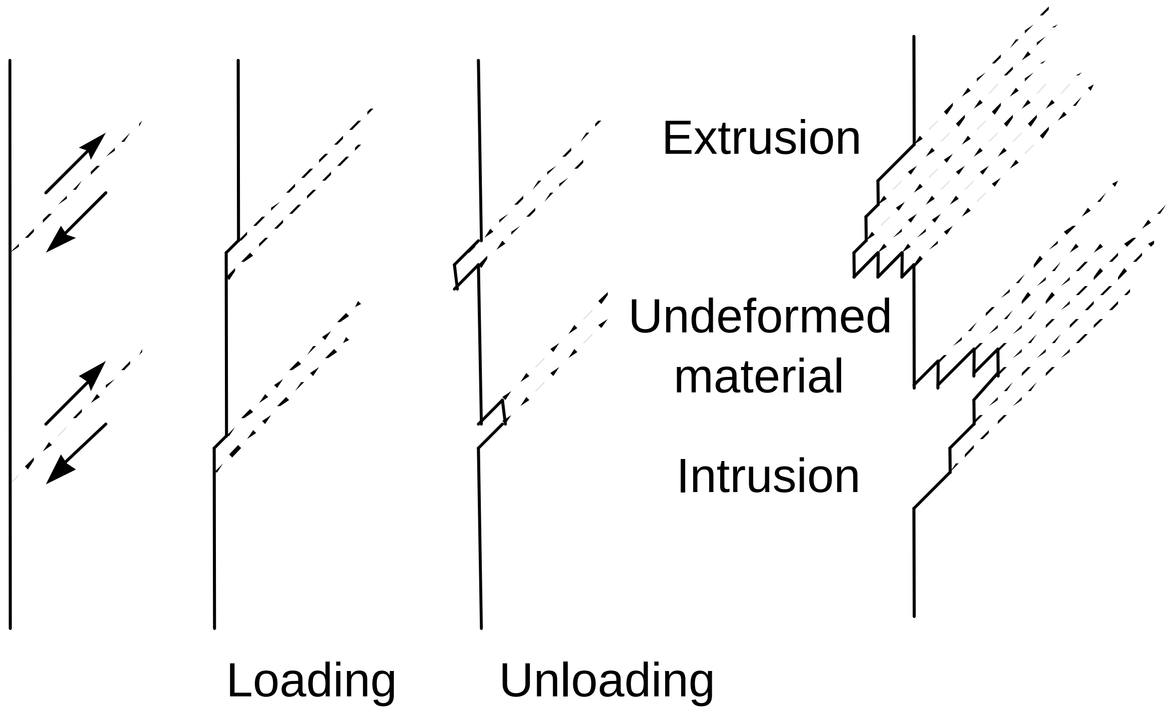
Fatigue Mechanisms

- Crack nucleation
- Fracture modes
- Crack growth
- State of stress effects

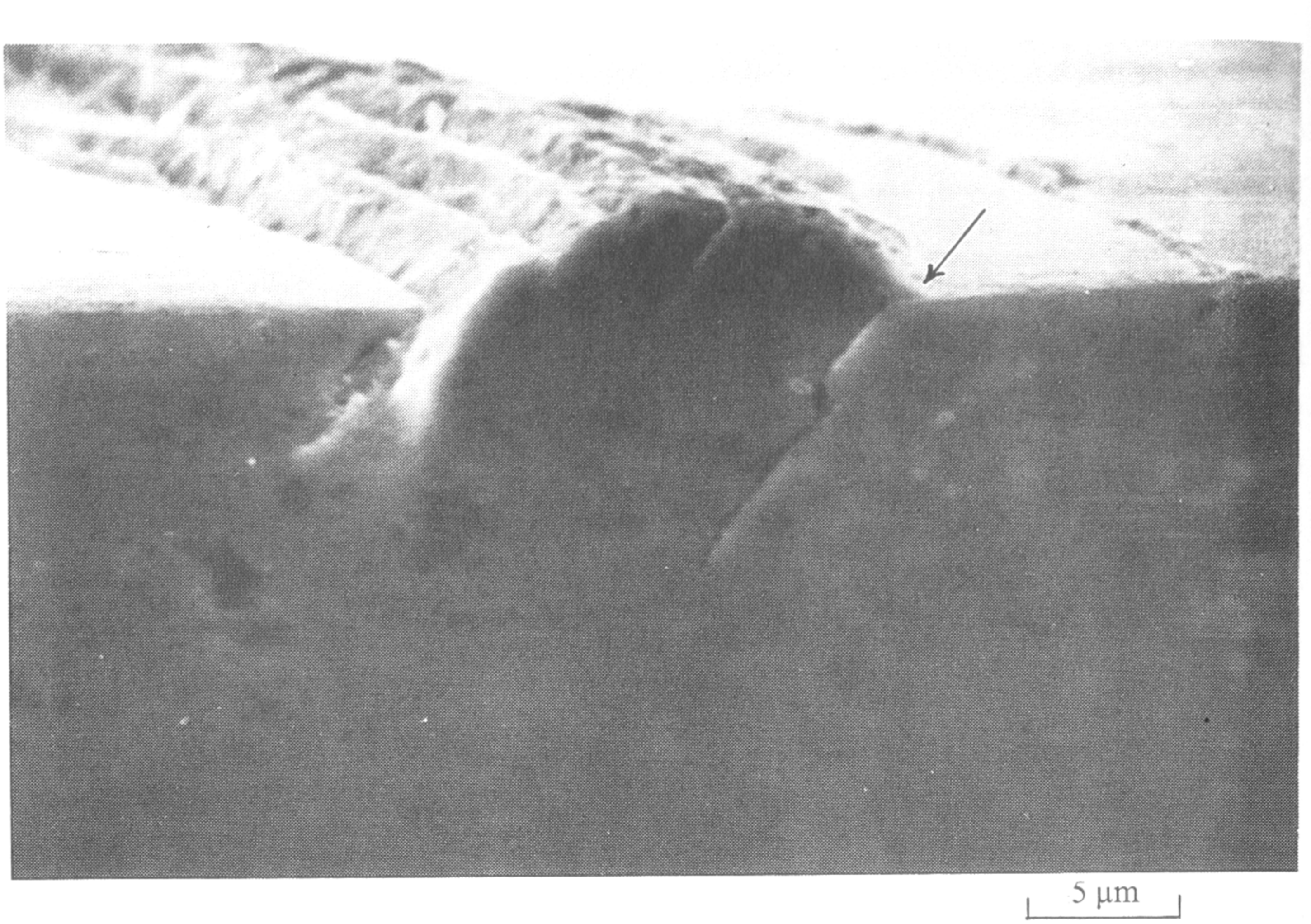
Crack Nucleation



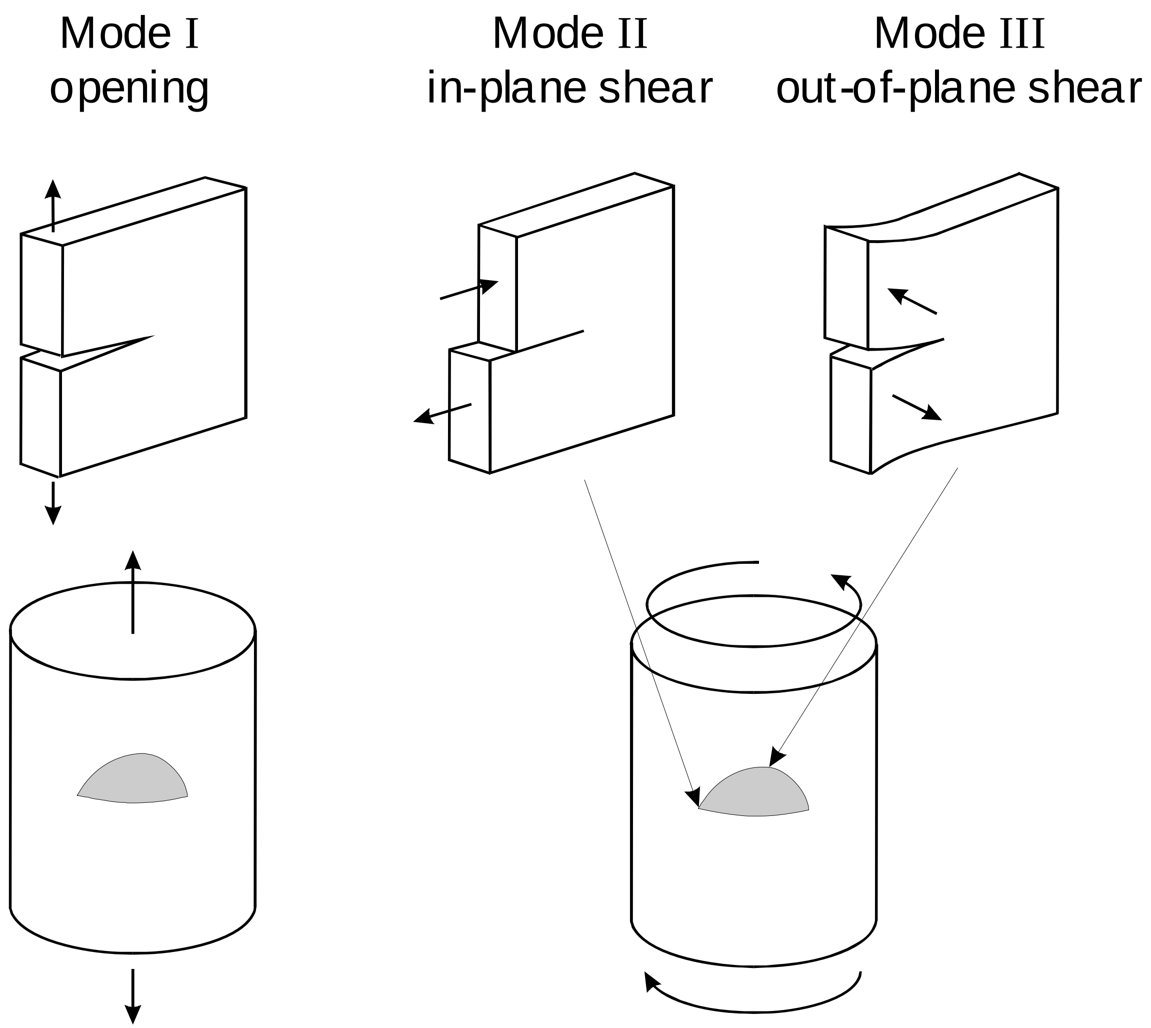
Slip Bands


Extrusion
Undeformed material
Intrusion
Loading
Unloading

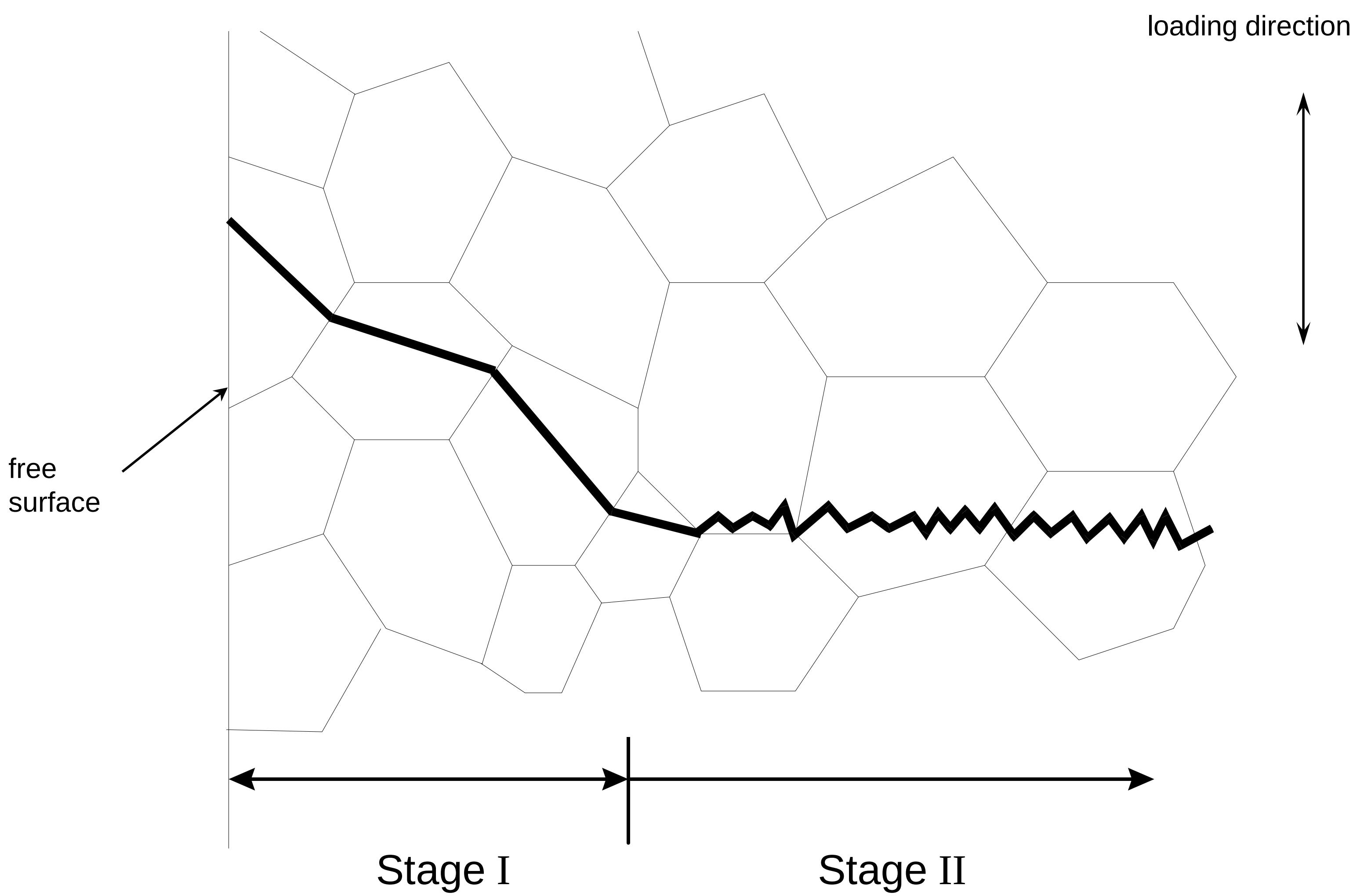
Slip Bands



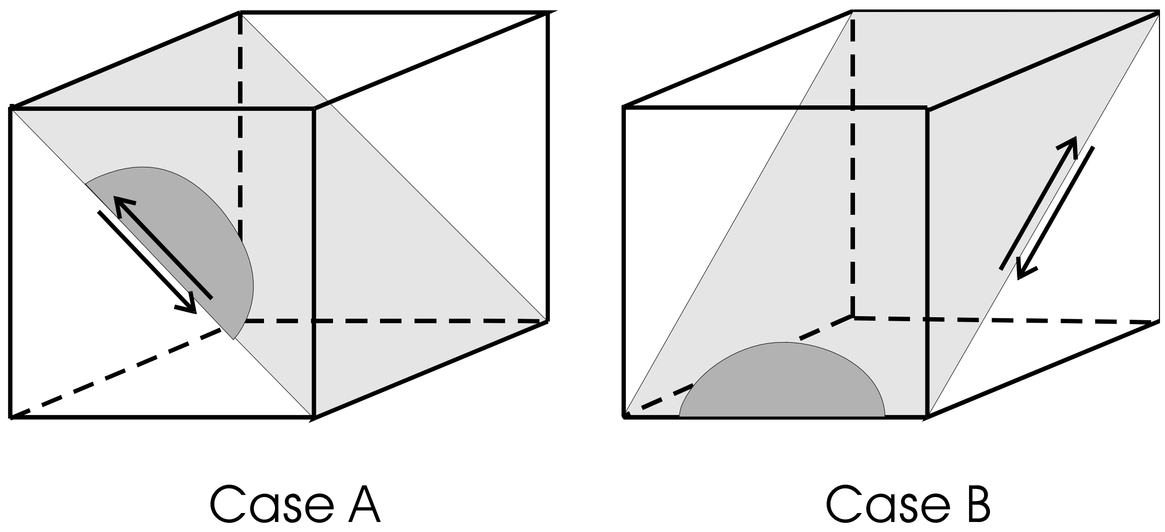
Mode I, Mode II, and Mode III



Stage I and Stage II



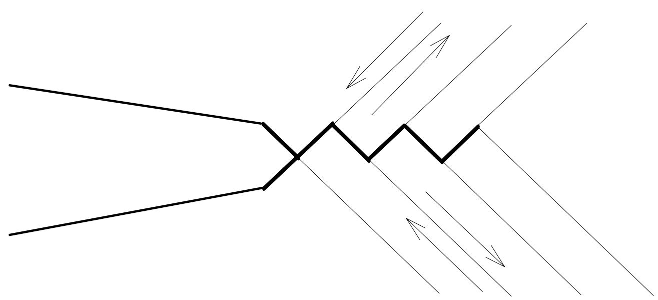
Case A and Case B

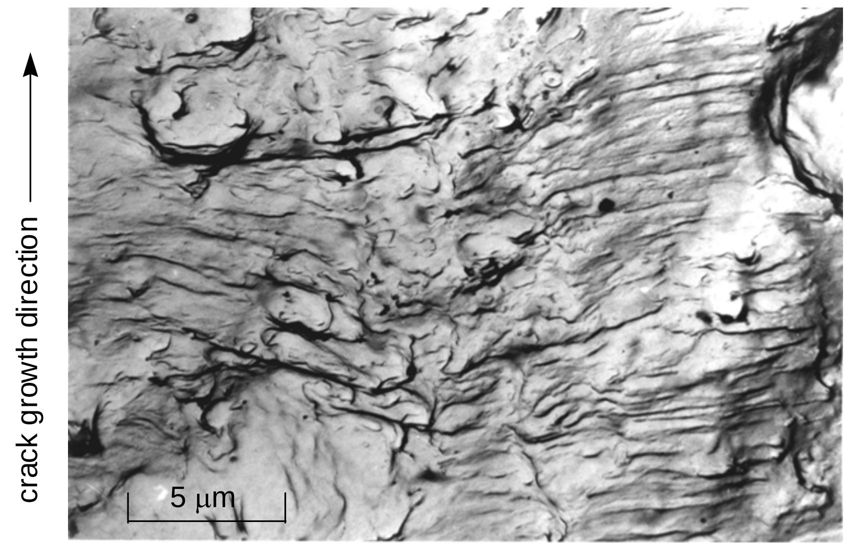


Growth along the surface

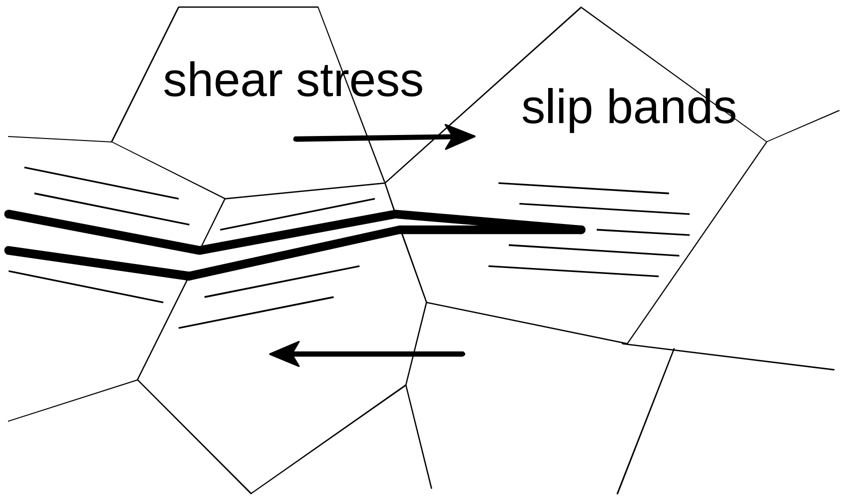
Growth into the surface

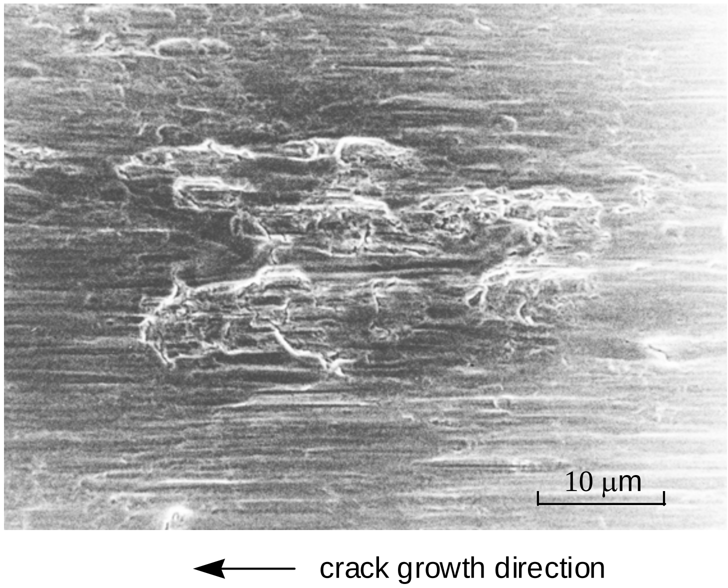
Mode I Growth



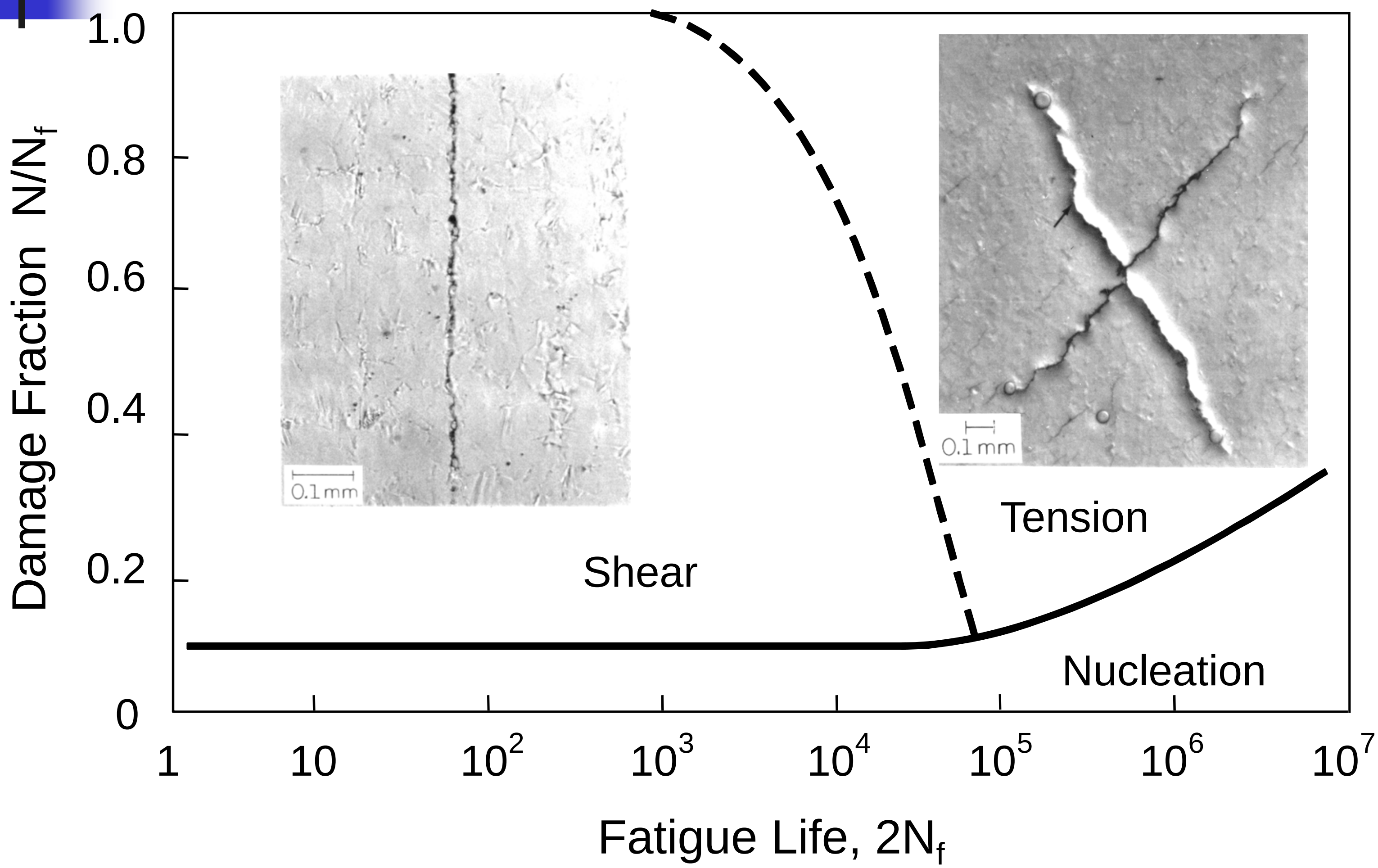


Mode II Growth

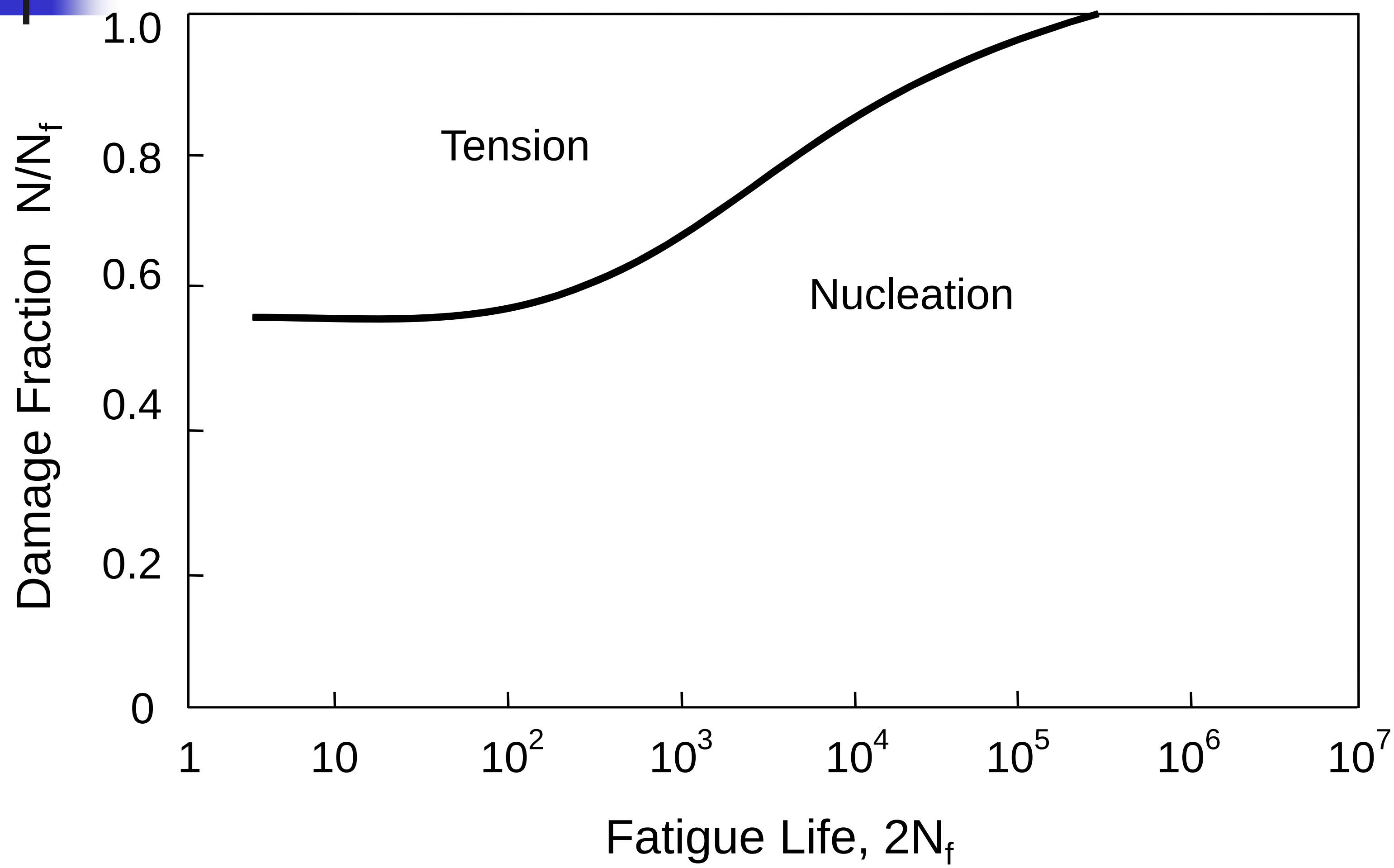




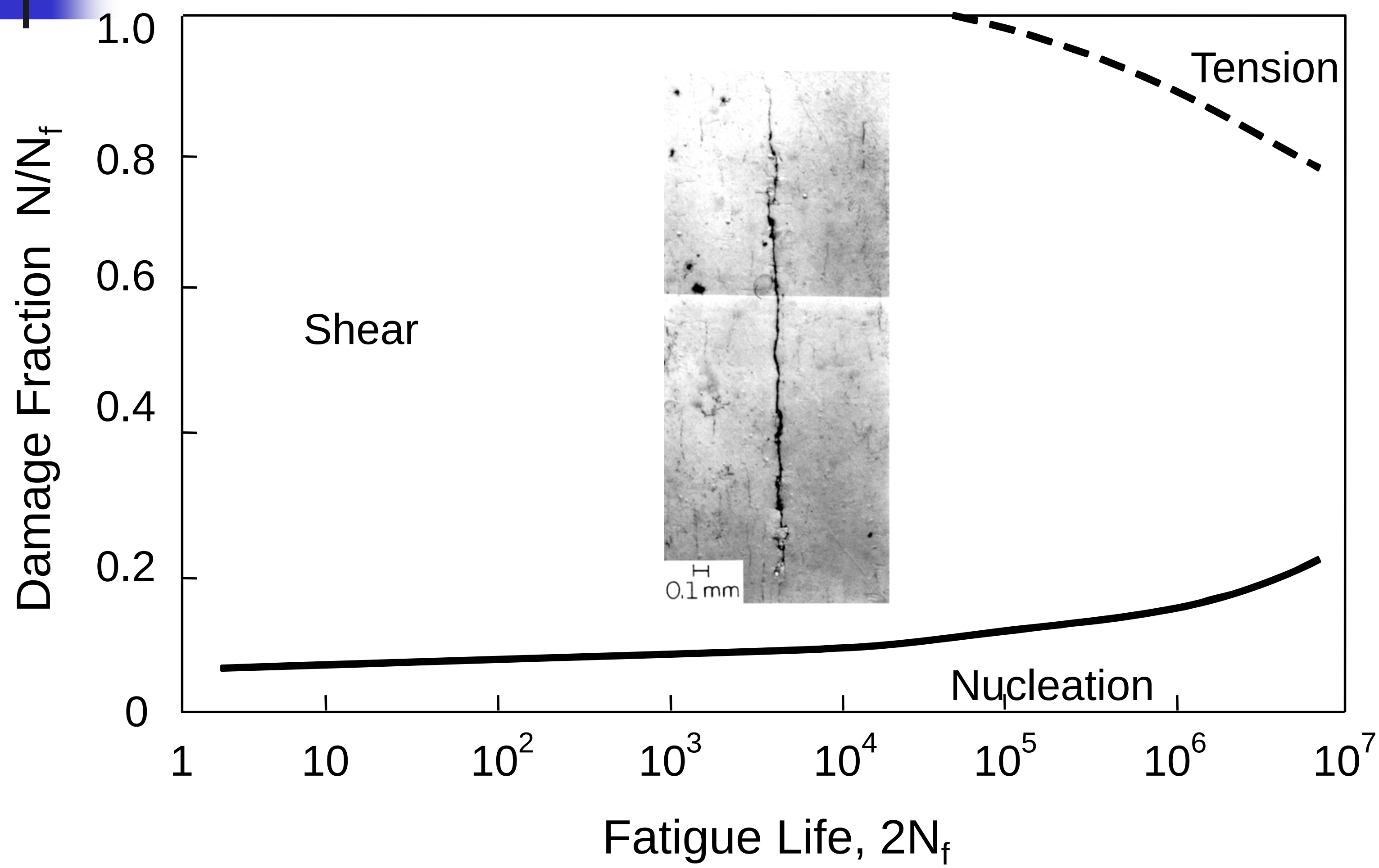
304 Stainless Steel - Torsion



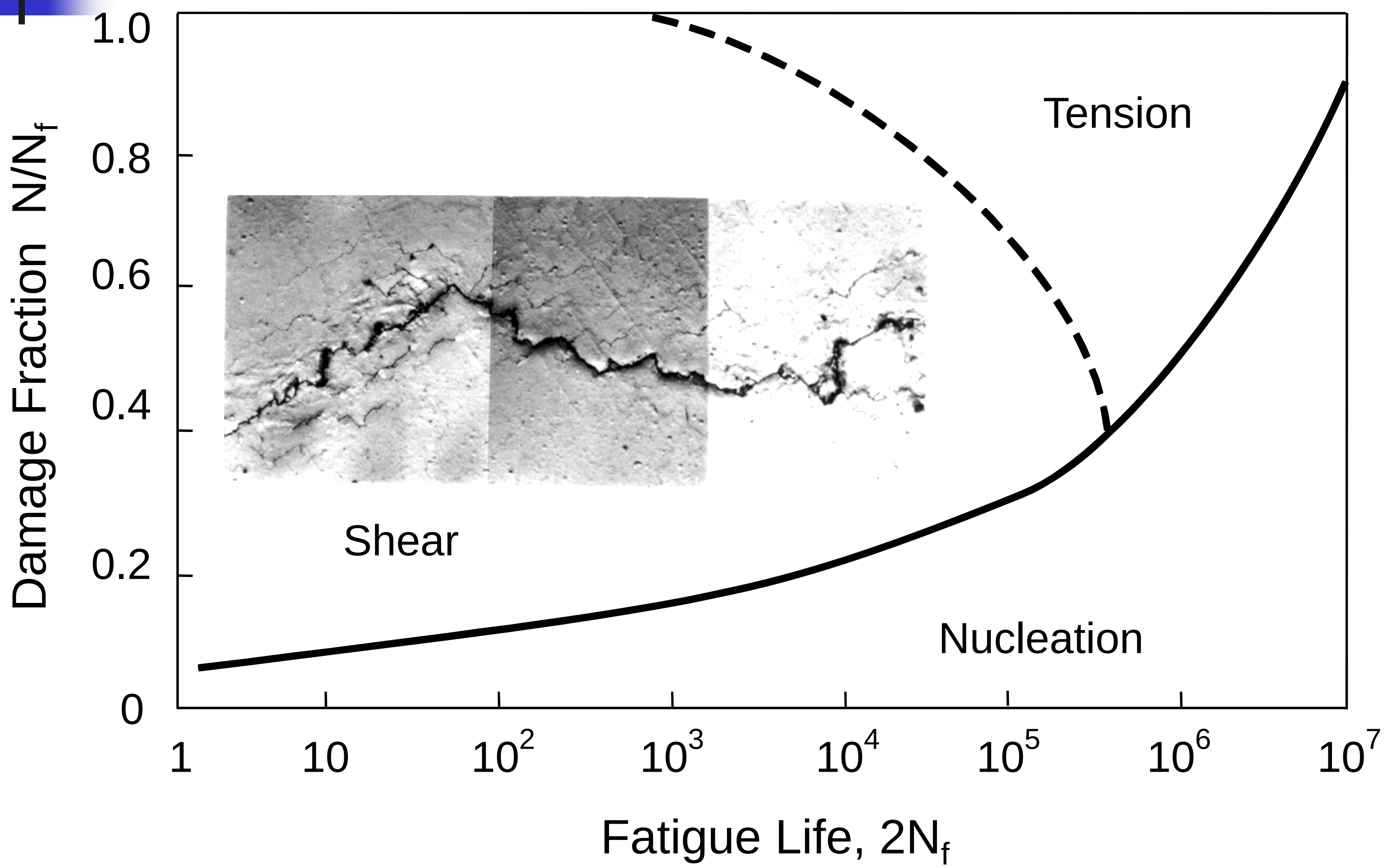
304 Stainless Steel - Tension



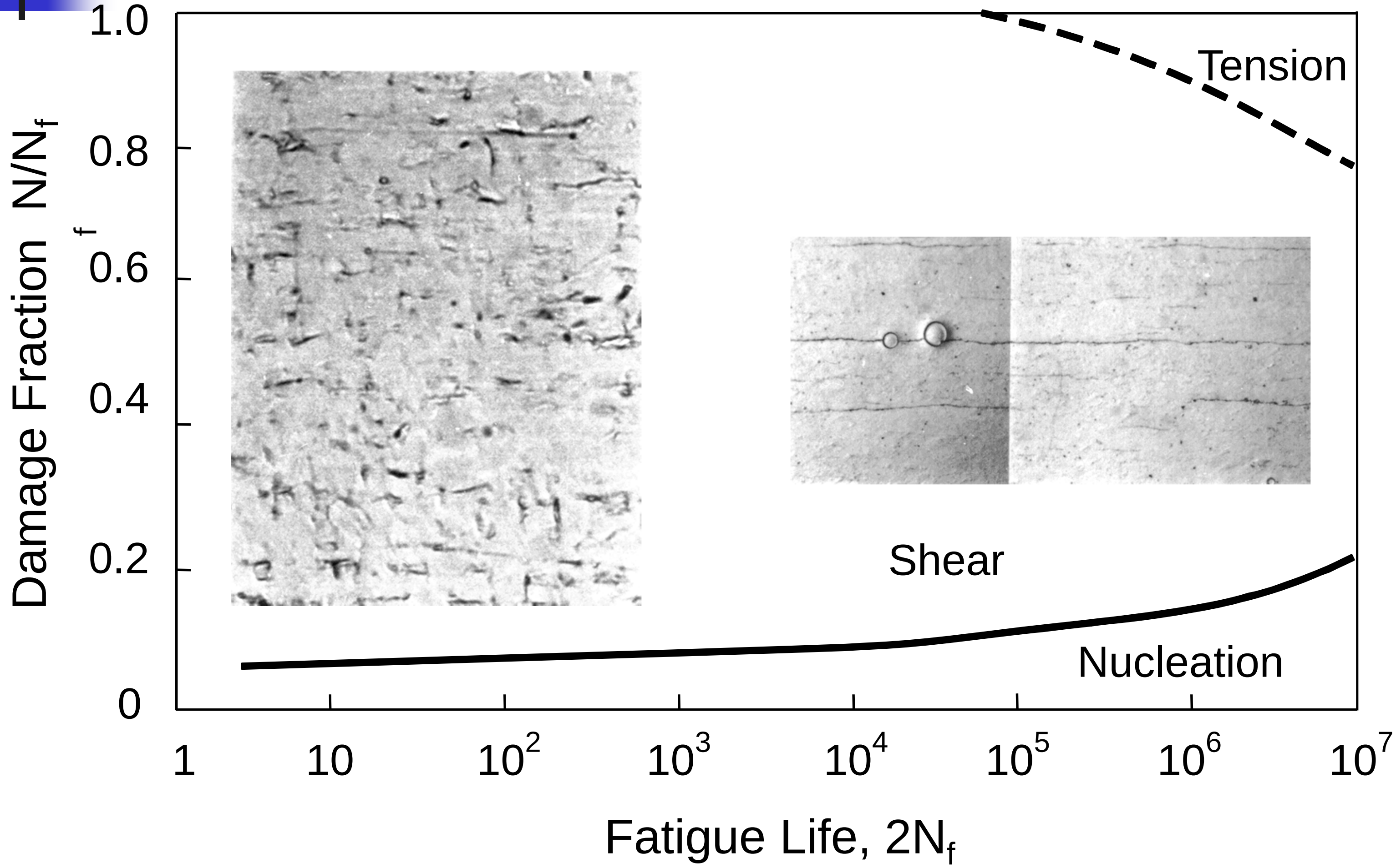
Inconel 718 - Torsion



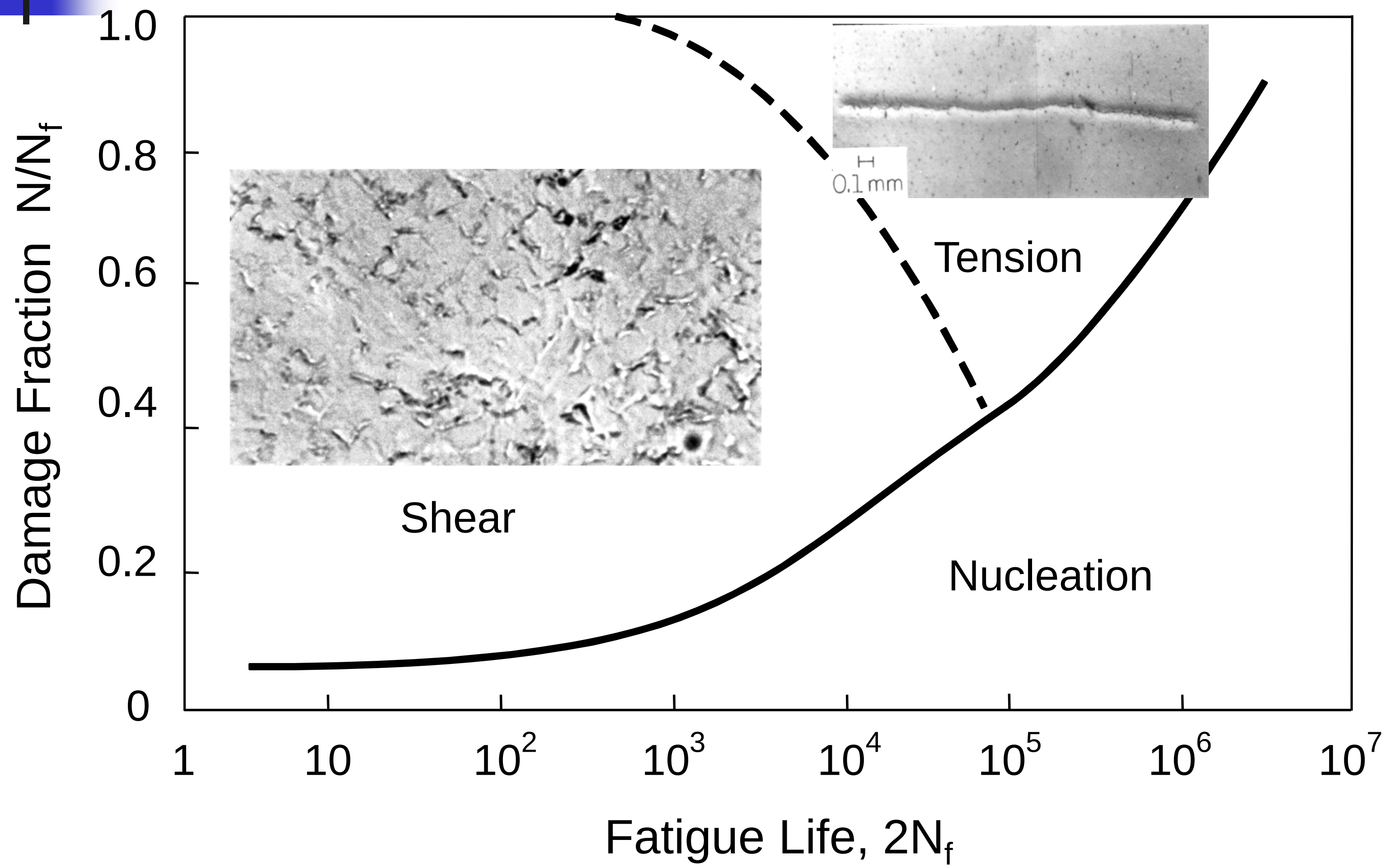
Inconel 718 - Tension



1045 Steel - Torsion



1045 Steel - Tension



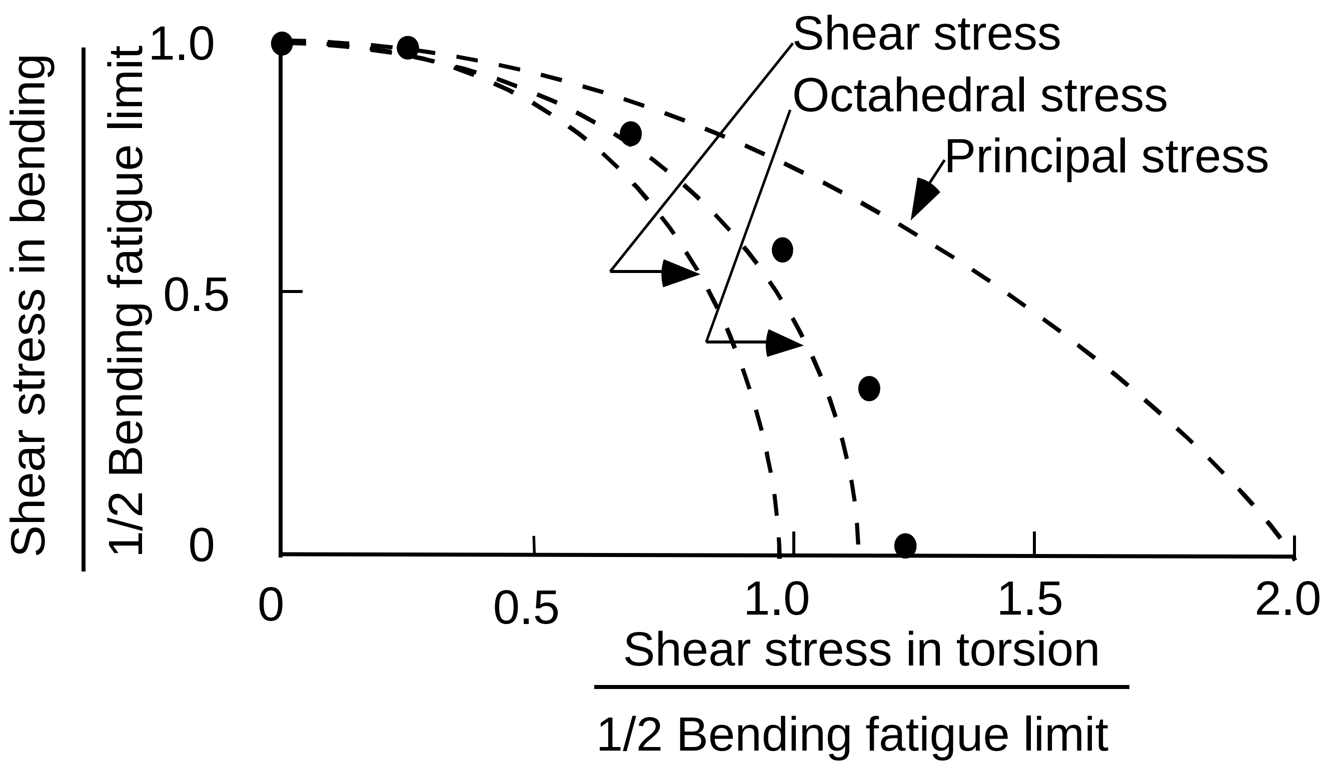
Fatigue Mechanisms Summary

- Fatigue cracks nucleate in shear
- Fatigue cracks grow in either shear or tension depending on material and state of stress

Stress Based Models

- Sines
- Findley
- Dang Van

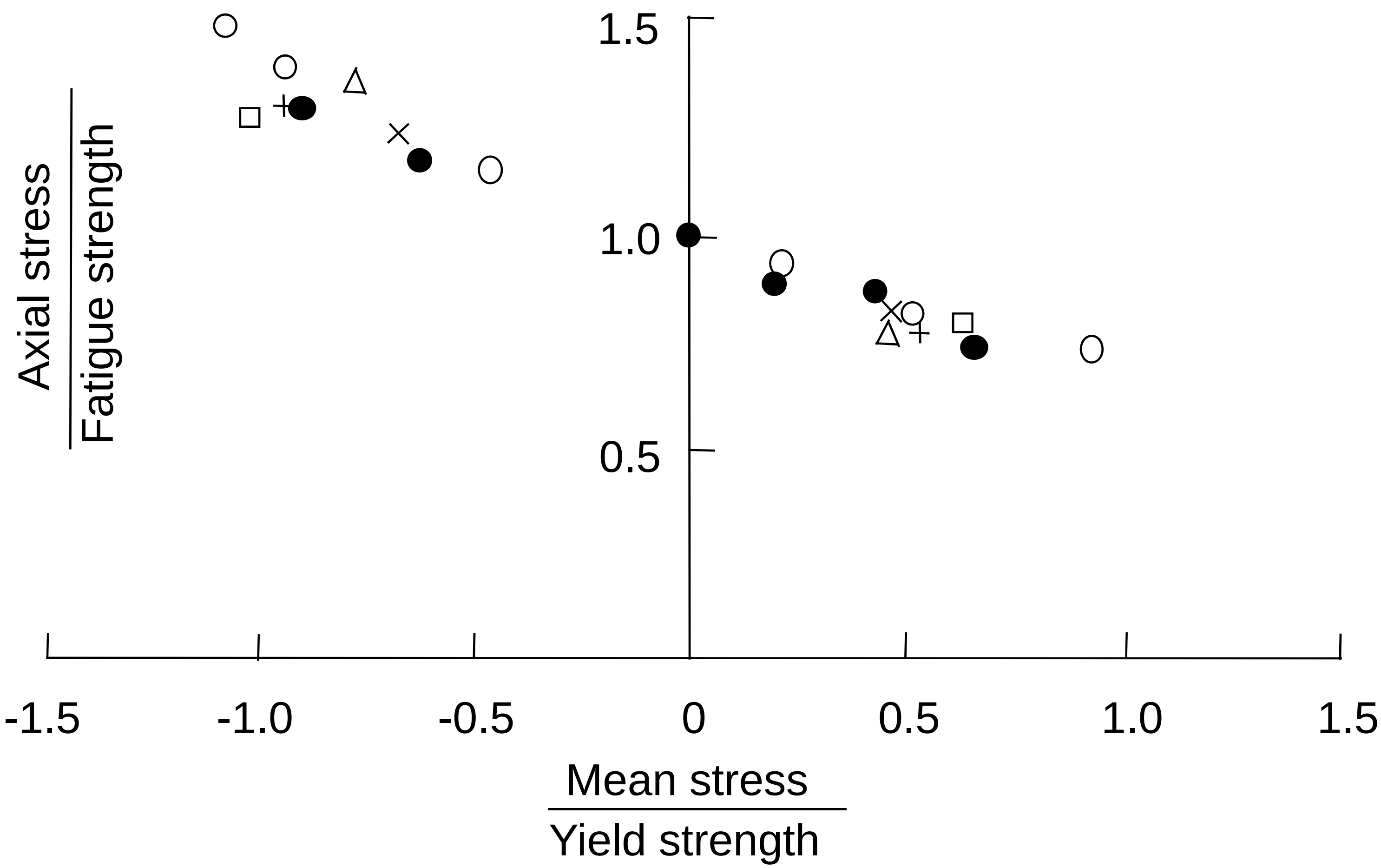
Bending Torsion Correlation



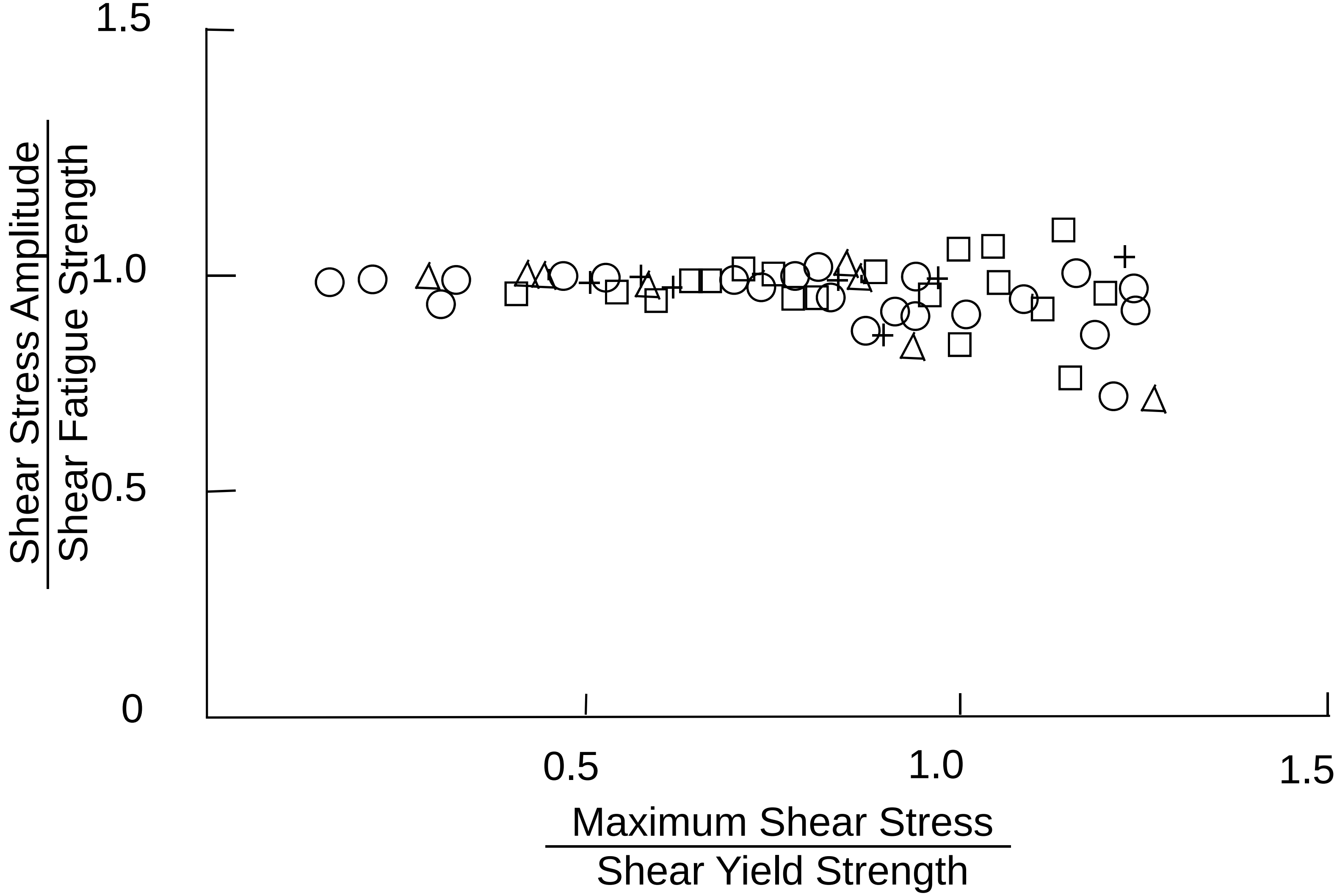
Test Results

- Cyclic tension with static tension
- Cyclic torsion with static torsion
- Cyclic tension with static torsion
- Cyclic torsion with static tension

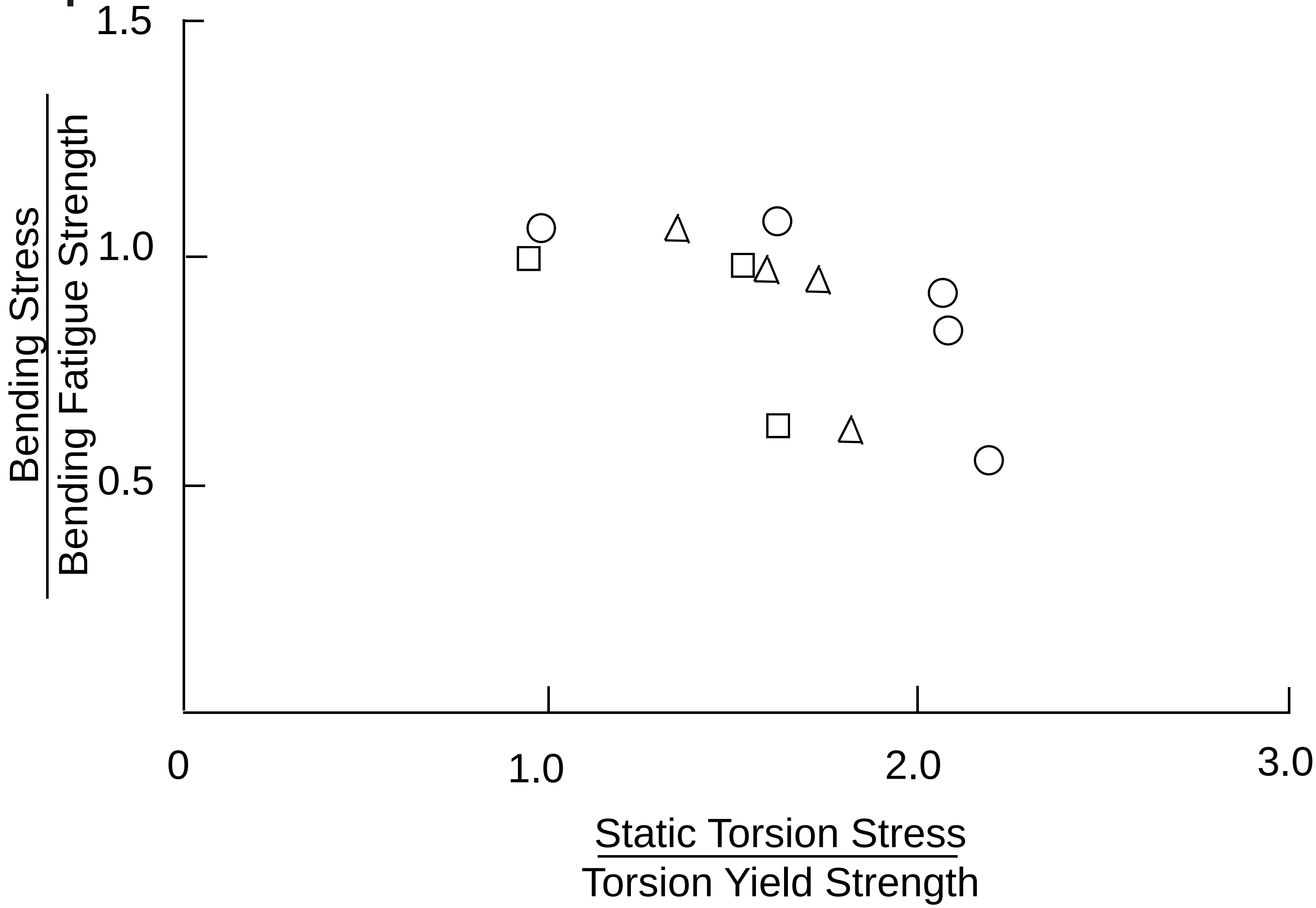
Cyclic Tension with Static Tension



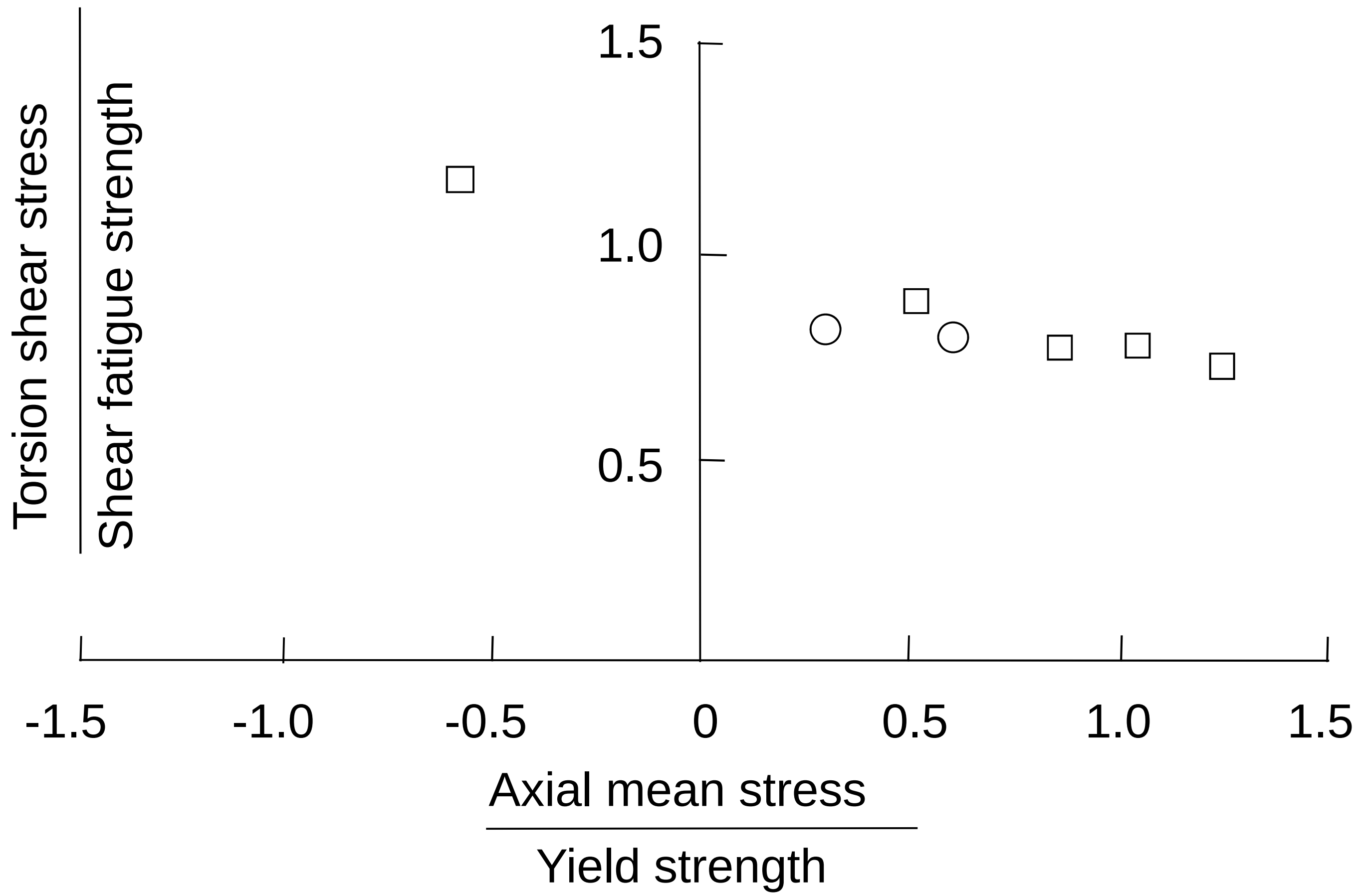
Cyclic Torsion with Static Torsion



Cyclic Tension with Static Torsion



Cyclic Torsion with Static Tension



Conclusions

- Tension mean stress affects both tension and torsion
- Torsion mean stress does not affect tension or torsion

Sines

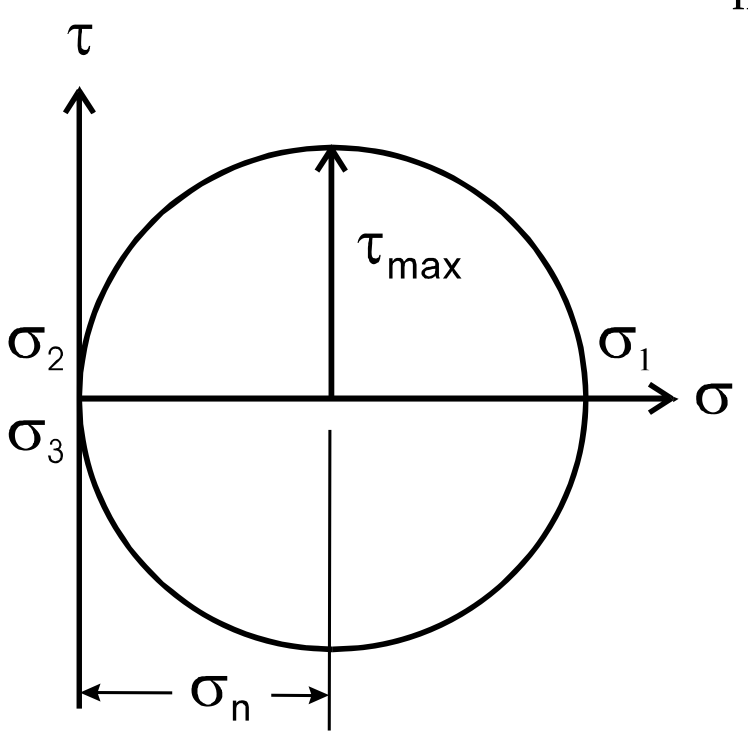
$$\frac{\Delta\tau_{oct}}{2} + \alpha(3\sigma_h) = \beta$$

$$\frac{1}{6}\sqrt{(\Delta\sigma_x - \Delta\sigma_y)^2 + (\Delta\sigma_x - \Delta\sigma_z)^2 + (\Delta\sigma_y - \Delta\sigma_z)^2 + 6(\Delta\tau_{xy}^2 + \Delta\tau_{xz}^2 + \Delta\tau_{yz}^2)} +$$

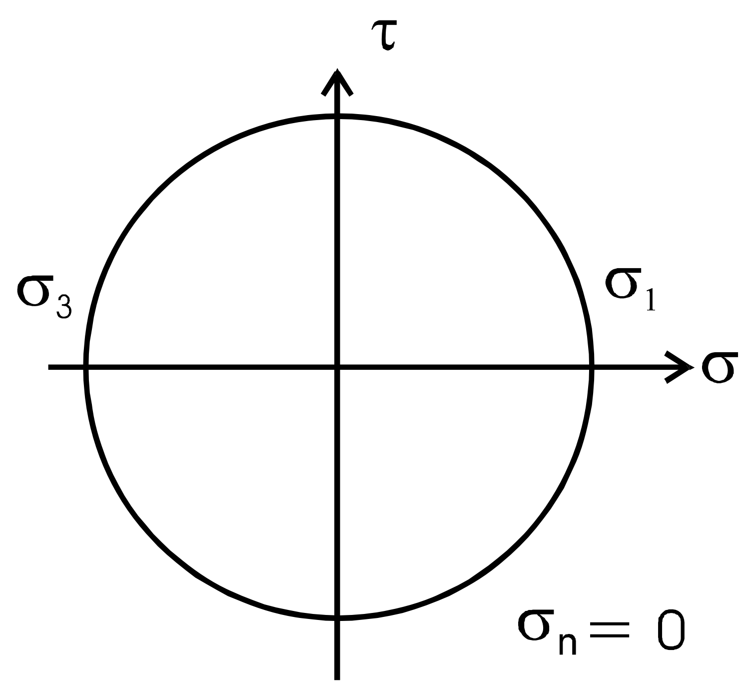
$$\alpha(\sigma_x^{mean} + \sigma_y^{mean} + \sigma_z^{mean}) = \beta$$

Findley

$$\left(\frac{\Delta\tau}{2} + k\sigma_n\right)_{max} = f$$

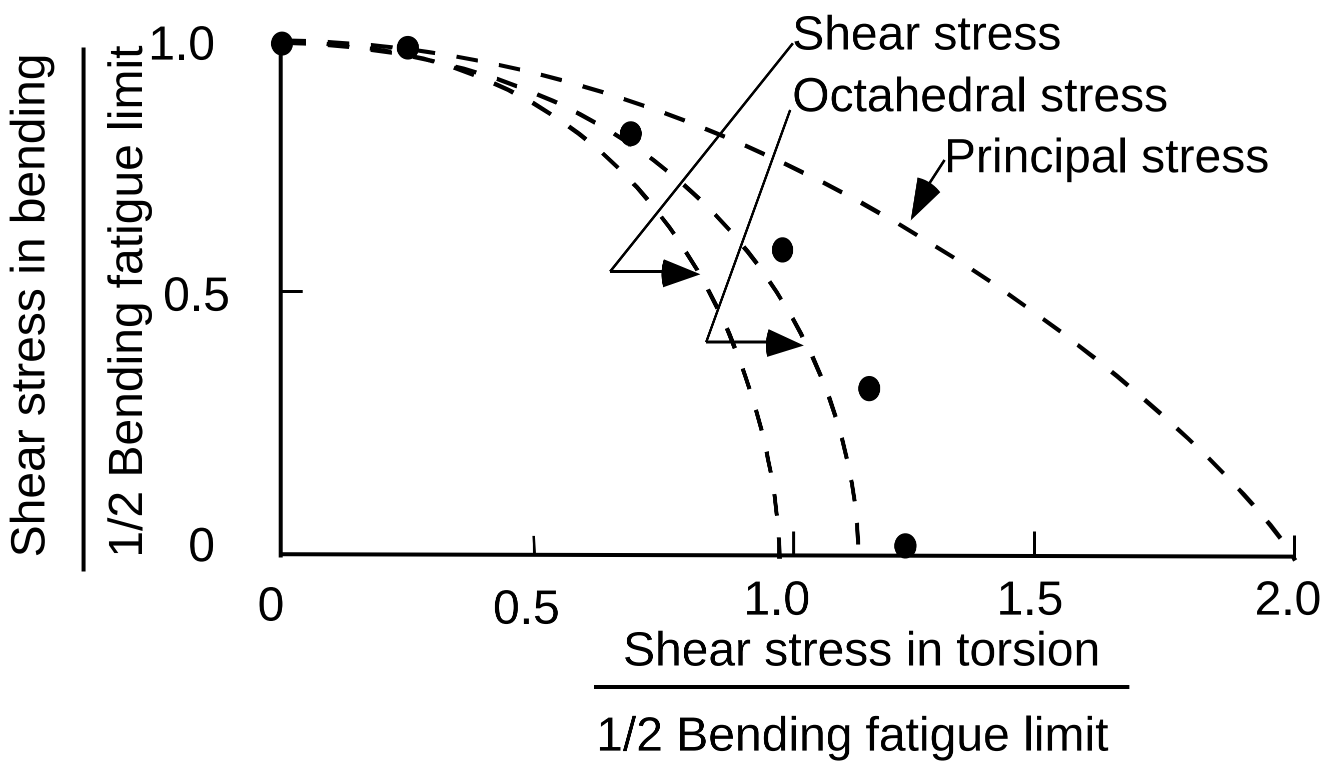


tension



torsion

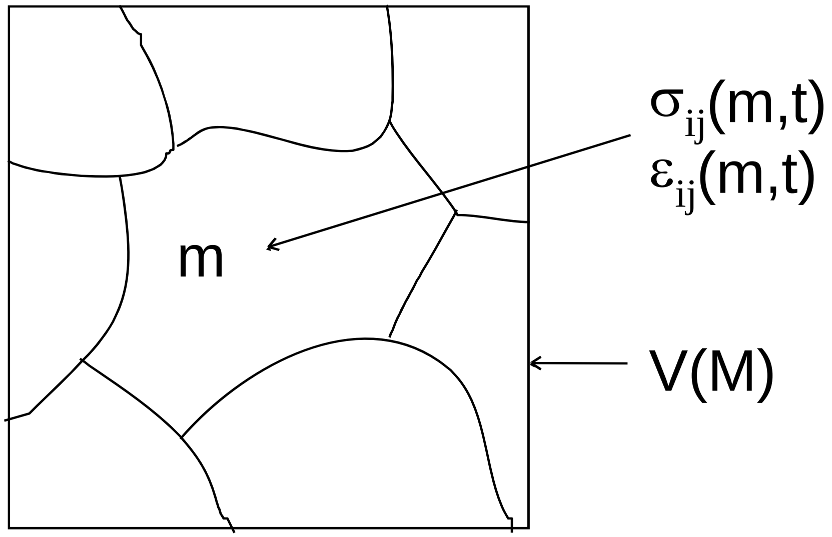
Bending Torsion Correlation



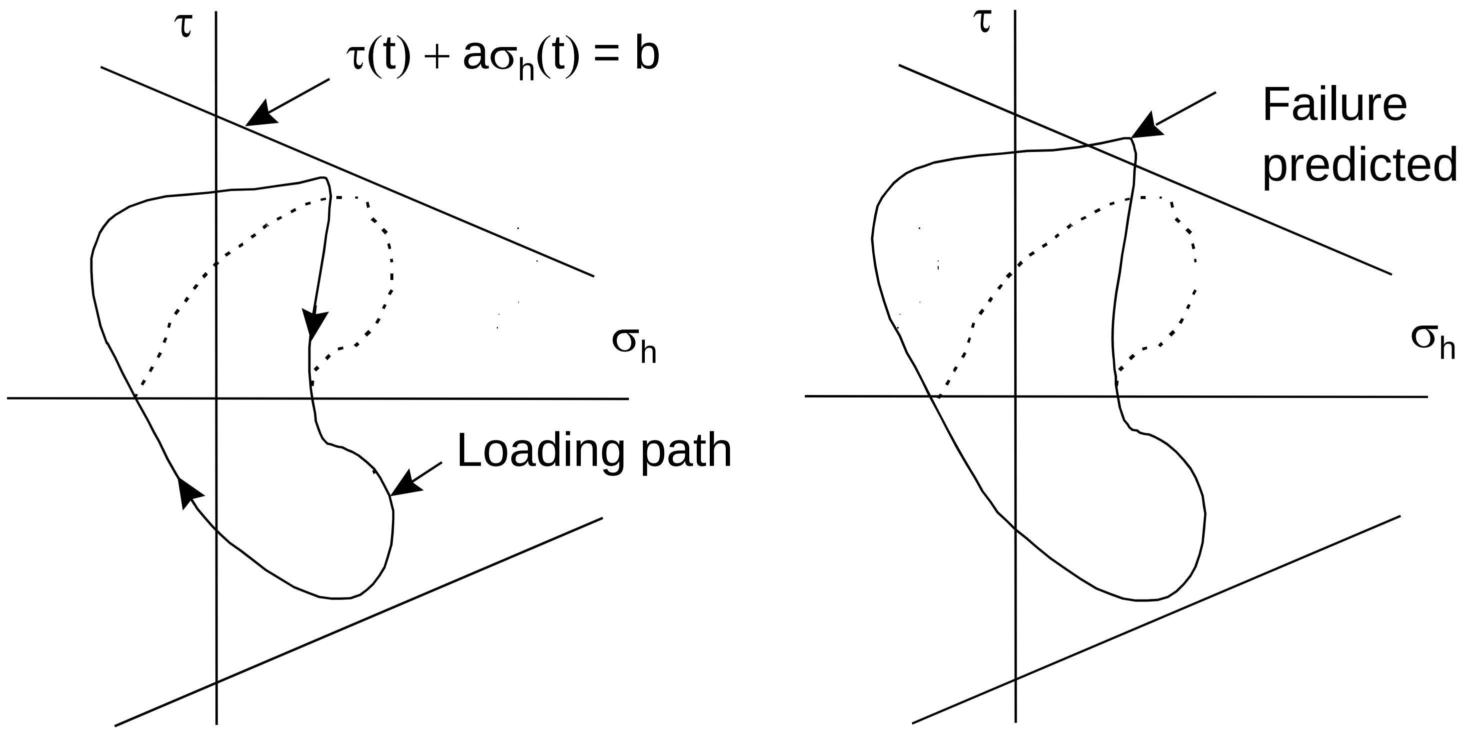
Dang Van

$$\tau(t) + a\sigma_h(t) = b$$

$\Sigma_{ij}(M,t)$ $E_{ij}(M,t)$



Dang Van (continued)



Stress Based Models Summary

Sines: $$\frac{\Delta\tau_{oct}}{2} + \alpha(3\sigma_h) = \beta$$

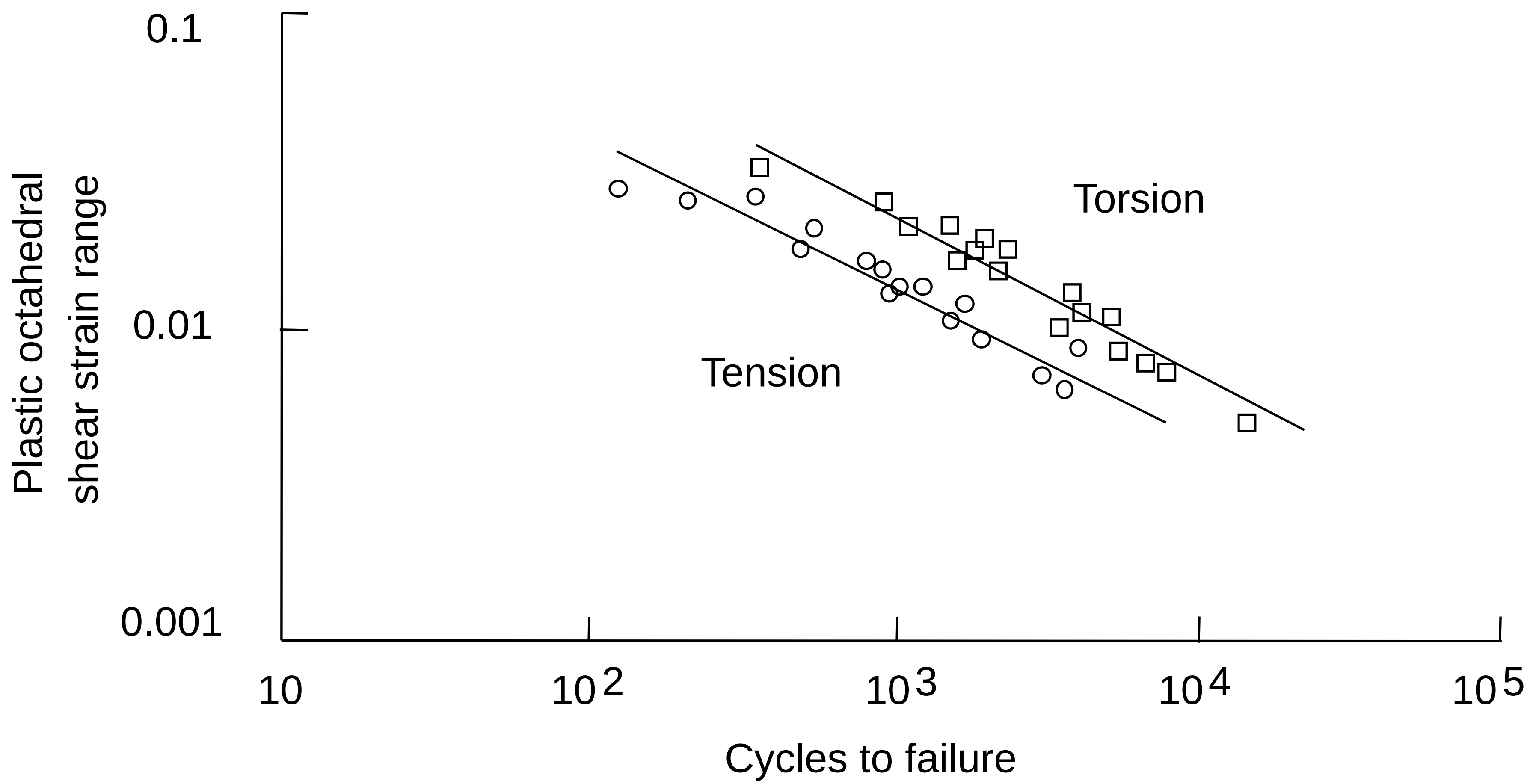
Findley: $$\left(\frac{\Delta\tau}{2} + k\sigma_n\right)_{max} = f$$

Dang Van: $$\tau(t) + a\sigma_h(t) = b$$

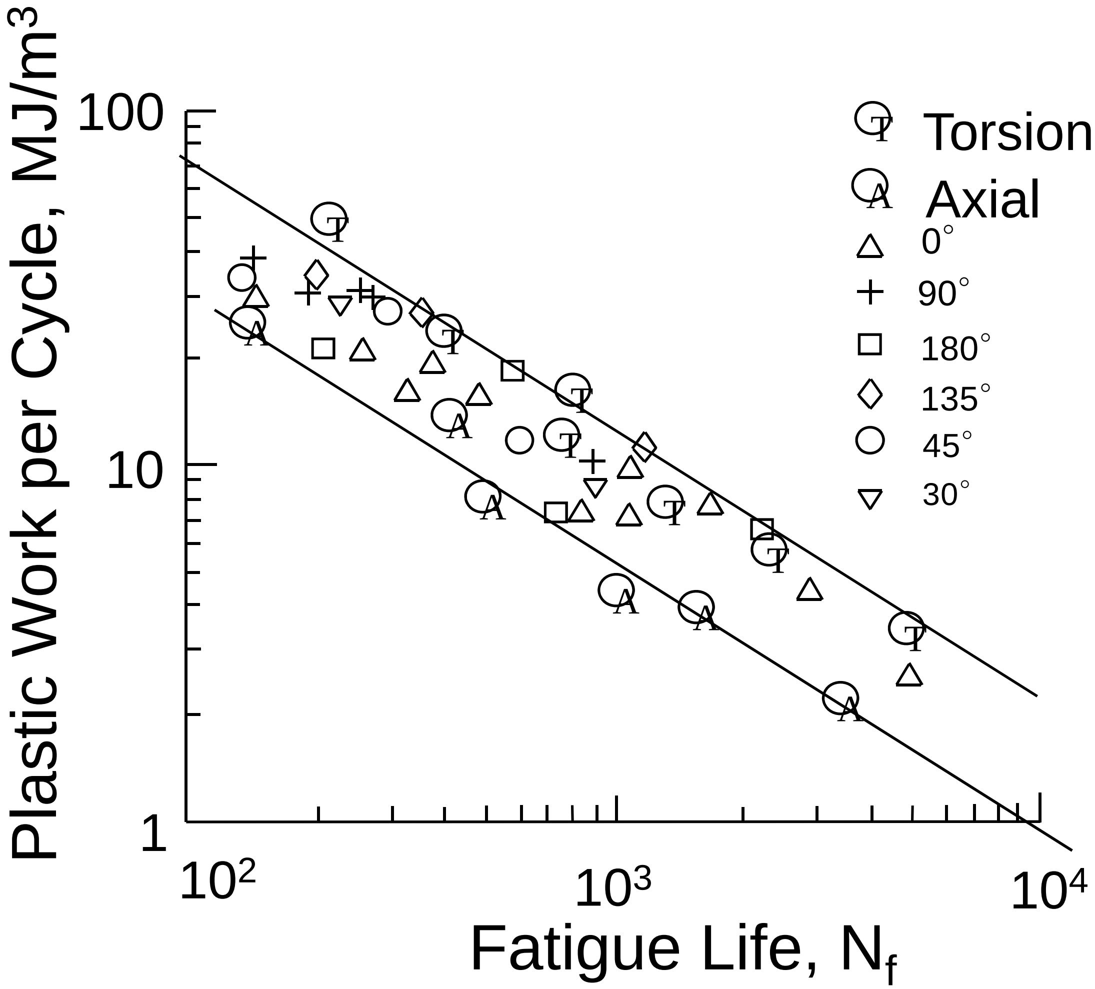
Strain Based Models

- Plastic Work
- Brown and Miller
- Fatemi and Socie
- Smith Watson and Topper
- Liu

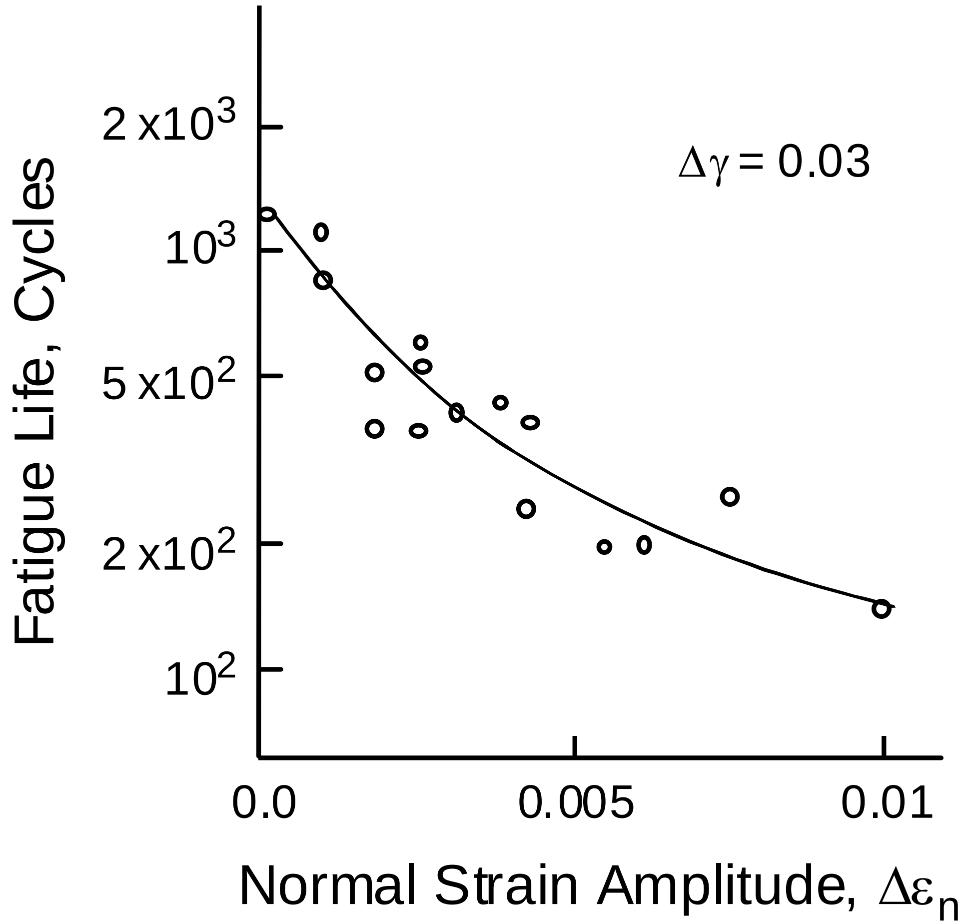
Octahedral Shear Strain



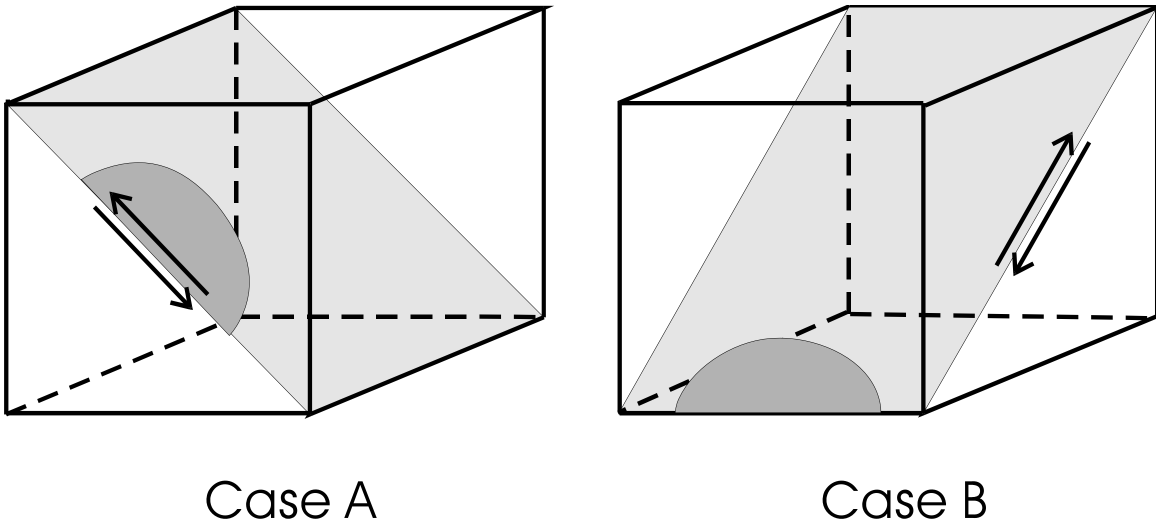
Plastic Work



Brown and Miller

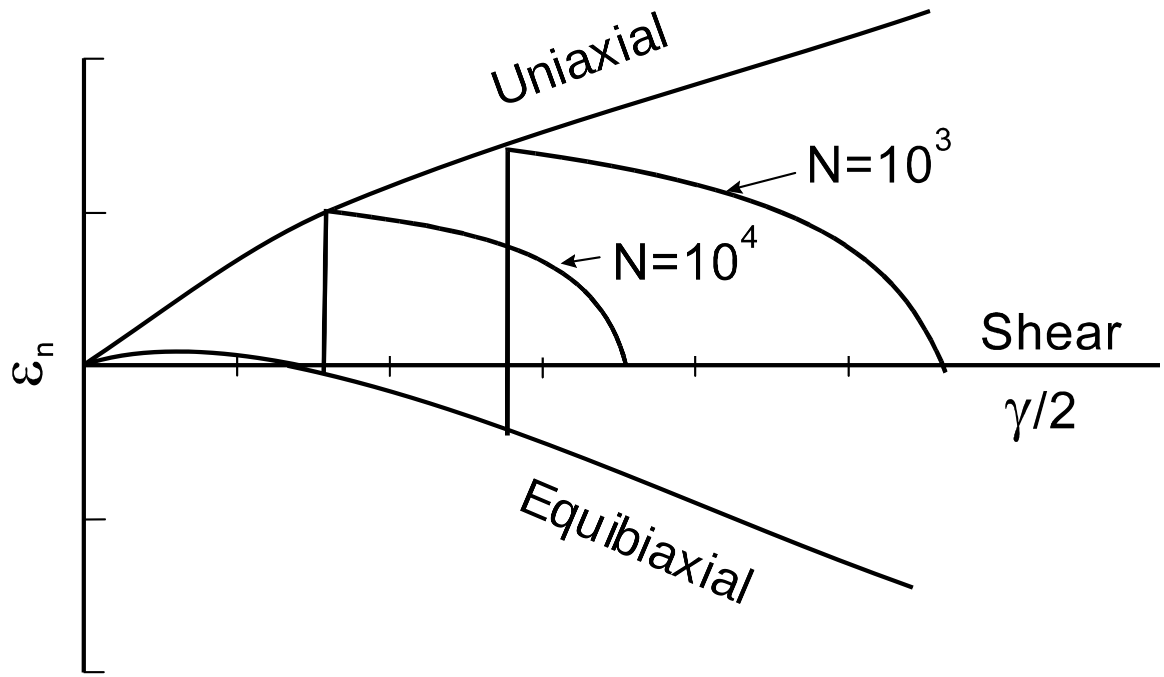


Case A and B



Growth along the surface | Growth into the surface

Brown and Miller (continued)

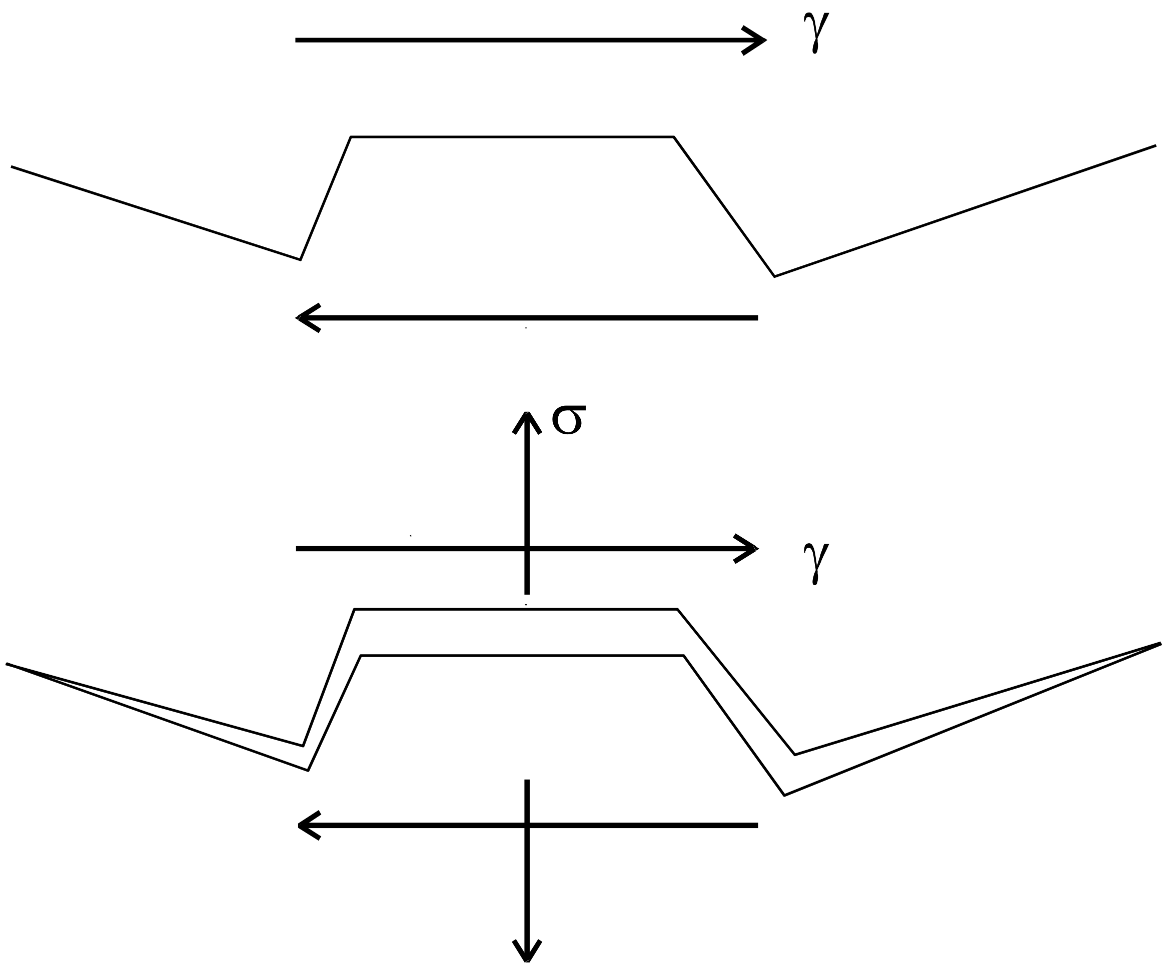


Brown and Miller (continued)

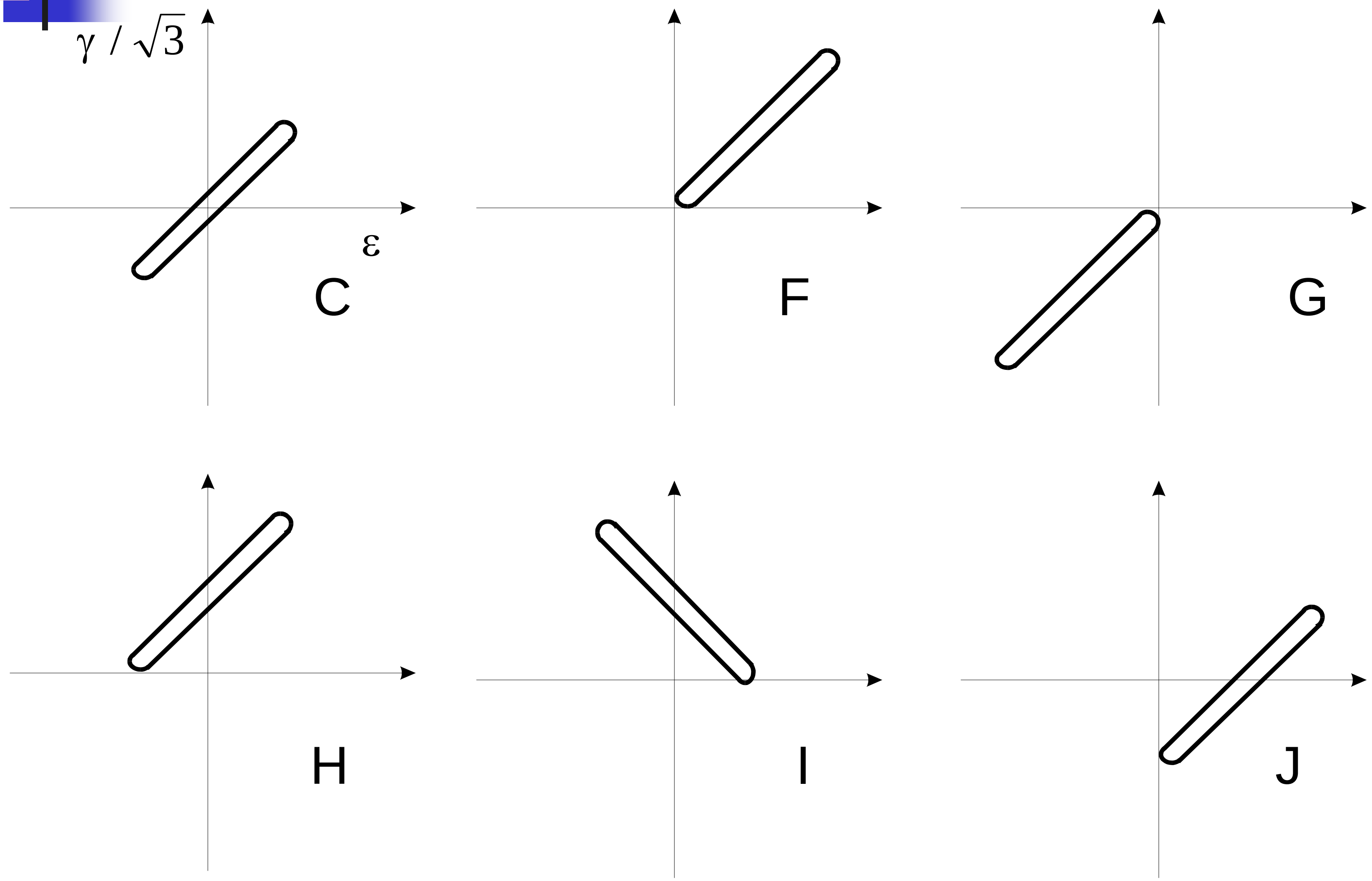
$$\Delta\hat{\gamma} = \left(\Delta\gamma_{max}^{\alpha} + S\Delta\varepsilon_{n}^{\alpha}\right)^{\frac{1}{\alpha}}$$

$$\frac{\Delta\gamma_{max}}{2} + S\,\Delta\varepsilon_{n} = A\,\frac{\sigma_{f}^{'} - 2\sigma_{n,mean}}{E}\left(2N_{f}\right)^{b} + B\,\varepsilon_{f}^{'}\left(2N_{f}\right)^{c}$$

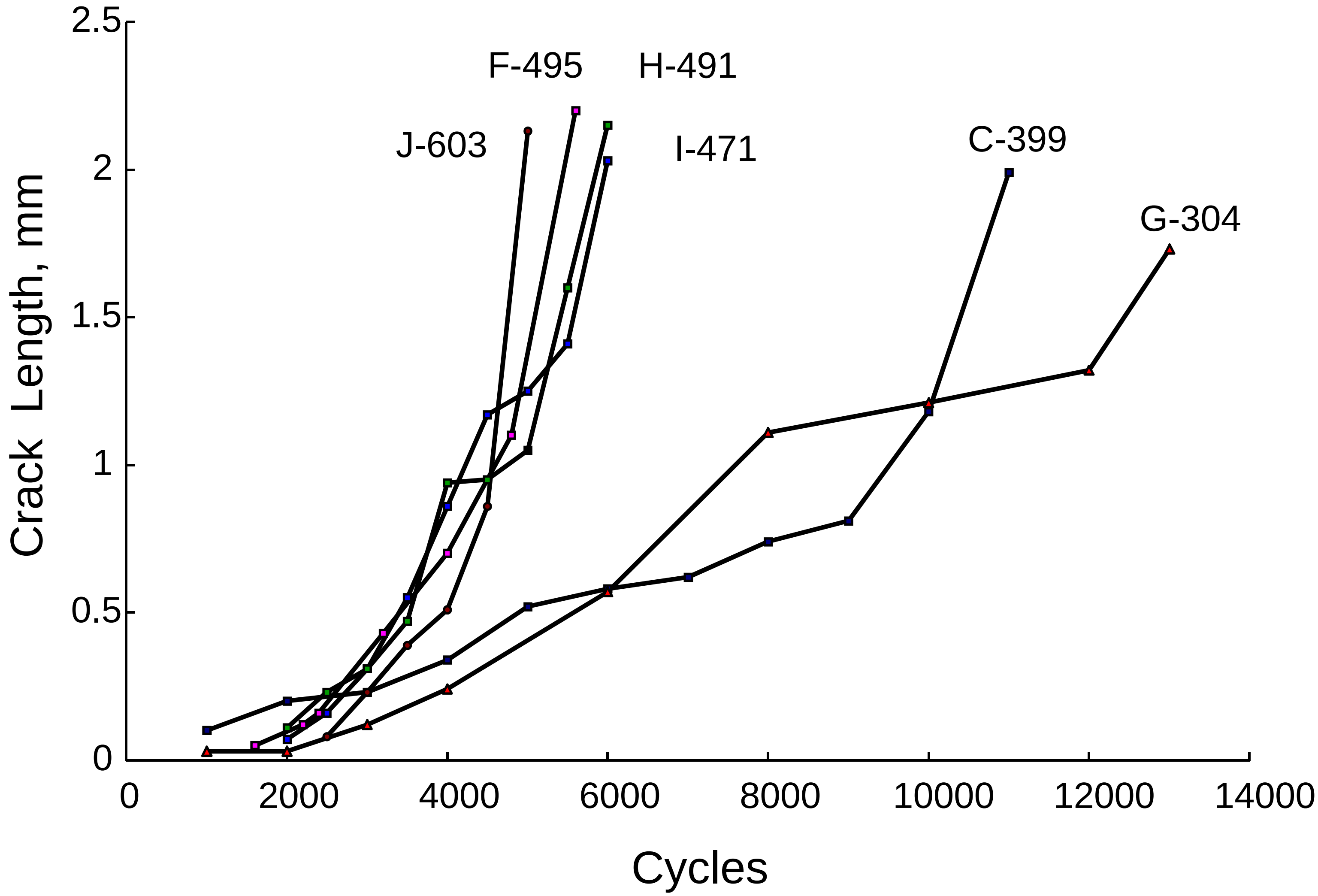
Fatemi and Socie



Loading Histories



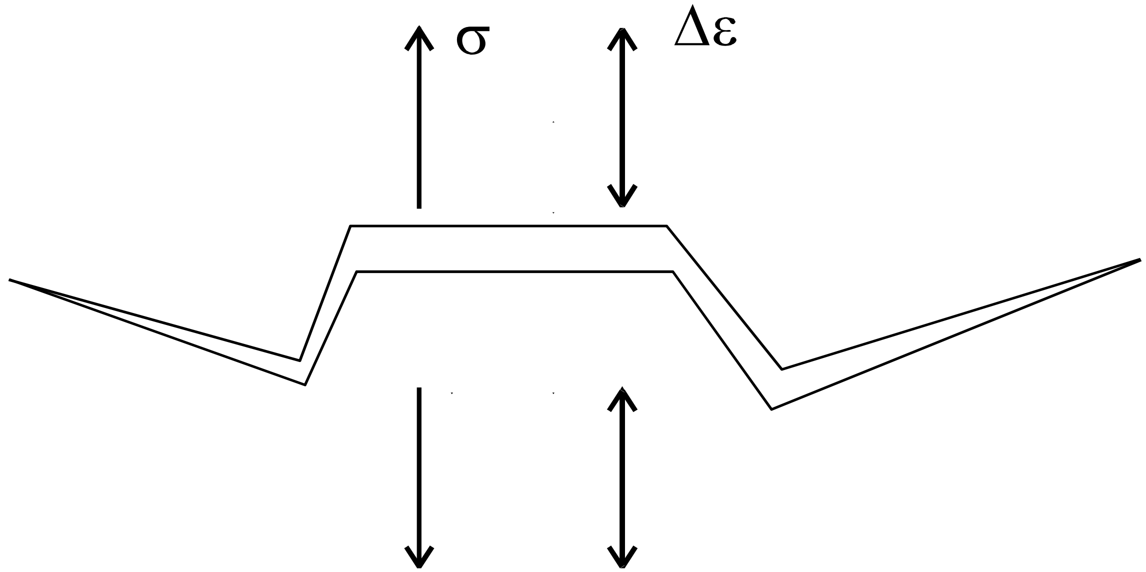
Crack Length Observations



Fatemi and Socie

$$\frac{\Delta\gamma}{2}\left(1+k\frac{\sigma_{n,max}}{\sigma_y}\right)=\frac{\tau_f^{'}}{G}(2N_f)^{bo}+\gamma_f^{'}(2N_f)^{co}$$

Smith Watson Topper



SWT

$$\sigma_n \frac{\Delta\varepsilon_1}{2} = \frac{\sigma_f'^{\,2}}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}$$

Liu

Virtual strain energy for both mode I and mode II cracking

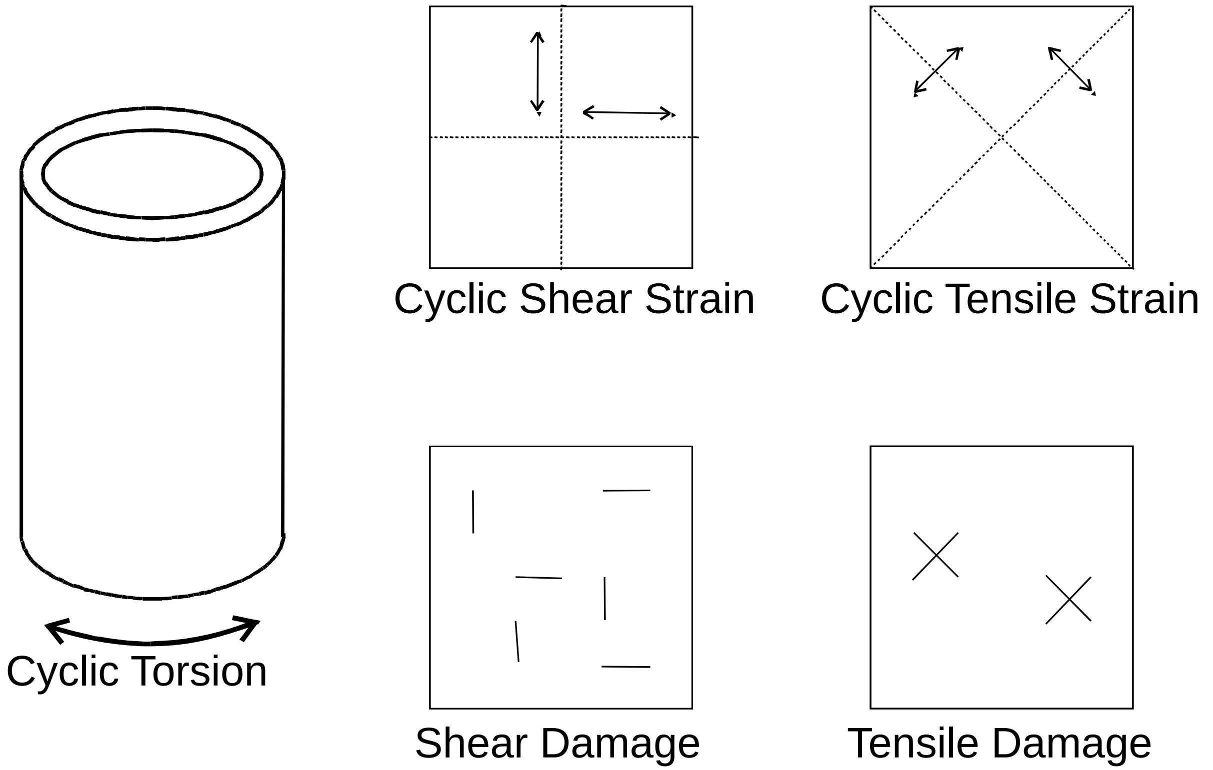
$$\Delta W_I = (\Delta\sigma_n \, \Delta\varepsilon_n)_{max} + (\Delta\tau \, \Delta\gamma)$$

$$\Delta W_I = 4\sigma'_f \varepsilon'_f (2N_f)^{b+c} + \frac{4\sigma'^{2}_f}{E}(2N_f)^{2b}$$

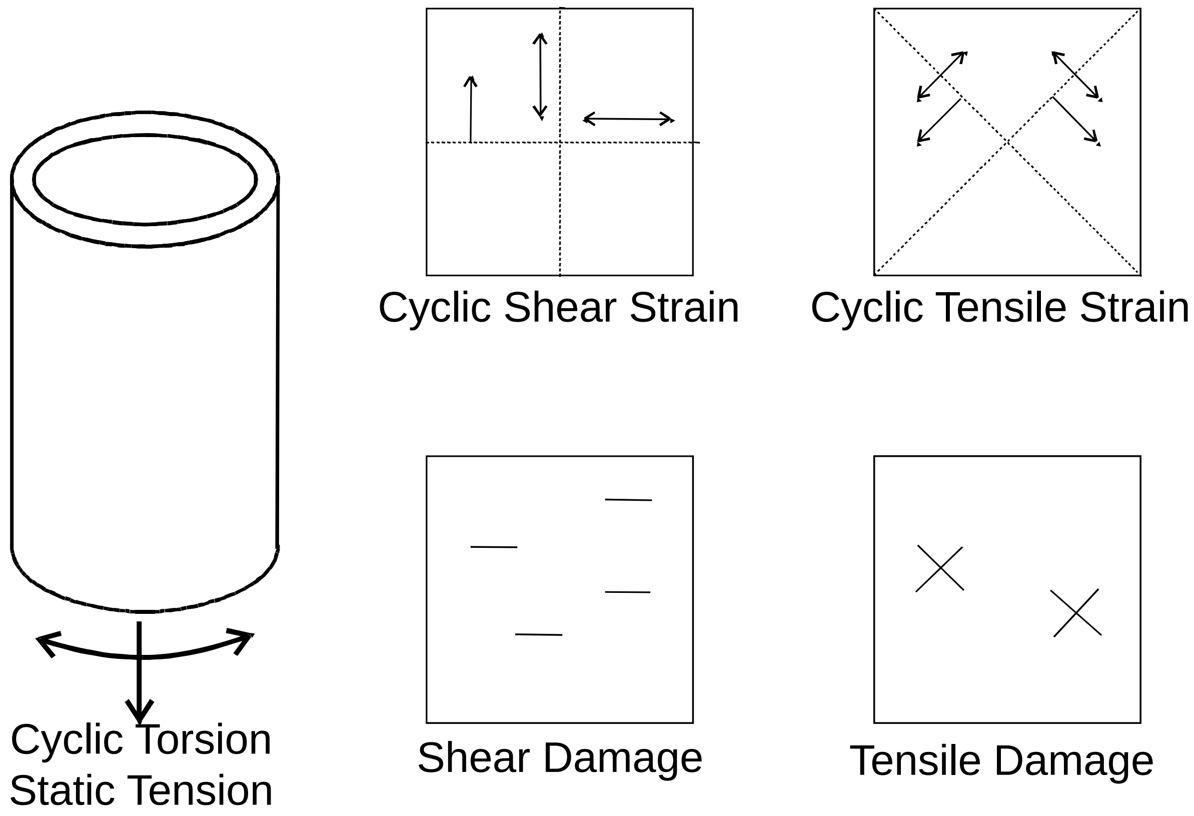
$$\Delta W_{II} = (\Delta\sigma_n \, \Delta\varepsilon_n) + (\Delta\tau \, \Delta\gamma)_{max}$$

$$\Delta W_{II} = 4\tau'_f \gamma'_f (2N_f)^{bo+co} + \frac{4\tau'^{2}_f}{G}(2N_f)^{2bo}$$

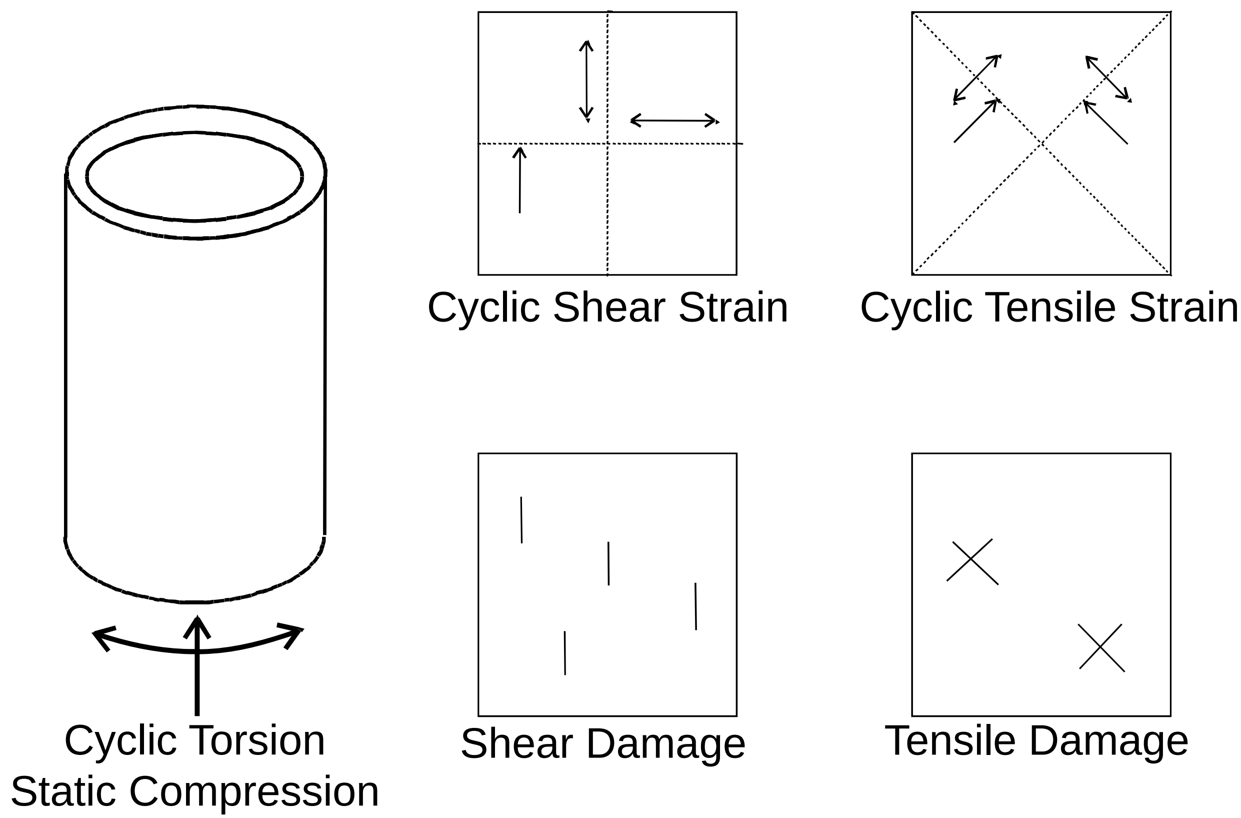
Cyclic Torsion



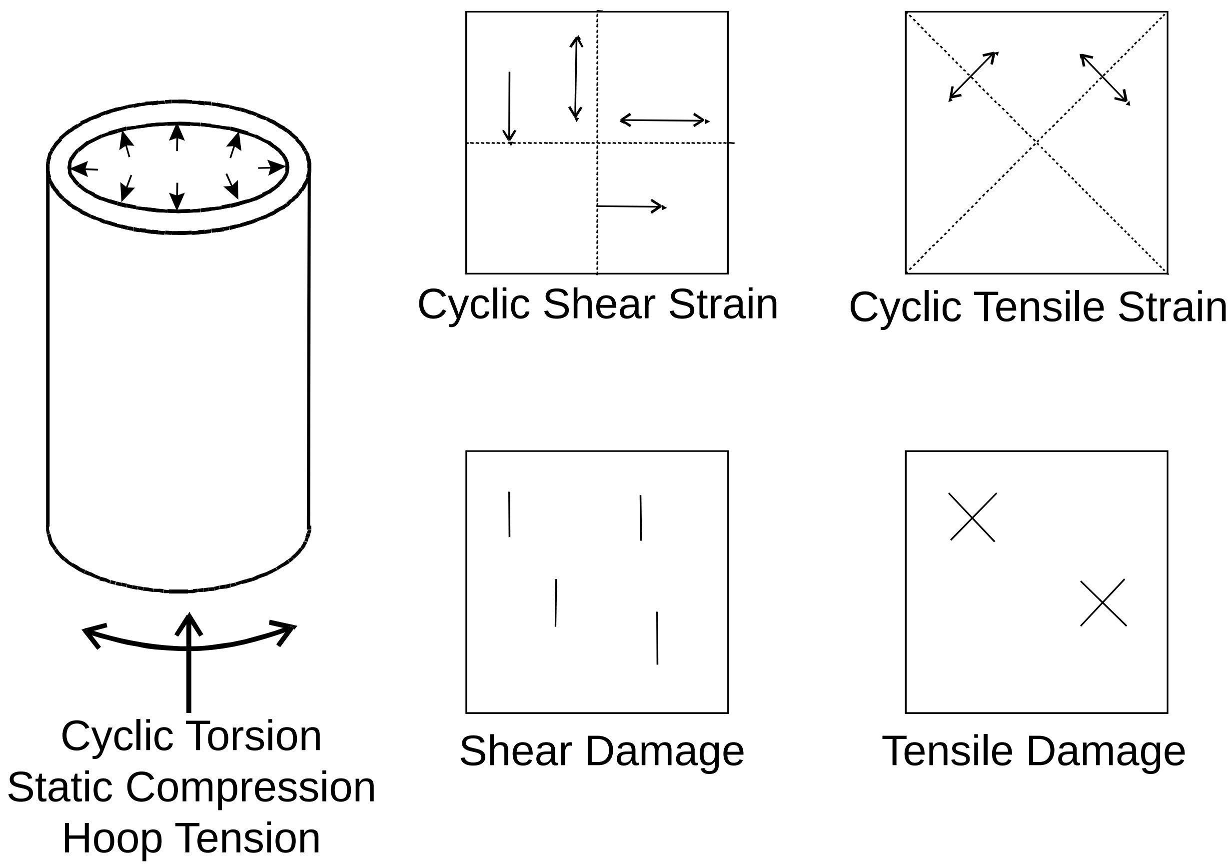
Cyclic Torsion with Static Tension



Cyclic Torsion with Compression



Cyclic Torsion with Tension and Compression



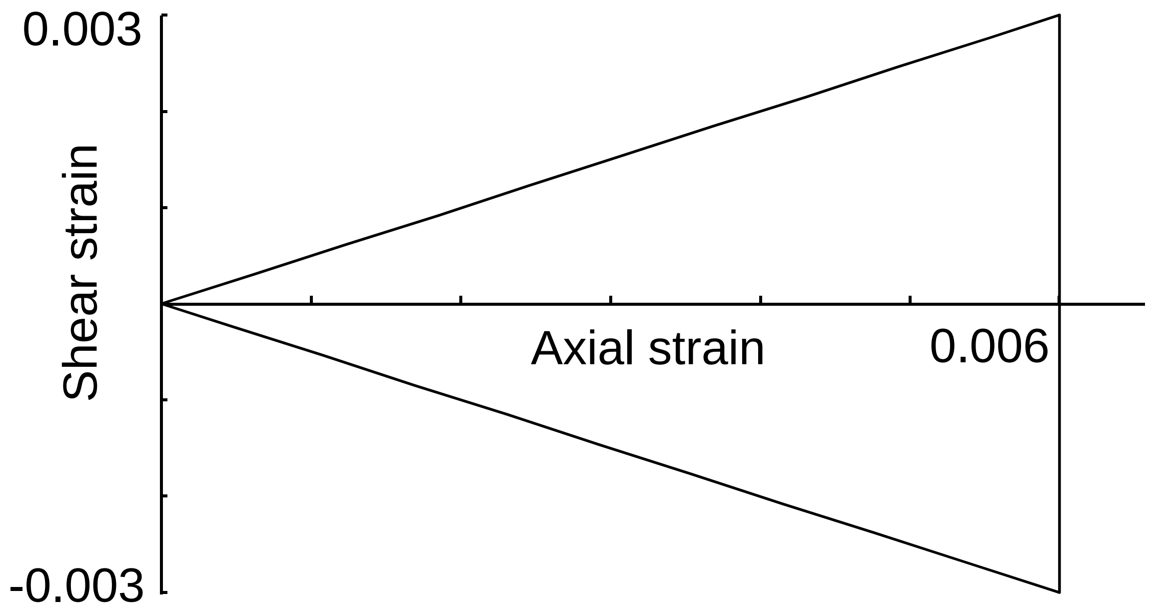
Test Results

Load Case	$\Delta\gamma/2$	σ_{hoop} MPa	σ_{axial} MPa	N_f
Torsion	0.0054	0	0	45,200
with tension	0.0054	0	450	10,300
with compression	0.0054	0	-500	50,000
with tension and compression	0.0054	450	-500	11,200

Conclusions

- All critical plane models correctly predict these results
- Hydrostatic stress models can not predict these results

Loading History



Model Comparison

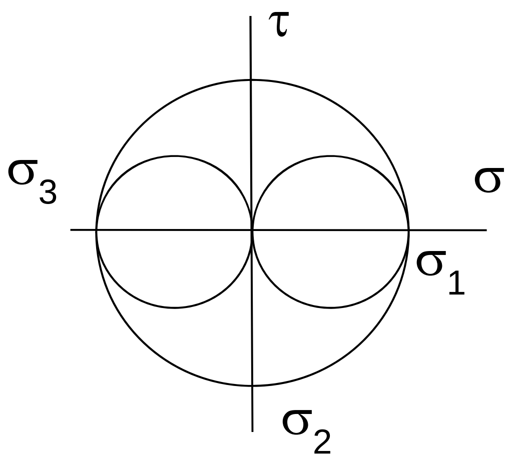
Summary of calculated fatigue lives

Model	Equation	Life
Epsilon	6.5	14,060
Garud	6.7	5,210
Ellyin	6.17	4,450
Brown-Miller	6.22	3,980
SWT	6.24	9,930
Liu I	6.41	4,280
Liu II	6.42	5,420
Chu	6.37	3,040
Gamma		26,775
Fatemi-Socie	6.23	10,350
Glinka	6.39	33,220

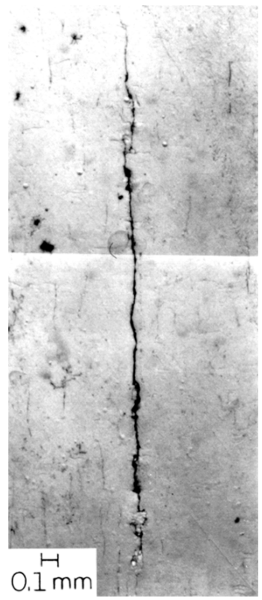
Strain Based Models Summary

- Two separate models are needed, one for tensile growth and one for shear growth
- Cyclic plasticity governs stress and strain ranges
- Mean stress effects are a result of crack closure on the critical plane

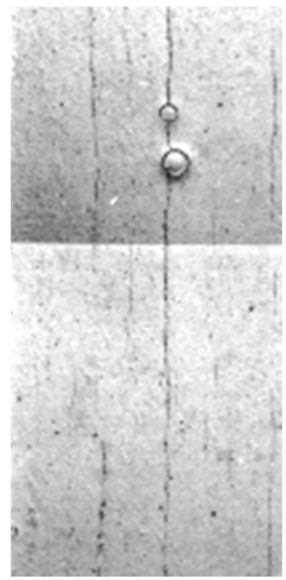
Separate Tensile and Shear Models



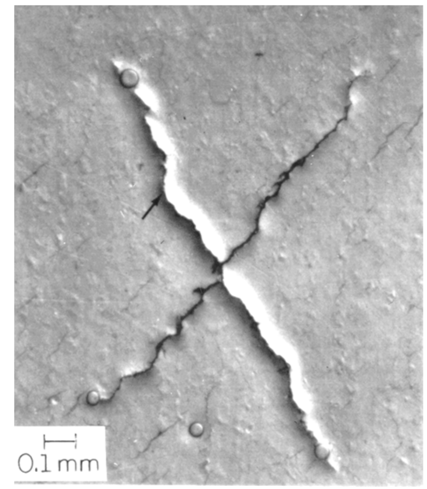
$\sigma_1 = \tau_{xy}$



Inconel



1045 steel



stainless steel

Cyclic Plasticity

$\Delta\varepsilon$

$\Delta\gamma$

$\Delta\varepsilon^{p}$

$\Delta\gamma^{p}$

$\Delta\varepsilon\Delta\sigma$

$\Delta\gamma\Delta\tau$

$\Delta\varepsilon^{p}\Delta\sigma$

$\Delta\gamma^{p}\Delta\tau$

Mean Stresses

$$\Delta\varepsilon_{eq} = \frac{\sigma'_f - \sigma_{mean}}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c$$

$$\frac{\Delta\gamma_{max}}{2} + S\Delta\varepsilon_n = (1.3 + 0.7S)\frac{\sigma'_f - 2\sigma_n}{E}(2N_f)^b + (1.5 + 0.5S)\,\varepsilon'_f(2N_f)^c$$

$$\frac{\Delta\gamma}{2}\left(1 + k\frac{\sigma_{n,max}}{\sigma_y}\right) = \frac{\tau'_f}{G}(2N_f)^{bo} + \gamma'_f(2N_f)^{co}$$

$$\sigma_n\frac{\Delta\varepsilon_1}{2} = \frac{\sigma'^{\,2}_f}{E}(2N_f)^{2b} + \sigma'_f\varepsilon'_f(2N_f)^{b+c}$$

$$\Delta W_I = \left[(\Delta\sigma_n\Delta\varepsilon_n)_{max} + (\Delta\tau\Delta\gamma)\right]\left(\frac{2}{1-R}\right)$$

Fracture Mechanics Models

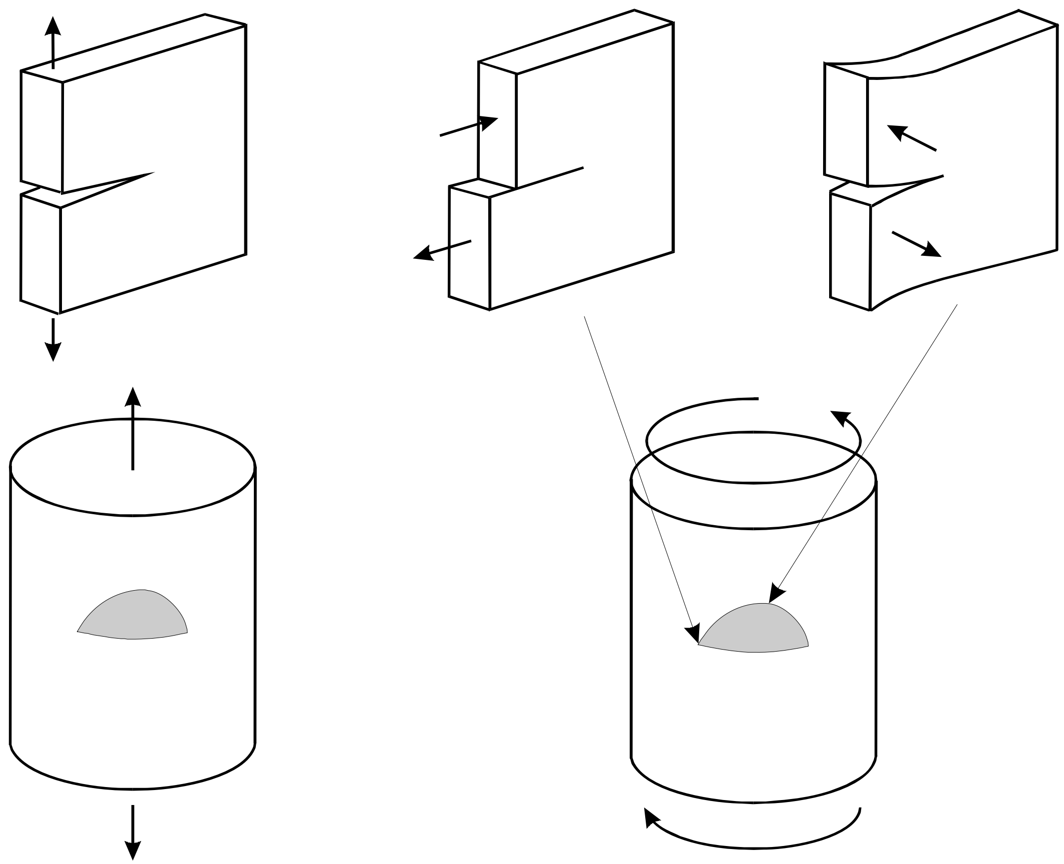
- Mode I growth
- Torsion
- Mode II growth
- Mode III growth

Mode I, Mode II, and Mode III

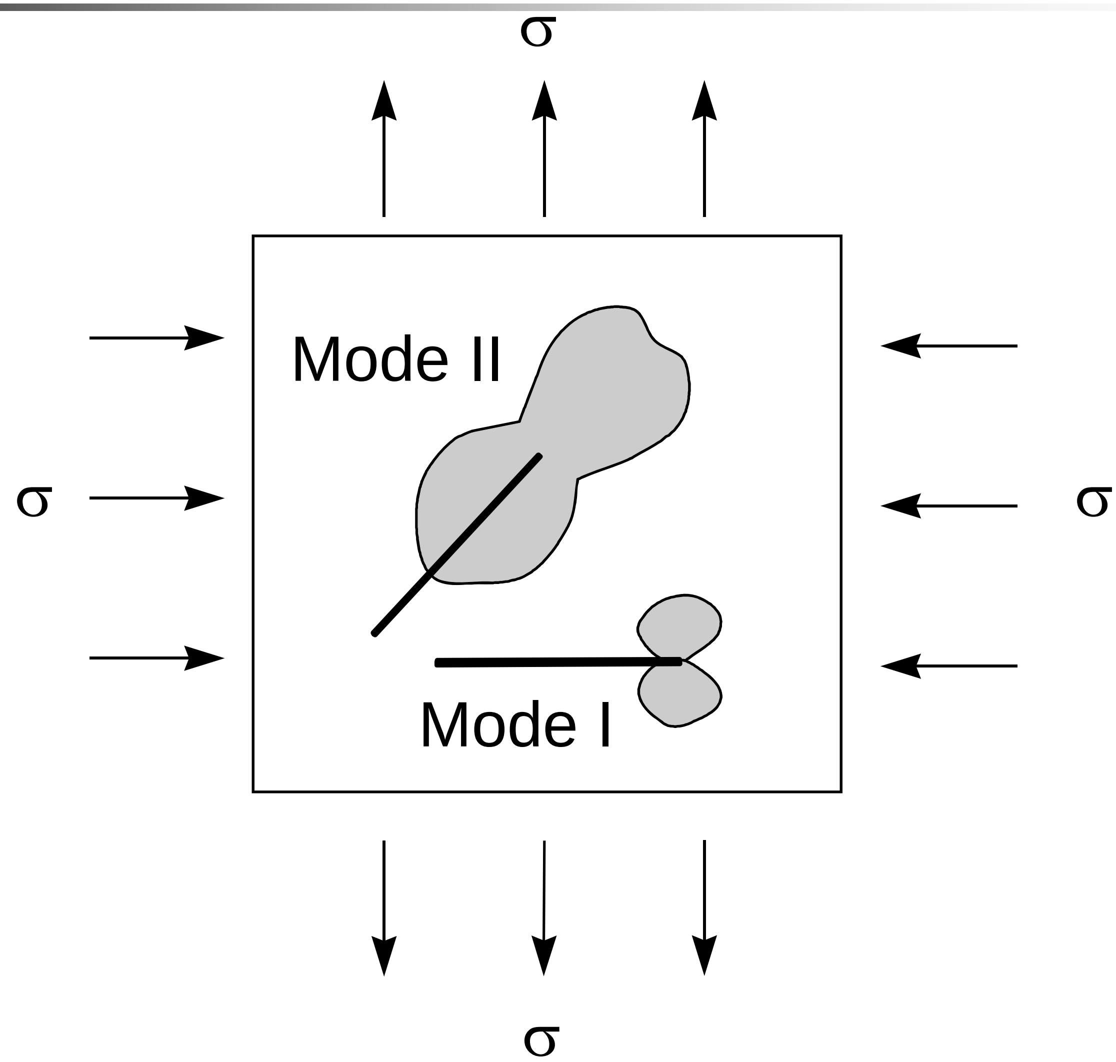
Mode I
opening

Mode II
in-plane shear

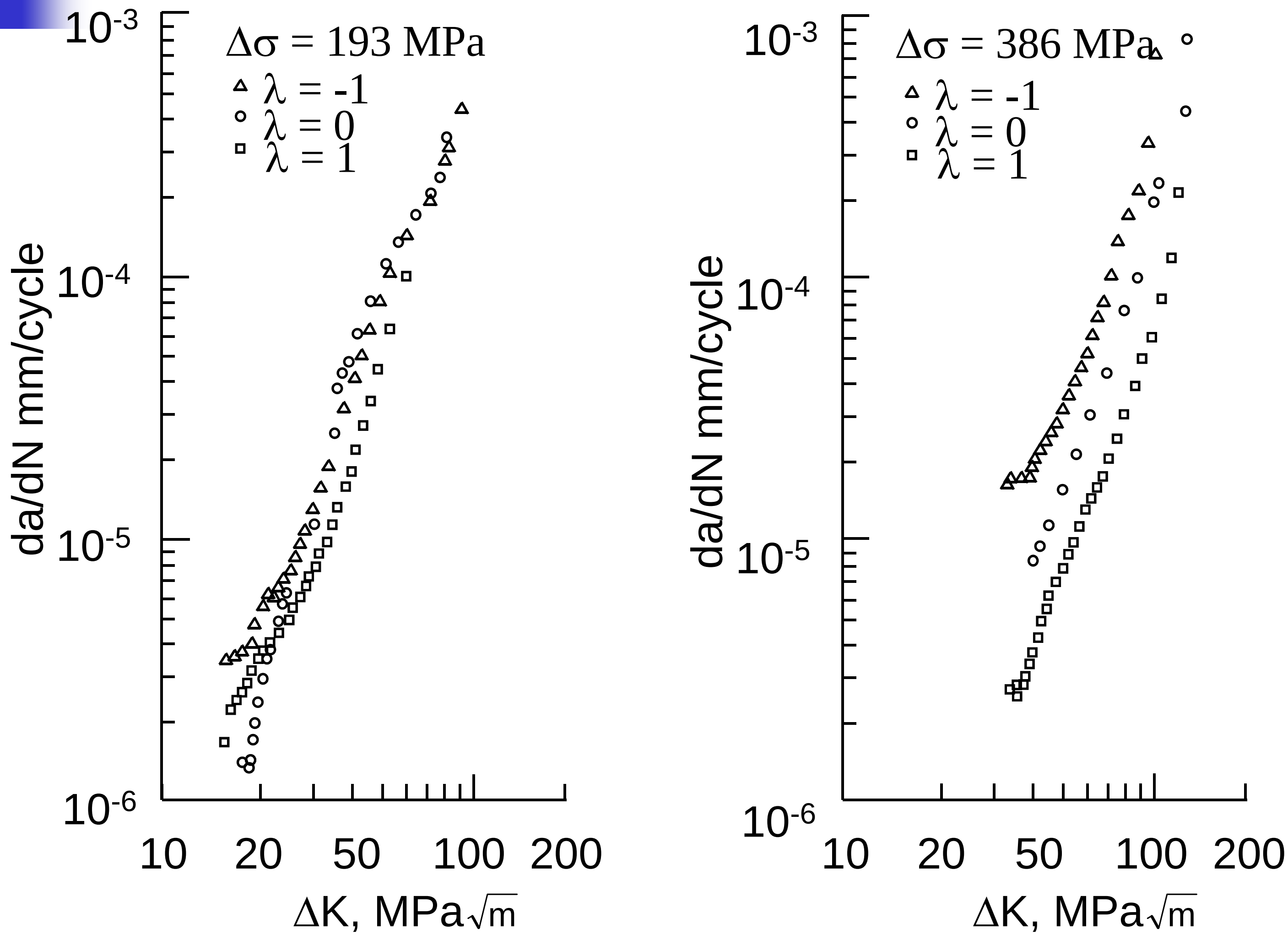
Mode III
out-of-plane shear



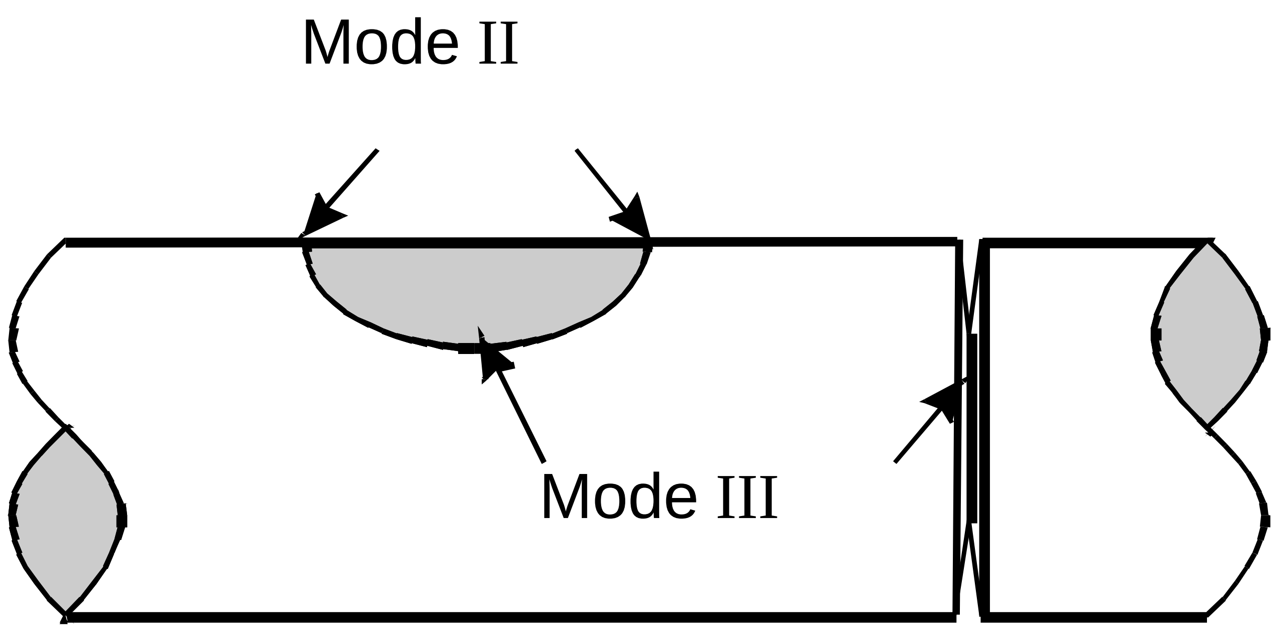
Mode I and Mode II Surface Cracks



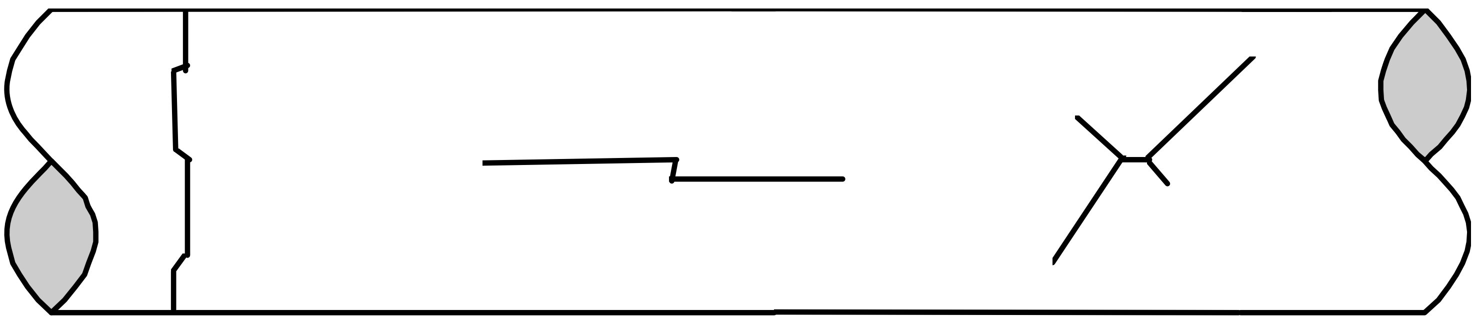
Biaxial Mode I Growth



Surface Cracks in Torsion

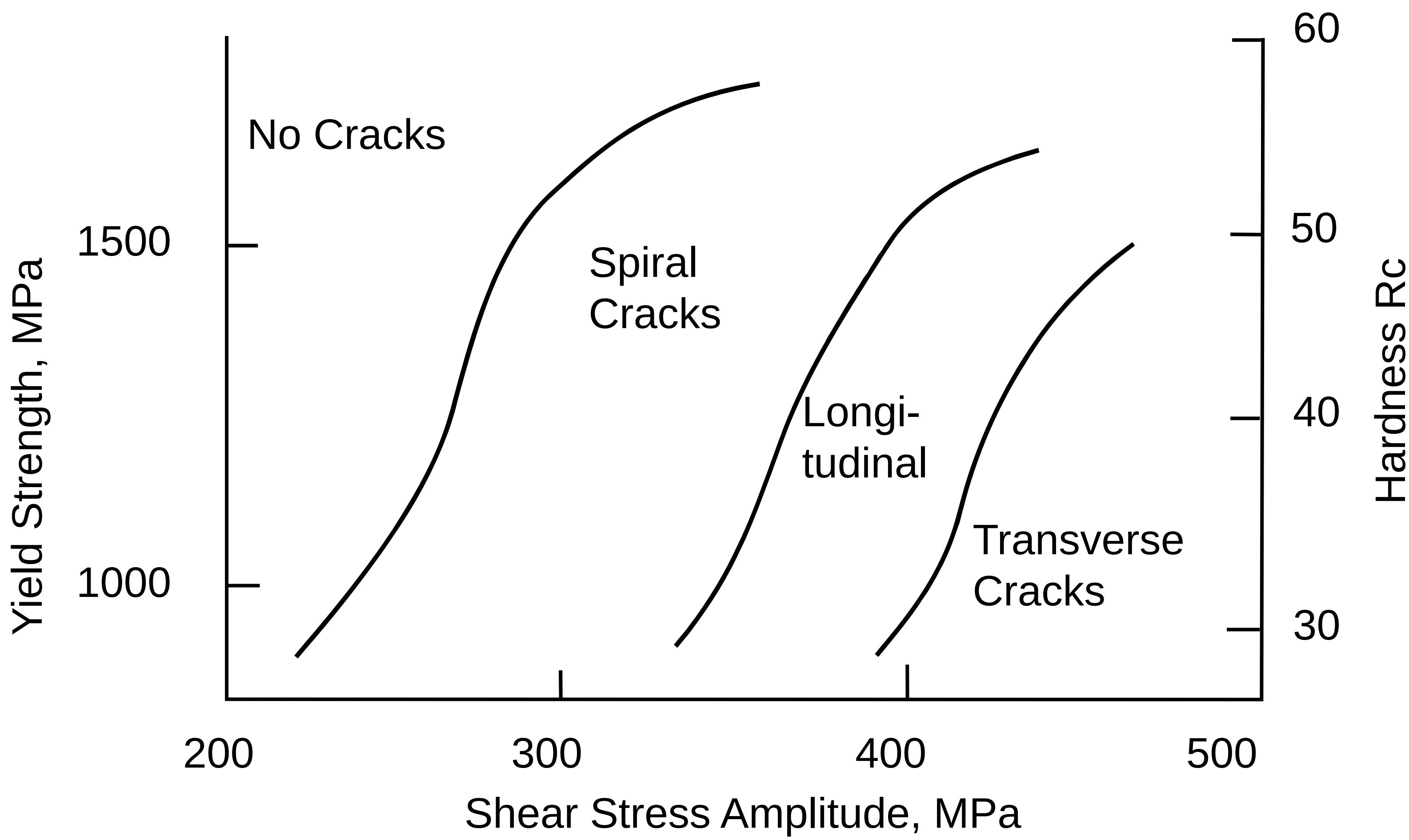


Failure Modes in Torsion

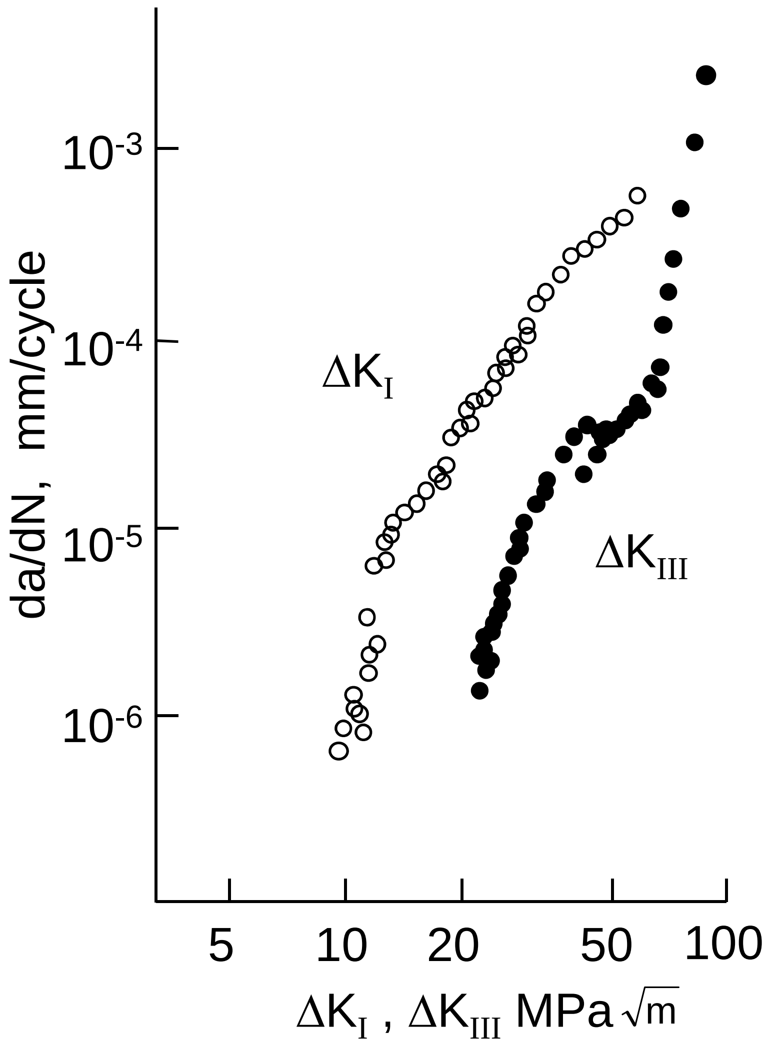


Transverse Longitudinal Spiral

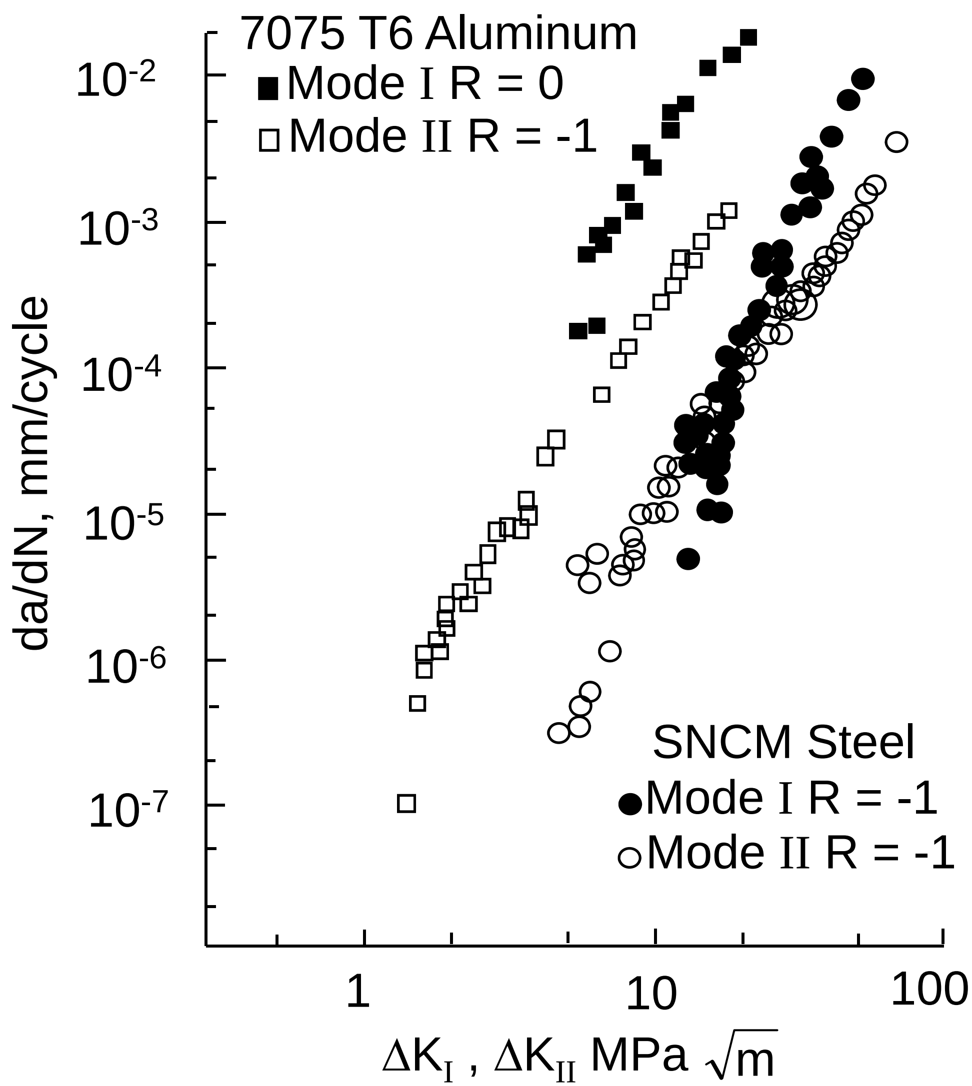
Fracture Mechanism Map



Mode I and Mode III Growth



Mode I and Mode II Growth



Fracture Mechanics Models

$$\frac{da}{dN} = C\left(\Delta K_{eq}\right)^m$$

$$\Delta K_{eq} = \left[\Delta K_I^4 + 8\Delta K_{II}^4 + 8\Delta K_{III}^4/(1-\nu)\right]^{0.25}$$

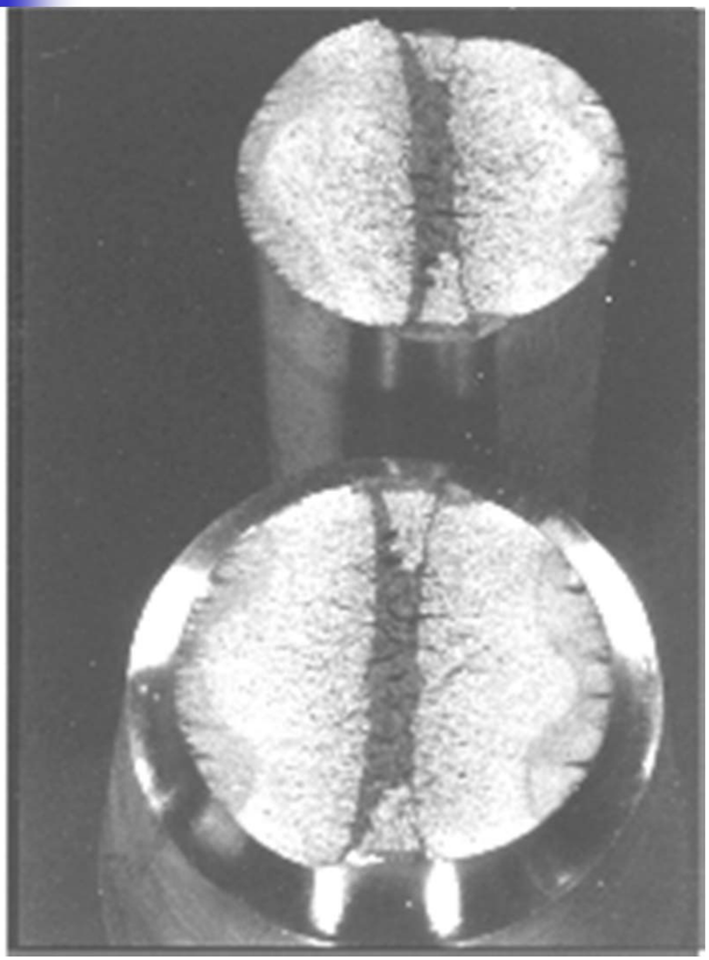
$$\Delta K_{eq} = \left[\Delta K_I^2 + \Delta K_{II}^2 + (1+\nu)\Delta K_{III}^2\right]^{0.5}$$

$$\Delta K_{eq} = \left[\Delta K_I^2 + \Delta K_I\,\Delta K_{II} + \Delta K_{II}^2\right]^{0.5}$$

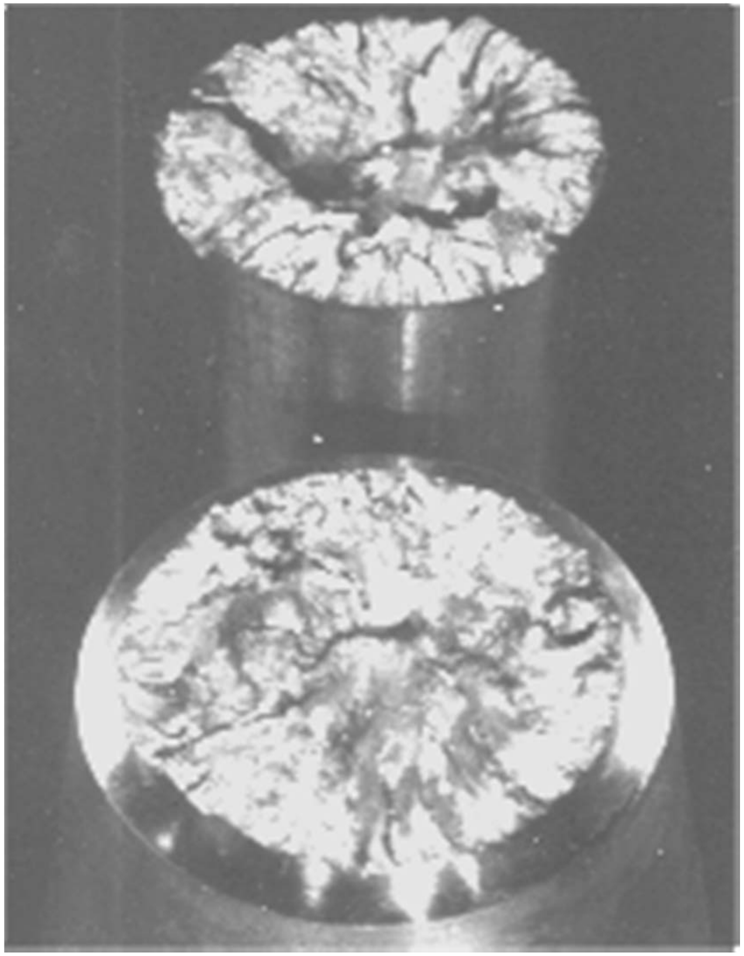
$$\Delta K_{eq}(\varepsilon) = \left[\left(F_{II}\frac{E}{2(1+\nu)}\Delta\gamma\right)^2 + \left(F_I E\Delta\varepsilon\right)^2\right]^{0.5}\sqrt{\pi a}$$

$$\Delta K_{eq}(\varepsilon) = FG\Delta\gamma\left(1 + k\frac{\sigma_{n,max}}{\sigma_{ys}}\right)\sqrt{\pi a}$$

Fracture Surfaces

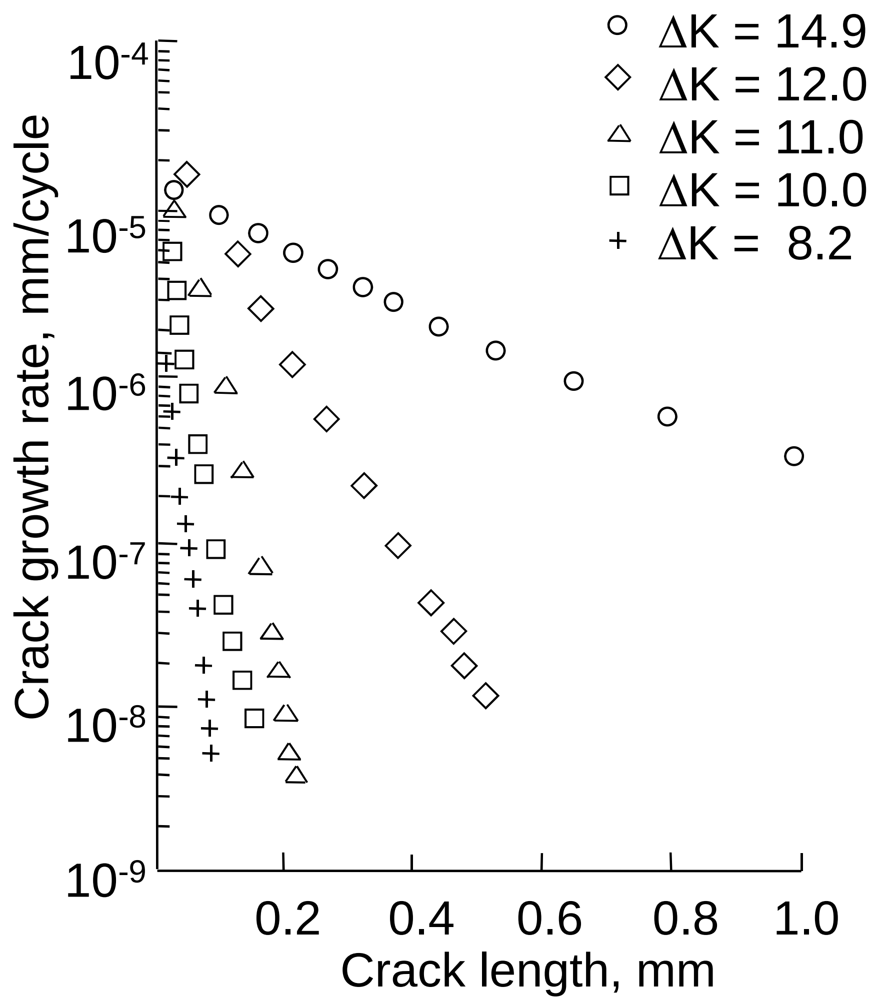


Bending



Torsion

Mode III Growth



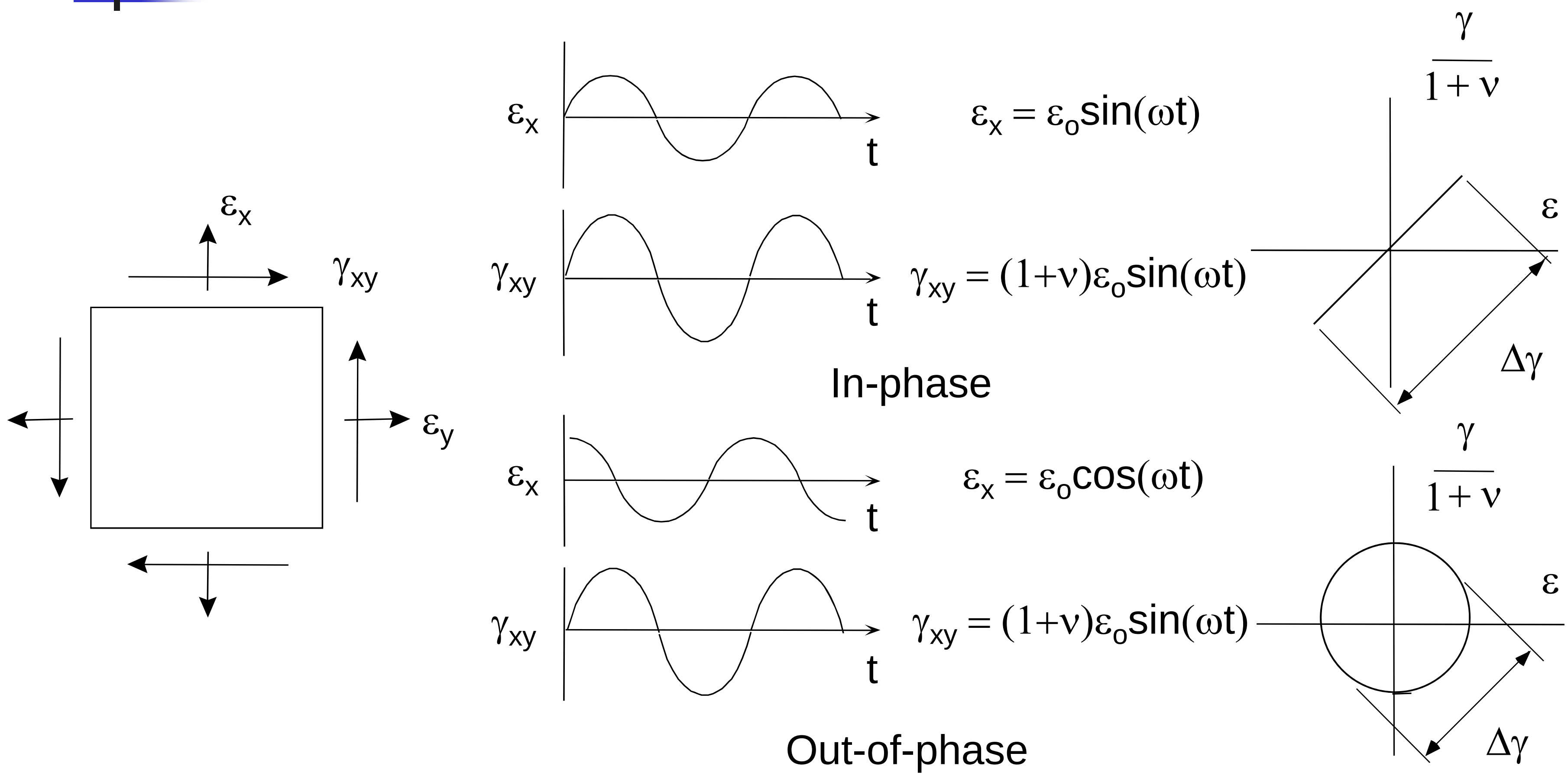
Fracture Mechanics Models Summary

- Multiaxial loading has little effect in Mode I
- Crack closure makes Mode II and Mode III calculations difficult

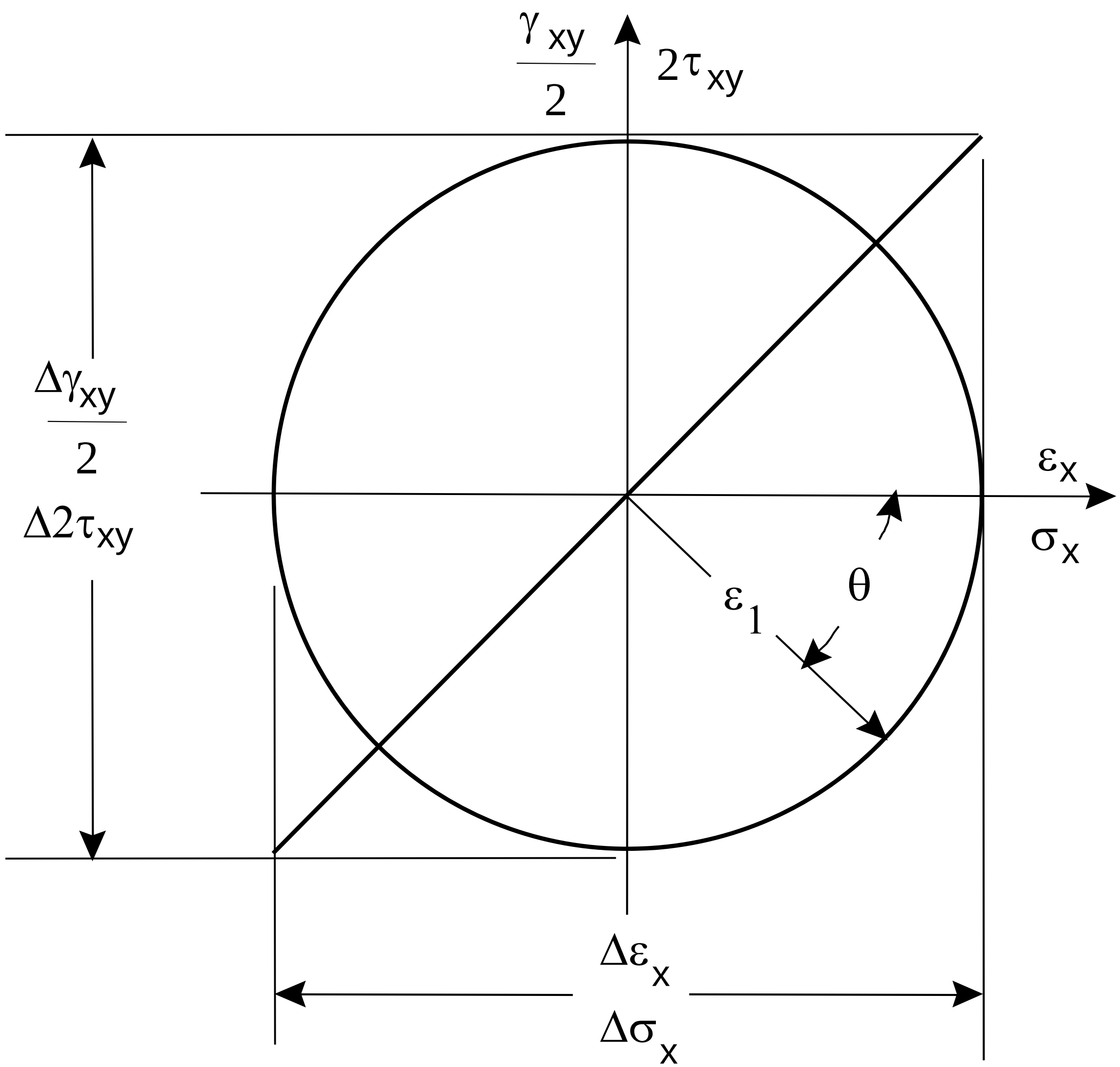
Nonproportional Loading

- In and Out-of-phase loading
- Nonproportional cyclic hardening
- Variable amplitude

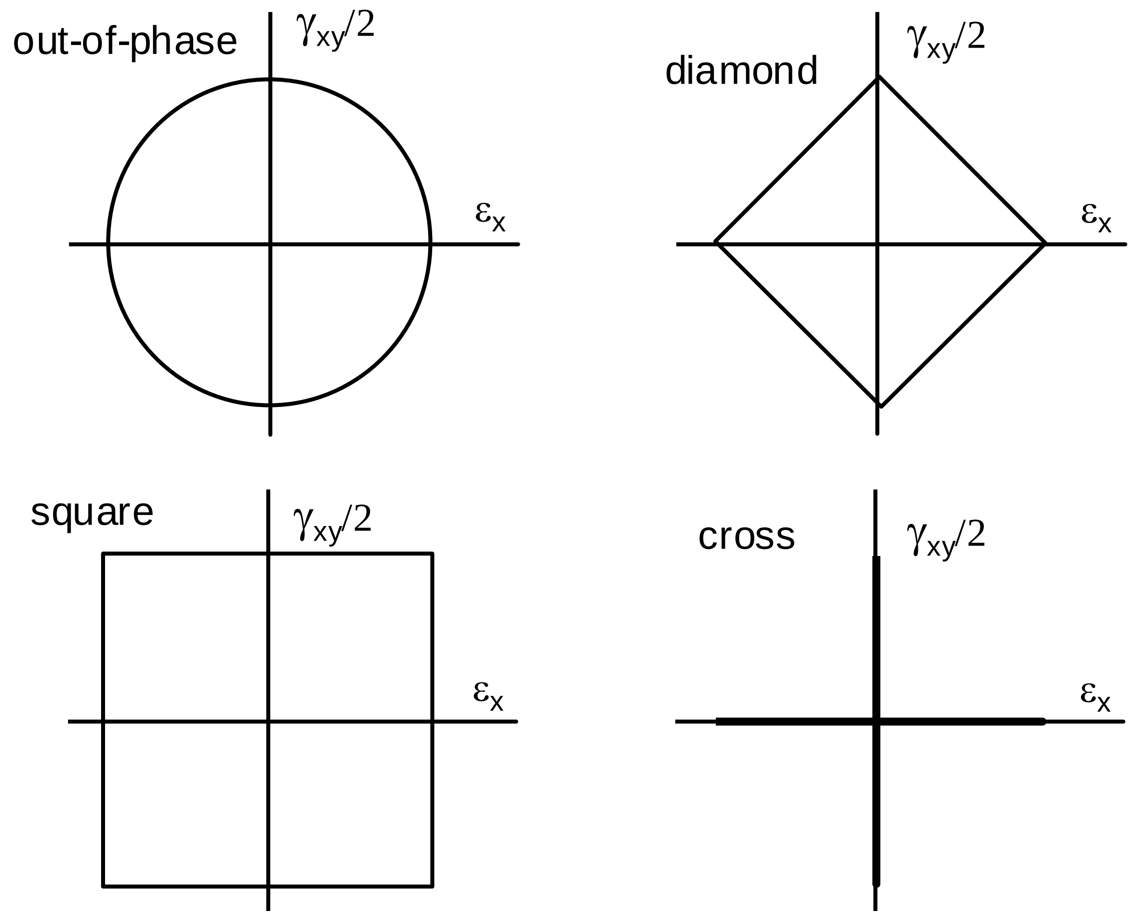
In and Out-of-Phase Loading



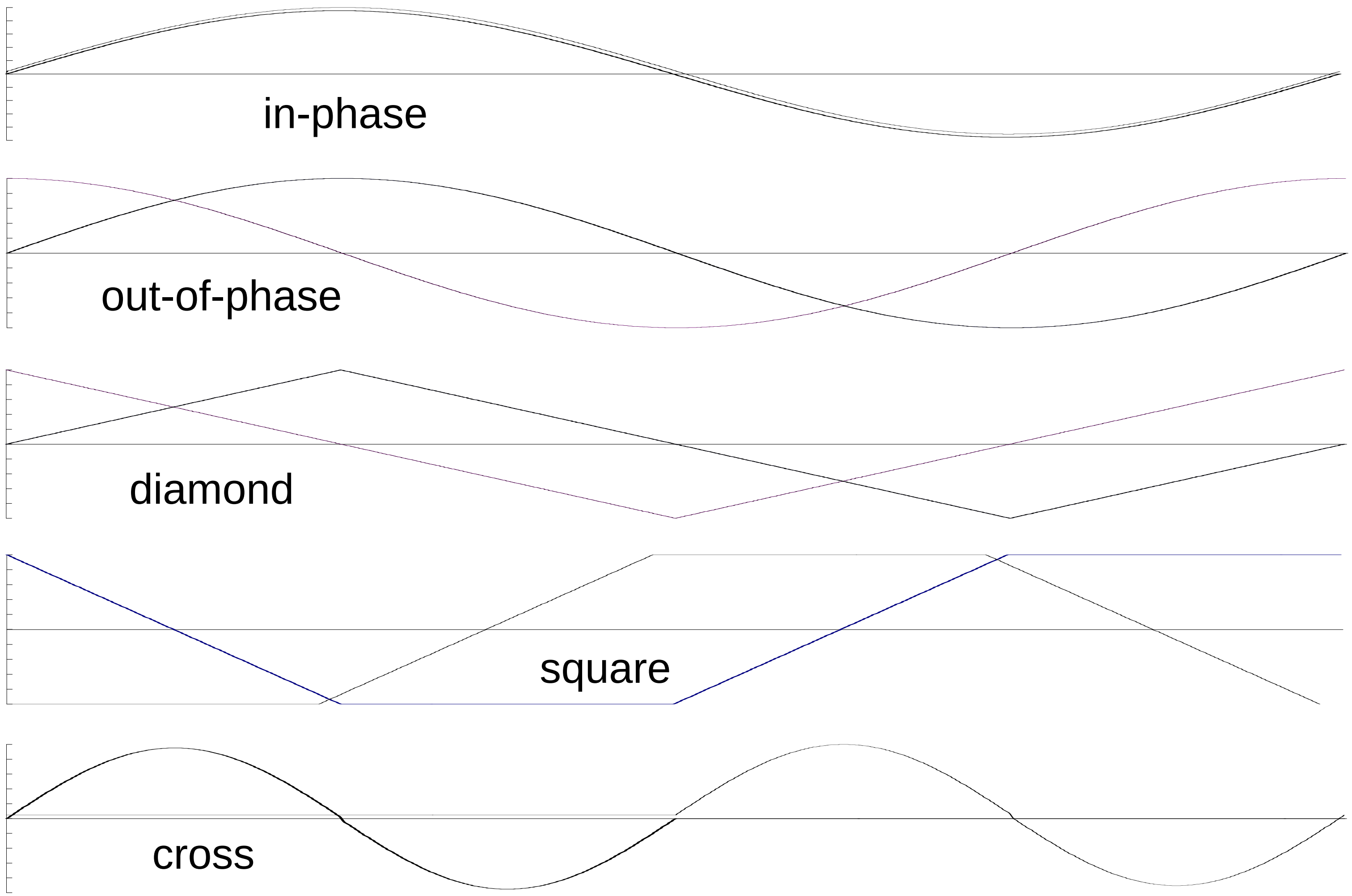
In-Phase and Out-of-Phase



Loading Histories

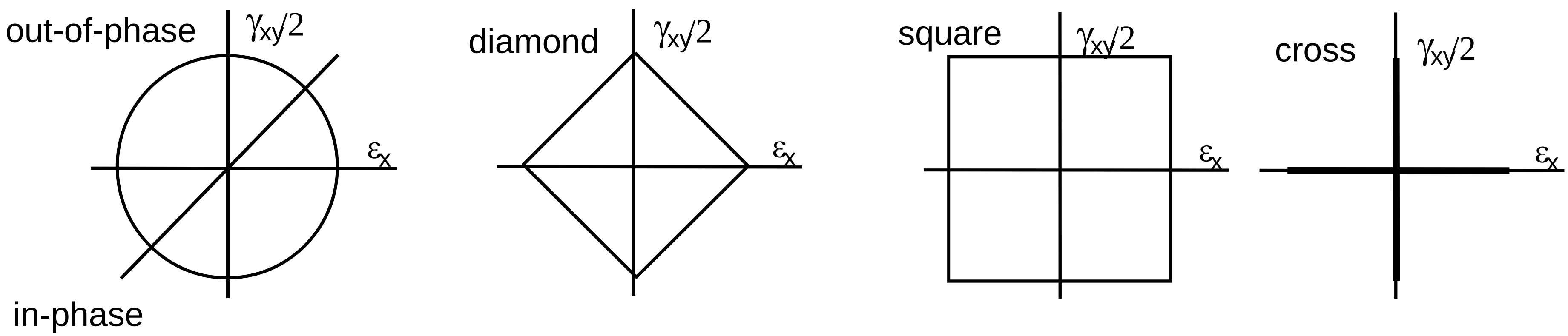


Loading Histories

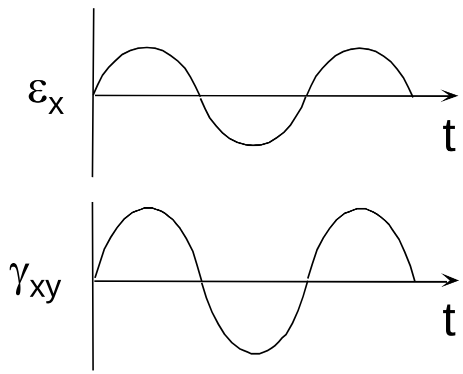


Findley Model Results

	$\Delta\tau/2$ MPa	$\sigma_{n.max}$ MPa	$\Delta\tau/2 + 0.3\ \sigma_{n.max}$	N/N_{ip}
in-phase	353	250	428	1.0
90° out-of-phase	250	500	400	2.0
diamond	250	500	400	2.0
square	353	603	534	0.11
cross - tension cycle	250	250	325	16
cross - torsion cycle	250	0	250	216



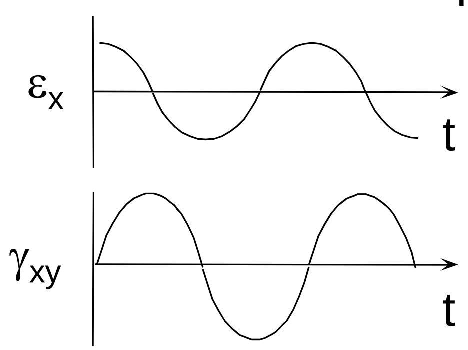
Nonproportional Hardening



$$\varepsilon_x = \varepsilon_o \sin(\omega t)$$

$$\gamma_{xy} = (1+\nu)\varepsilon_o \sin(\omega t)$$

In-phase

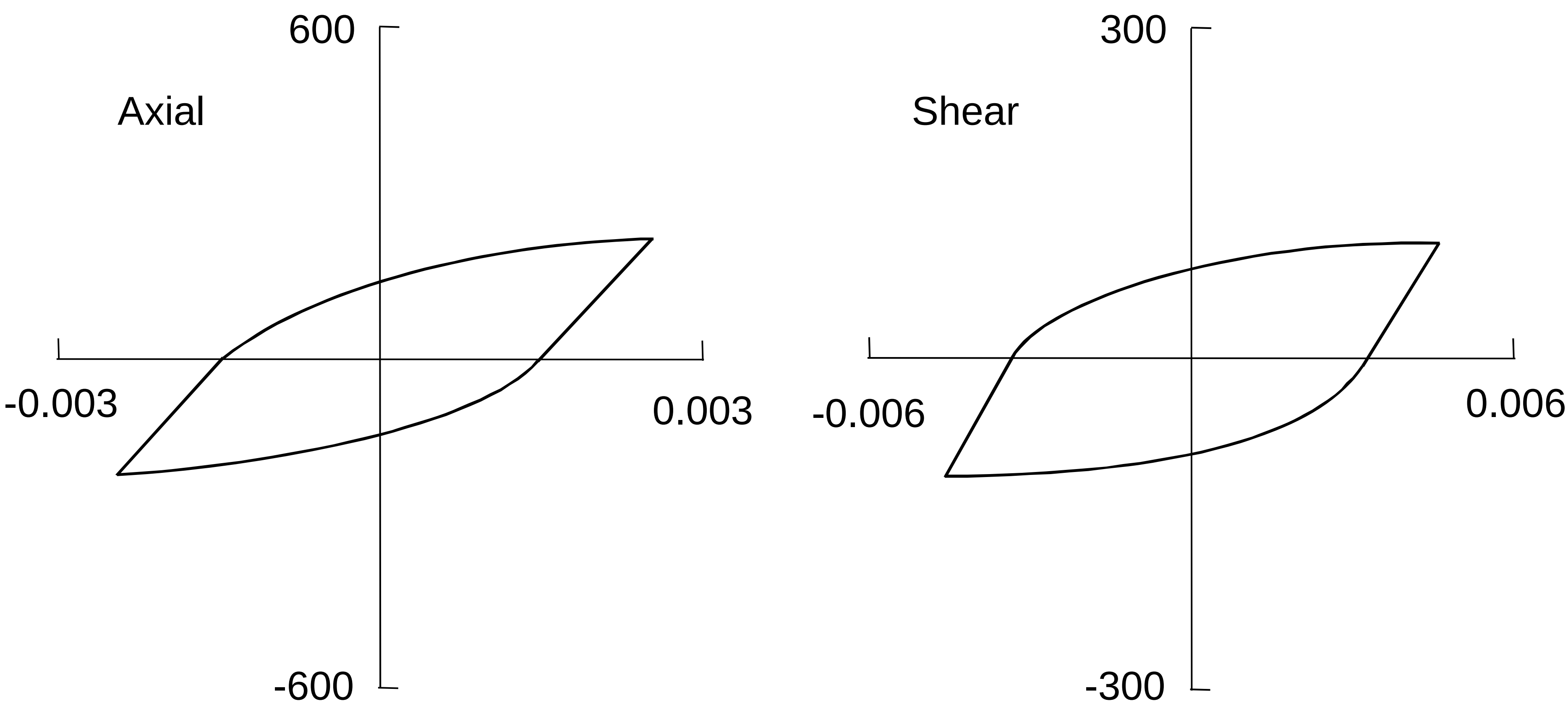


$$\varepsilon_x = \varepsilon_o \cos(\omega t)$$

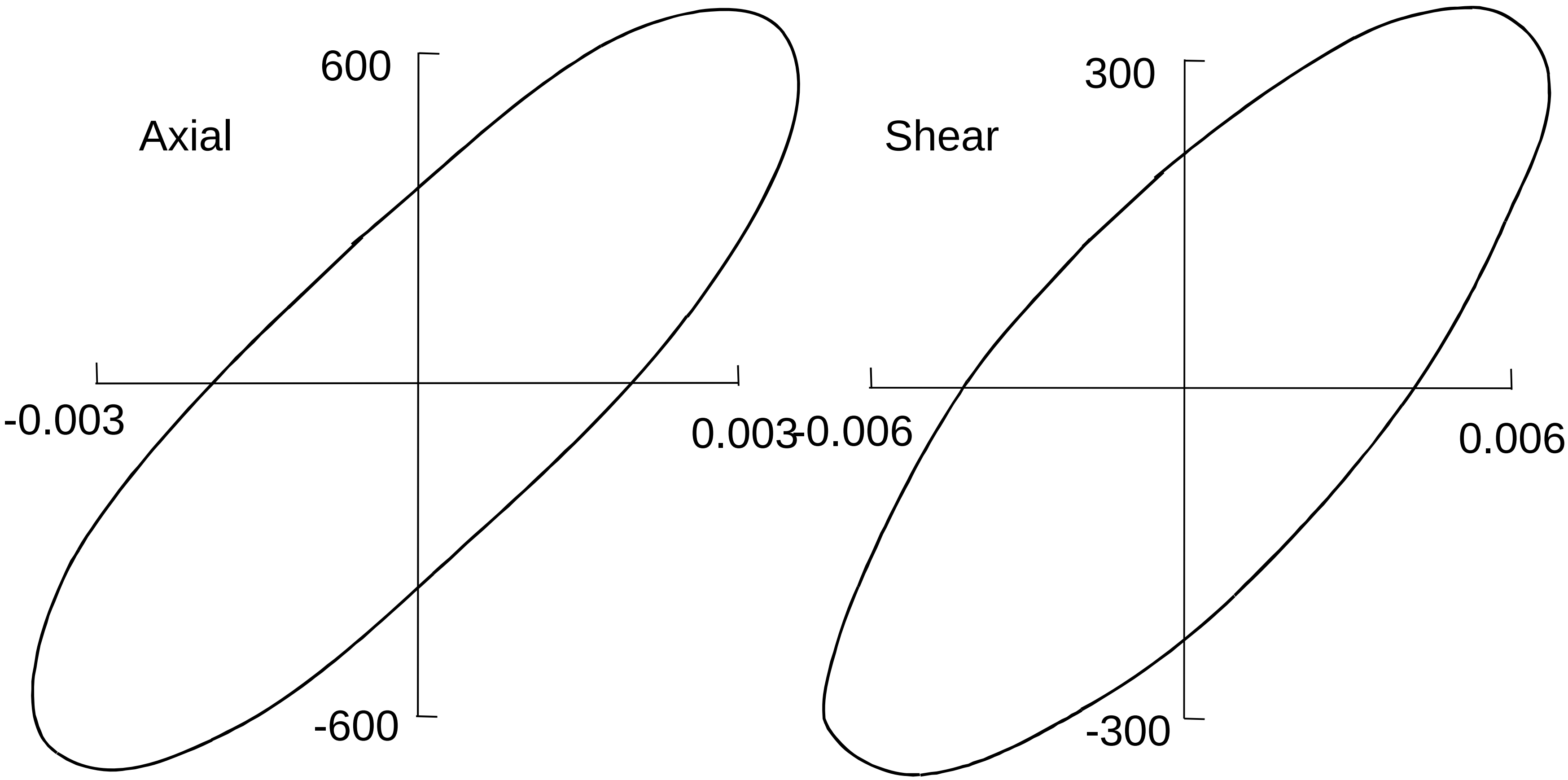
$$\gamma_{xy} = (1+\nu)\varepsilon_o \sin(\omega t)$$

Out-of-phase

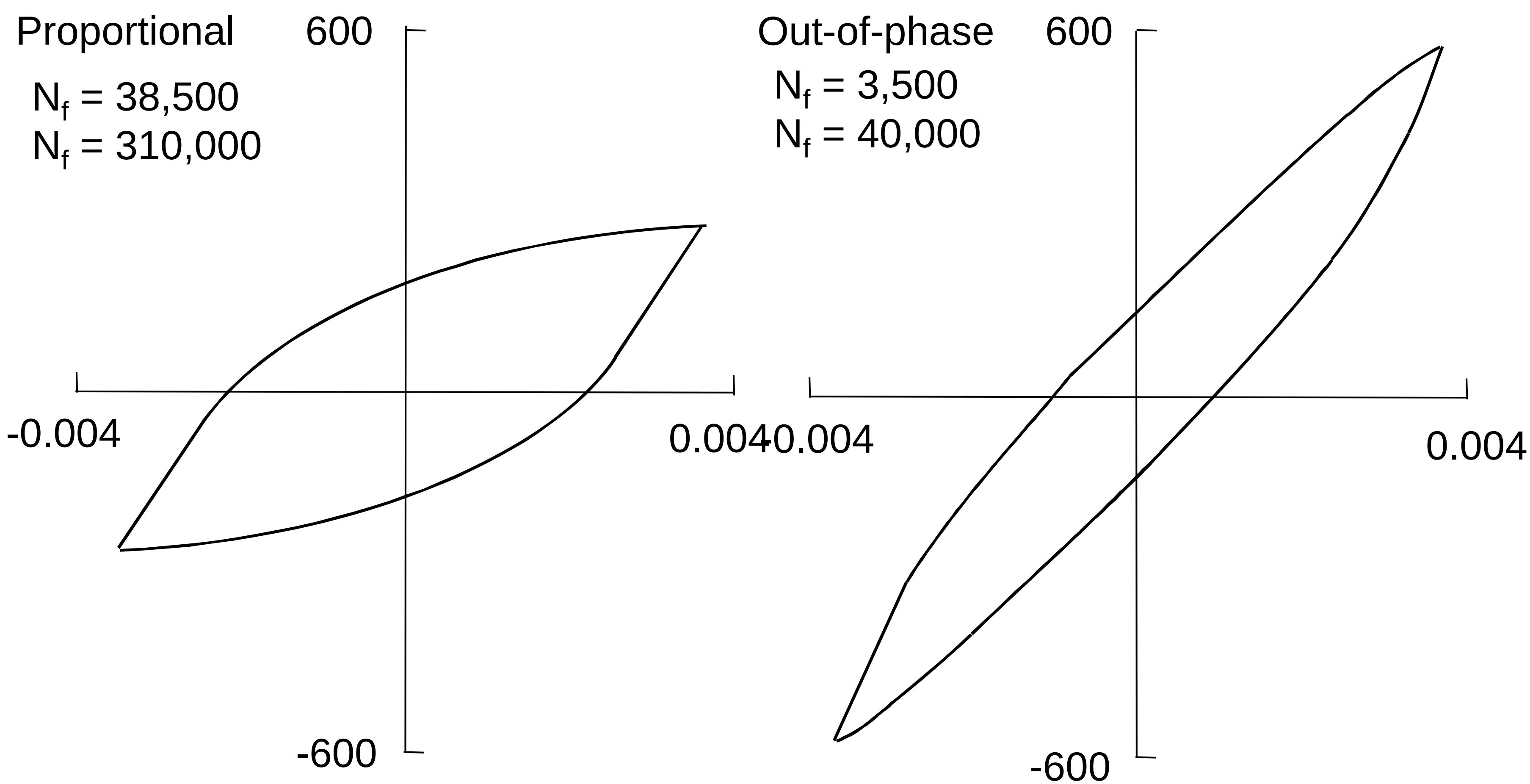
In-Phase



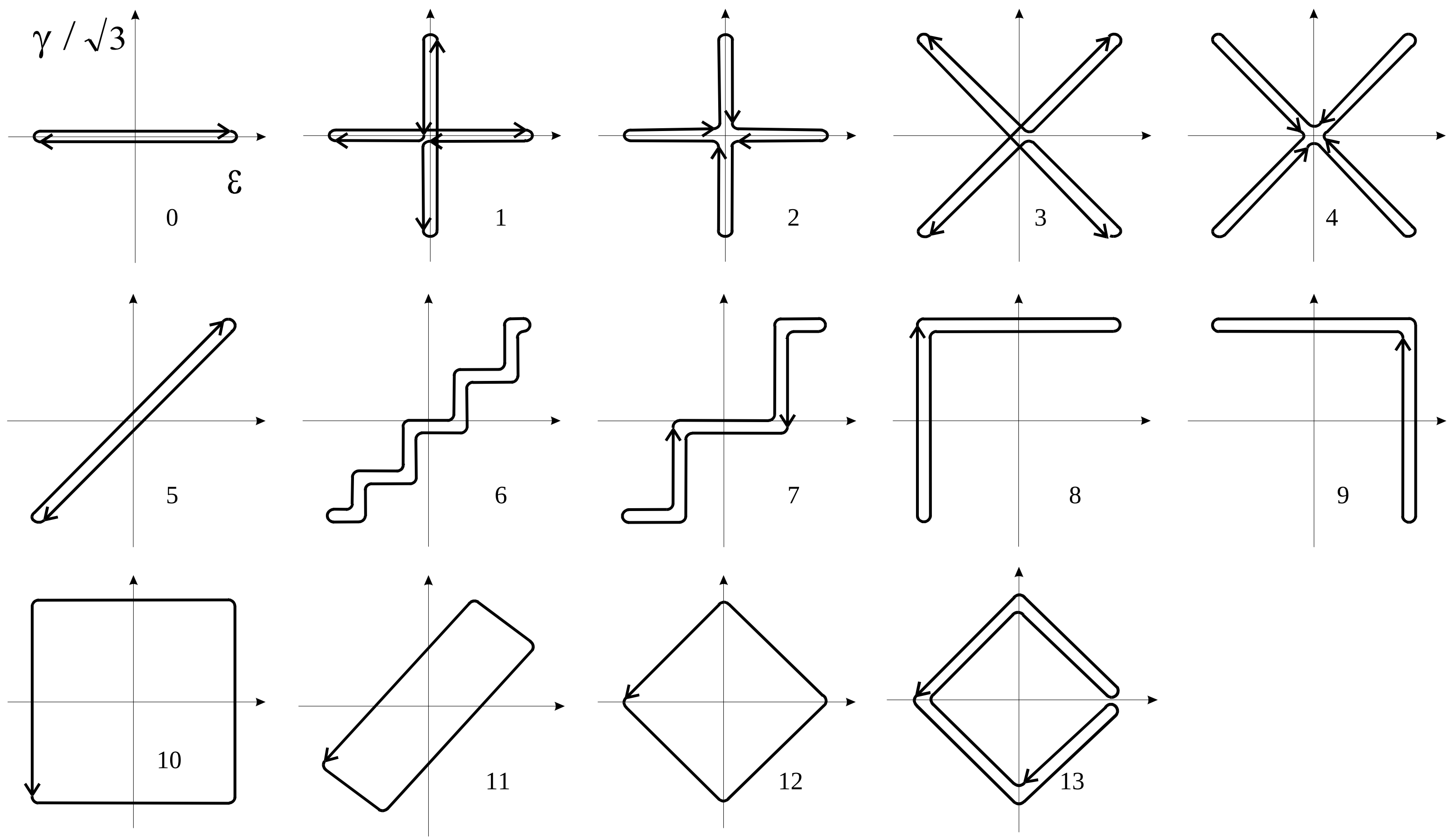
90° Out-of-Phase



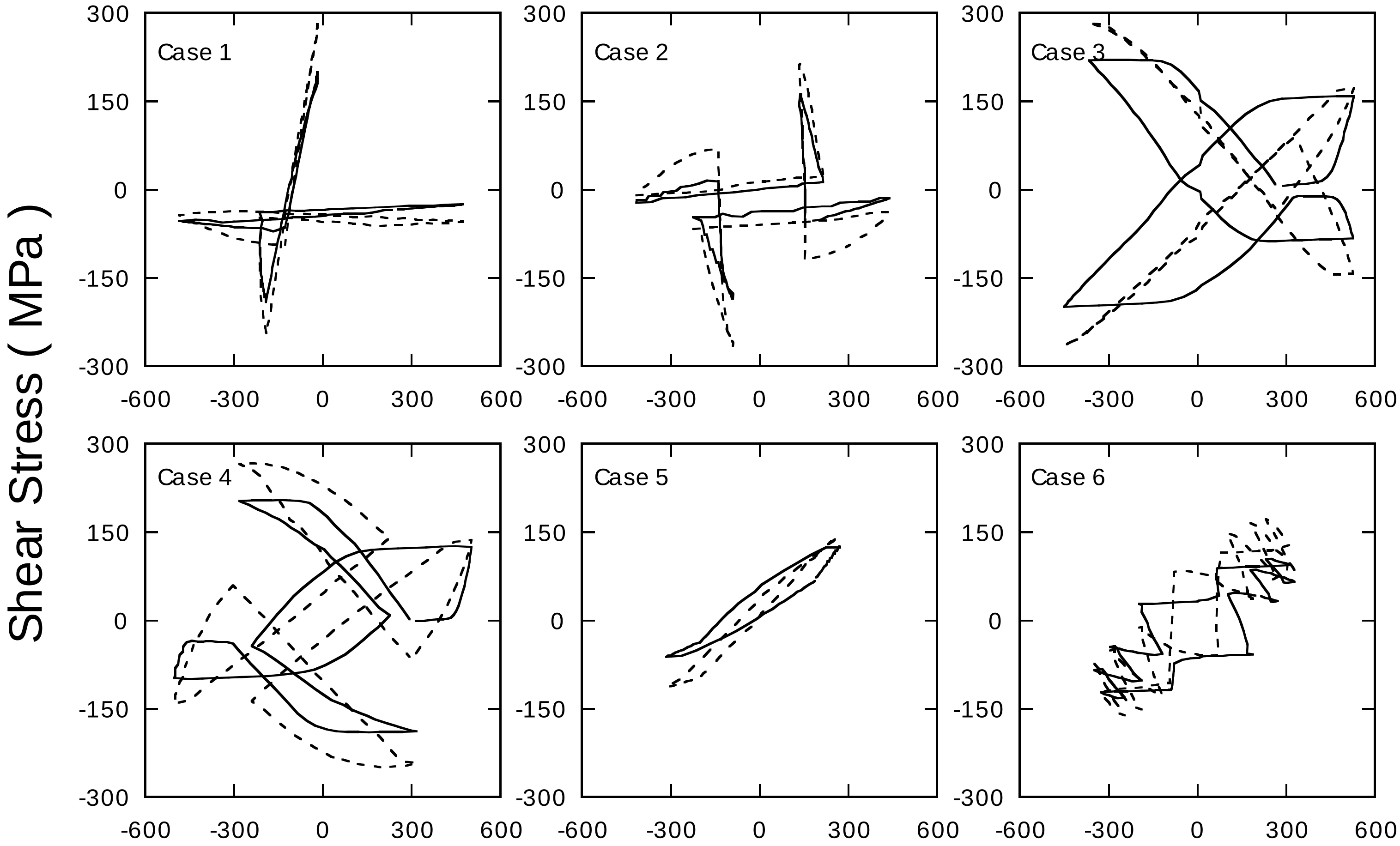
Critical Plane



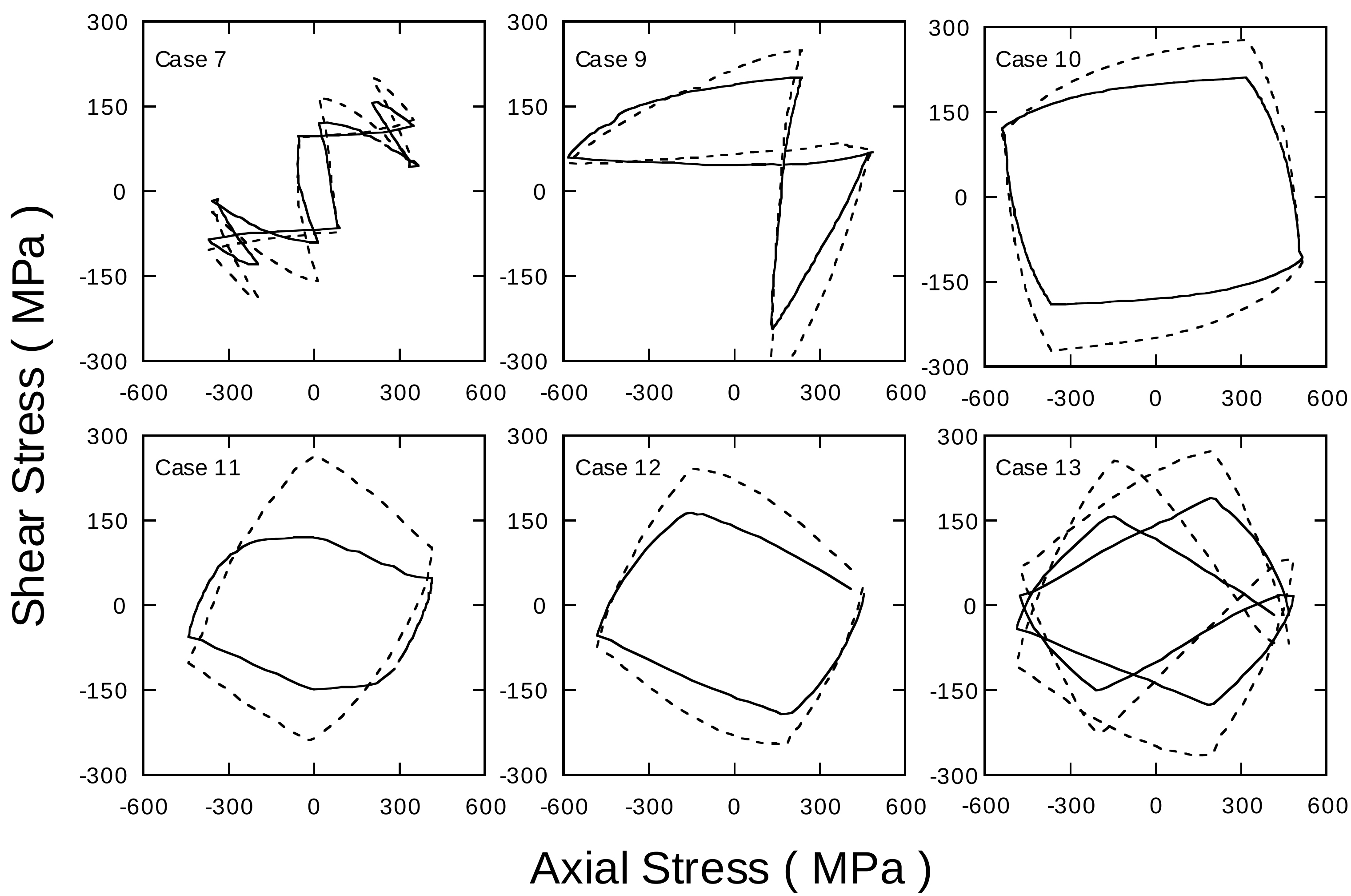
Loading Histories



Stress-Strain Response

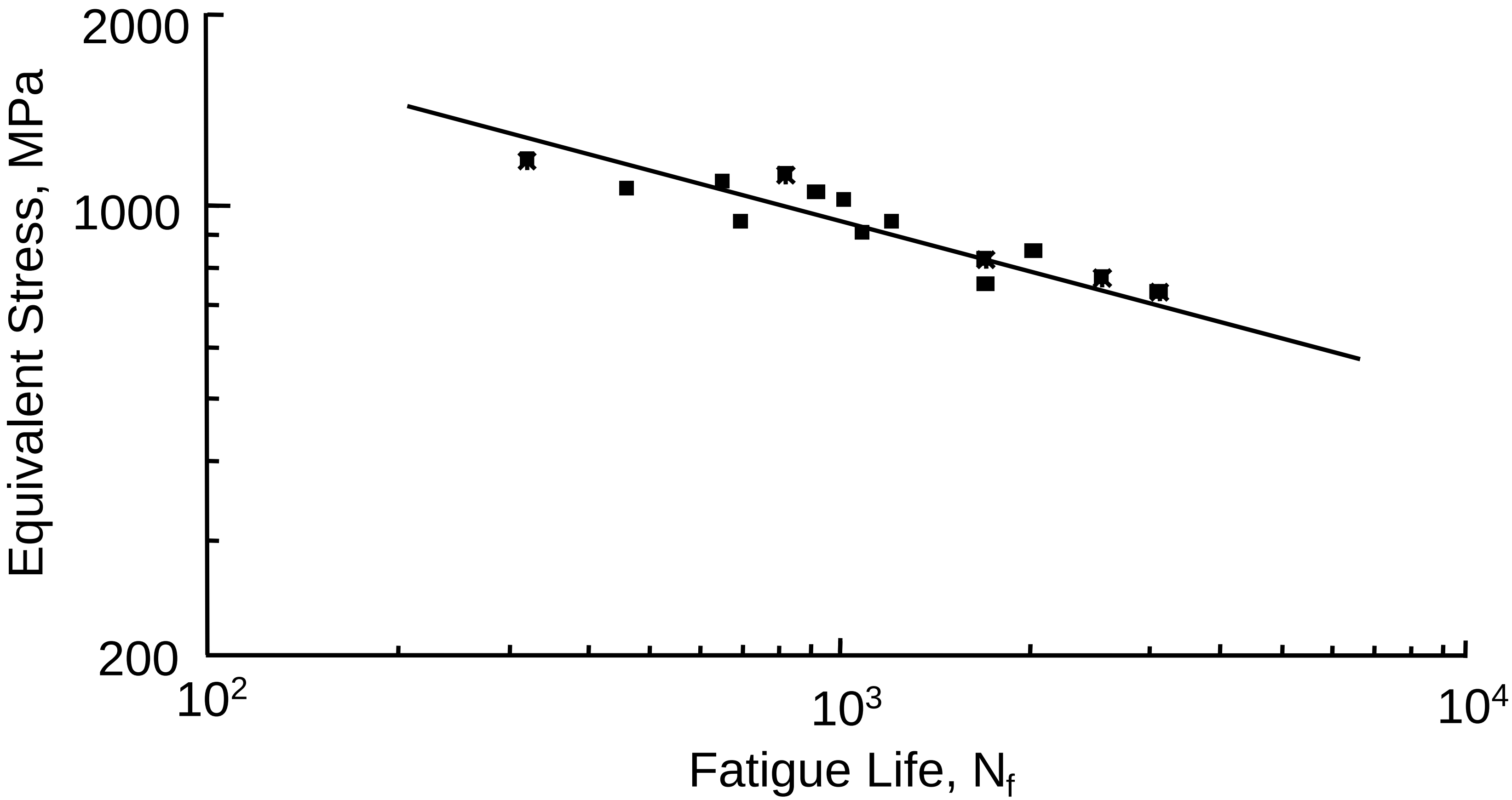


Stress-Strain Response (continued)



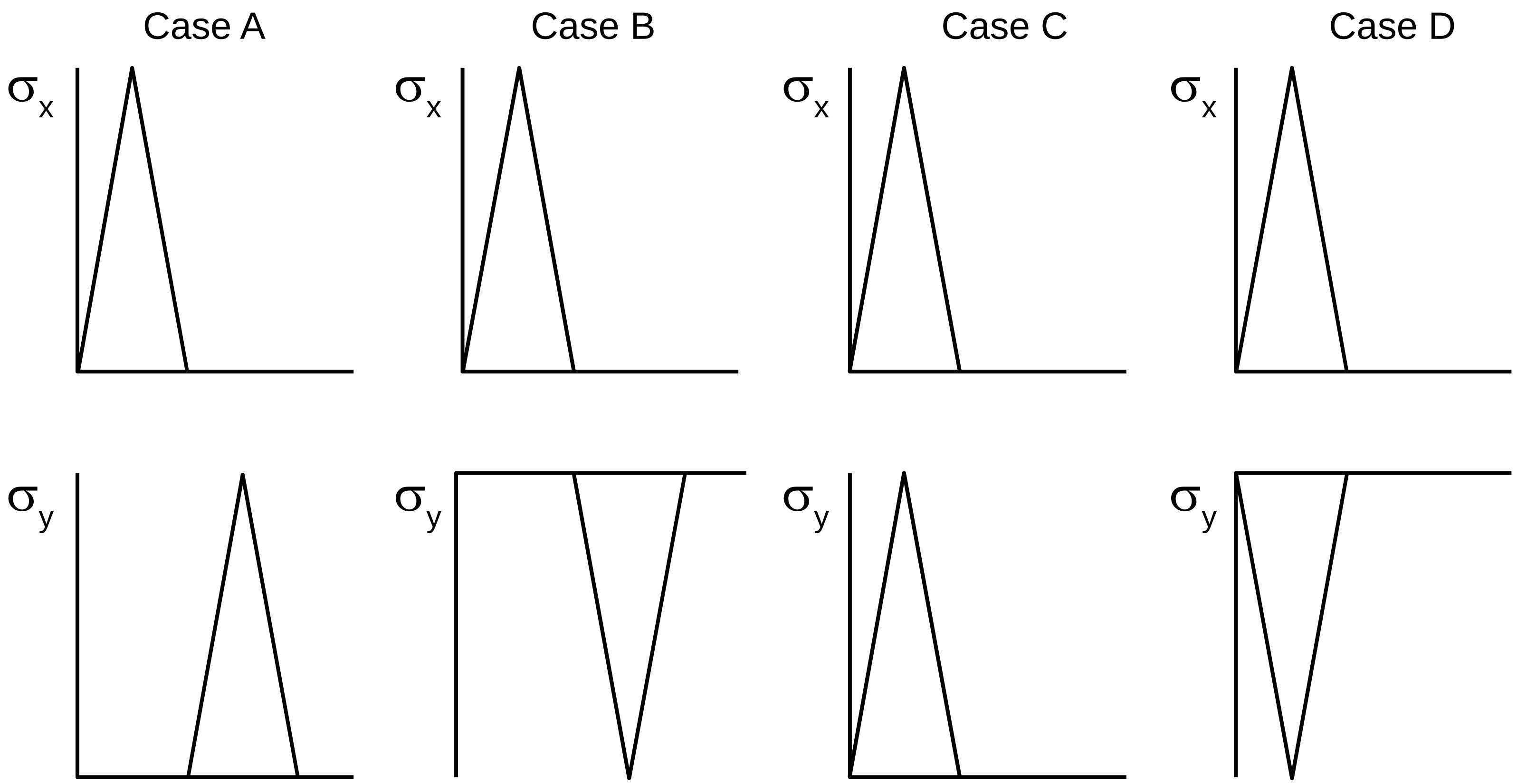
Maximum Stress

All tests have the same strain ranges

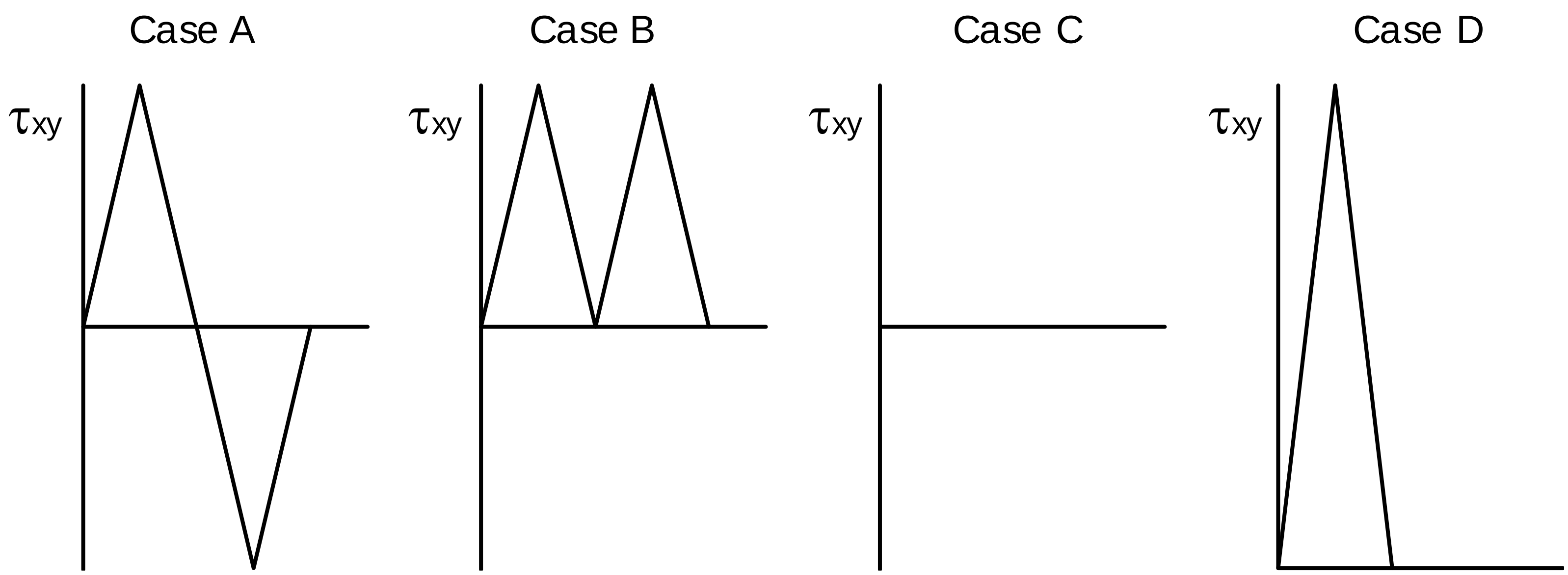


Nonproportional hardening results in lower fatigue lives

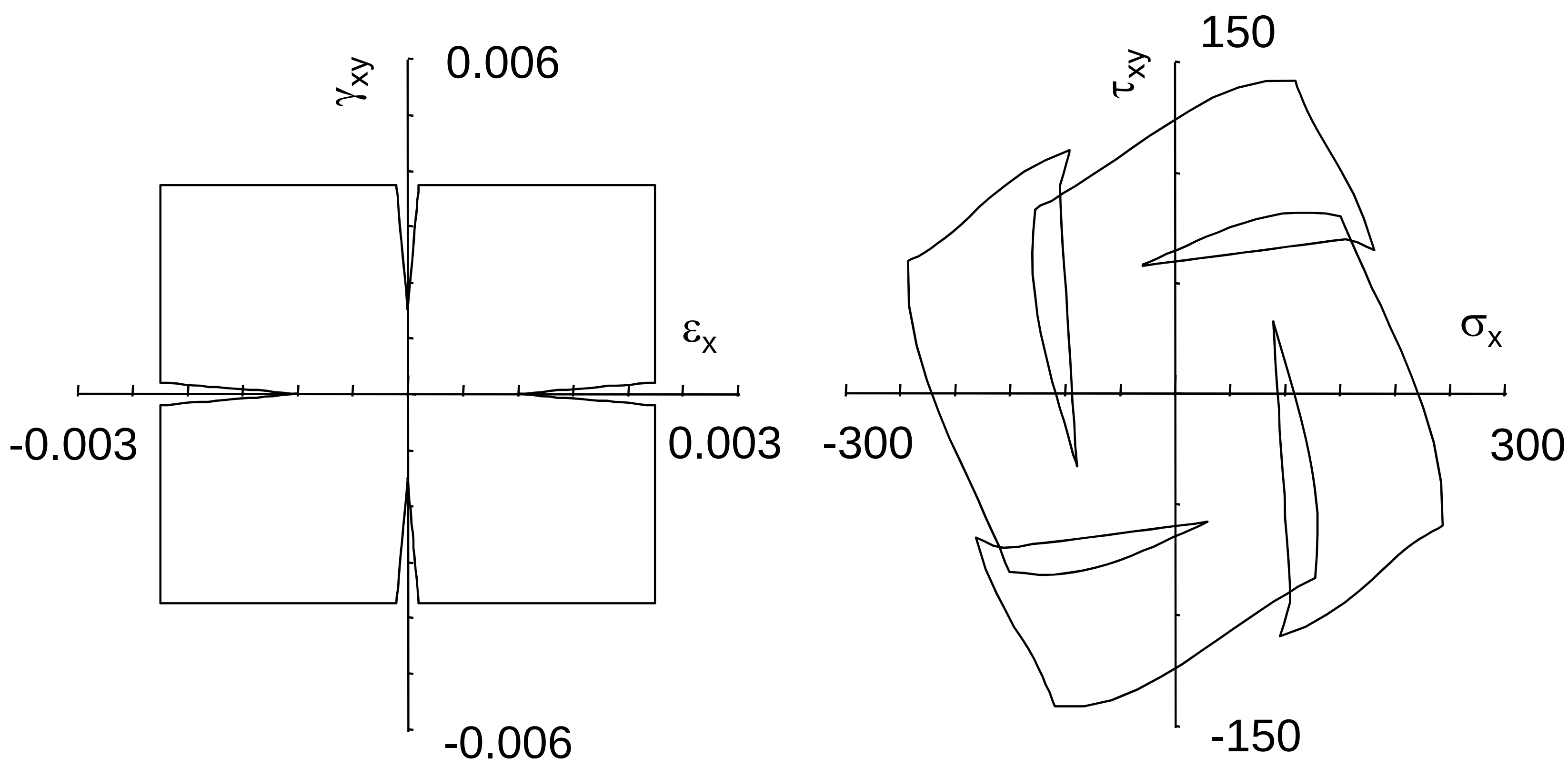
Nonproportional Example



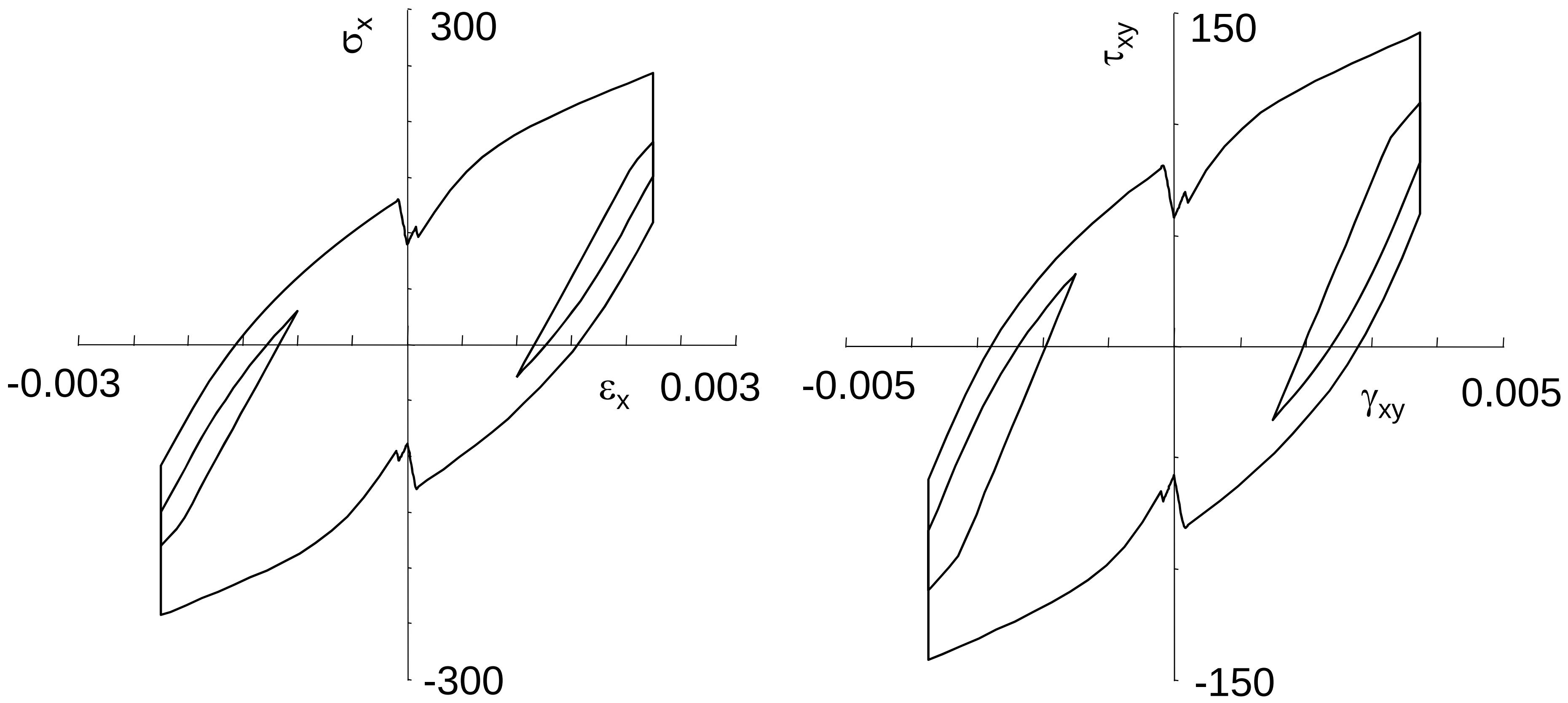
Shear Stresses



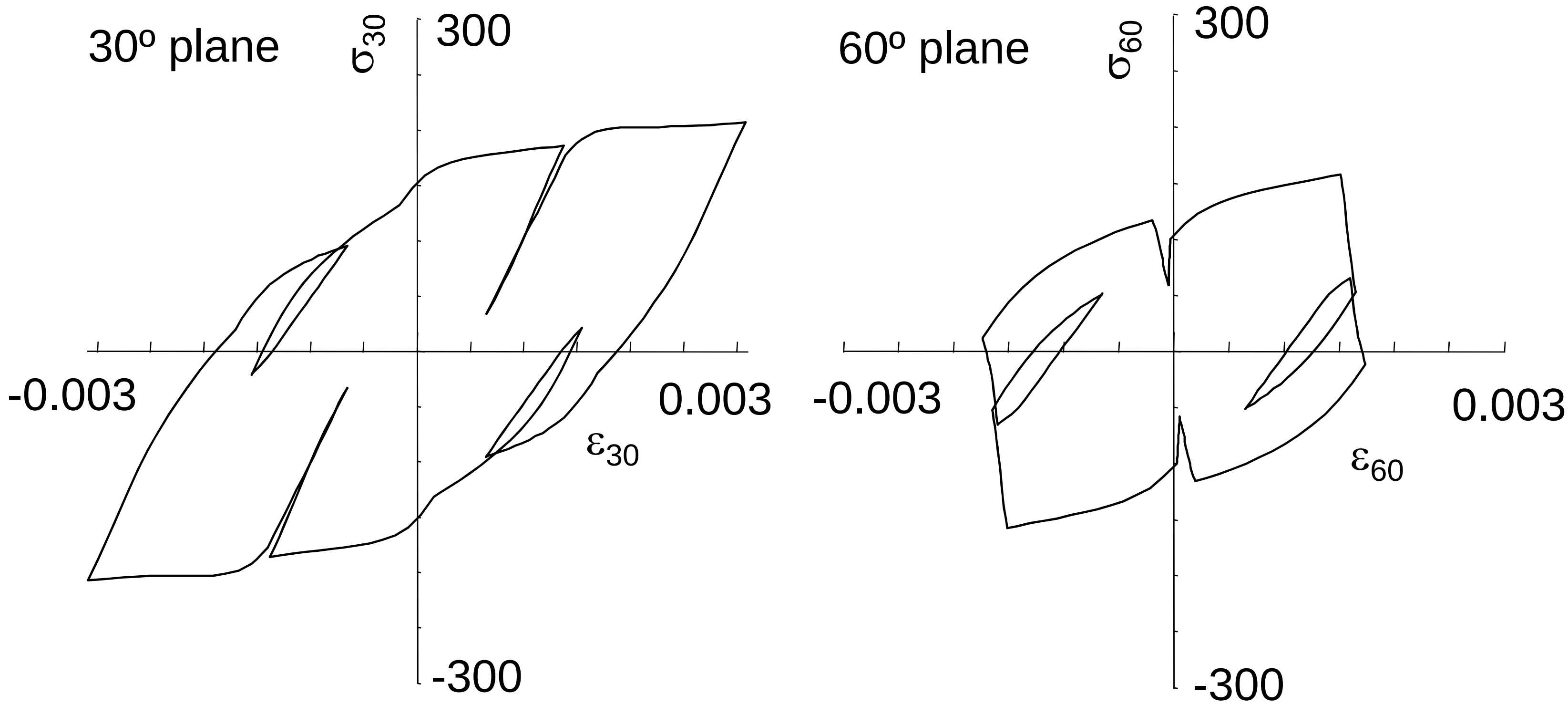
Simple Variable Amplitude History



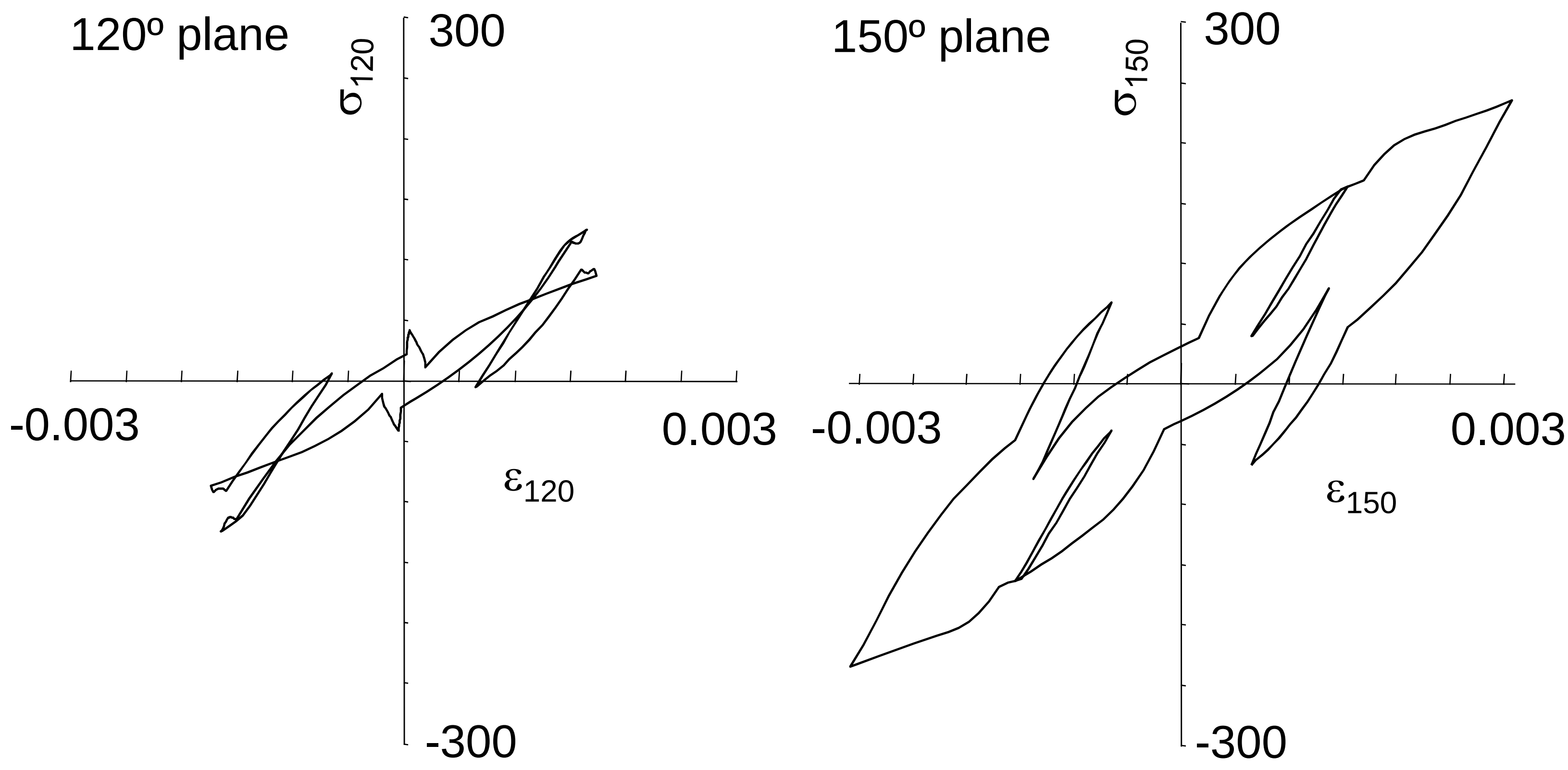
Stress-Strain on 0° Plane



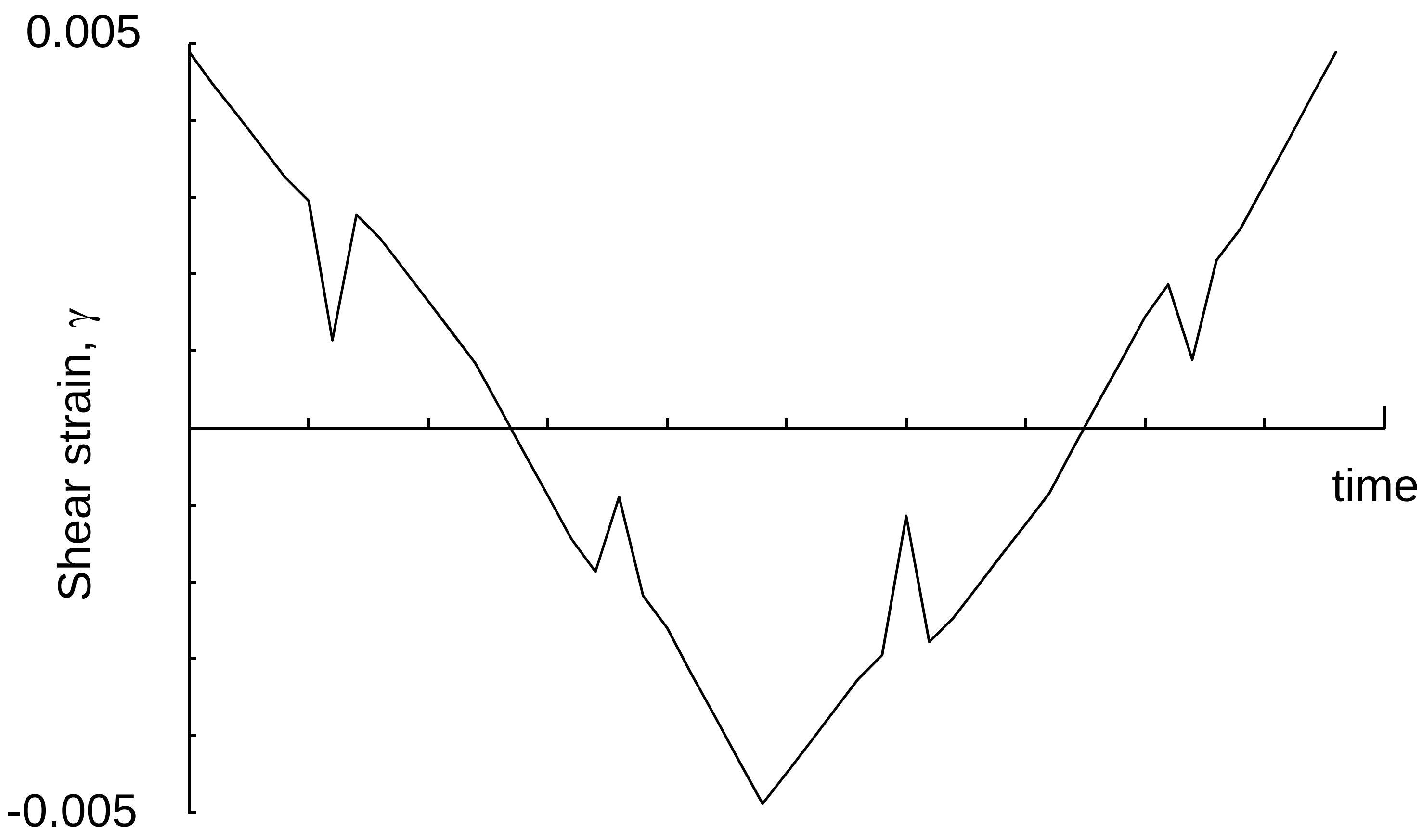
Stress-Strain on 30° and 60° Planes



Stress-Strain on 120° and 150° Planes



Shear Strain History on Critical Plane



Fatigue Calculations

Load or strain history

Cyclic plasticity model

Stress and strain tensor

Search for critical plane

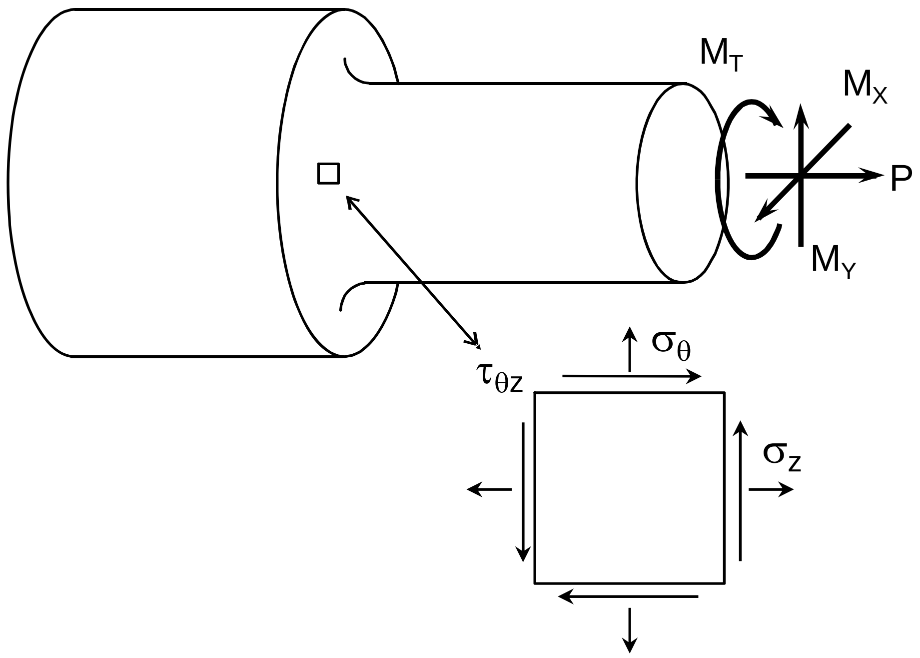
Nonproportional Loading Summary

- Nonproportional cyclic hardening increases stress levels
- Critical plane models are used to assess fatigue damage

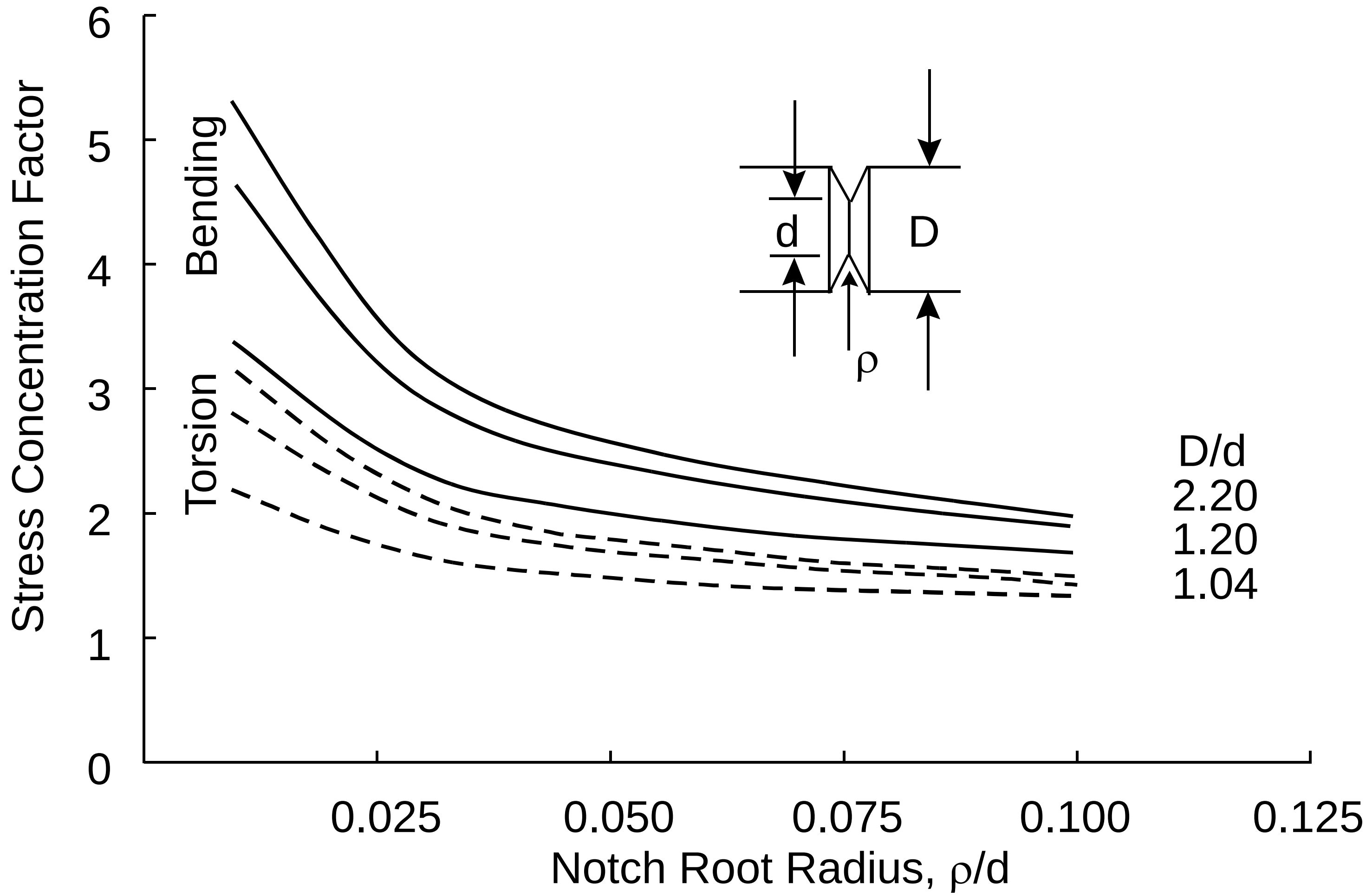
Notches

- Stress and strain concentrations
- Nonproportional loading and stressing
- Fatigue notch factors
- Cracks at notches

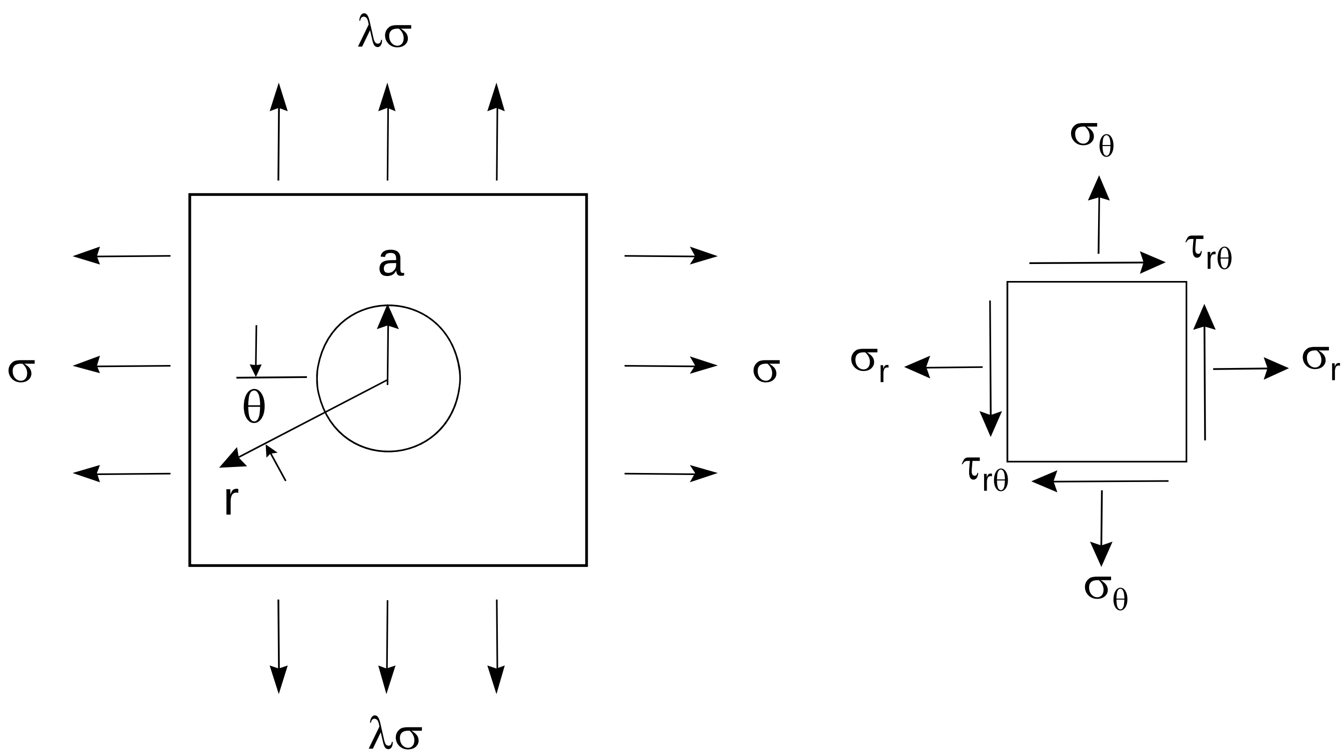
Notched Shaft Loading



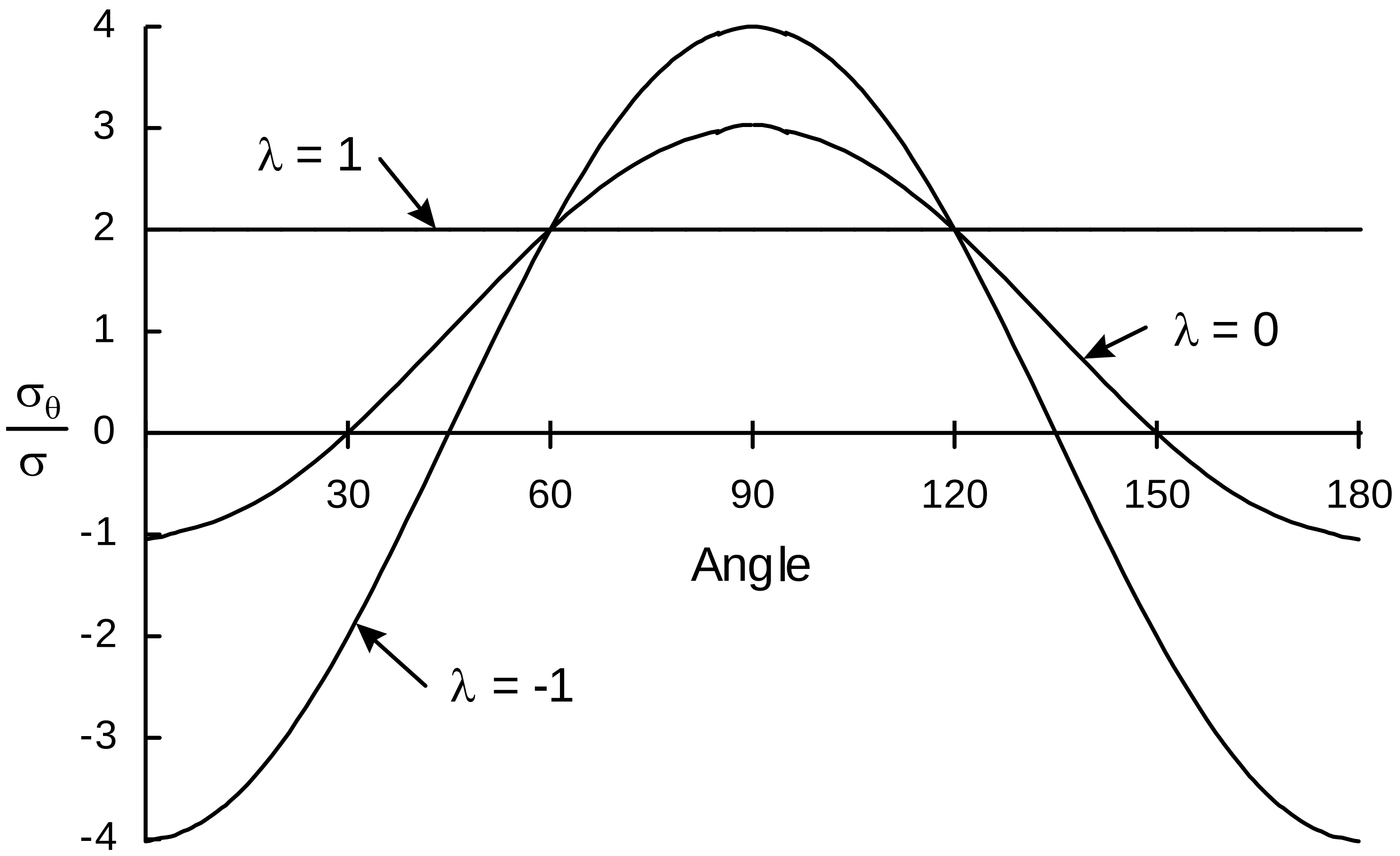
Stress Concentration Factors



Hole in a Plate

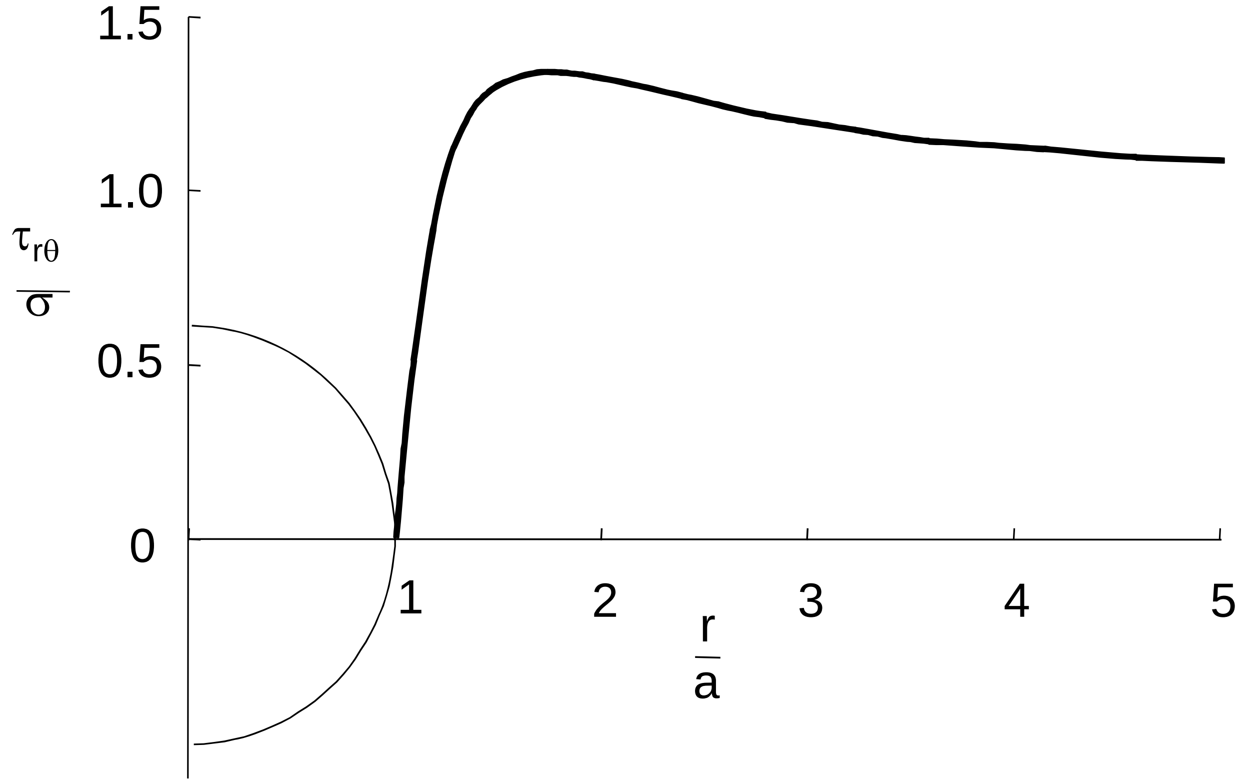


Stresses at the Hole

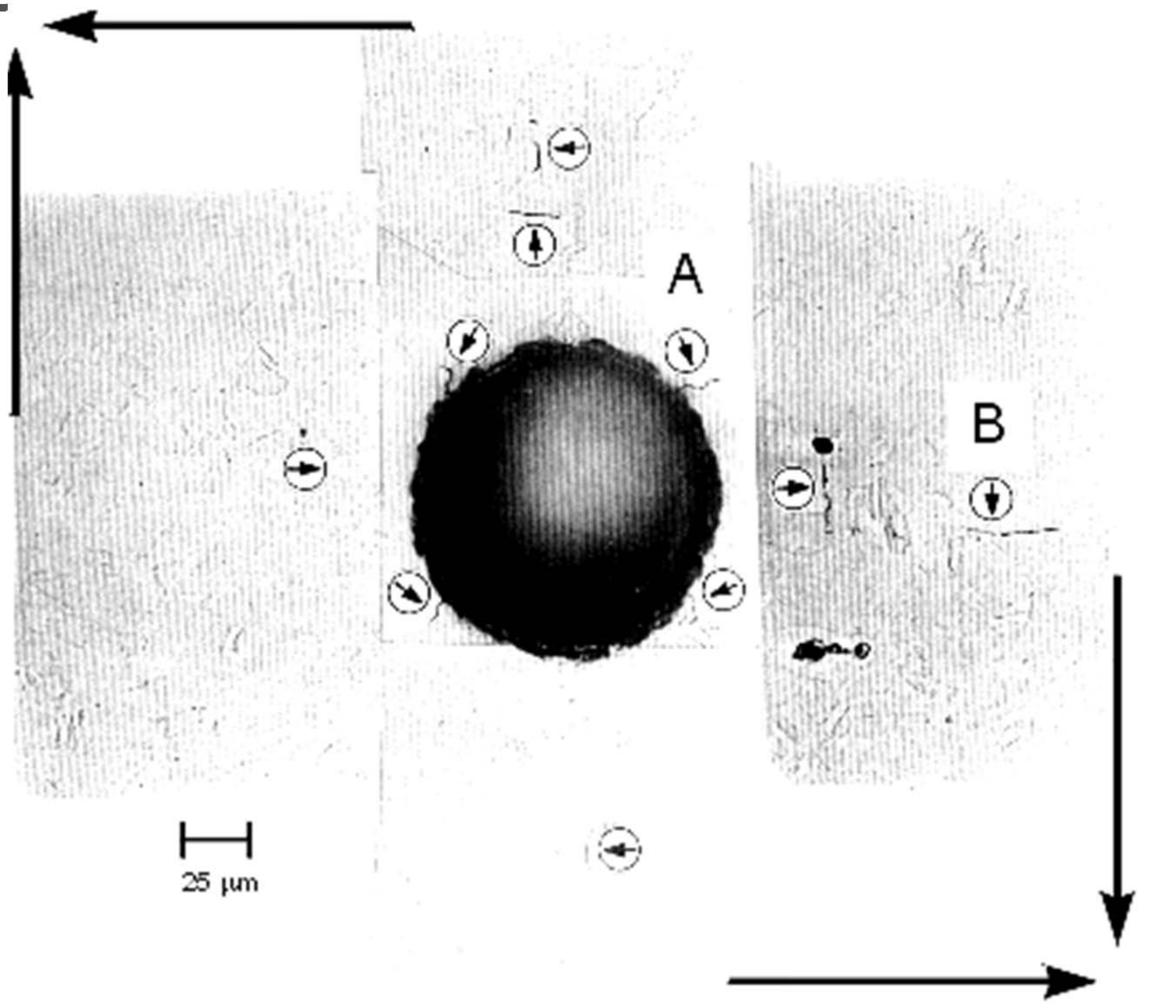


Stress concentration factor depends on type of loading

Shear Stresses during Torsion



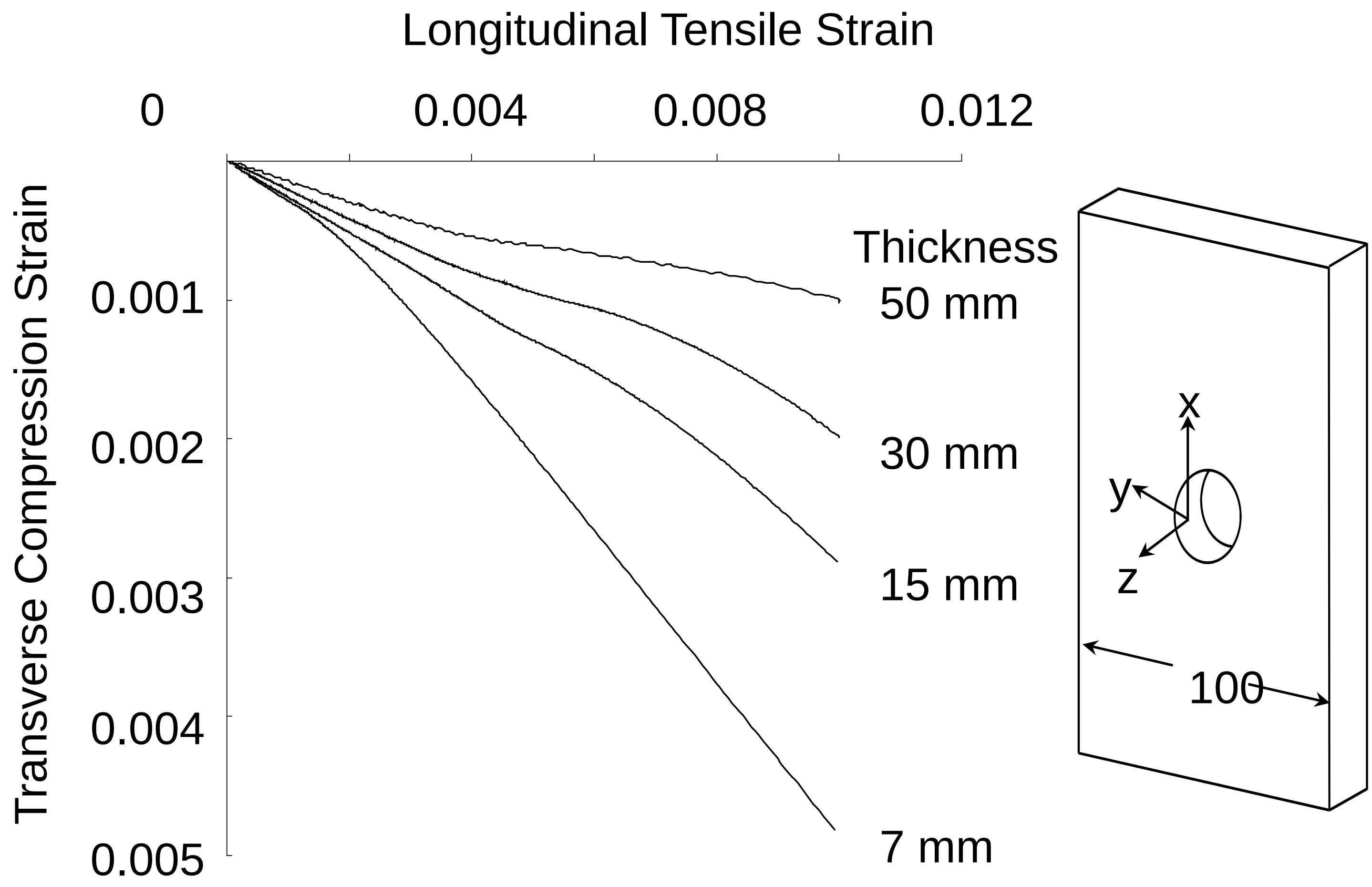
Torsion Experiments



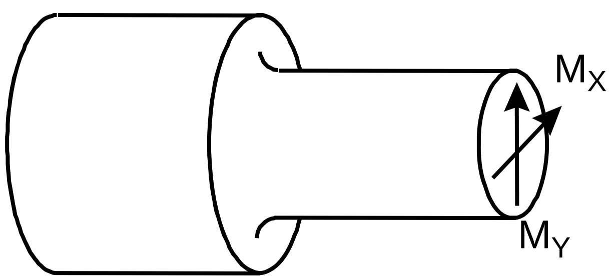
Multiaxial Loading

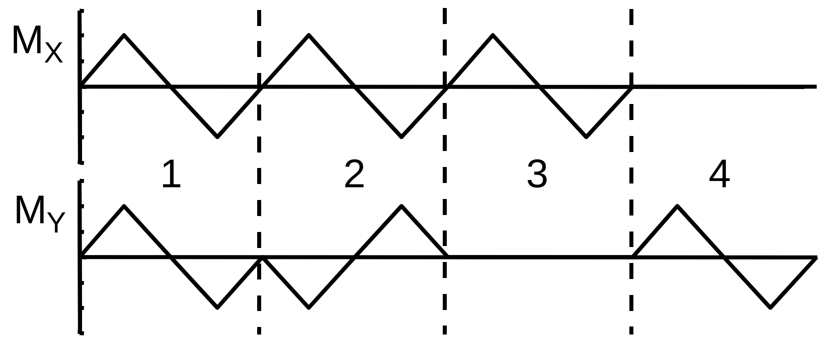
- Uniaxial loading that produces multiaxial stresses at notches
- Multiaxial loading that produces uniaxial stresses at notches
- Multiaxial loading that produces multiaxial stresses at notches

Thickness Effects

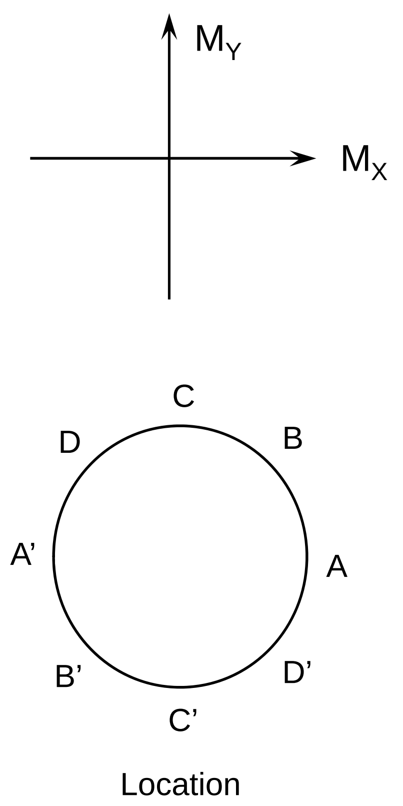


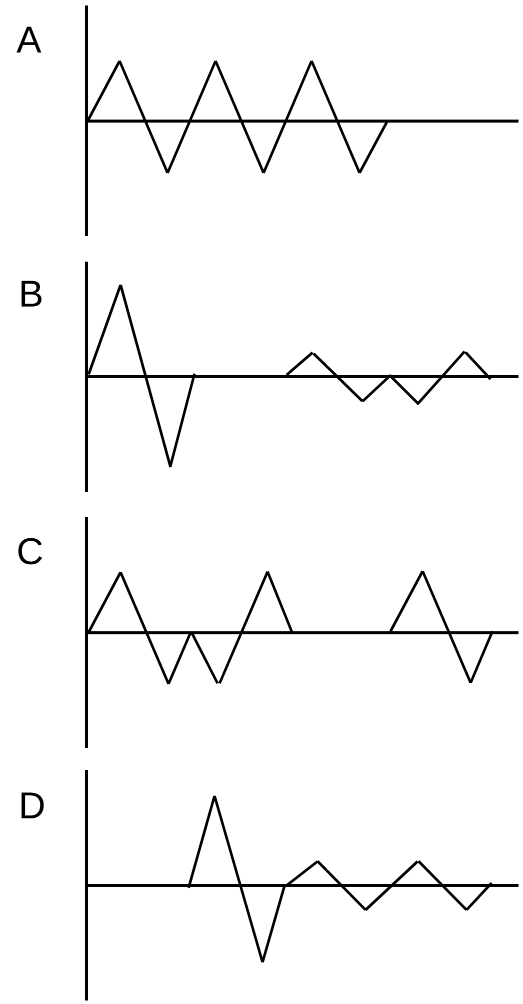
Applied Bending Moments





Bending Moments on the Shaft





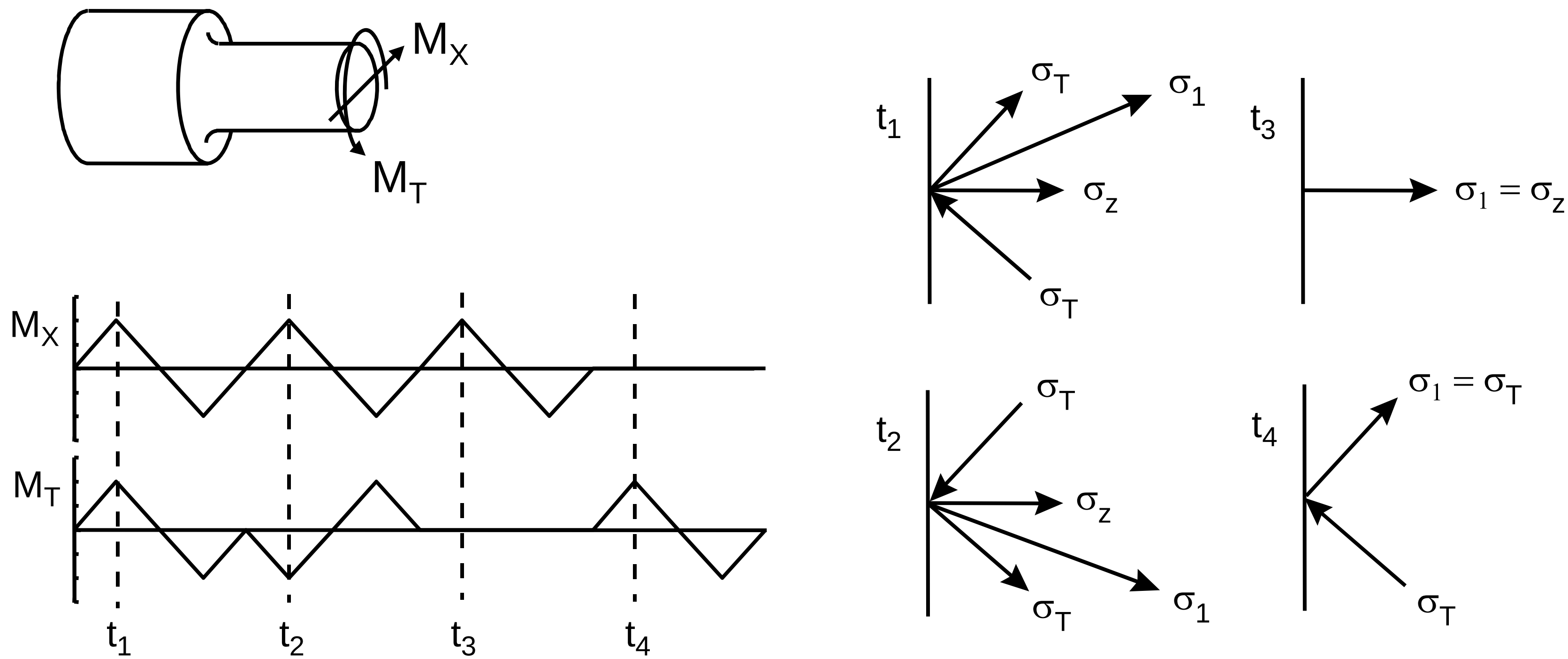
Bending Moments

ΔM	A	B	C	D
2.82		1		1
2.00	3		2	
1.41		2		1
1.00			2	
0.71				2

$$\Delta \overline{M} = \sqrt[5]{\sum \Delta M^5}$$

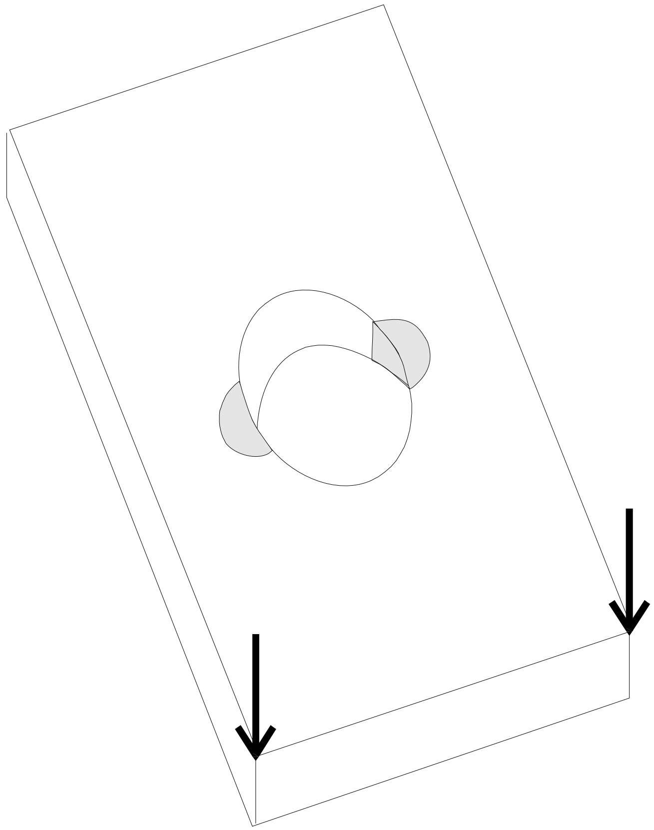
	A	B	C	D
$\Delta \overline{M}$	2.49	2.85	2.31	2.84

Torsion Loading

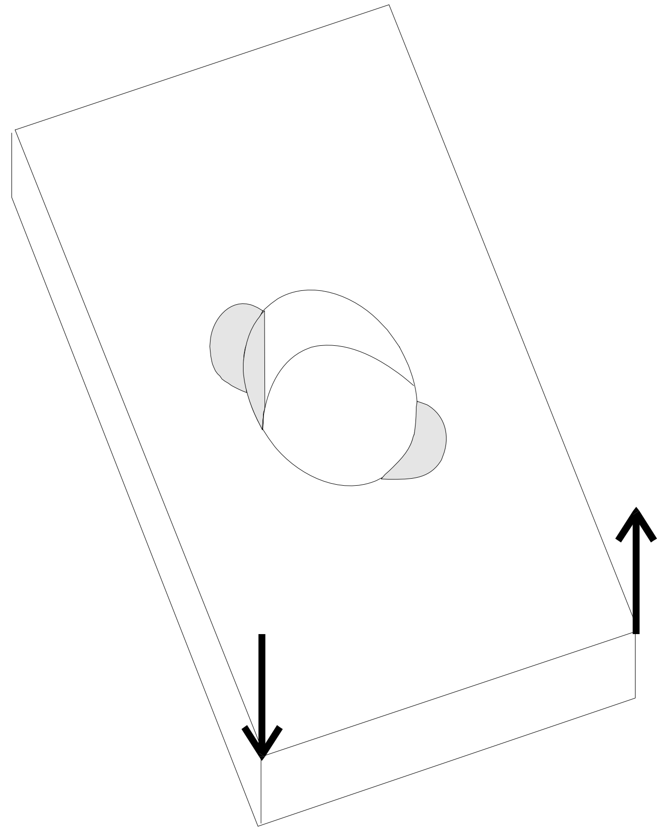


Out-of-phase shear loading is needed to produce nonproportional stressing

Plate and Shell Structures

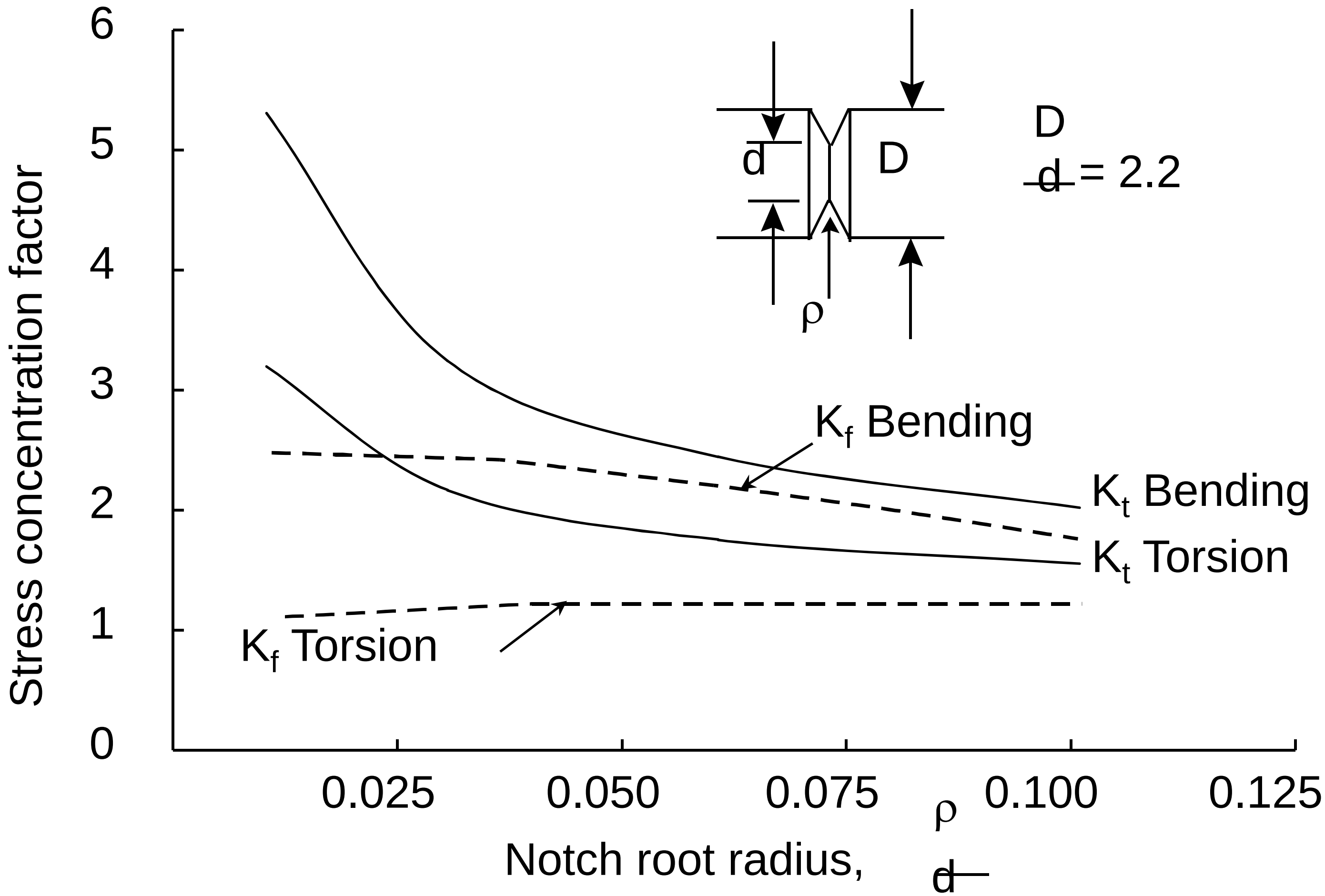


$K_t = 3$

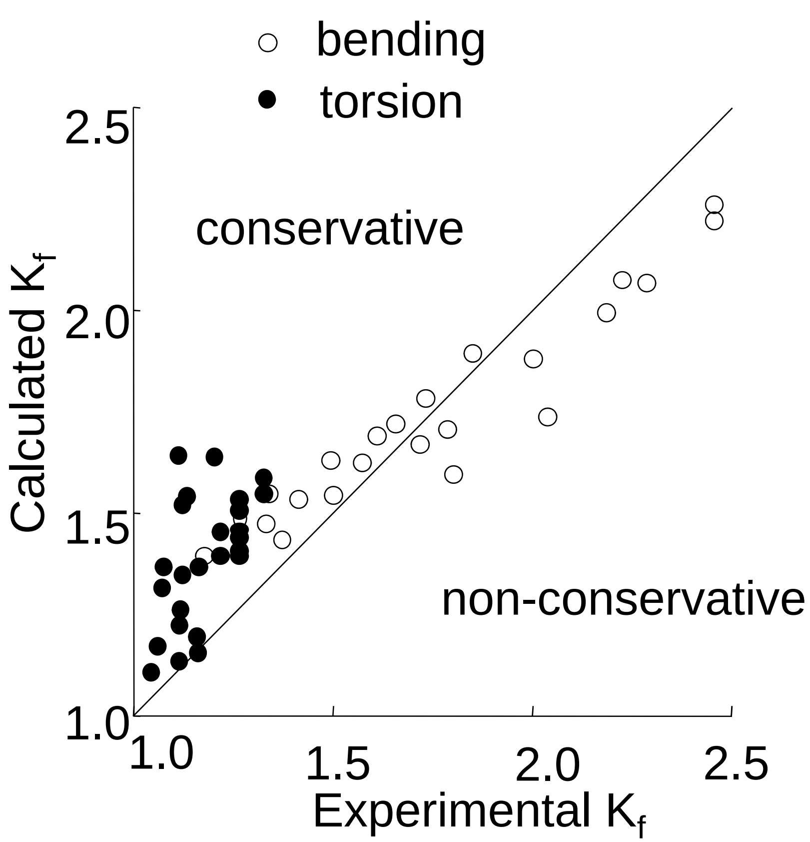


$K_t = 4$

Fatigue Notch Factors



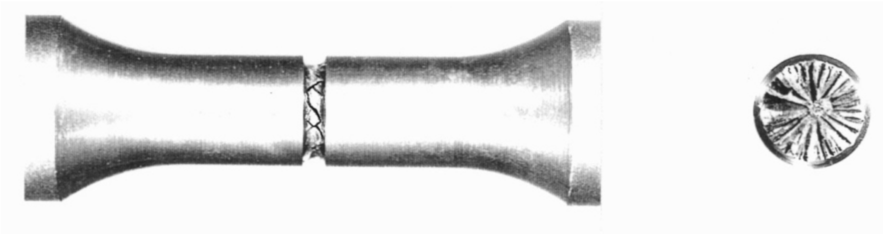
Fatigue Notch Factors (continued)



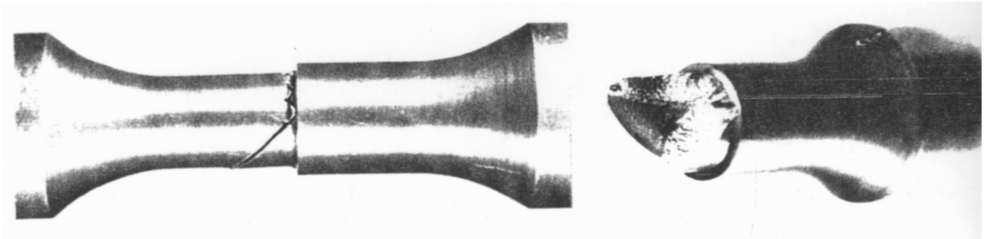
Peterson's Equation

$$K_f = 1 + \frac{K_T - 1}{1 + \frac{a}{r}}$$

Fracture Surfaces in Torsion

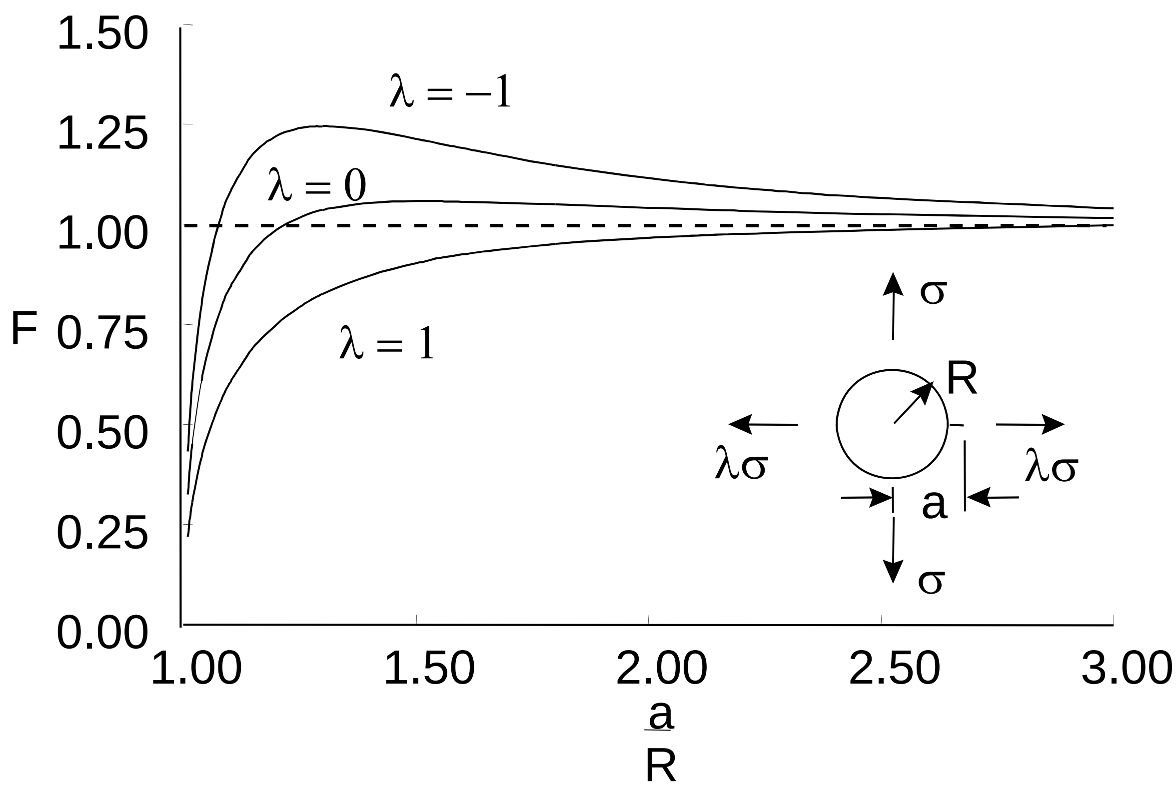


Circumferencial Notch



Shoulder Fillet

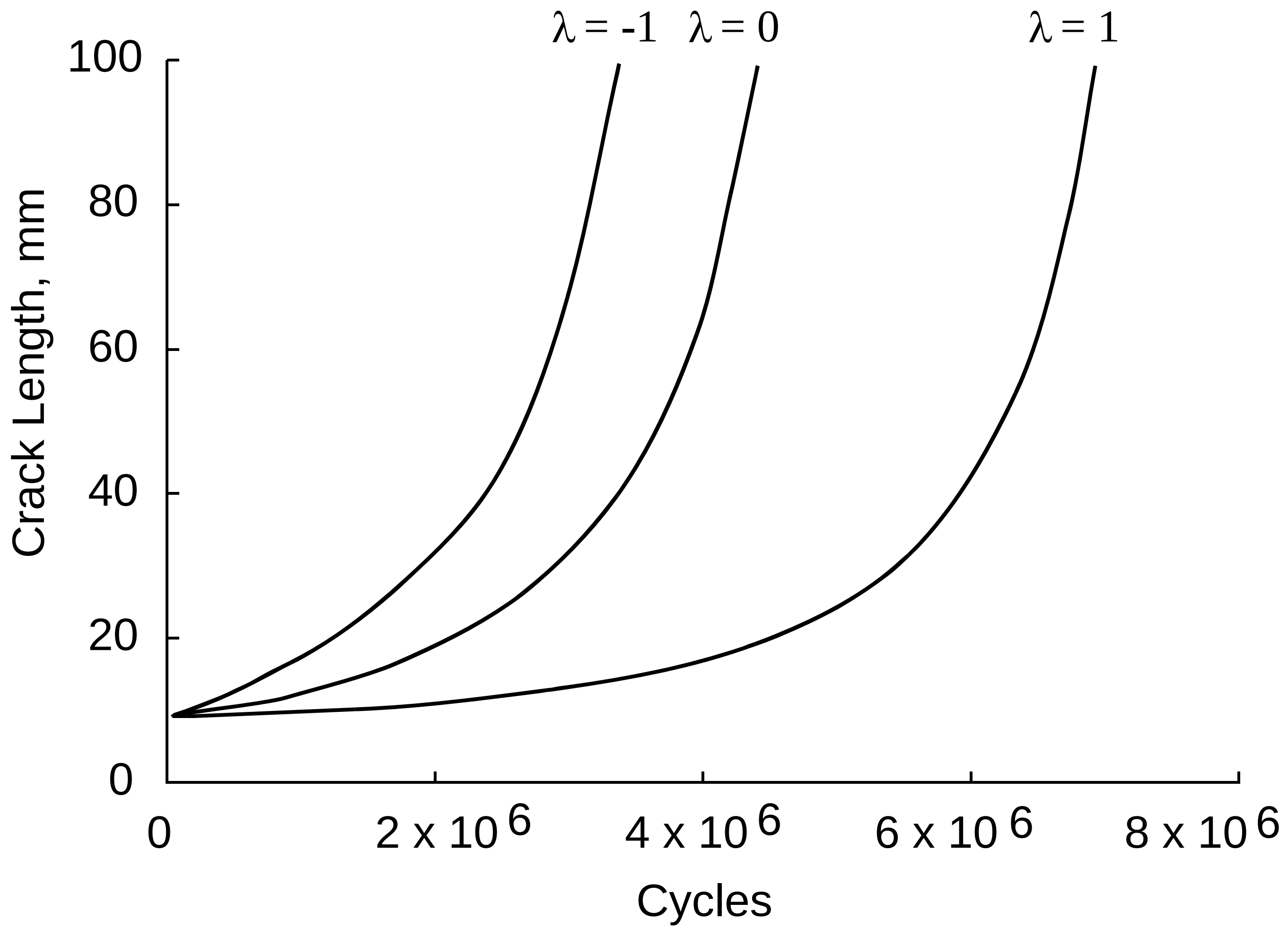
Stress Intensity Factors



$$\frac{da}{dN} = C\left(\Delta K_{eq}\right)^{m}$$

$$K_I = F \Delta\sigma \sqrt{\pi a}$$

Crack Growth From a Hole



Notches Summary

- Uniaxial loading can produce multiaxial stresses at notches
- Multiaxial loading can produce uniaxial stresses at notches
- Multiaxial stresses are not very important in thin plate and shell structures
- Multiaxial stresses are not very important in crack growth

Final Summary

- Fatigue is a planar process involving the growth of cracks on many size scales
- Critical plane models provide reasonable estimates of fatigue damage

Multiaxial Fatigue

University of Illinois at Urbana-Champaign