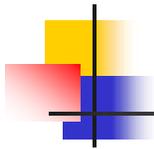


Multiaxial Fatigue

Professor Darrell F. Socie
Department of Mechanical and Industrial Engineering
University of Illinois at Urbana-Champaign

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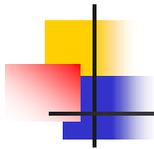
Contact Information

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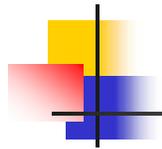
Tel: 217 333 7630

Fax: 217 333 5634



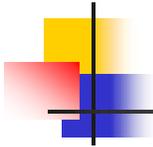
Outline

- State of Stress (Chapter 1)
- Fatigue Mechanisms (Chapter 3)
- Stress Based Models (Chapter 5)
- Strain Based Models (Chapter 6)
- Fracture Mechanics Models (Chapter 7)
- Nonproportional Loading (Chapter 8)
- Notches (Chapter 9)

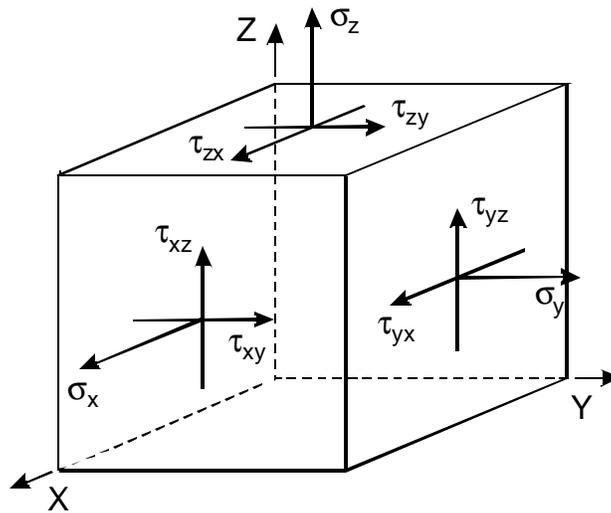


State of Stress

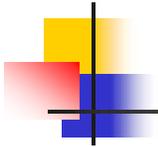
- Stress components
- Common states of stress
- Shear stresses



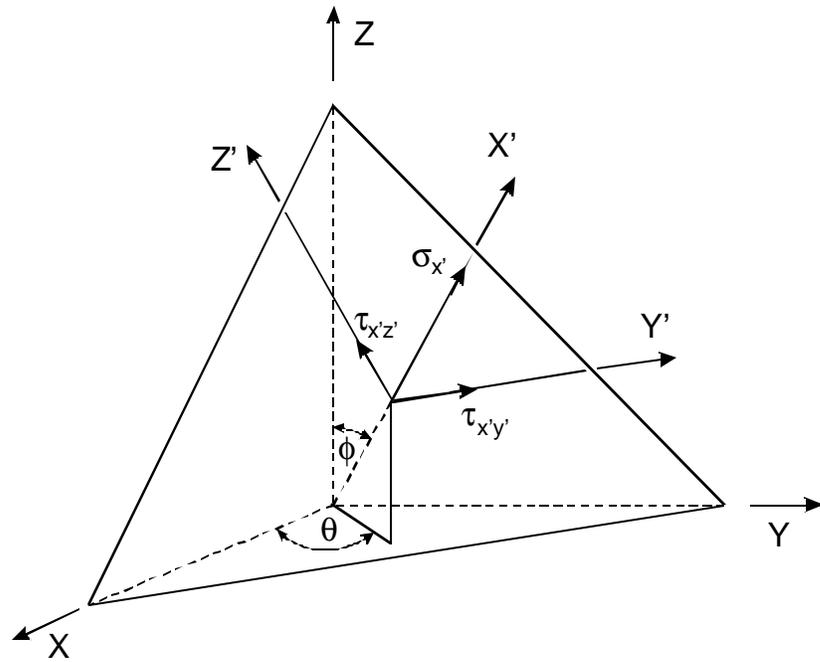
Stress Components

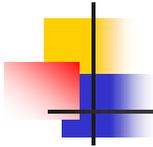


Six stresses and six strains

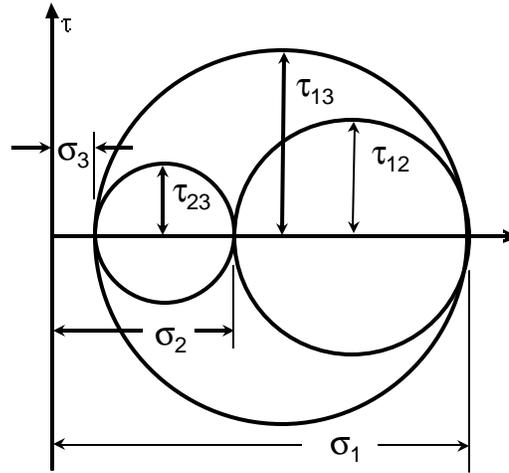


Stresses Acting on a Plane

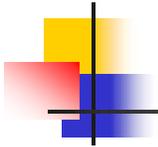




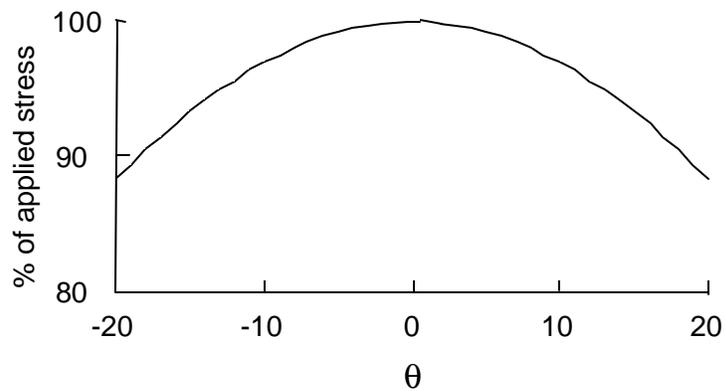
Principal Stresses



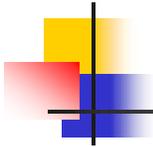
$$\sigma^3 - \sigma^2(\sigma_X + \sigma_Y + \sigma_Z) + \sigma(\sigma_X\sigma_Y + \sigma_Y\sigma_Z + \sigma_X\sigma_Z - \tau_{XY}^2 - \tau_{YZ}^2 - \tau_{XZ}^2) - (\sigma_X\sigma_Y\sigma_Z + 2\tau_{XY}\tau_{YZ}\tau_{XZ} - \sigma_X\tau_{YZ}^2 - \sigma_Y\tau_{ZX}^2 - \sigma_Z\tau_{XY}^2) = 0$$



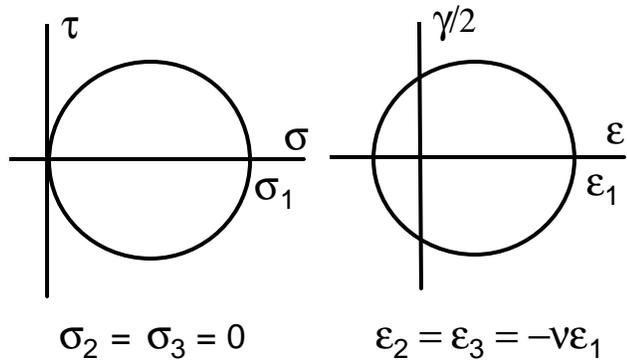
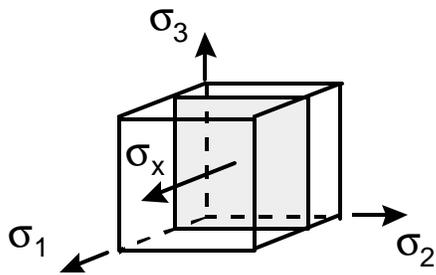
Stress and Strain Distributions

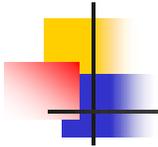


Stresses are nearly the same over a 10° range of angles

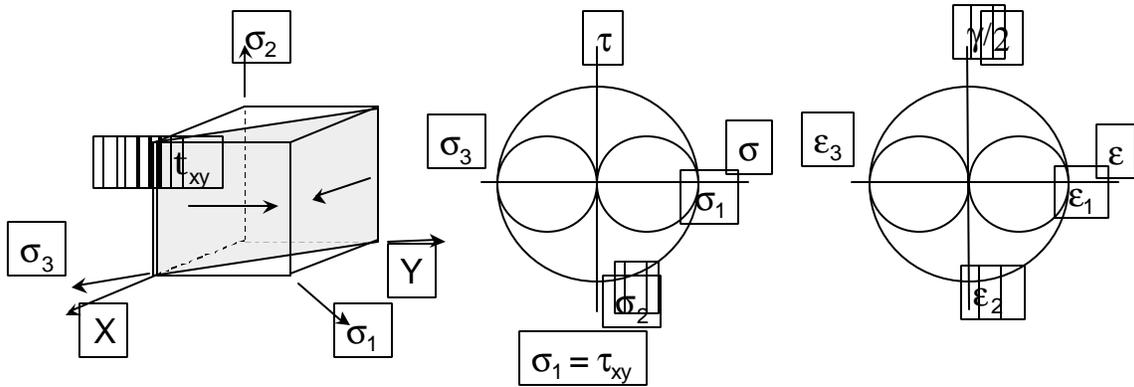


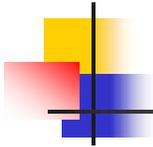
Tension



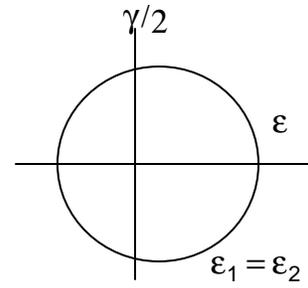
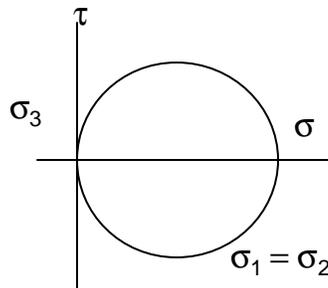
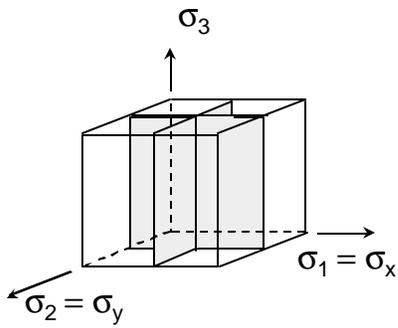


Torsion

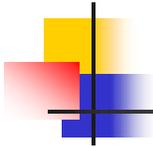




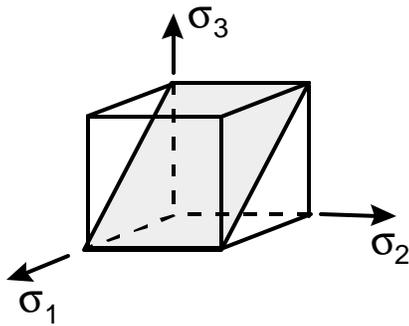
Biaxial Tension



$$\epsilon_3 = -\frac{2\nu}{1-\nu}\epsilon$$



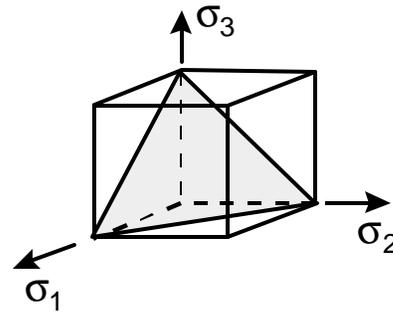
Shear Stresses



Maximum shear stress

$$\tau_{13} = \frac{|\sigma_1 - \sigma_3|}{2}$$

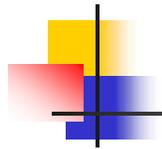
Mises: $\bar{\sigma} = \frac{3}{\sqrt{2}} \tau_{\text{oct}}$



Octahedral shear stress

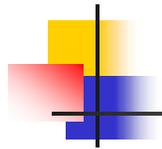
$$\tau_{\text{oct}} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2}$$

$$\tau_{\text{oct}} = \frac{3}{2\sqrt{2}} \tau_{13} = 0.94 \tau_{13}$$



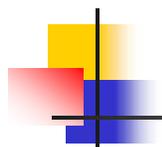
State of Stress Summary

- Stresses acting on a plane
- Principal stress
- Maximum shear stress
- Octahedral shear stress

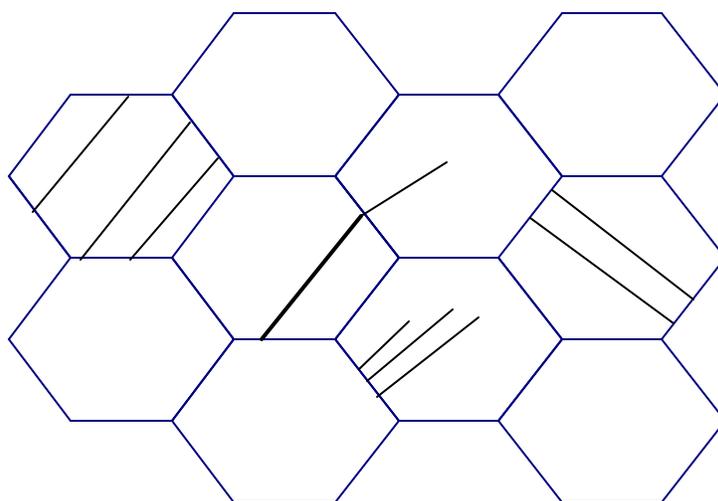


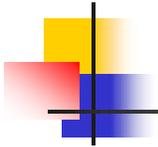
Fatigue Mechanisms

- Crack nucleation
- Fracture modes
- Crack growth
- State of stress effects

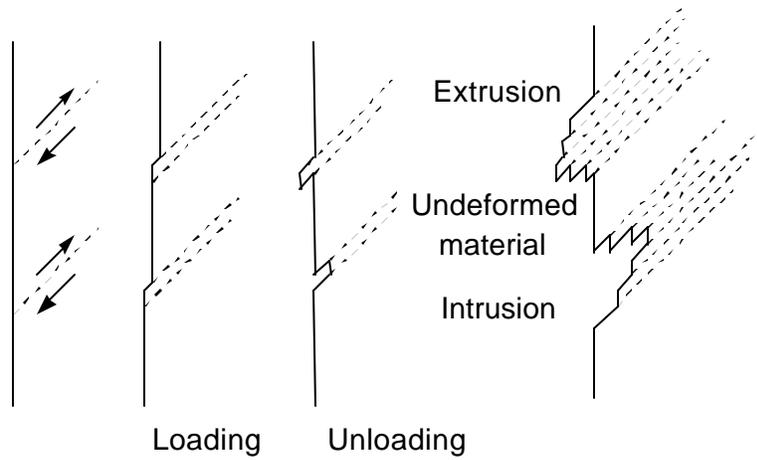


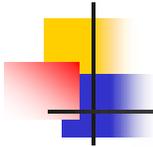
Crack Nucleation



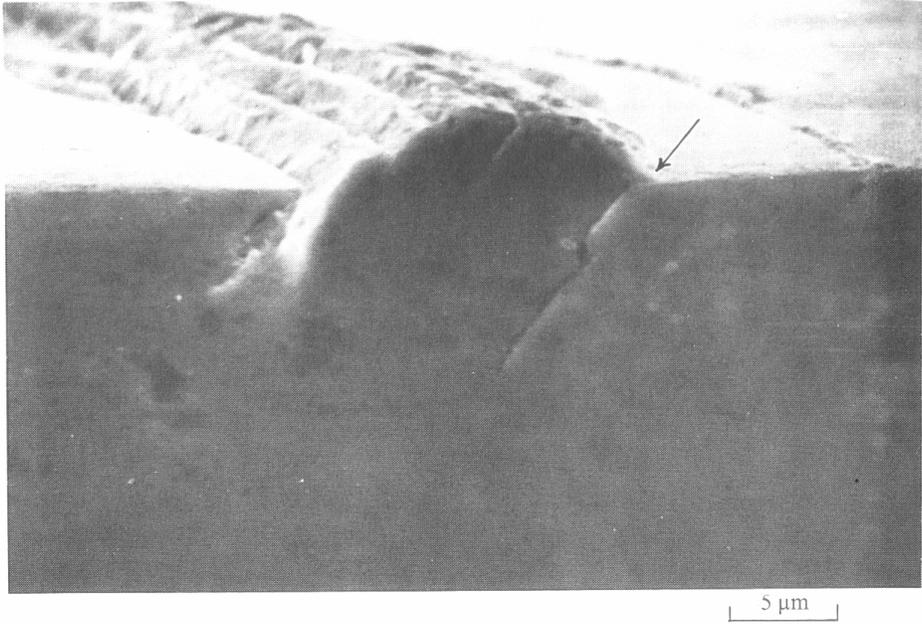


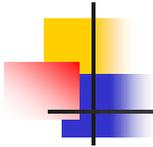
Slip Bands





Slip Bands



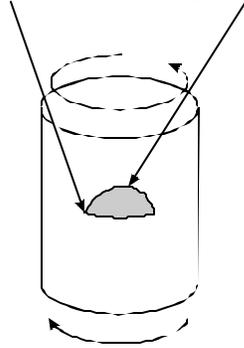
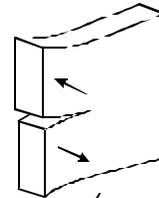
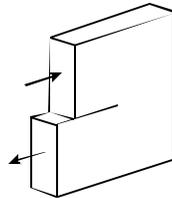
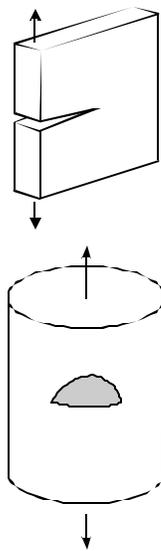


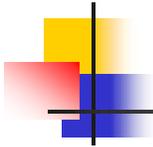
Mode I, Mode II, and Mode III

Mode I
opening

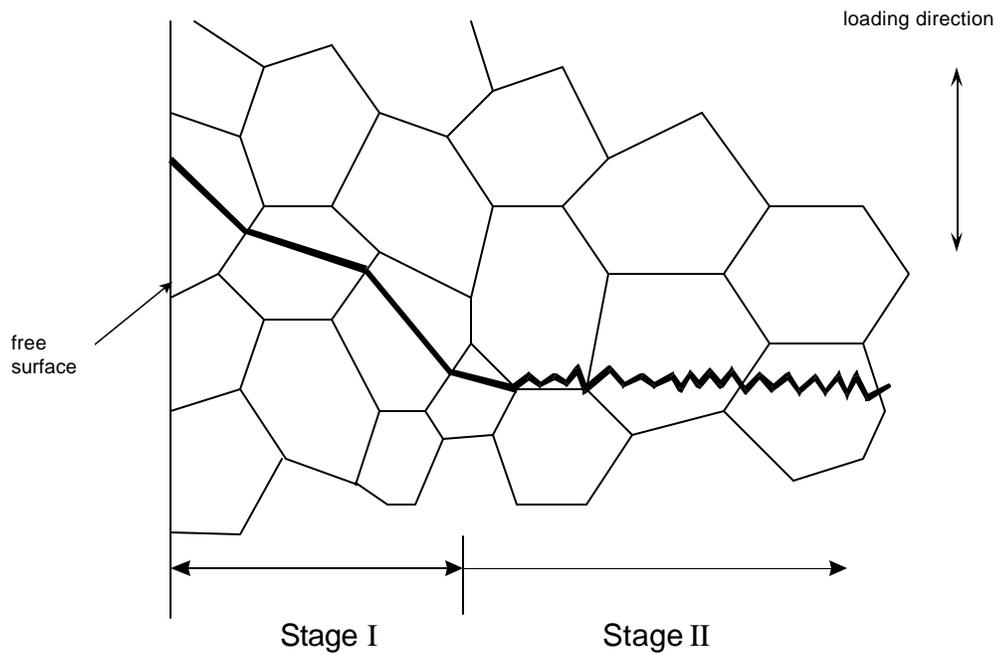
Mode II
in-plane shear

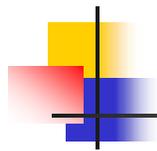
Mode III
out-of-plane shear



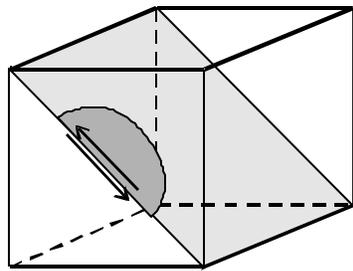


Stage I and Stage II



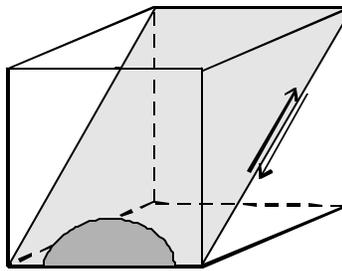


Case A and Case B



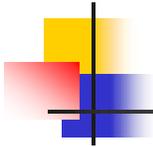
Case A

Growth along the surface

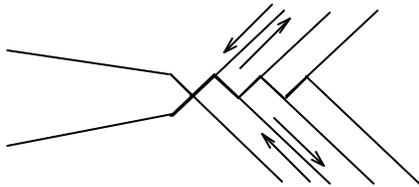


Case B

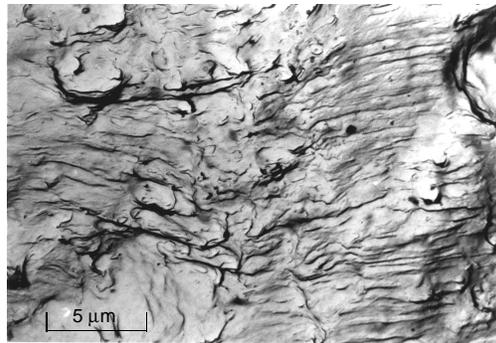
Growth into the surface

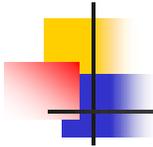


Mode I Growth

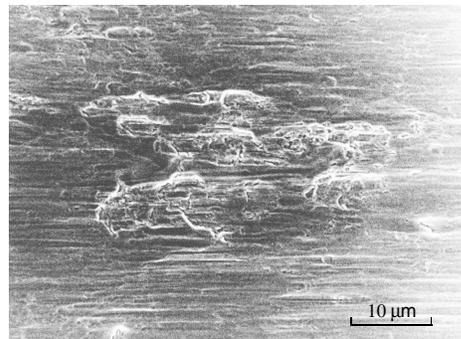
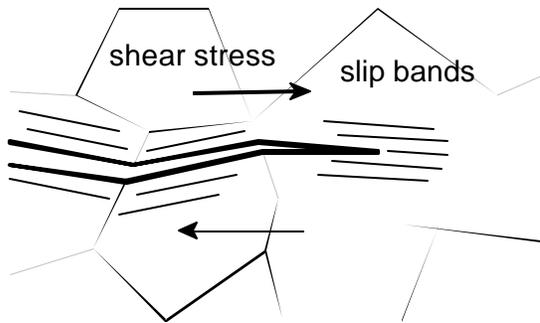


crack growth direction ↑



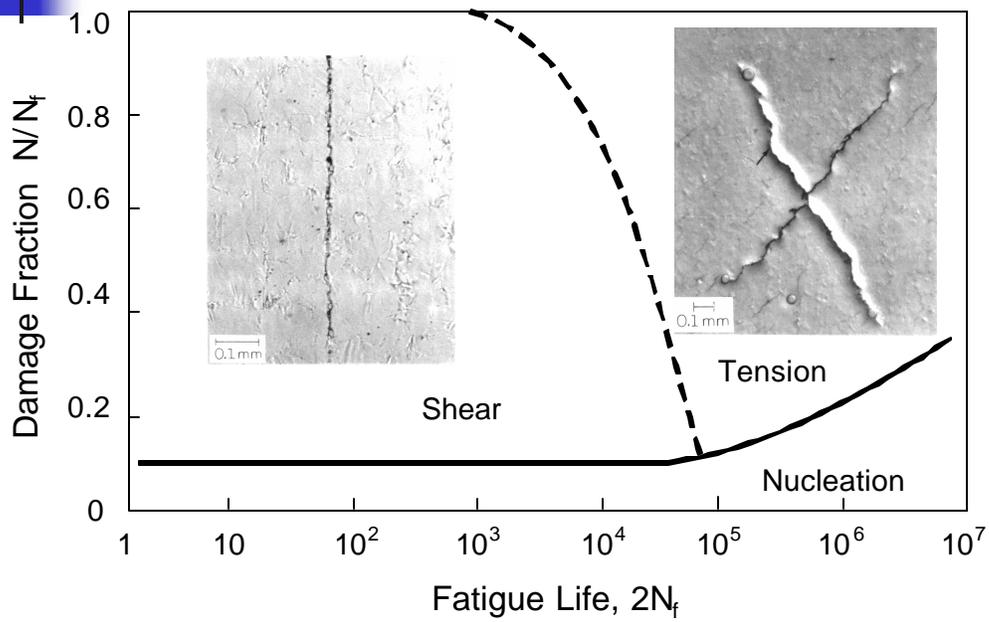


Mode II Growth

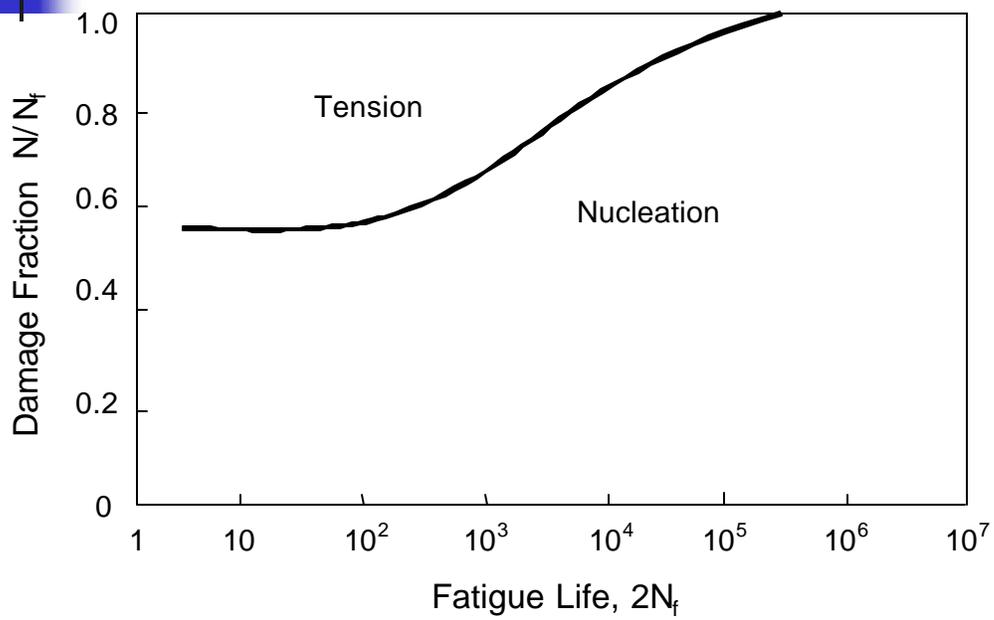


← crack growth direction

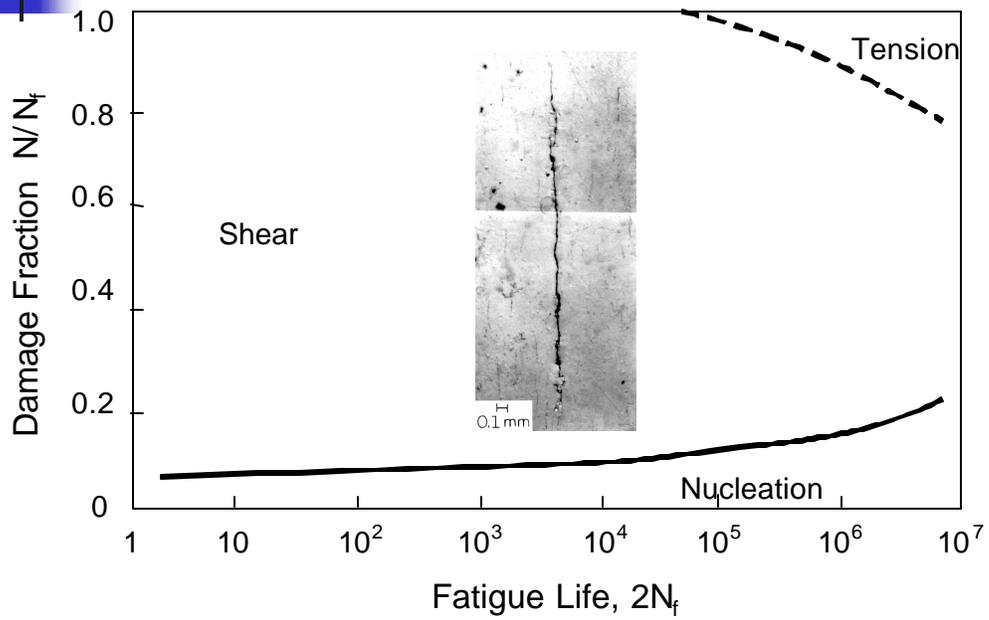
304 Stainless Steel - Torsion



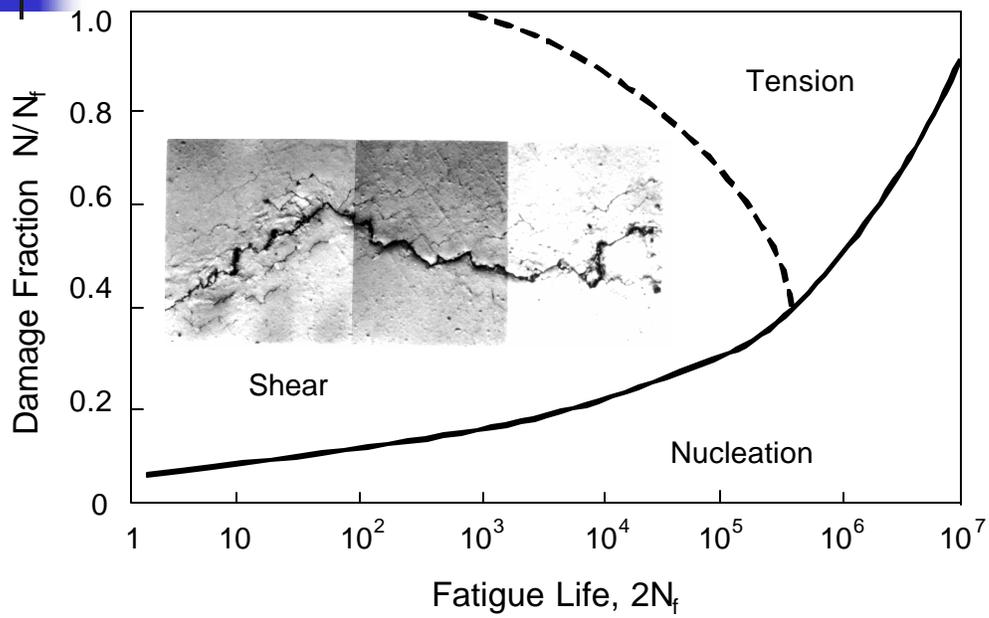
304 Stainless Steel - Tension

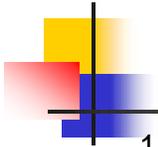


Inconel 718 - Torsion

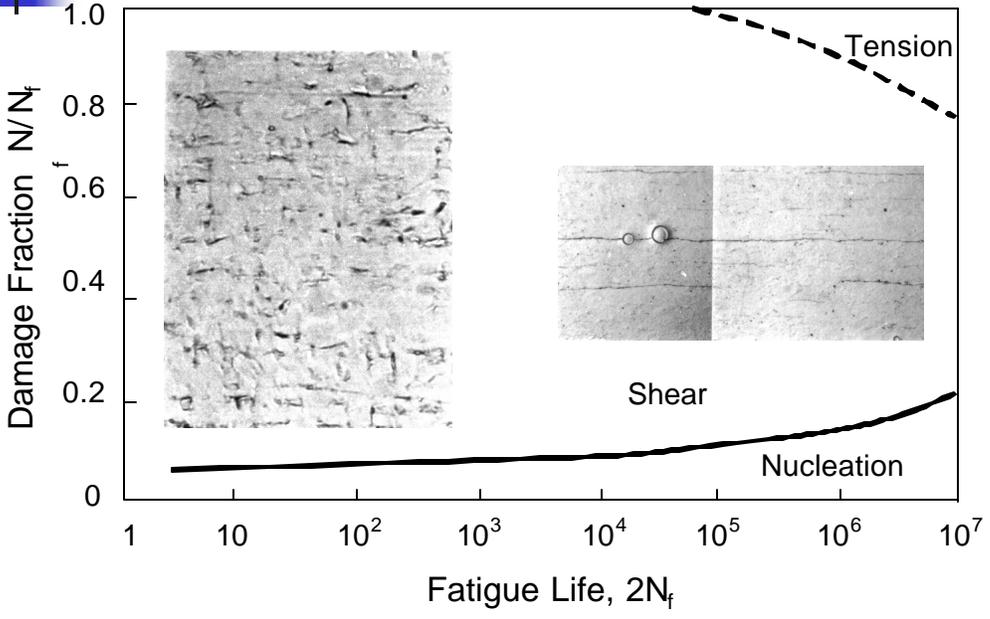


Inconel 718 - Tension

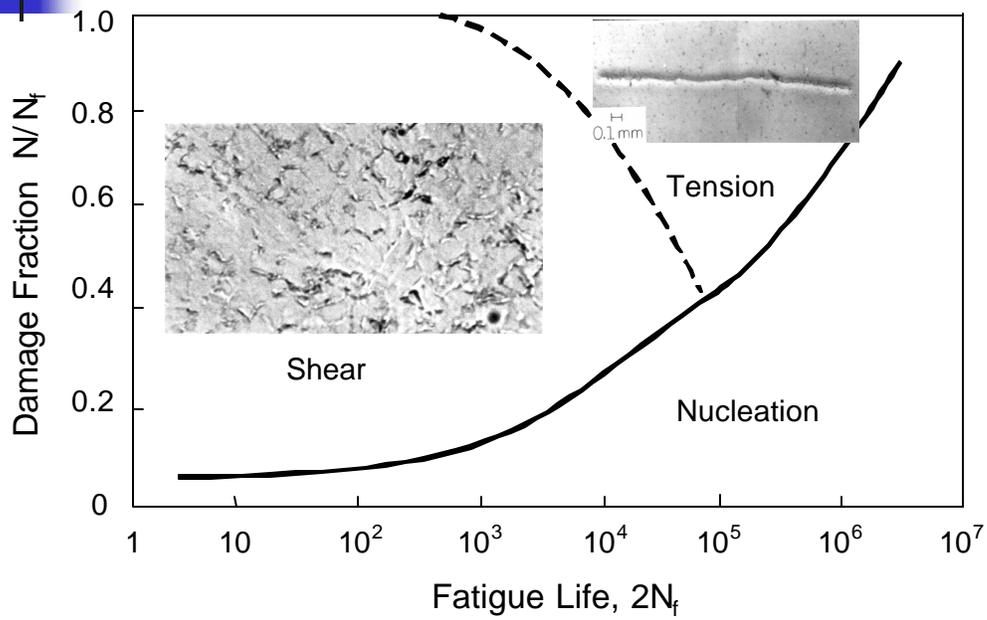


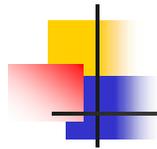


1045 Steel - Torsion



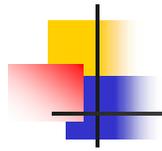
1045 Steel - Tension





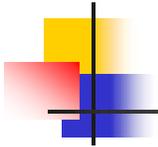
Fatigue Mechanisms Summary

- Fatigue cracks nucleate in shear
- Fatigue cracks grow in either shear or tension depending on material and state of stress

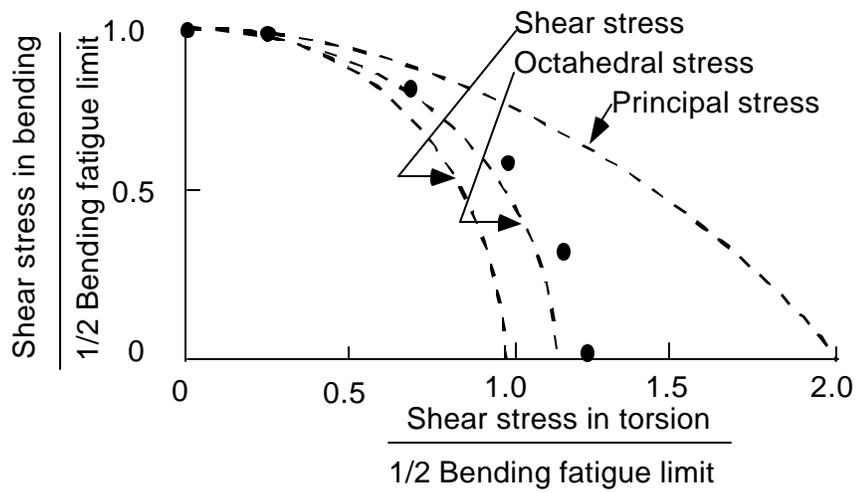


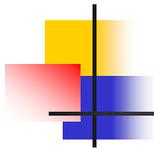
Stress Based Models

- Sines
- Findley
- Dang Van



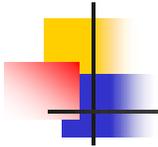
Bending Torsion Correlation



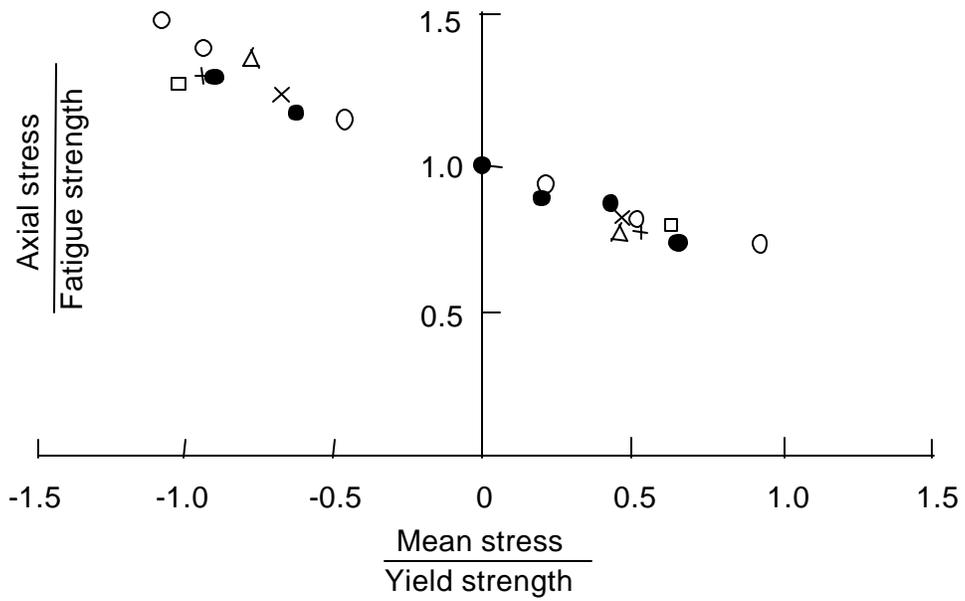


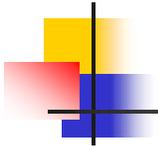
Test Results

- Cyclic tension with static tension
- Cyclic torsion with static torsion
- Cyclic tension with static torsion
- Cyclic torsion with static tension

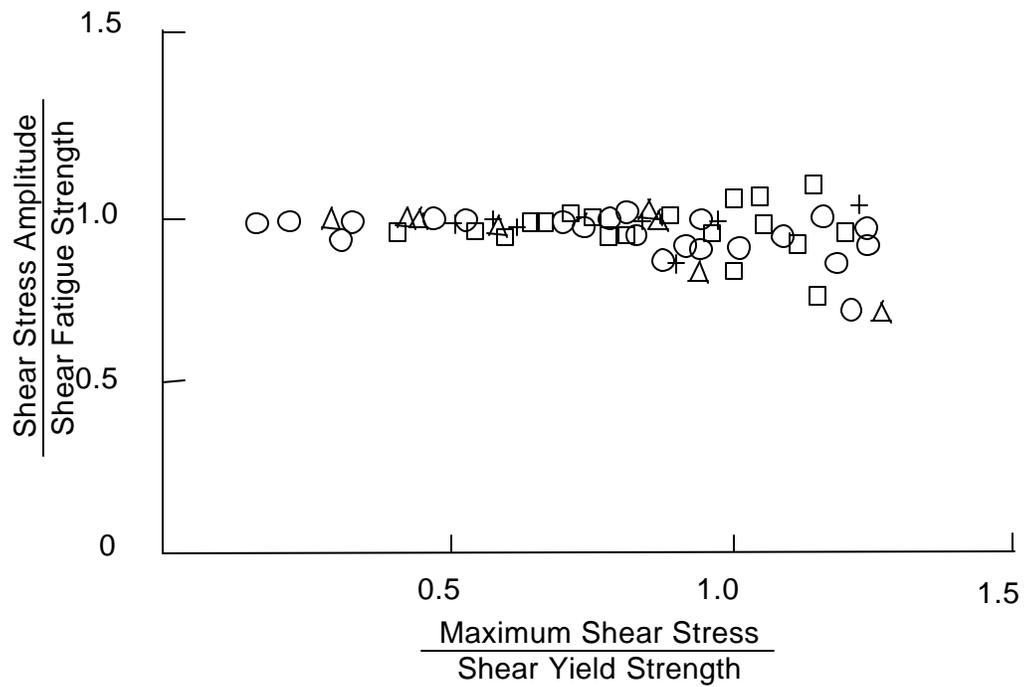


Cyclic Tension with Static Tension

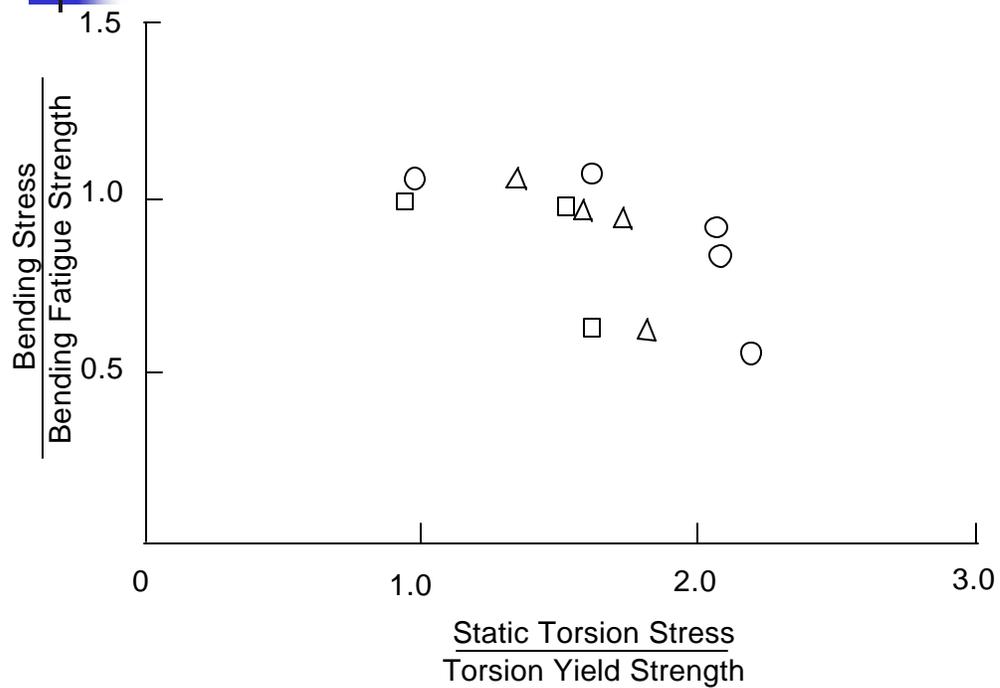


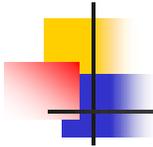


Cyclic Torsion with Static Torsion

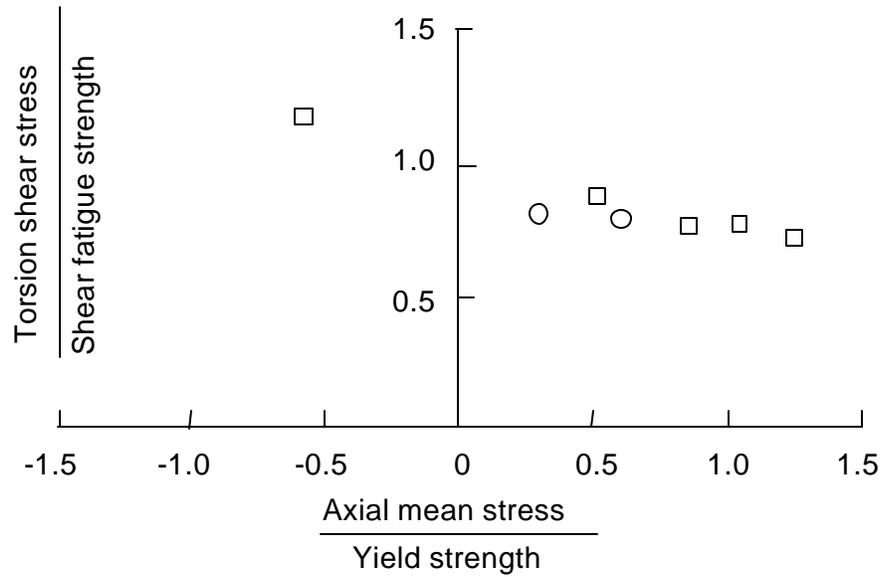


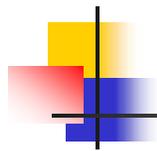
Cyclic Tension with Static Torsion





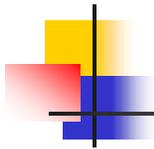
Cyclic Torsion with Static Tension





Conclusions

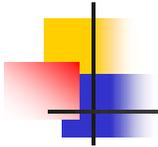
- Tension mean stress affects both tension and torsion
- Torsion mean stress does not affect tension or torsion



Sines

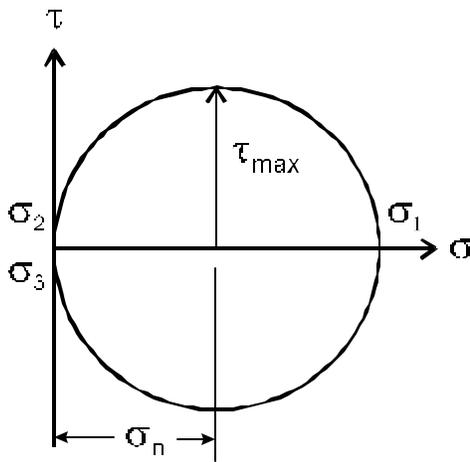
$$\frac{\Delta\tau_{\text{oct}}}{2} + \alpha(3\sigma_h) = \beta$$

$$\frac{1}{6}\sqrt{(\Delta\sigma_x - \Delta\sigma_y)^2 + (\Delta\sigma_x - \Delta\sigma_z)^2 + (\Delta\sigma_y - \Delta\sigma_z)^2 + 6(\Delta\tau_{xy}^2 + \Delta\tau_{xz}^2 + \Delta\tau_{yz}^2)} + \alpha(\sigma_x^{\text{mean}} + \sigma_y^{\text{mean}} + \sigma_z^{\text{mean}}) = \beta$$

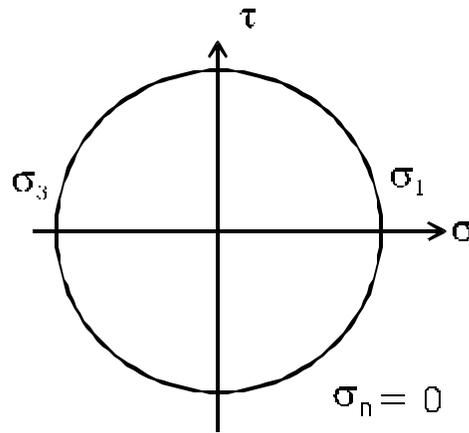


Findley

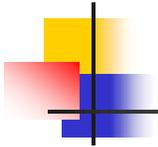
$$\left(\frac{\Delta\tau}{2} + k\sigma_n \right)_{\max} = f$$



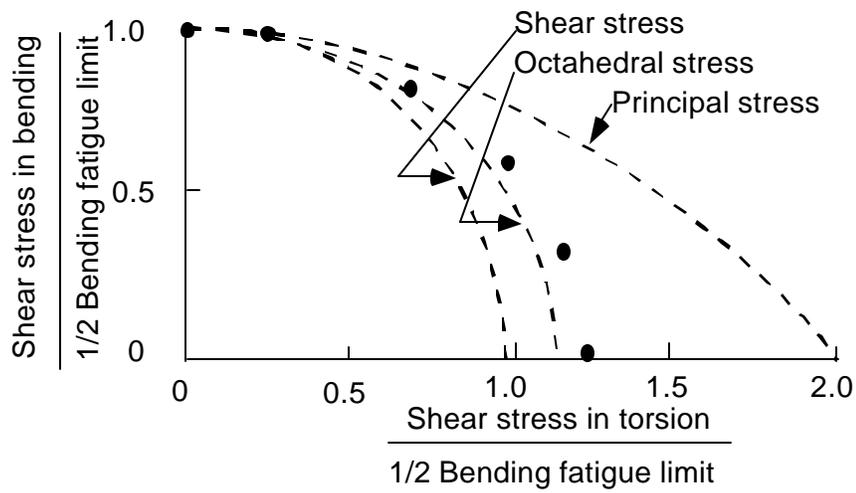
tension

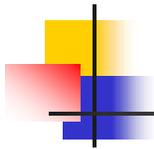


torsion



Bending Torsion Correlation

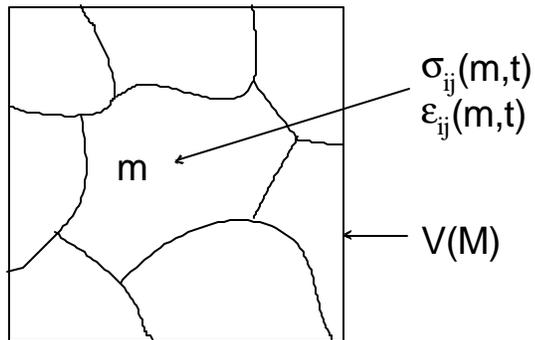


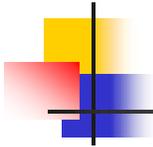


Dang Van

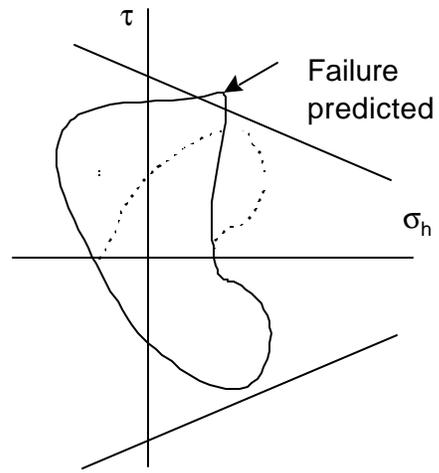
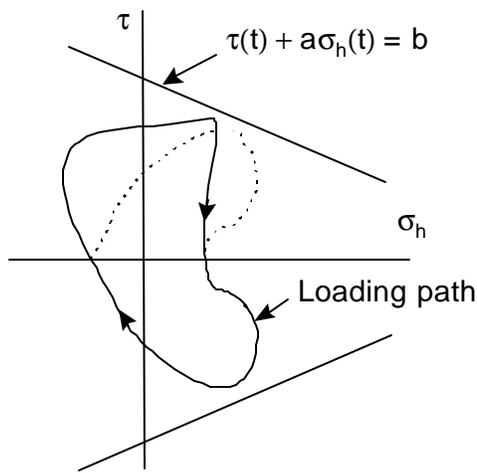
$$\tau(t) + a\sigma_h(t) = b$$

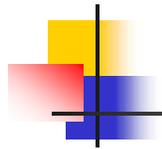
$$\Sigma_{ij}(M,t) \quad E_{ij}(M,t)$$





Dang Van (continued)



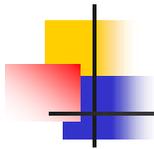


Stress Based Models Summary

Sines:
$$\frac{\Delta\tau_{\text{oct}}}{2} + \alpha(3\sigma_h) = \beta$$

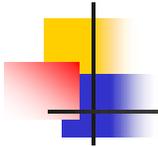
Findley:
$$\left(\frac{\Delta\tau}{2} + k\sigma_n \right)_{\text{max}} = f$$

Dang Van:
$$\tau(t) + a\sigma_h(t) = b$$

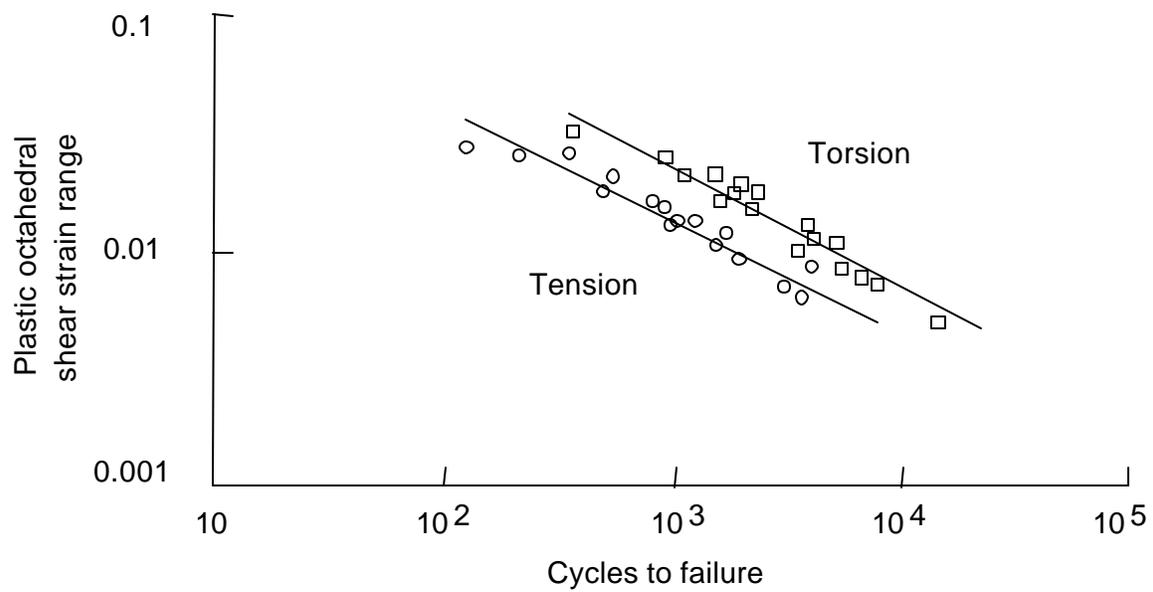


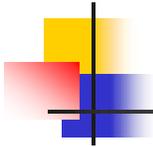
Strain Based Models

- Plastic Work
- Brown and Miller
- Fatemi and Socie
- Smith Watson and Topper
- Liu

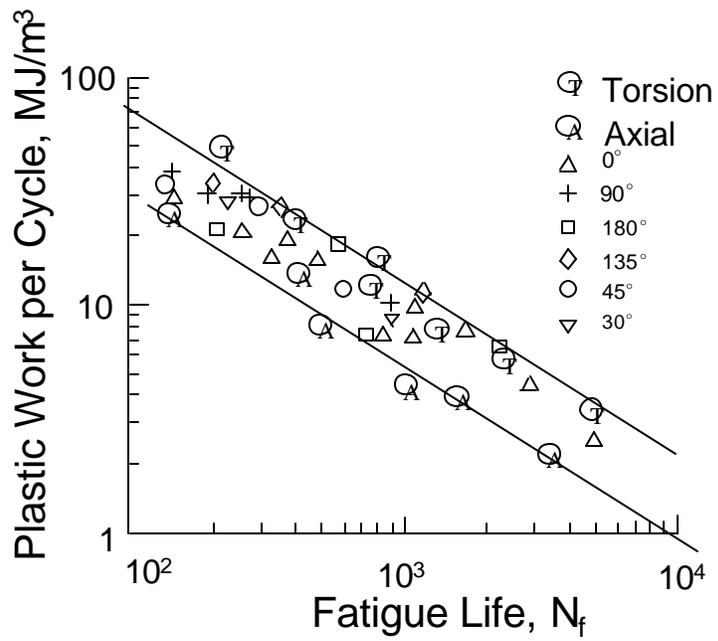


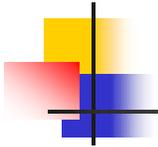
Octahedral Shear Strain



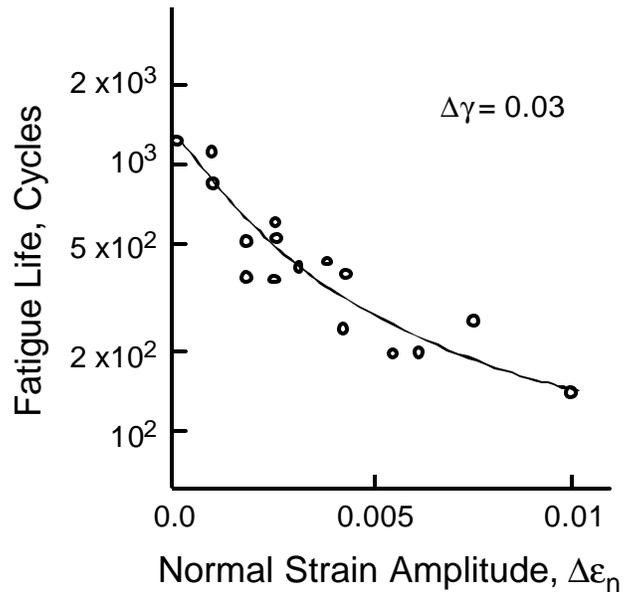


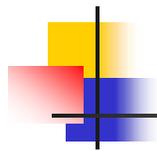
Plastic Work



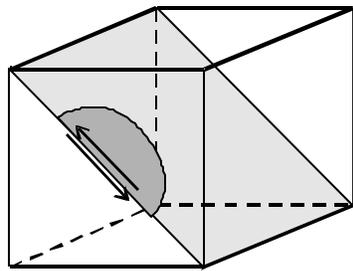


Brown and Miller



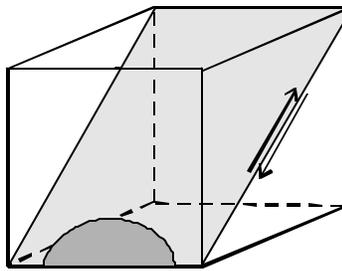


Case A and B



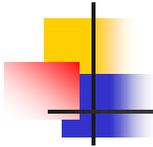
Case A

Growth along the surface

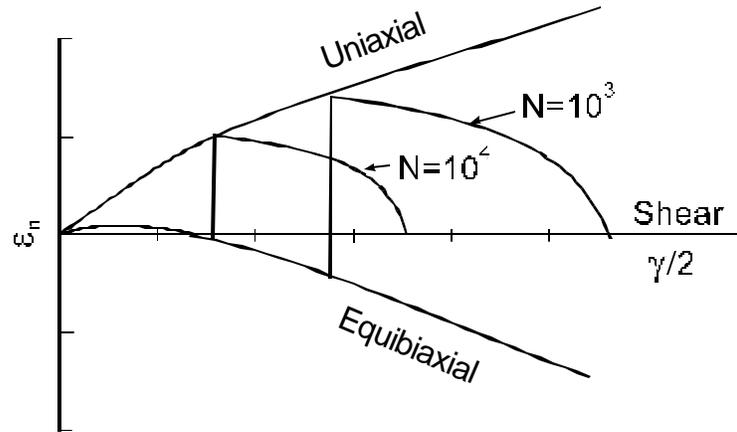


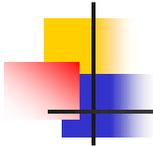
Case B

Growth into the surface



Brown and Miller (continued)

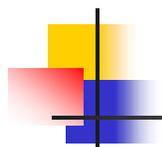




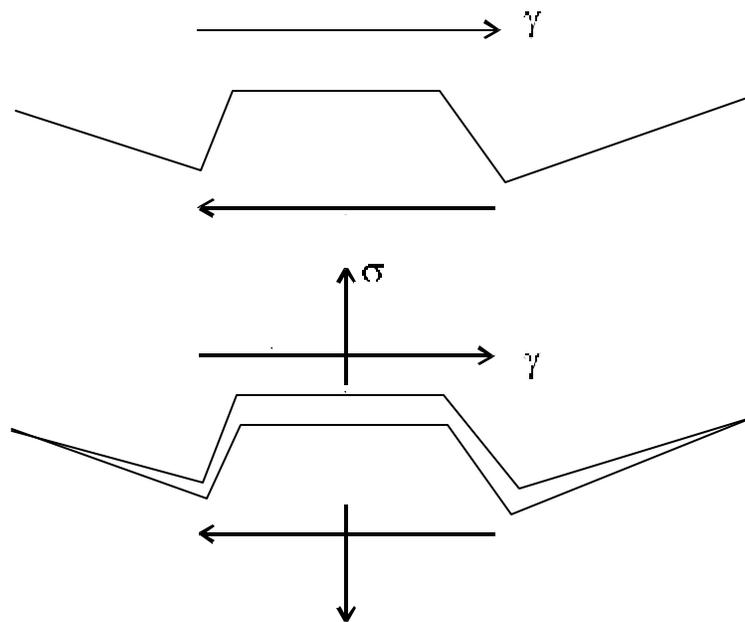
Brown and Miller (continued)

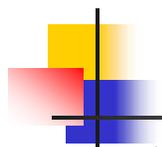
$$\Delta \hat{\gamma} = \left(\Delta \gamma_{\max}^{\alpha} + S \Delta \epsilon_n^{\alpha} \right)^{\frac{1}{\alpha}}$$

$$\frac{\Delta \gamma_{\max}}{2} + S \Delta \epsilon_n = A \frac{\sigma_f' - 2\sigma_{n,\text{mean}}}{E} (2N_f)^b + B \epsilon_f' (2N_f)^c$$

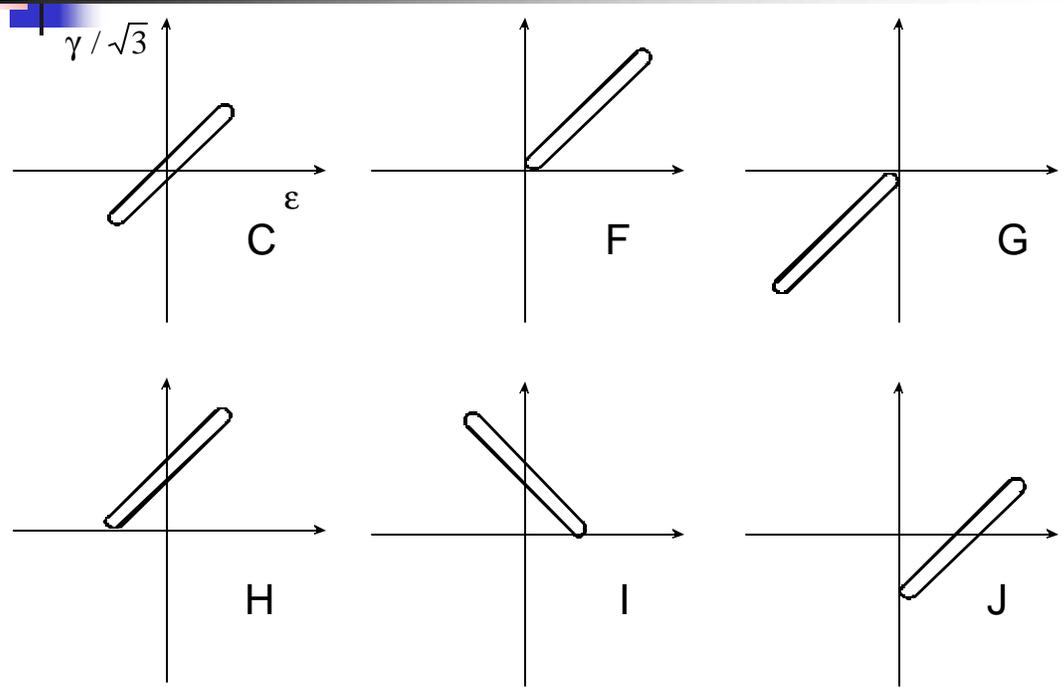


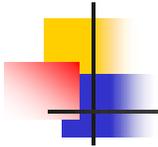
Fatemi and Socie



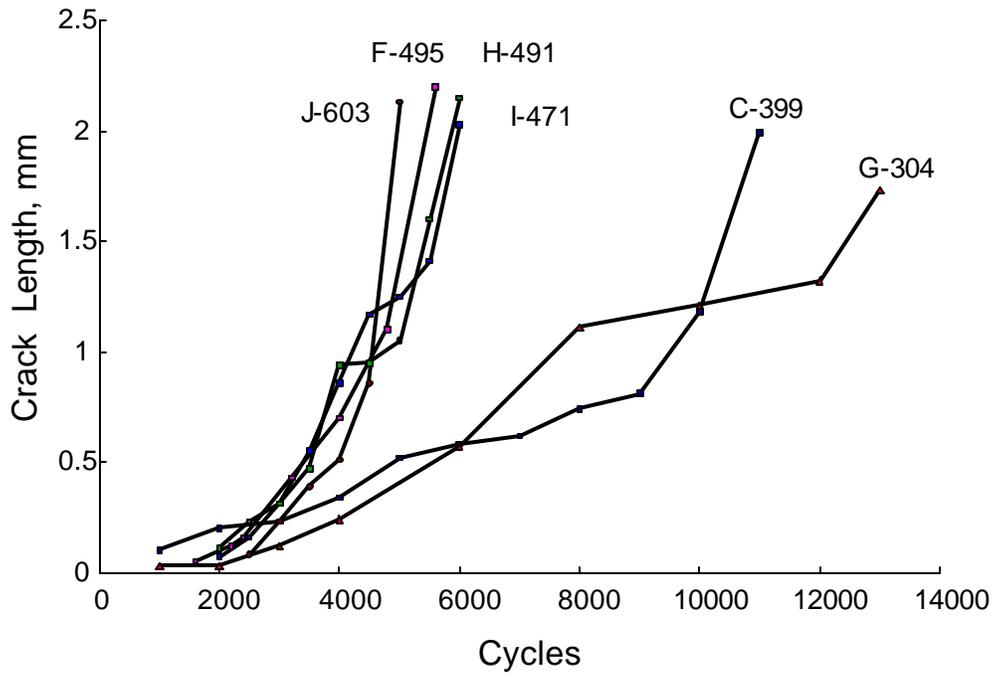


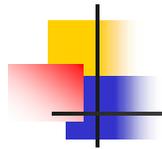
Loading Histories





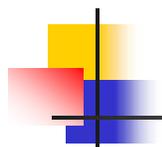
Crack Length Observations



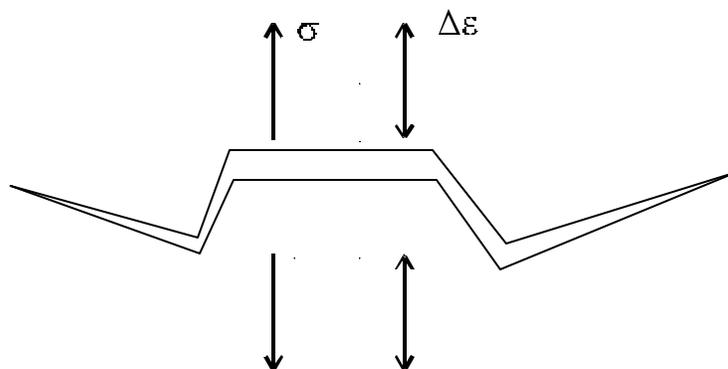


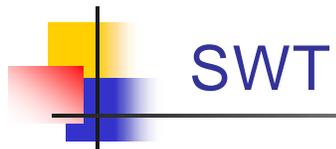
Fatemi and Socie

$$\frac{\Delta\gamma}{2} \left(1 + k \frac{\sigma_{n,\max}}{\sigma_y} \right) = \frac{\tau_f'}{G} (2N_f)^{b_0} + \gamma_f' (2N_f)^{c_0}$$

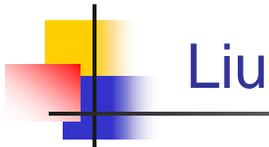


Smith Watson Topper





$$\sigma_n \frac{\Delta \epsilon_1}{2} = \frac{\sigma_f'^2}{E} (2N_f)^{2b} + \sigma_f' \epsilon_f' (2N_f)^{b+c}$$



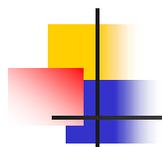
Virtual strain energy for both mode I and mode II cracking

$$\Delta W_{\text{I}} = (\Delta\sigma_{\text{n}} \Delta\varepsilon_{\text{n}})_{\text{max}} + (\Delta\tau \Delta\gamma)$$

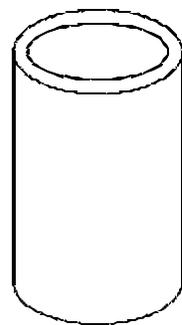
$$\Delta W_{\text{I}} = 4\sigma_{\text{f}}' \varepsilon_{\text{f}}' (2N_{\text{f}})^{b+c} + \frac{4\sigma_{\text{f}}'^2}{E} (2N_{\text{f}})^{2b}$$

$$\Delta W_{\text{II}} = (\Delta\sigma_{\text{n}} \Delta\varepsilon_{\text{n}}) + (\Delta\tau \Delta\gamma)_{\text{max}}$$

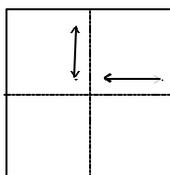
$$\Delta W_{\text{II}} = 4\tau_{\text{f}}' \gamma_{\text{f}}' (2N_{\text{f}})^{b_0+c_0} + \frac{4\tau_{\text{f}}'^2}{G} (2N_{\text{f}})^{2b_0}$$



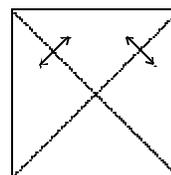
Cyclic Torsion



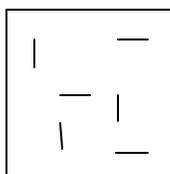
Cyclic Torsion



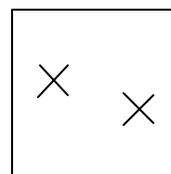
Cyclic Shear Strain



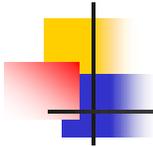
Cyclic Tensile Strain



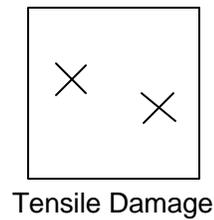
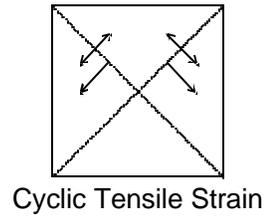
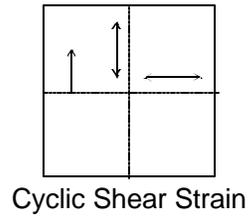
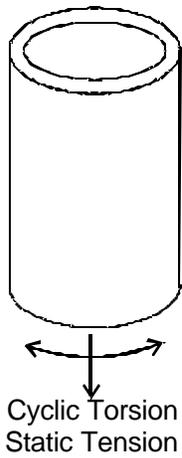
Shear Damage

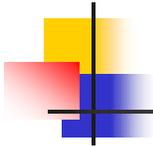


Tensile Damage

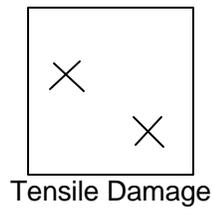
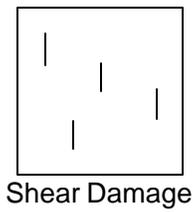
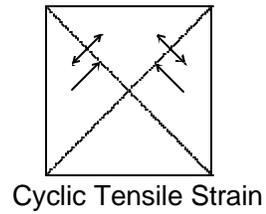
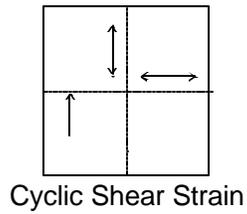
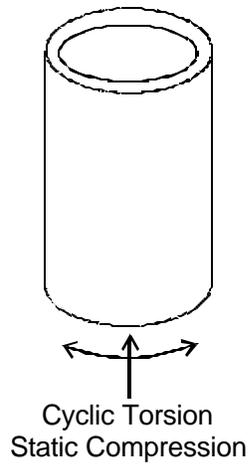


Cyclic Torsion with Static Tension

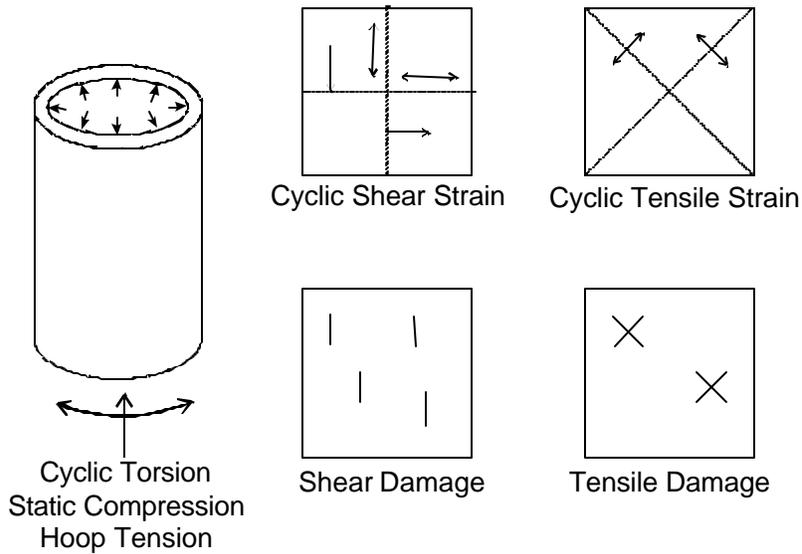
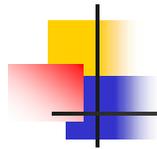


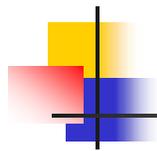


Cyclic Torsion with Compression



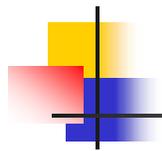
Cyclic Torsion with Tension and Compression





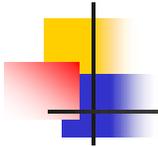
Test Results

Load Case	$\Delta\gamma/2$	σ_{hoop} MPa	σ_{axial} MPa	N_f
Torsion	0.0054	0	0	45,200
with tension	0.0054	0	450	10,300
with compression	0.0054	0	-500	50,000
with tension and compression	0.0054	450	-500	11,200

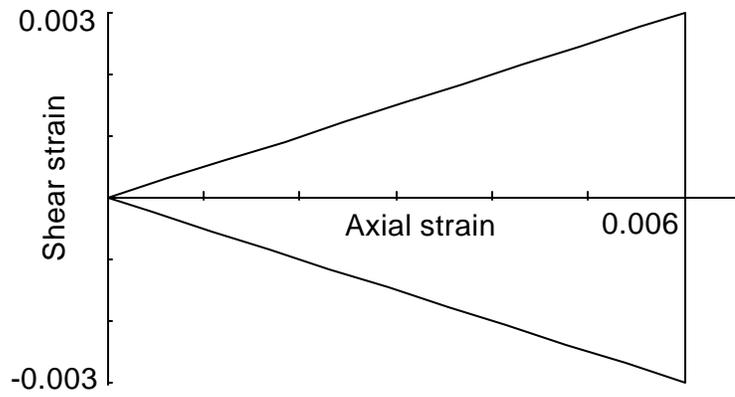


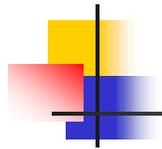
Conclusions

- All critical plane models correctly predict these results
- Hydrostatic stress models can not predict these results



Loading History

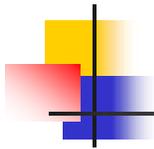




Model Comparison

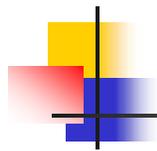
Summary of calculated fatigue lives

Model	Equation	Life
Epsilon	6.5	14,060
Garud	6.7	5,210
Ellyin	6.17	4,450
Brown-Miller	6.22	3,980
SWT	6.24	9,930
Liu I	6.41	4,280
Liu II	6.42	5,420
Chu	6.37	3,040
Gamma		26,775
Fatemi-Socie	6.23	10,350
Glinka	6.39	33,220

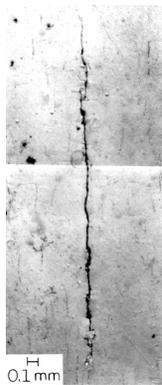
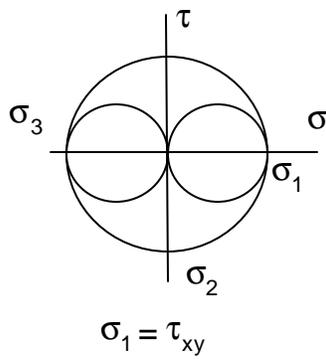


Strain Based Models Summary

- Two separate models are needed, one for tensile growth and one for shear growth
- Cyclic plasticity governs stress and strain ranges
- Mean stress effects are a result of crack closure on the critical plane



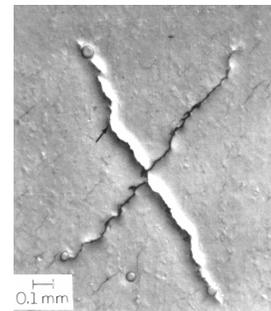
Separate Tensile and Shear Models



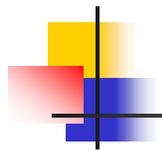
Inconel



1045 steel



stainless steel



Cyclic Plasticity

$$\Delta\varepsilon$$

$$\Delta\gamma$$

$$\Delta\varepsilon^p$$

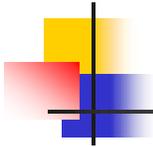
$$\Delta\gamma^p$$

$$\Delta\varepsilon\Delta\sigma$$

$$\Delta\gamma\Delta\tau$$

$$\Delta\varepsilon^p\Delta\sigma$$

$$\Delta\gamma^p\Delta\tau$$



Mean Stresses

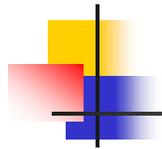
$$\Delta \varepsilon_{eq} = \frac{\sigma_f' - \sigma_{mean}}{E} (2N_f)^b + \varepsilon_f' (2N_f)^c$$

$$\frac{\Delta \gamma_{max}}{2} + S \Delta \varepsilon_n = (1.3 + 0.7S) \frac{\sigma_f' - 2\sigma_n}{E} (2N_f)^b + (1.5 + 0.5S) \varepsilon_f' (2N_f)^c$$

$$\frac{\Delta \gamma}{2} \left(1 + k \frac{\sigma_{n,max}}{\sigma_y} \right) = \frac{\tau_f'}{G} (2N_f)^{b_0} + \gamma_f' (2N_f)^{c_0}$$

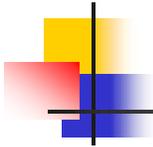
$$\sigma_n \frac{\Delta \varepsilon_1}{2} = \frac{\sigma_f'^2}{E} (2N_f)^{2b} + \sigma_f' \varepsilon_f' (2N_f)^{b+c}$$

$$\Delta W_I = [(\Delta \sigma_n \Delta \varepsilon_n)_{max} + (\Delta \tau \Delta \gamma)] \left(\frac{2}{1-R} \right)$$



Fracture Mechanics Models

- Mode I growth
- Torsion
- Mode II growth
- Mode III growth

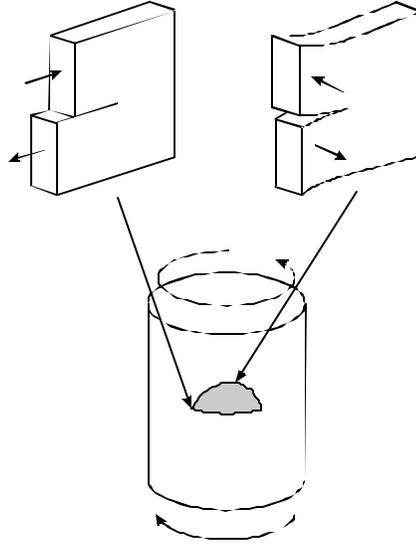
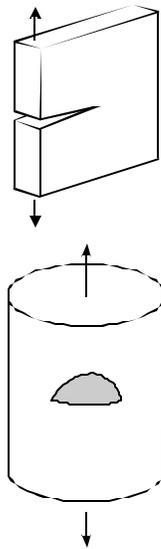


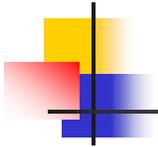
Mode I, Mode II, and Mode III

Mode I
opening

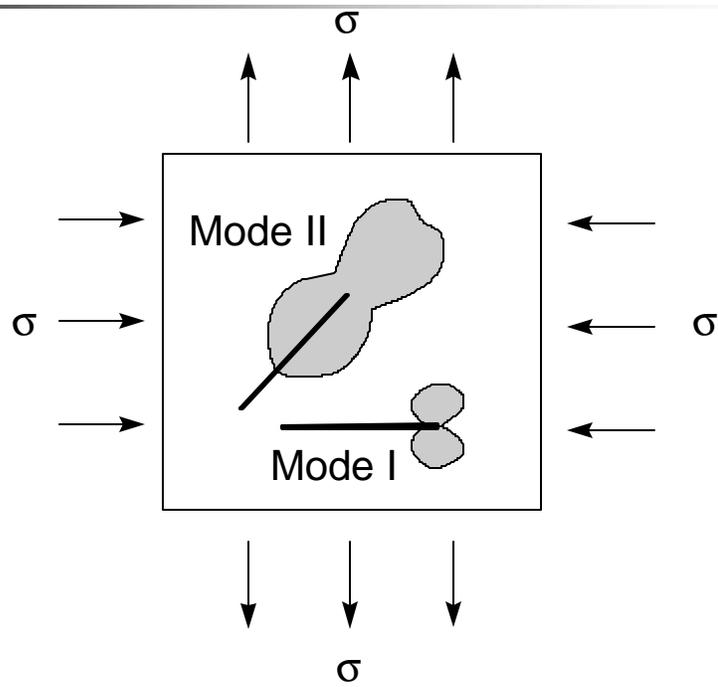
Mode II
in-plane shear

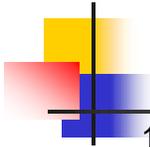
Mode III
out-of-plane shear



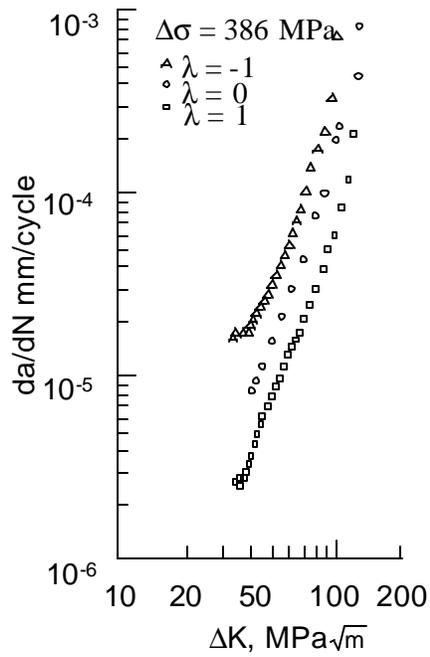
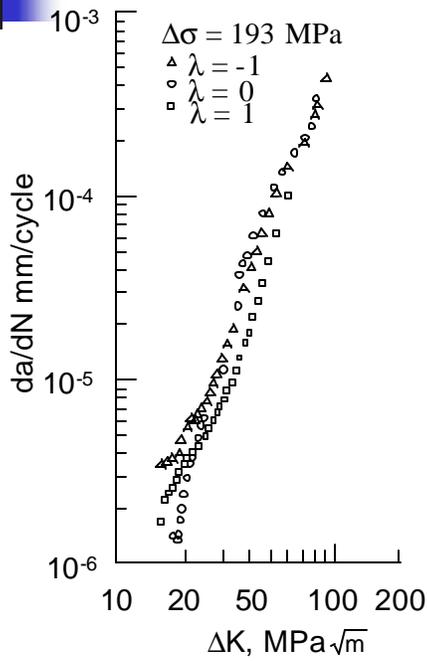


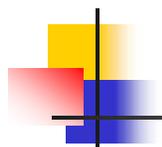
Mode I and Mode II Surface Cracks



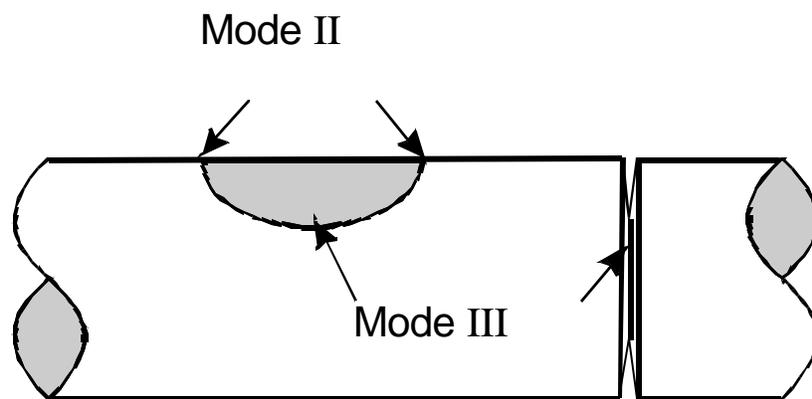


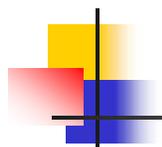
Biaxial Mode I Growth



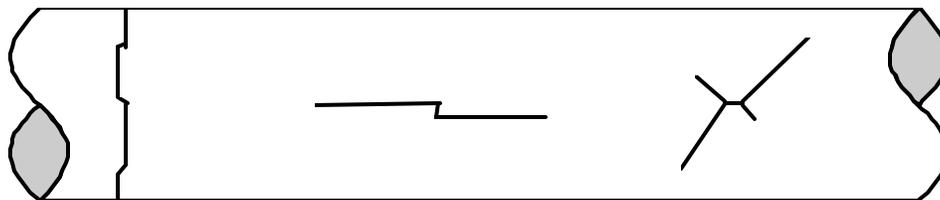


Surface Cracks in Torsion





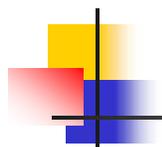
Failure Modes in Torsion



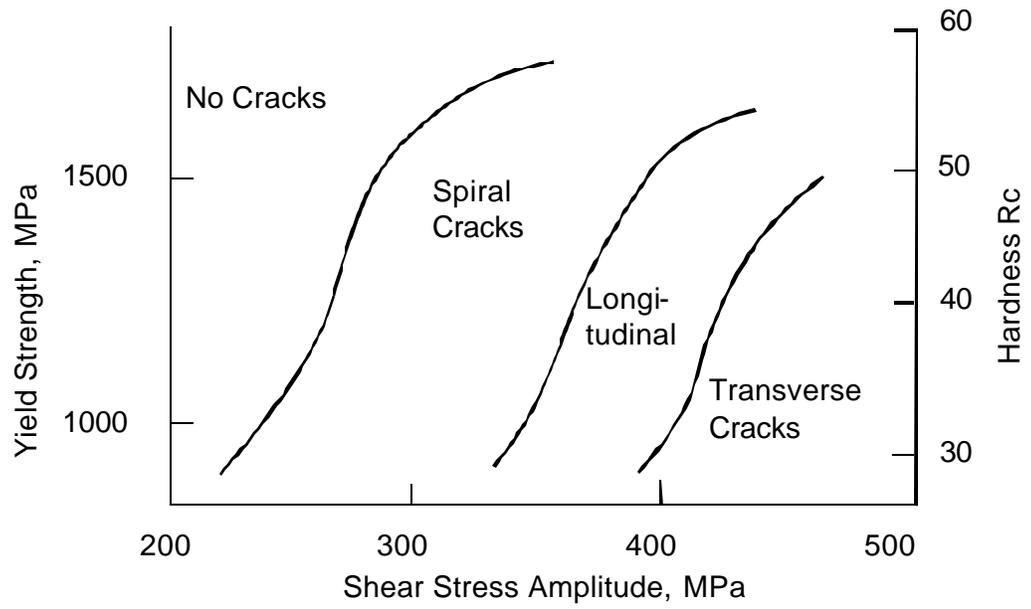
Transverse

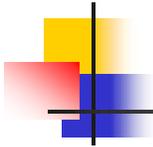
Longitudinal

Spiral

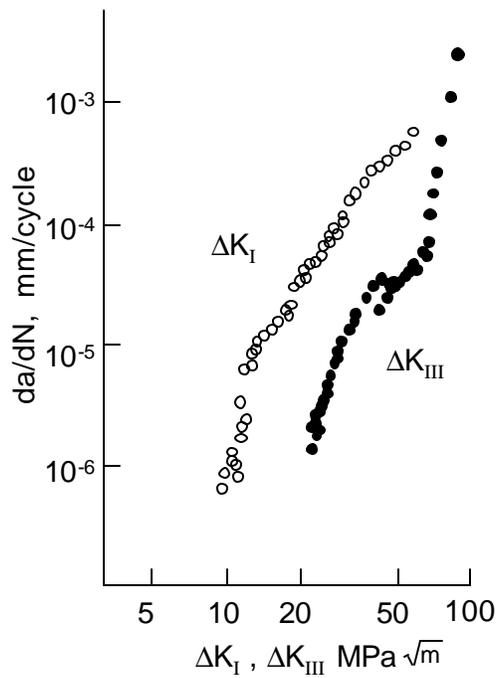


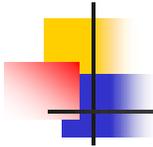
Fracture Mechanism Map



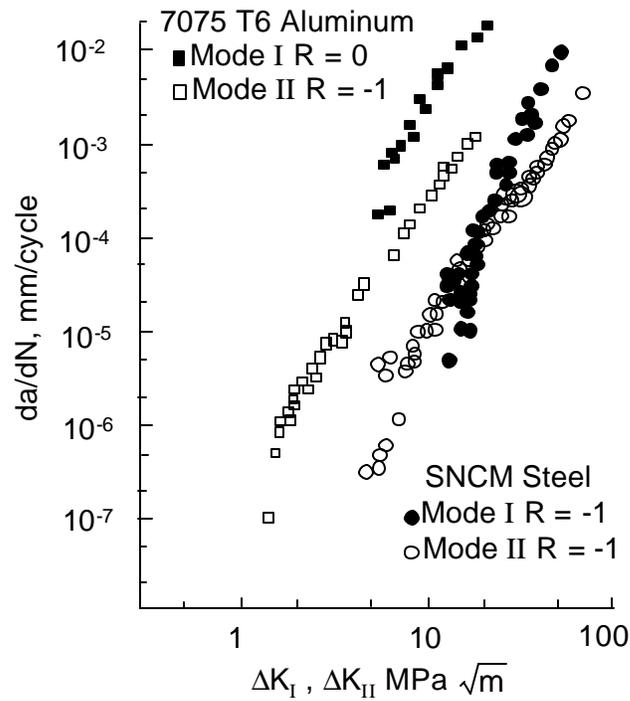


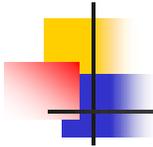
Mode I and Mode III Growth





Mode I and Mode II Growth





Fracture Mechanics Models

$$\frac{da}{dN} = C(\Delta K_{eq})^m$$

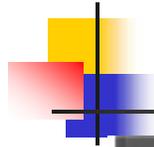
$$\Delta K_{eq} = [\Delta K_I^4 + 8\Delta K_{II}^4 + 8\Delta K_{III}^4 / (1-\nu)]^{0.25}$$

$$\Delta K_{eq} = [\Delta K_I^2 + \Delta K_{II}^2 + (1+\nu)\Delta K_{III}^2]^{0.5}$$

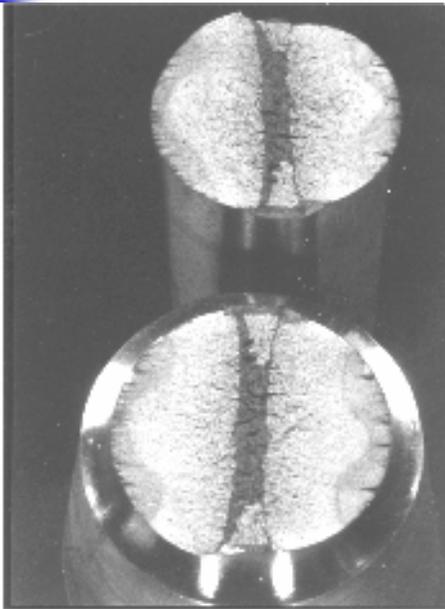
$$\Delta K_{eq} = [\Delta K_I^2 + \Delta K_I \Delta K_{II} + \Delta K_{II}^2]^{0.5}$$

$$\Delta K_{eq}(\varepsilon) = \left[\left(F_{II} \frac{E}{2(1+\nu)} \Delta\gamma \right)^2 + (F_I E \Delta\varepsilon)^2 \right]^{0.5} \sqrt{\pi a}$$

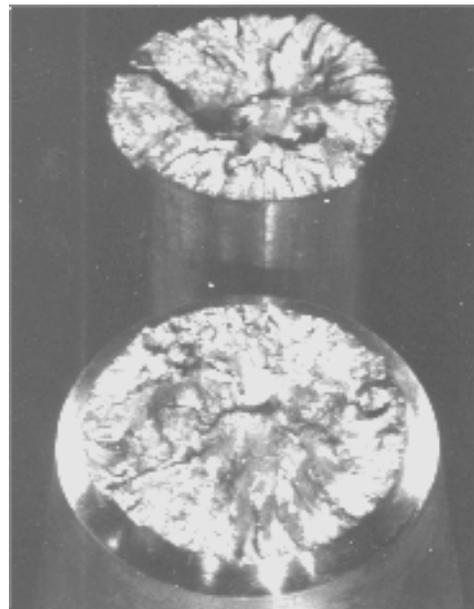
$$\Delta K_{eq}(\varepsilon) = FG \Delta\gamma \left(1 + k \frac{\sigma_{n,max}}{\sigma_{ys}} \right) \sqrt{\pi a}$$



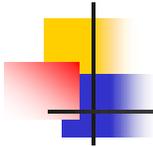
Fracture Surfaces



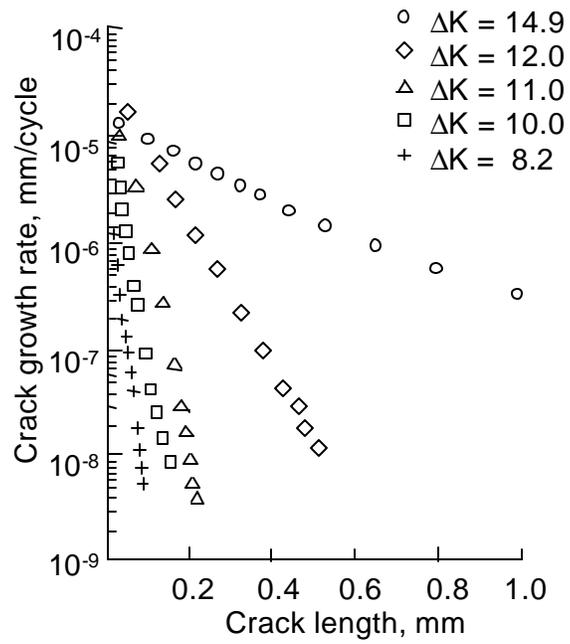
Bending

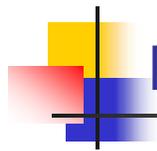


Torsion



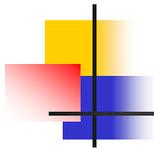
Mode III Growth





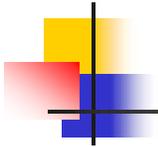
Fracture Mechanics Models Summary

- Multiaxial loading has little effect in Mode I
- Crack closure makes Mode II and Mode III calculations difficult

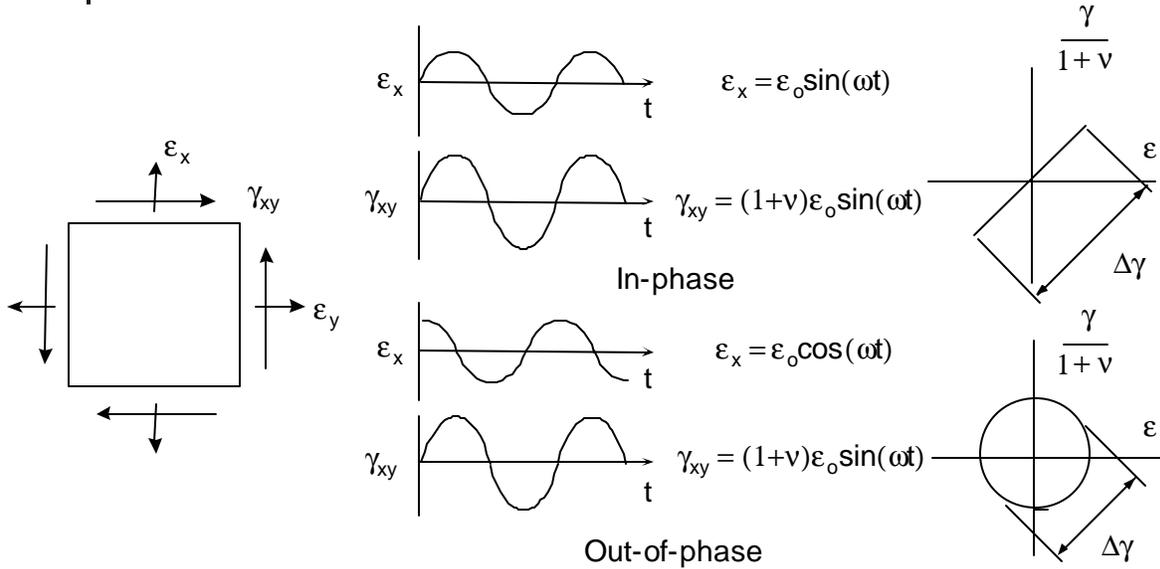


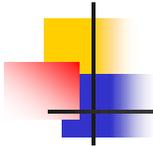
Nonproportional Loading

- In and Out-of-phase loading
- Nonproportional cyclic hardening
- Variable amplitude

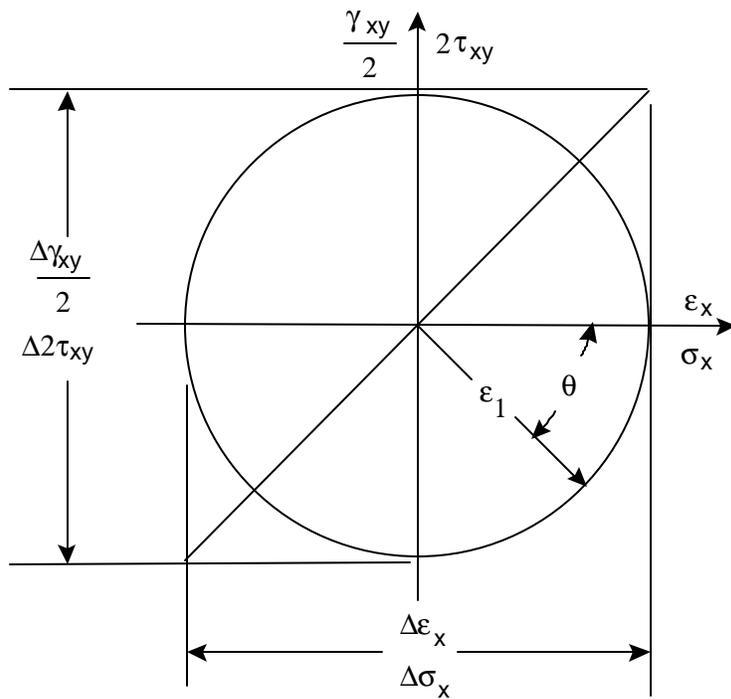


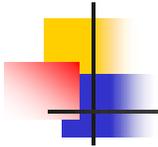
In and Out-of-Phase Loading



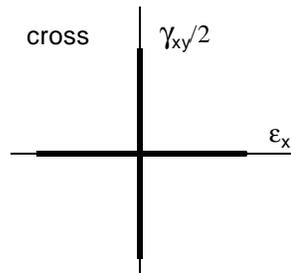
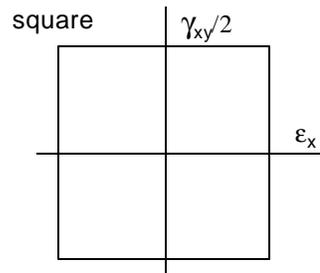
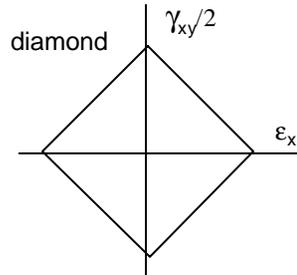
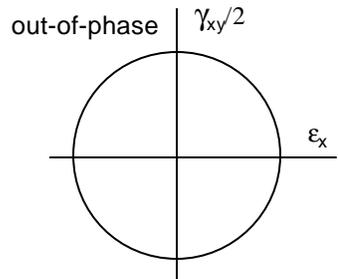


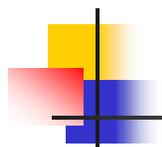
In-Phase and Out-of-Phase



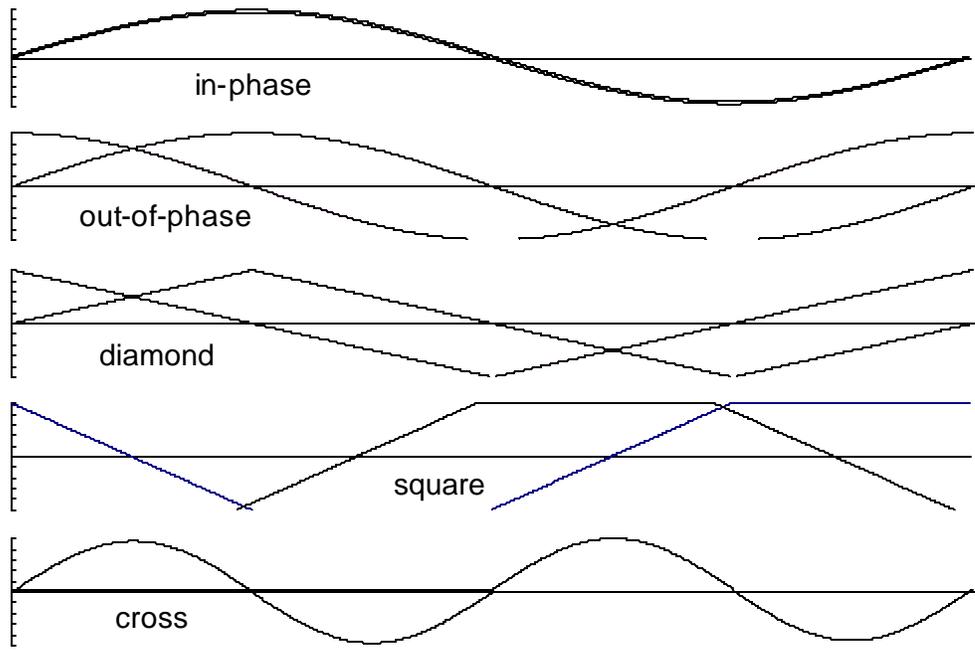


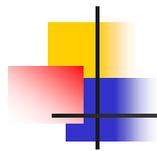
Loading Histories





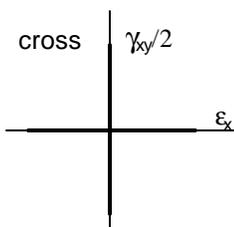
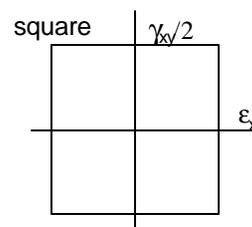
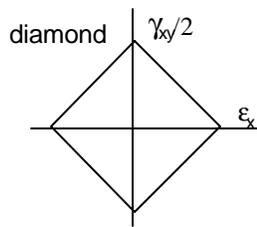
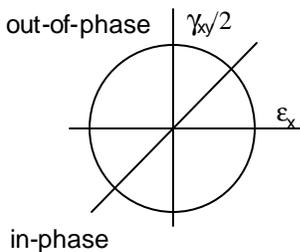
Loading Histories

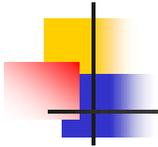




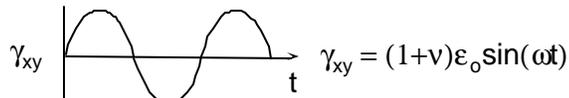
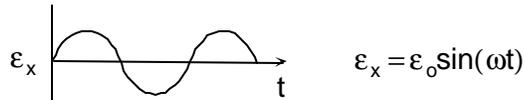
Findley Model Results

	$\Delta\tau/2$ MPa	$\sigma_{n,max}$ MPa	$\Delta\tau/2 + 0.3 \sigma_{n,max}$	N/N_{ip}
in-phase	353	250	428	1.0
90° out-of-phase	250	500	400	2.0
diamond	250	500	400	2.0
square	353	603	534	0.11
cross - tension cycle	250	250	325	16
cross - torsion cycle	250	0	250	216

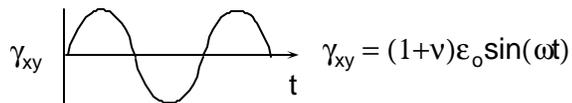
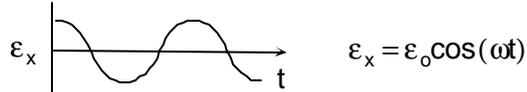




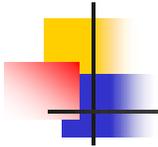
Nonproportional Hardening



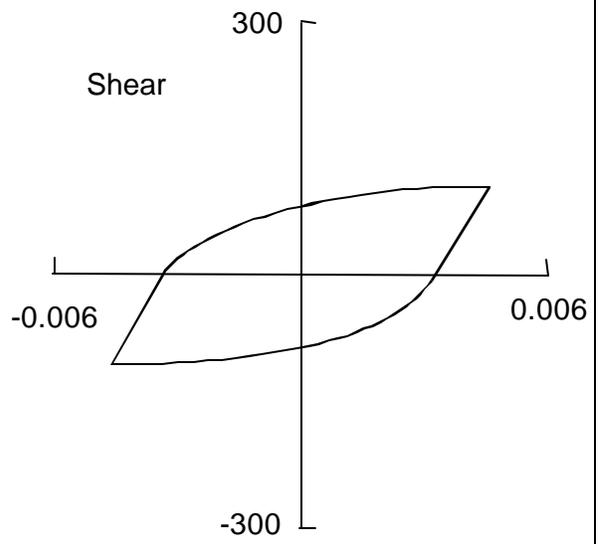
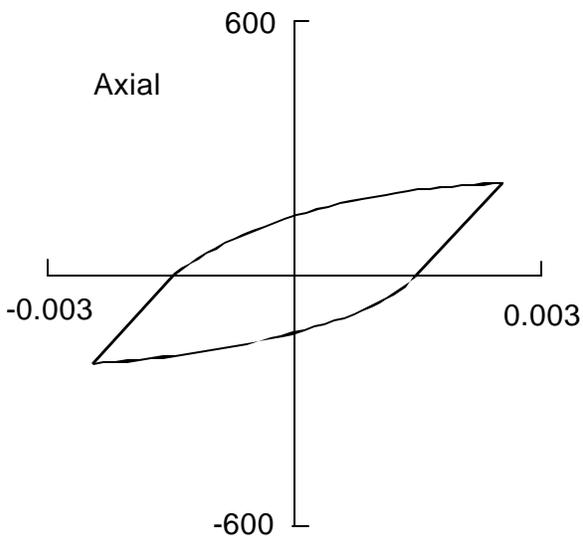
In-phase

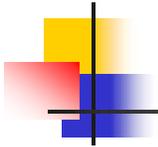


Out-of-phase

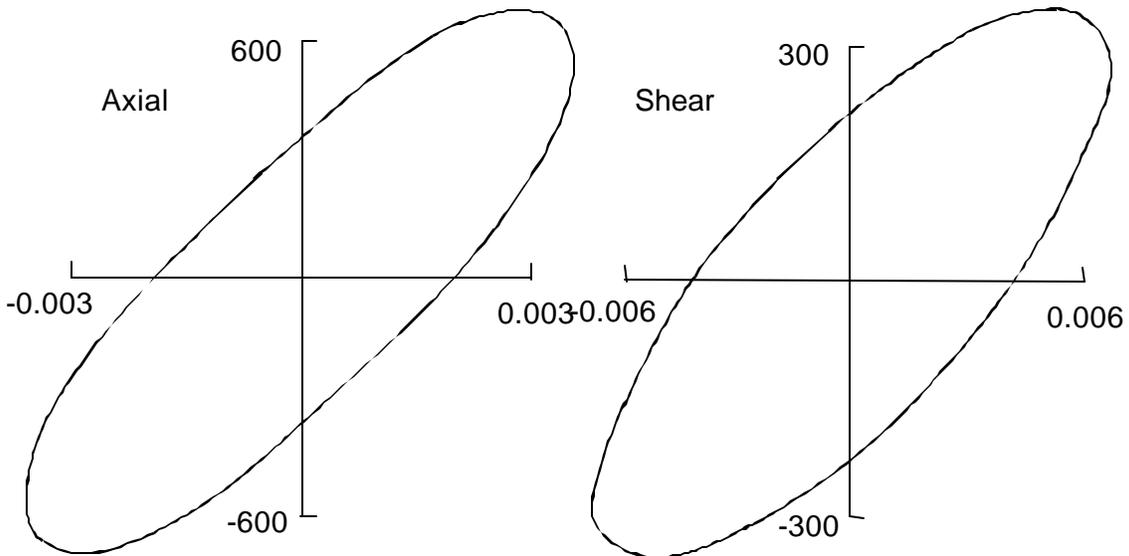


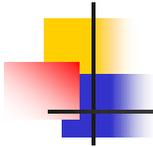
In-Phase





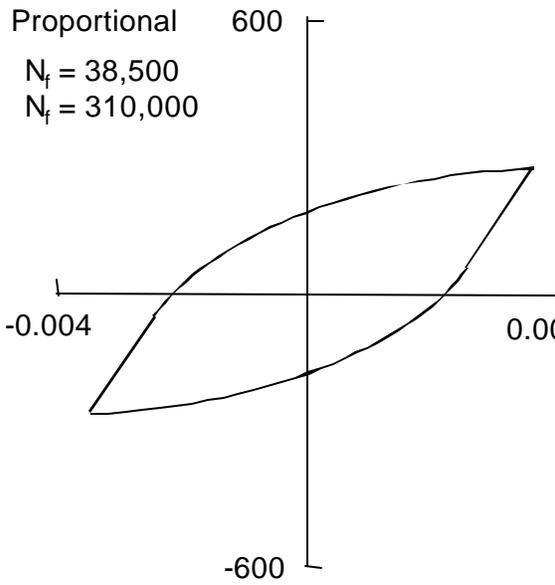
90° Out-of-Phase



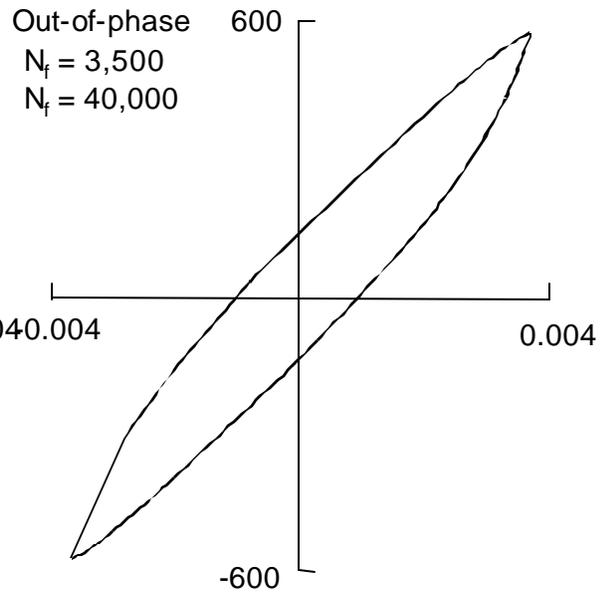


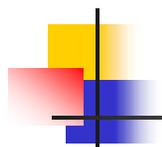
Critical Plane

Proportional
 $N_f = 38,500$
 $N_f = 310,000$

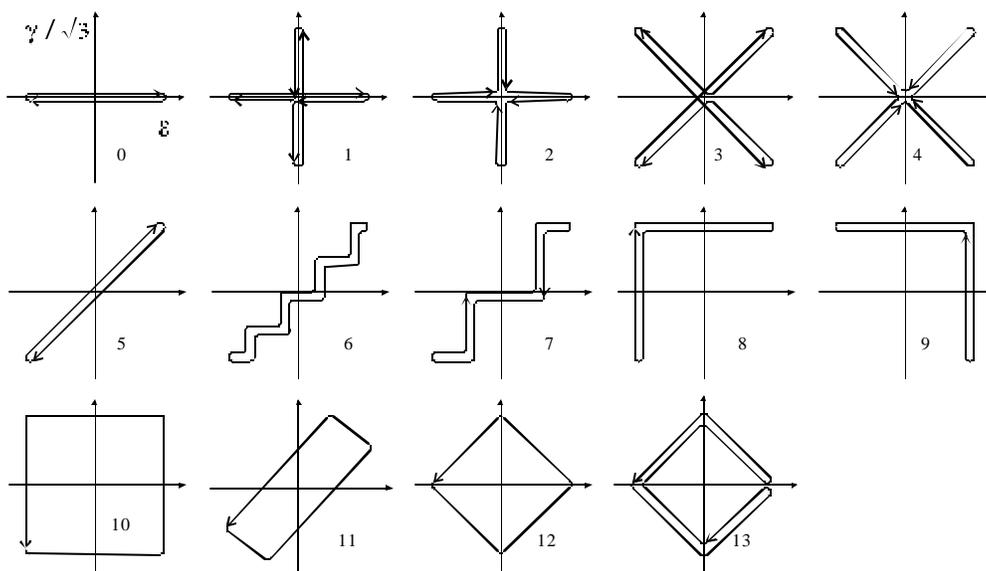


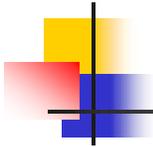
Out-of-phase
 $N_f = 3,500$
 $N_f = 40,000$



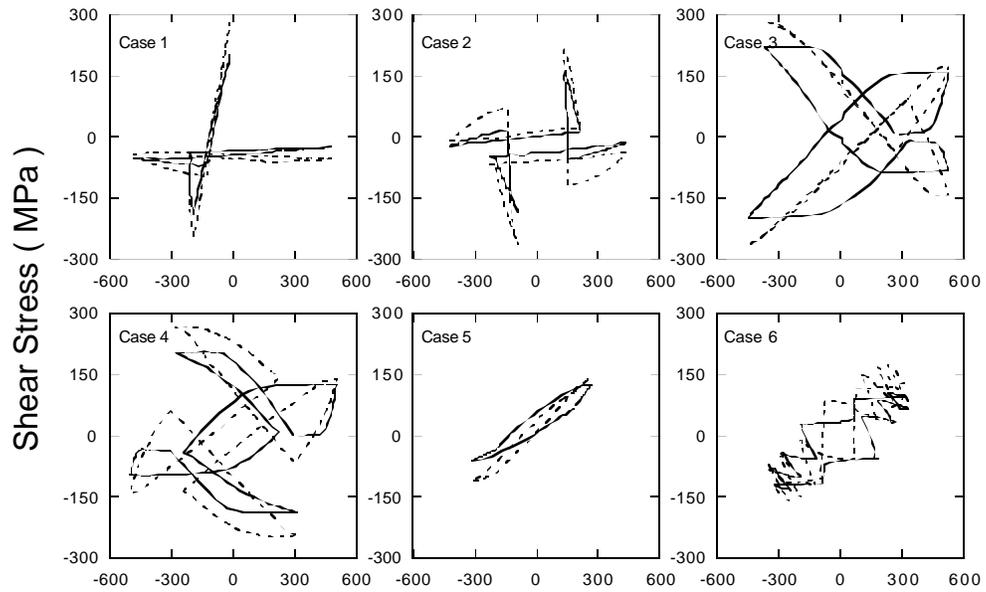


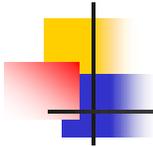
Loading Histories



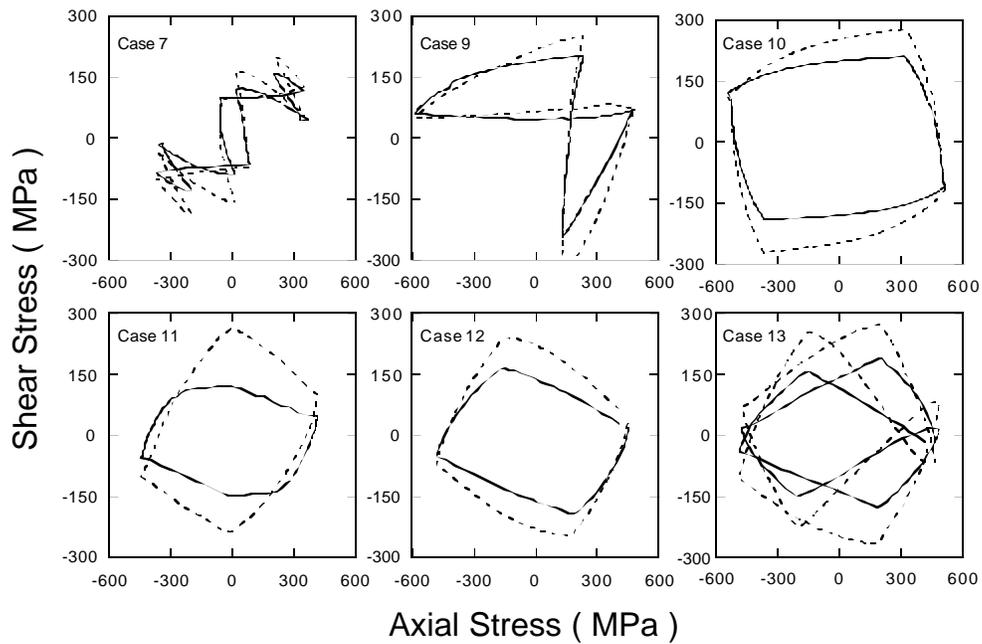


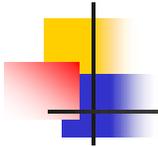
Stress-Strain Response





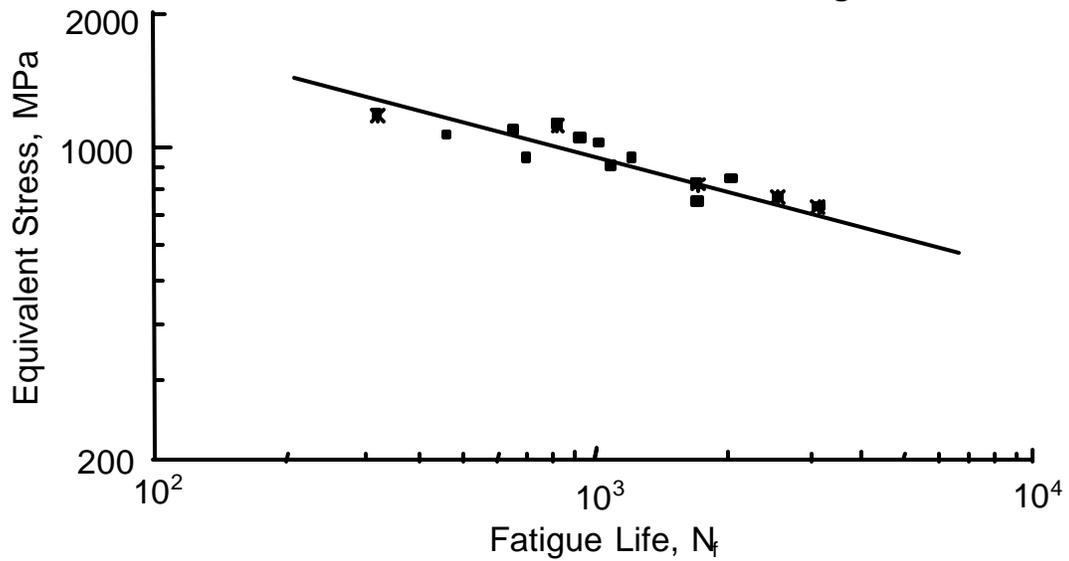
Stress-Strain Response (continued)



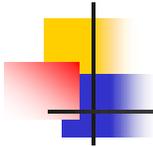


Maximum Stress

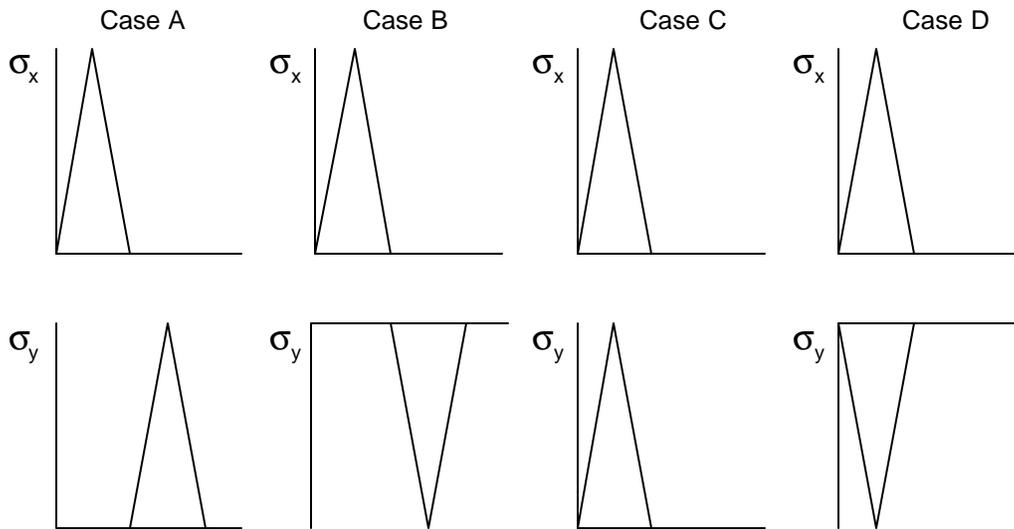
All tests have the same strain ranges

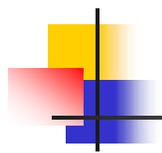


Nonproportional hardening results in lower fatigue lives

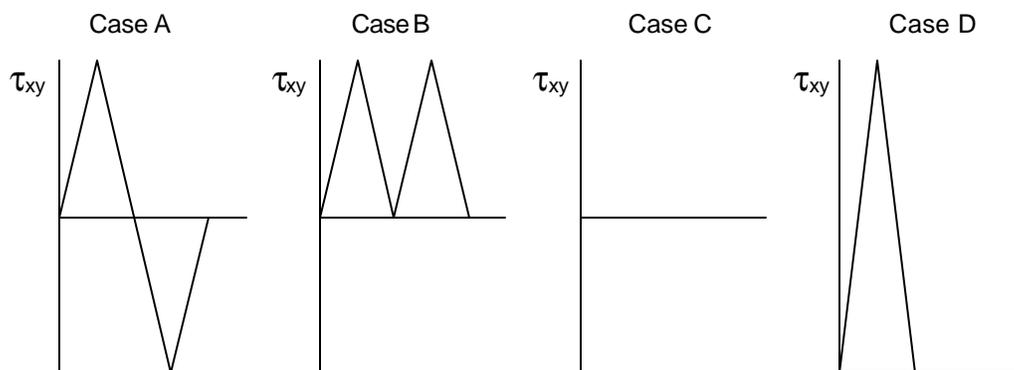


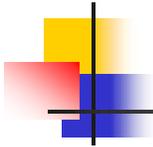
Nonproportional Example



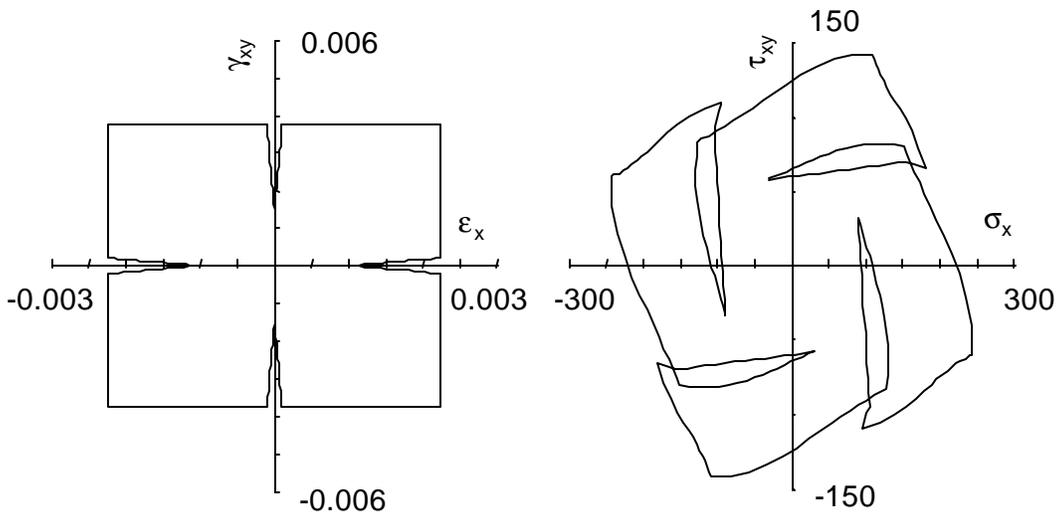


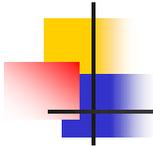
Shear Stresses



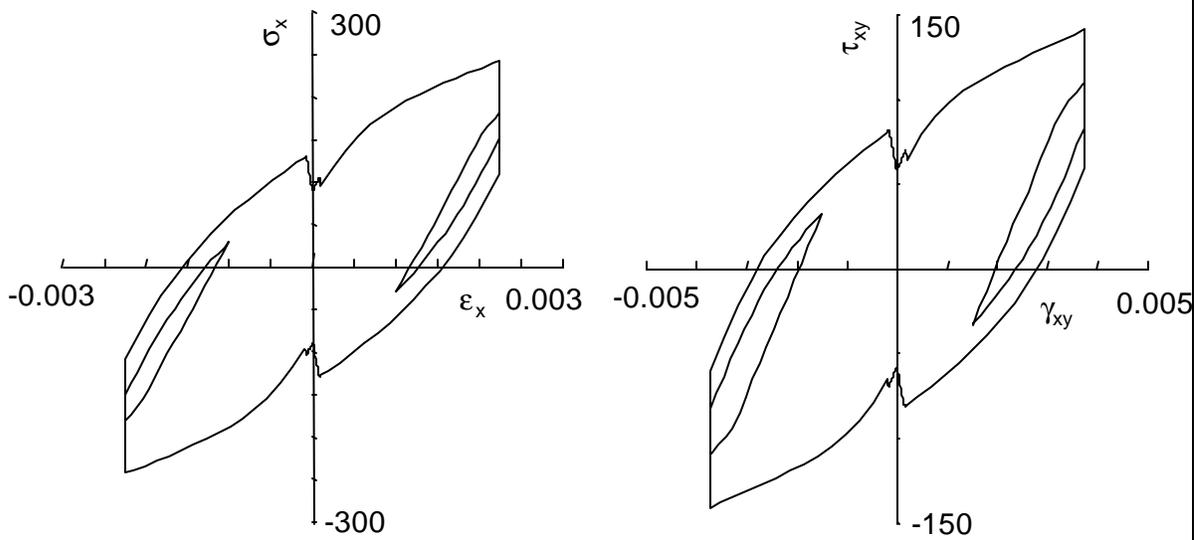


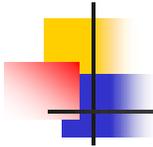
Simple Variable Amplitude History



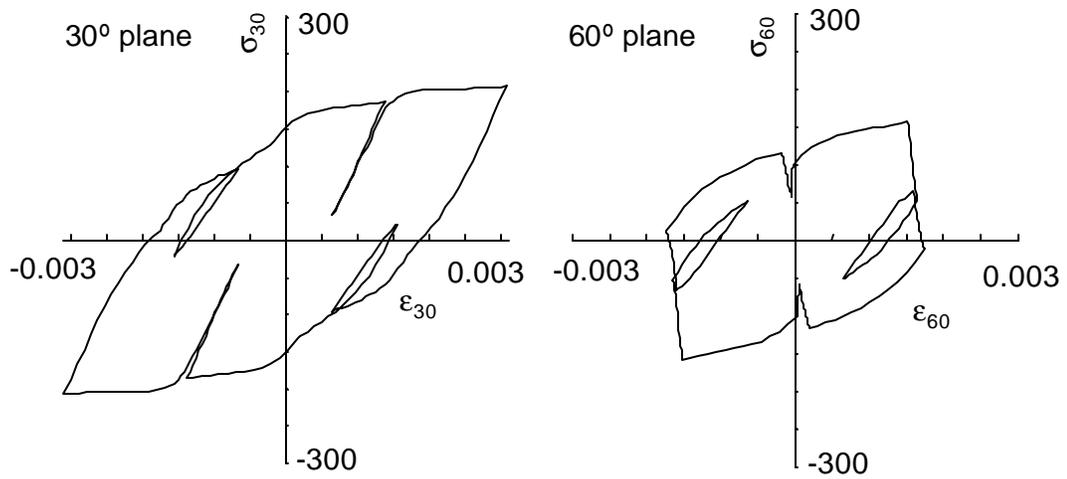


Stress-Strain on 0° Plane



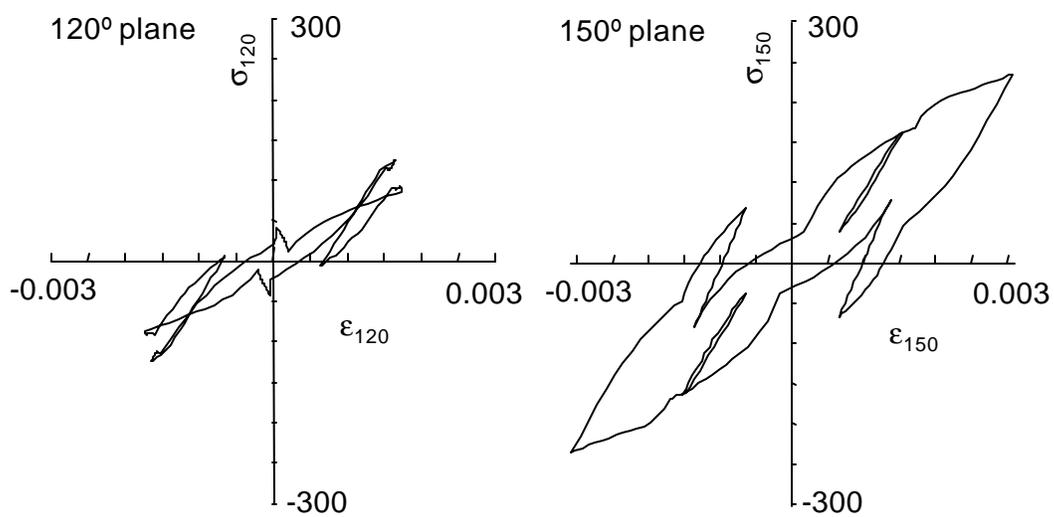


Stress-Strain on 30° and 60° Planes



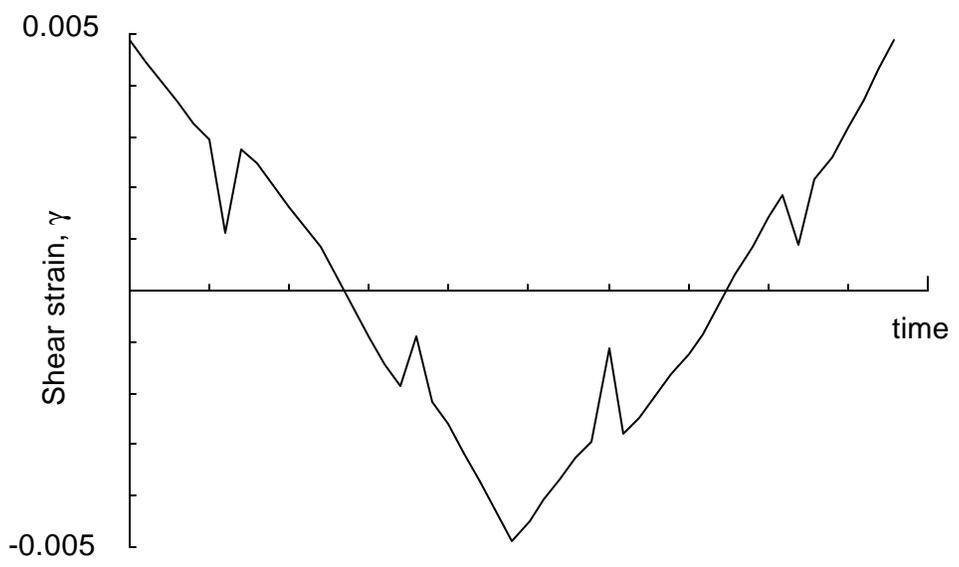


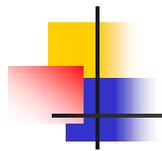
Stress-Strain on 120° and 150° Planes





Shear Strain History on Critical Plane





Fatigue Calculations

Load or strain history



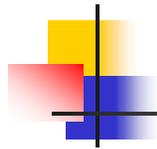
Cyclic plasticity model



Stress and strain tensor

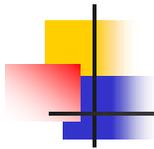


Search for critical plane



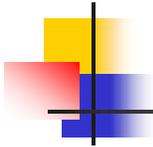
Nonproportional Loading Summary

- Nonproportional cyclic hardening increases stress levels
- Critical plane models are used to assess fatigue damage

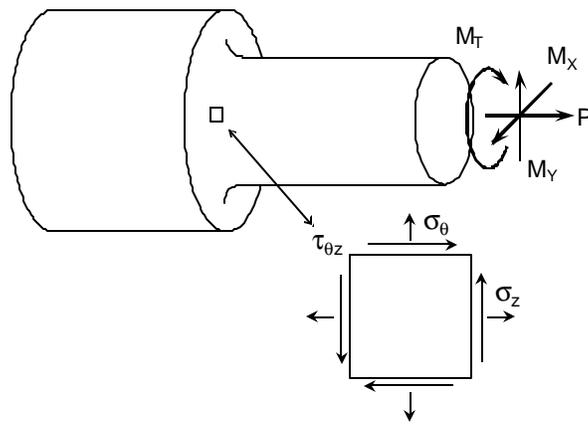


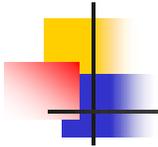
Notches

- Stress and strain concentrations
- Nonproportional loading and stressing
- Fatigue notch factors
- Cracks at notches

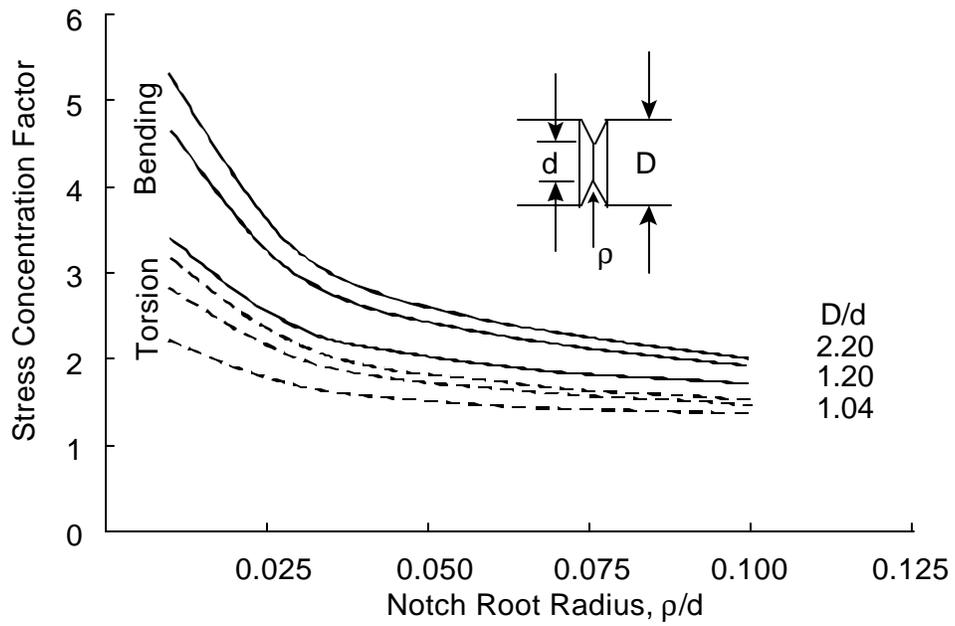


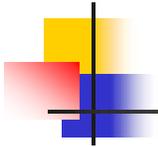
Notched Shaft Loading



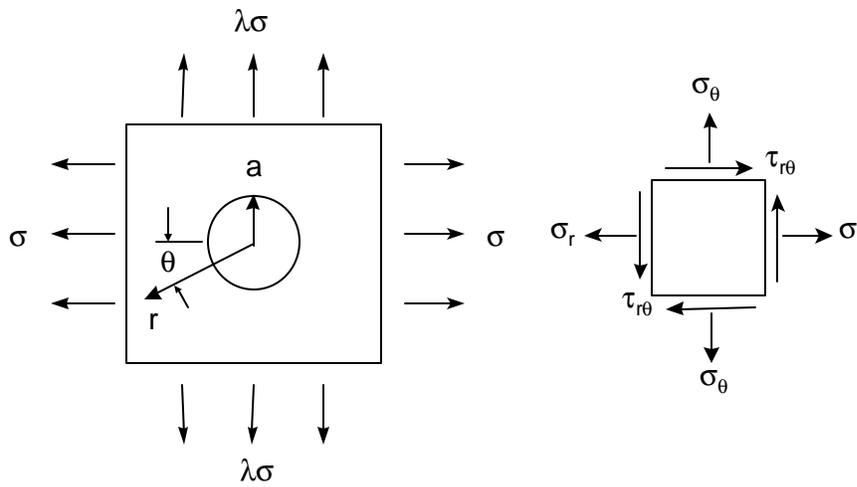


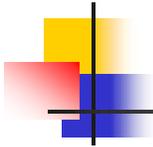
Stress Concentration Factors



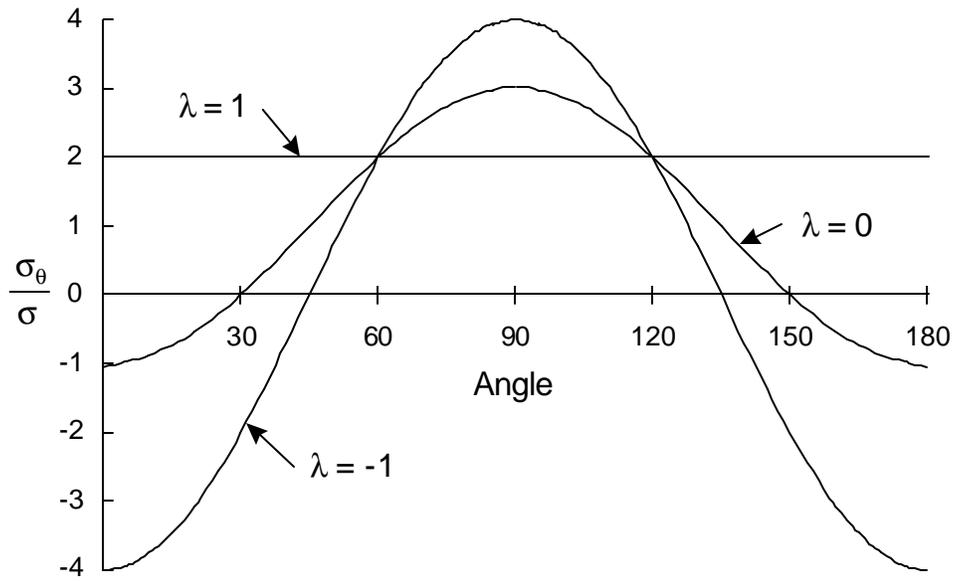


Hole in a Plate

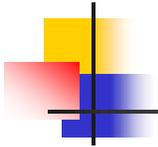




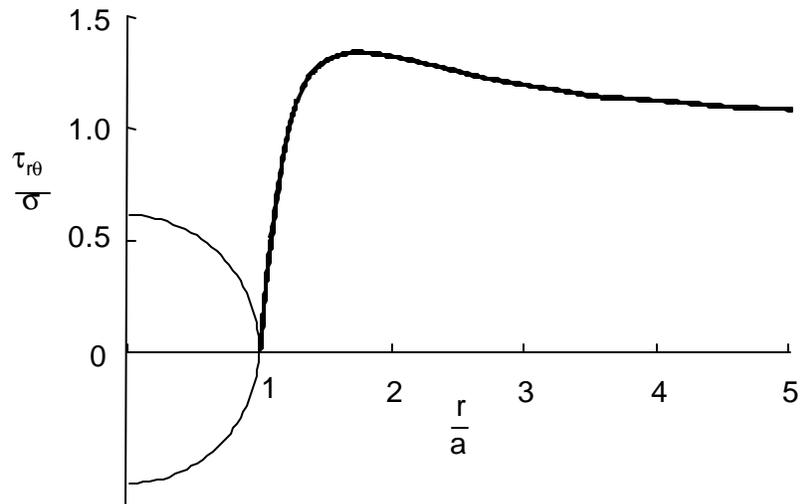
Stresses at the Hole

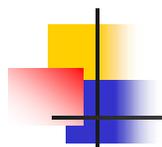


Stress concentration factor depends on type of loading

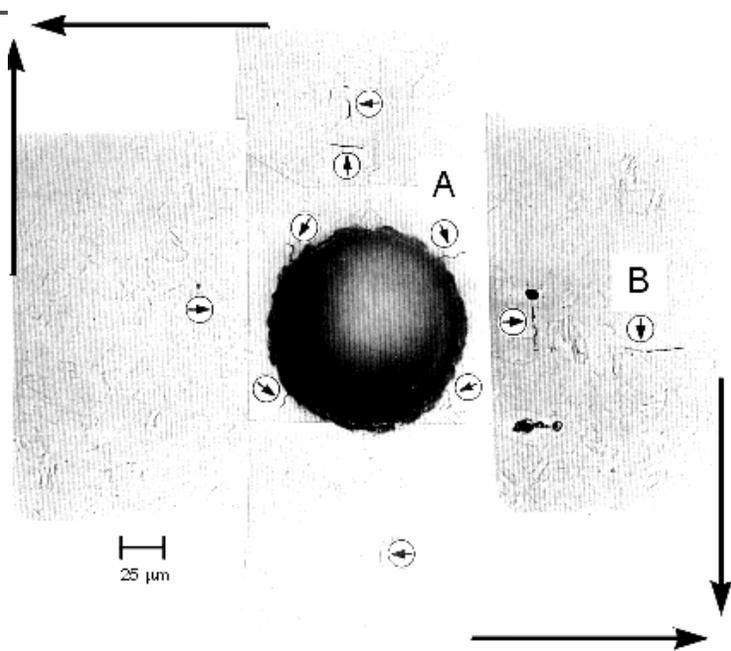


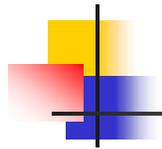
Shear Stresses during Torsion





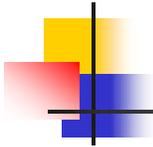
Torsion Experiments



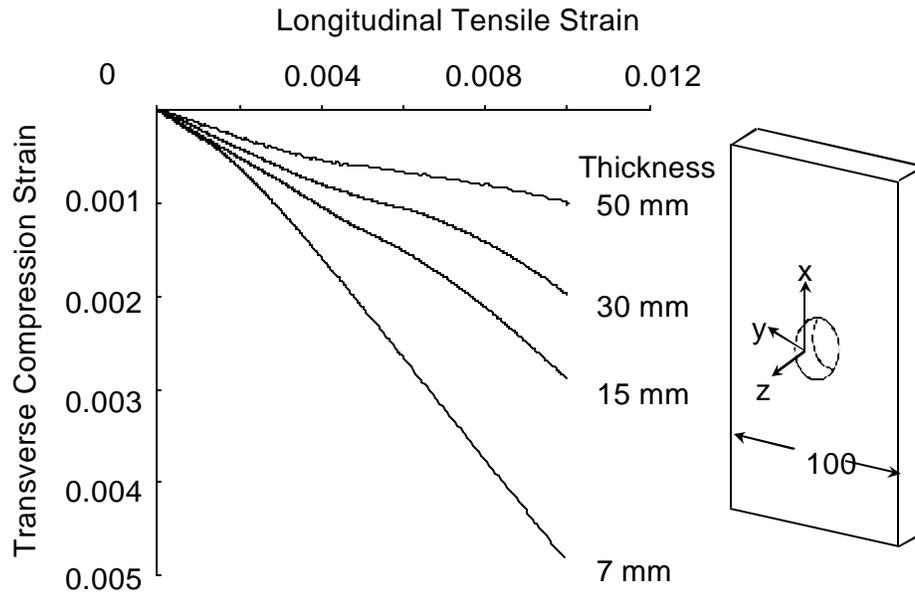


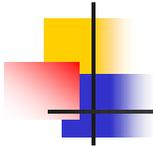
Multiaxial Loading

- Uniaxial loading that produces multiaxial stresses at notches
- Multiaxial loading that produces uniaxial stresses at notches
- Multiaxial loading that produces multiaxial stresses at notches

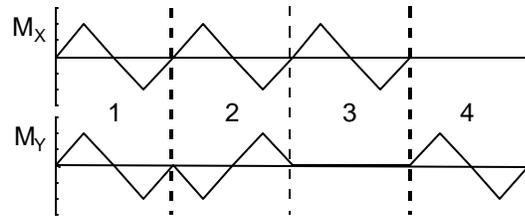
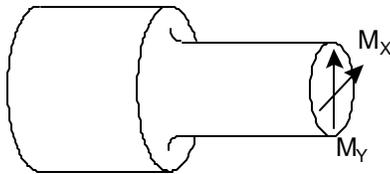


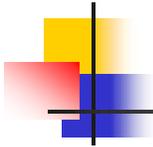
Thickness Effects



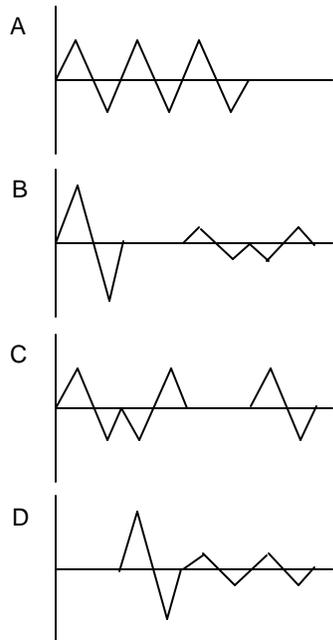
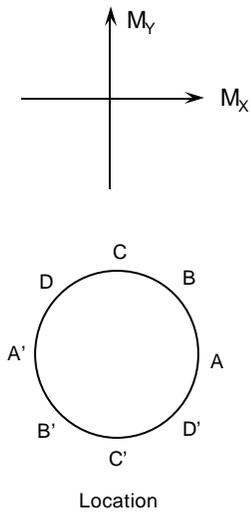


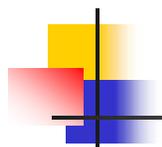
Applied Bending Moments





Bending Moments on the Shaft



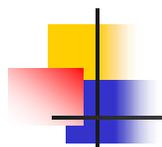


Bending Moments

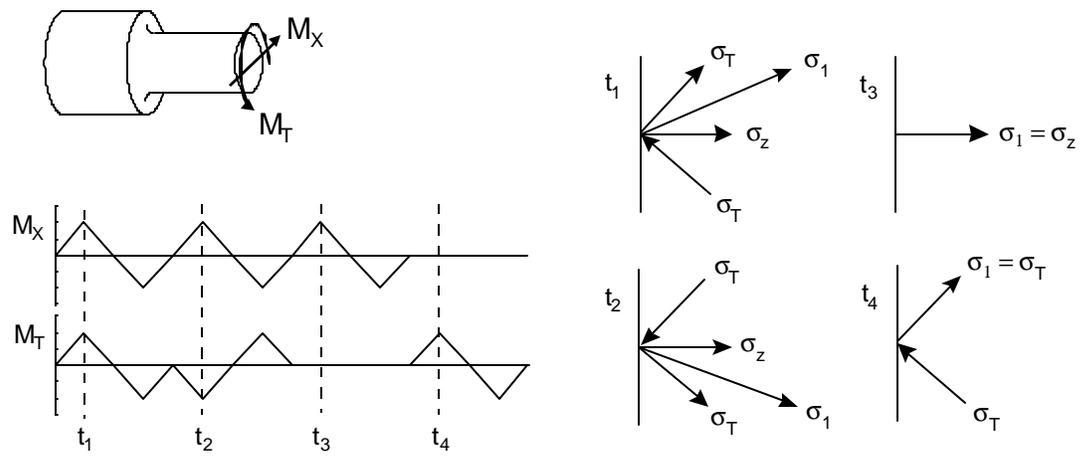
ΔM	A	B	C	D
2.82		1		1
2.00	3		2	
1.41		2		1
1.00			2	
0.71				2

$$\Delta \bar{M} = \sqrt[5]{\sum \Delta M^5}$$

	A	B	C	D
$\Delta \bar{M}$	2.49	2.85	2.31	2.84



Torsion Loading



Out-of-phase shear loading is needed to produce nonproportional stressing

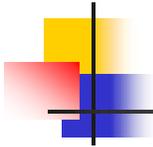
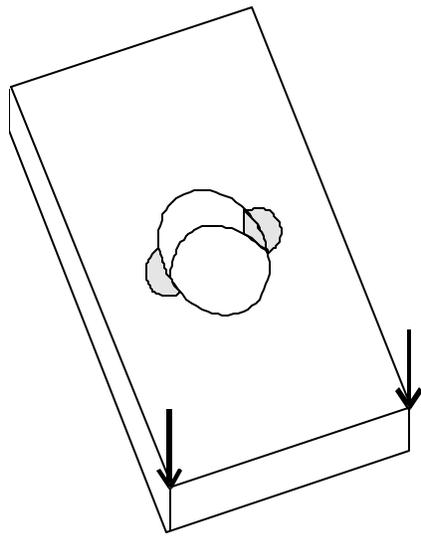
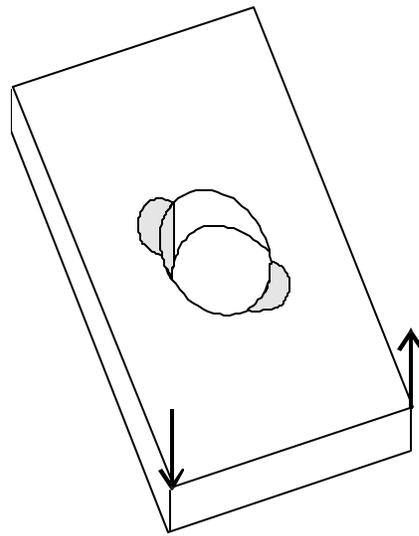


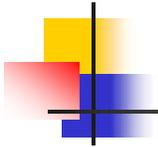
Plate and Shell Structures



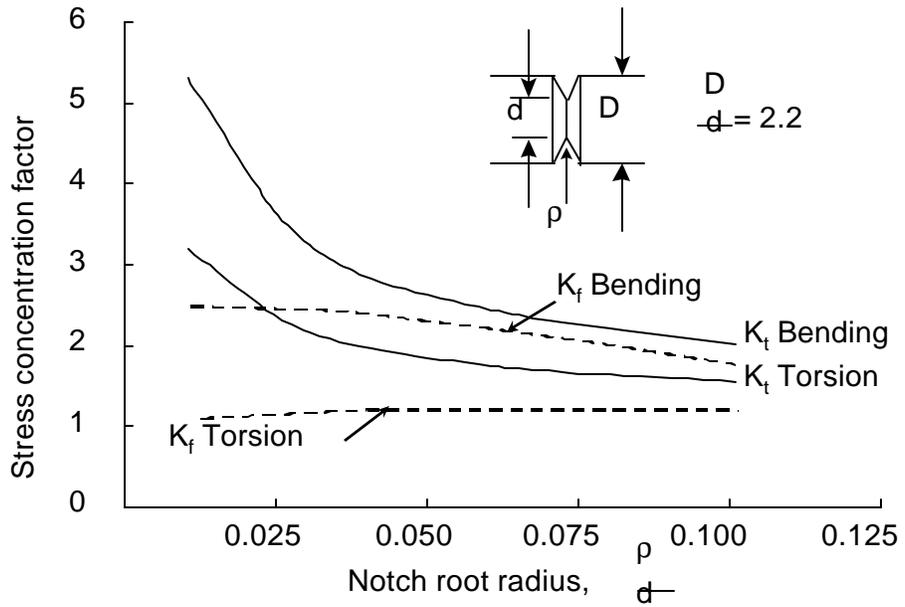
$$K_t = 3$$

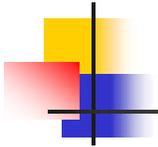


$$K_t = 4$$

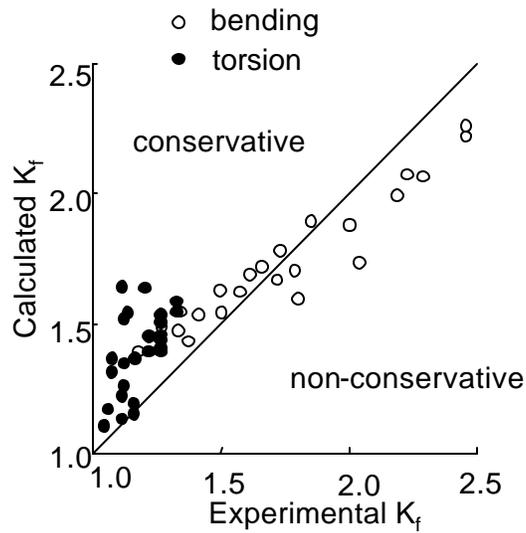


Fatigue Notch Factors



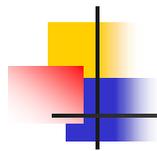


Fatigue Notch Factors (continued)



Peterson's Equation

$$K_f = 1 + \frac{K_T - 1}{1 + \frac{a}{r}}$$



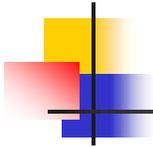
Fracture Surfaces in Torsion



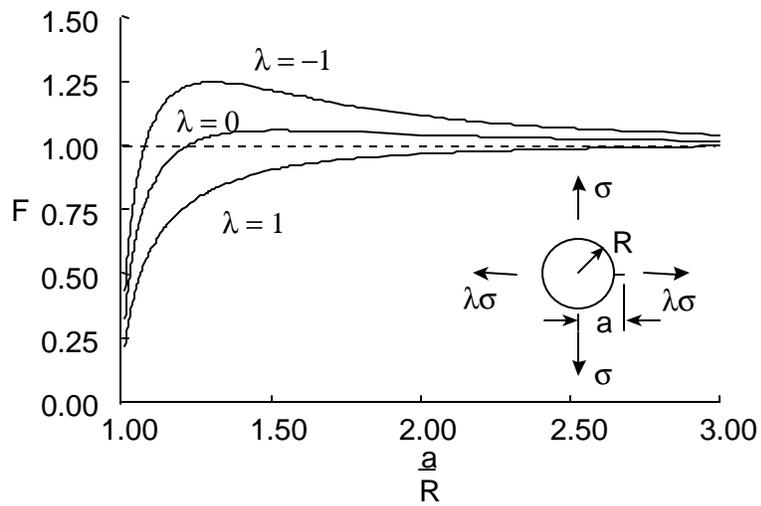
Circumferencial Notch



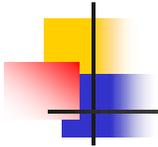
Shoulder Fillet



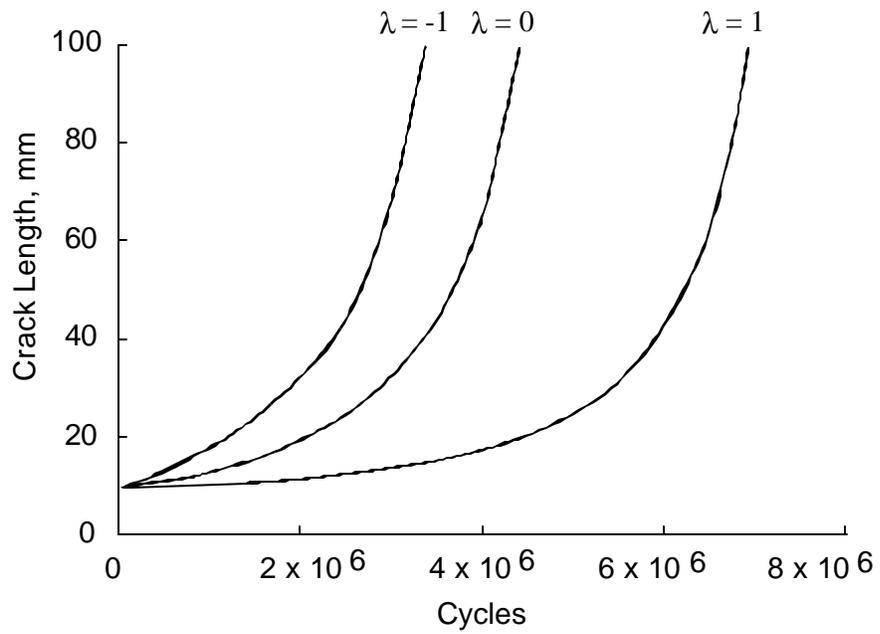
Stress Intensity Factors

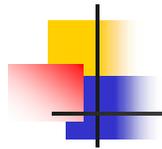


$$\frac{da}{dN} = C (\Delta K_{eq})^m \quad K_I = F \Delta \sigma \sqrt{\pi a}$$



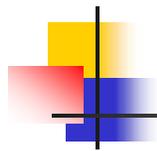
Crack Growth From a Hole





Notches Summary

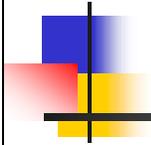
- Uniaxial loading can produce multiaxial stresses at notches
- Multiaxial loading can produce uniaxial stresses at notches
- Multiaxial stresses are not very important in thin plate and shell structures
- Multiaxial stresses are not very important in crack growth



Final Summary

- Fatigue is a planar process involving the growth of cracks on many size scales
- Critical plane models provide reasonable estimates of fatigue damage

Multiaxial Fatigue



University of Illinois at Urbana-Champaign