Multiaxial Fatigue

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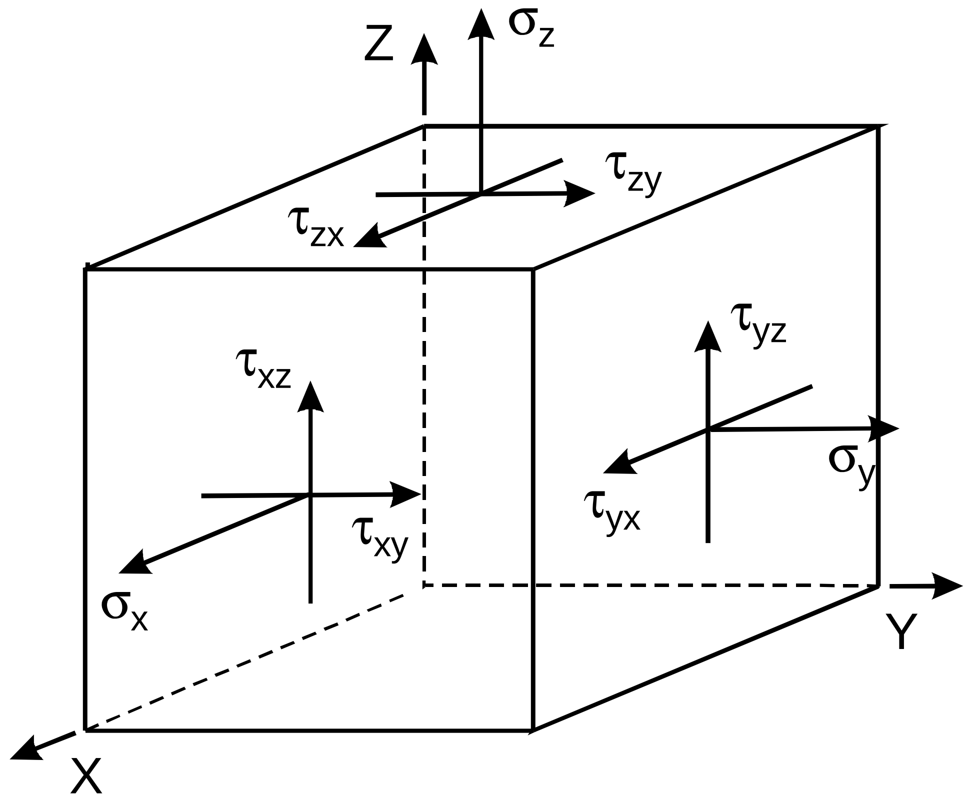
Outline

- State of Stress (Chapter 1)
- Fatigue Mechanisms (Chapter 3)
- Stress Based Models (Chapter 5)
- Strain Based Models (Chapter 6)
- Fracture Mechanics Models (Chapter 7)
- Nonproportional Loading (Chapter 8)
- Notches (Chapter 9)

State of Stress

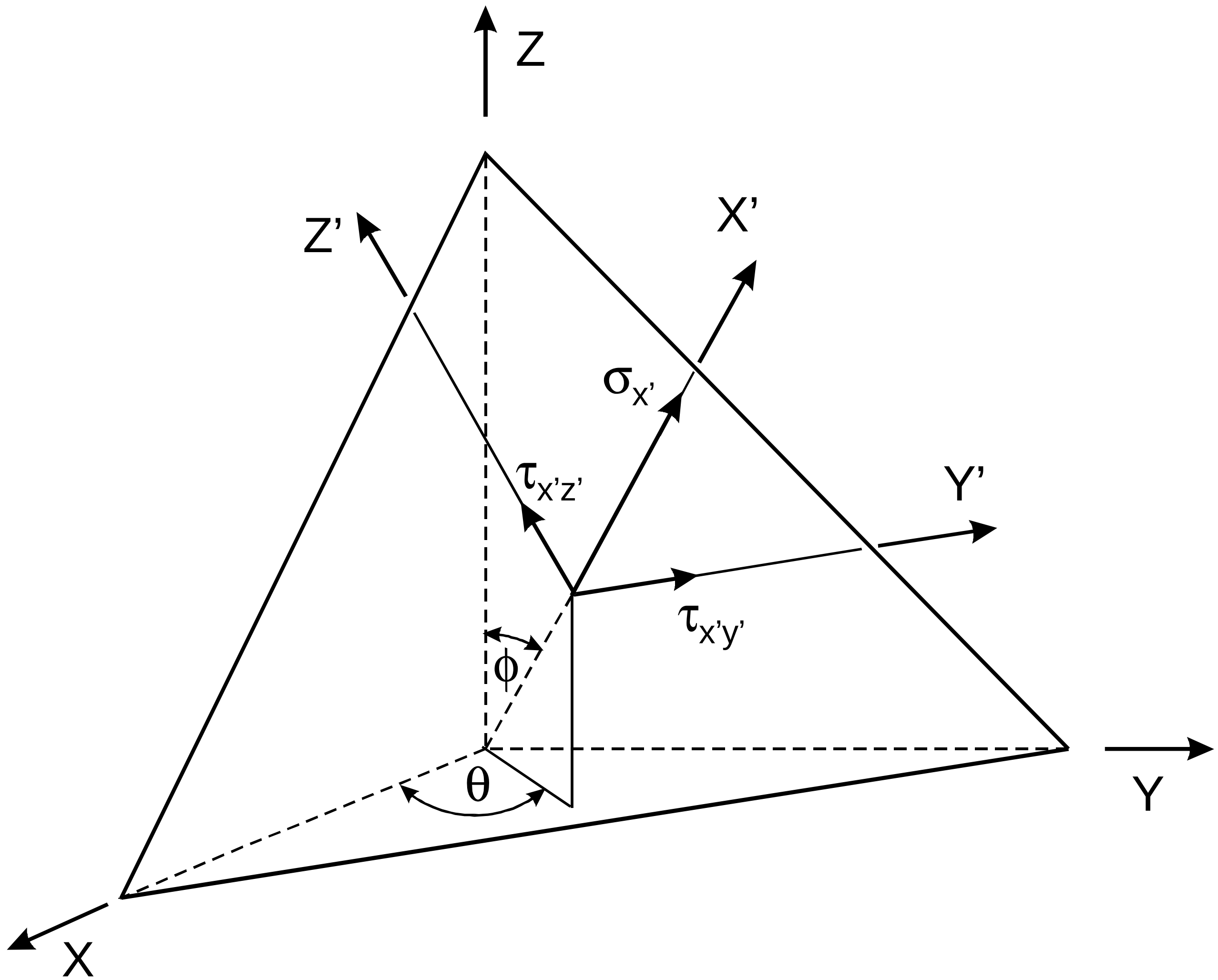
- Stress components
- Common states of stress
- Shear stresses

Stress Components

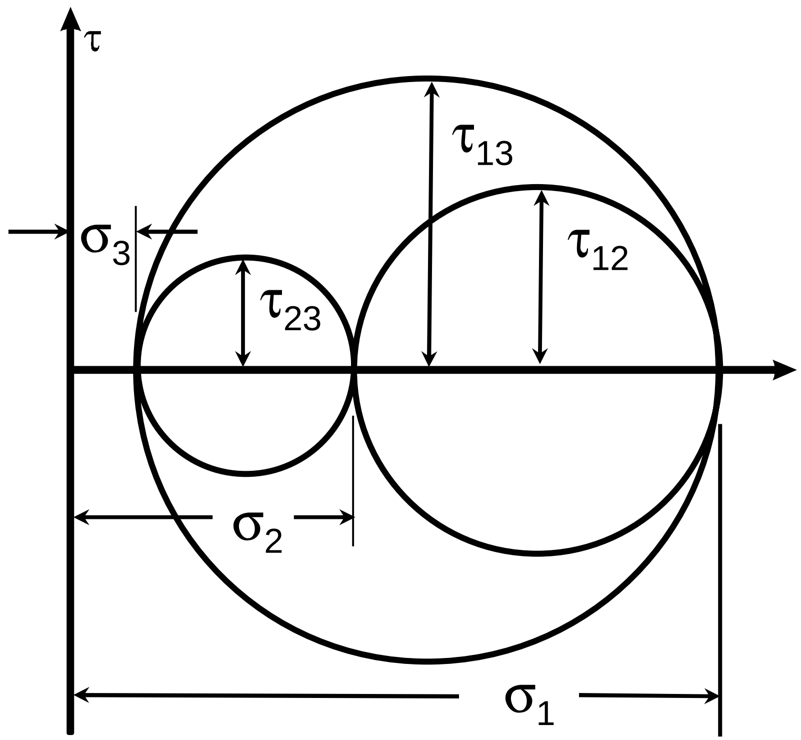


Six stresses and six strains

Stresses Acting on a Plane

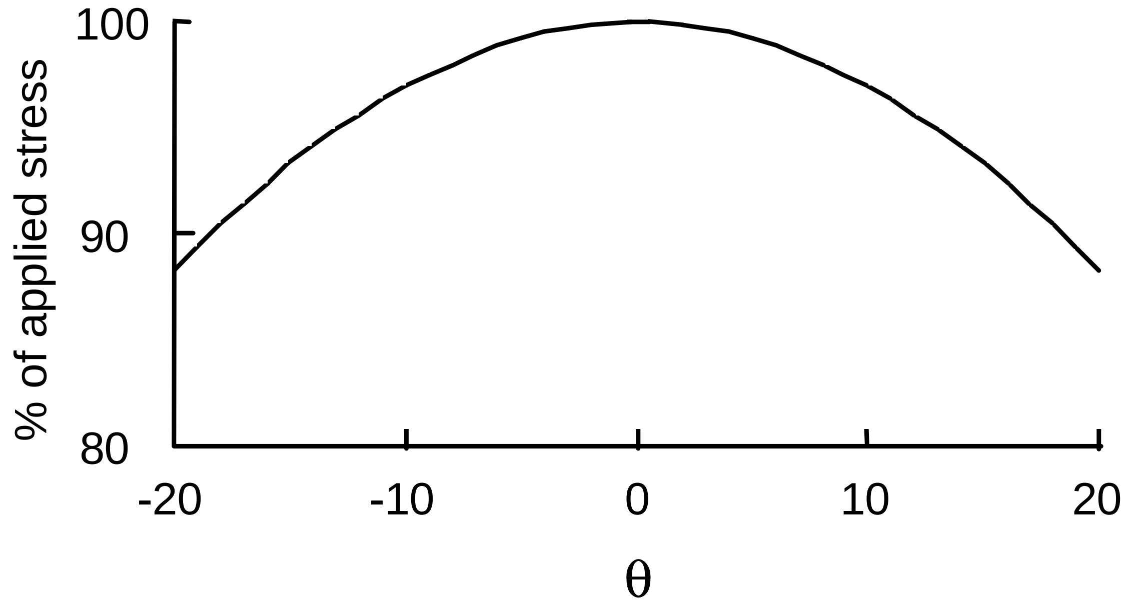


Principal Stresses



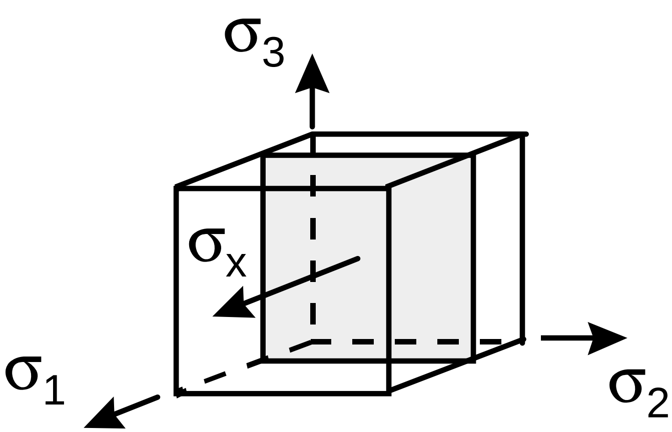
$$\sigma^3 - \sigma^2(\sigma_X + \sigma_Y + \sigma_Z) + \sigma(\sigma_X\sigma_Y + \sigma_Y\sigma_Z\sigma_X\sigma_Z - \tau^2_{XY} - \tau^2_{YZ} - \tau^2_{XZ})$$
$$- (\sigma_X\sigma_Y\sigma_Z + 2\tau_{XY}\tau_{YZ}\tau_{XZ} - \sigma_X\tau^2_{YZ} - \sigma_Y\tau^2_{ZX} - \sigma_Z\tau^2_{XY}) = 0$$

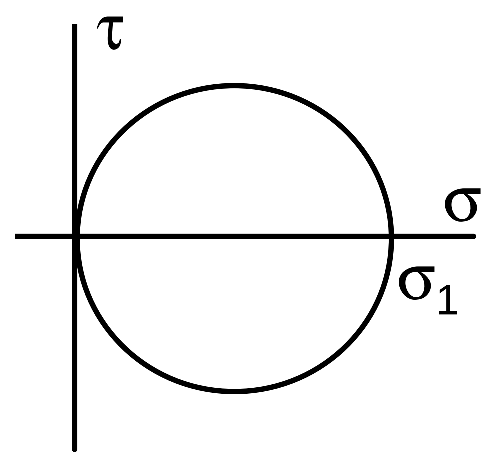
Stress and Strain Distributions

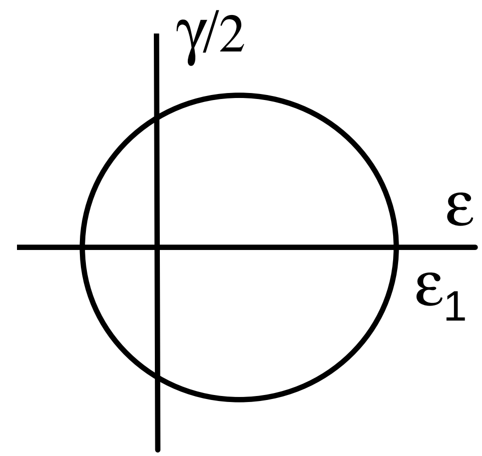


Stresses are nearly the same over a 10° range of angles

Tension



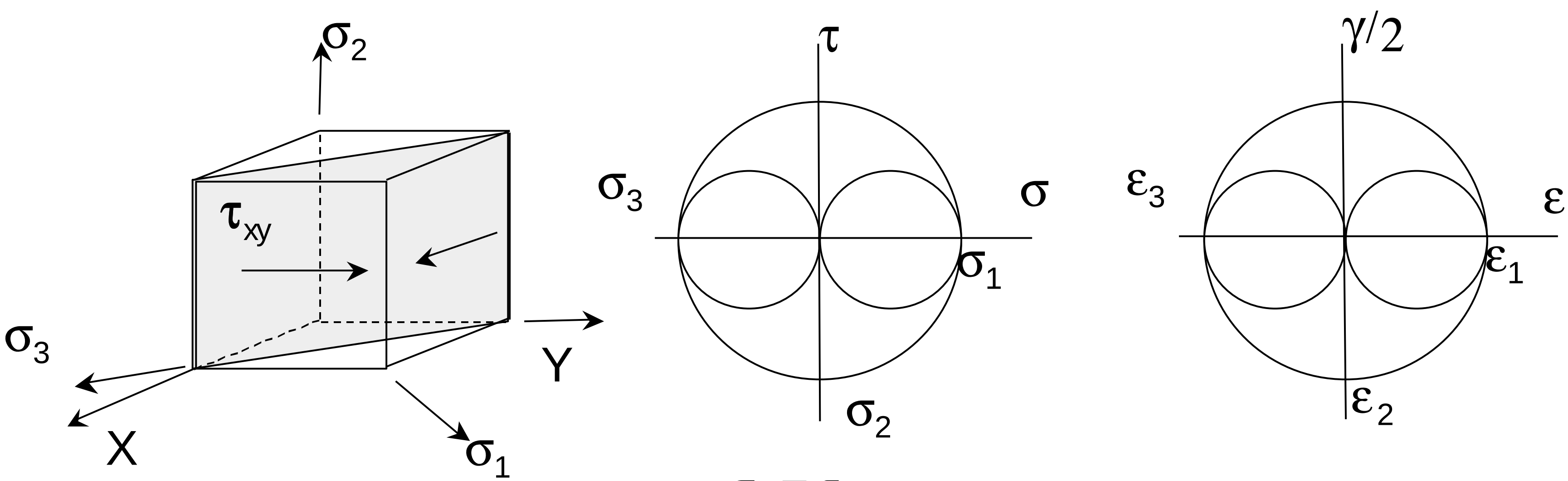




$$\sigma_2 = \sigma_3 = 0$$

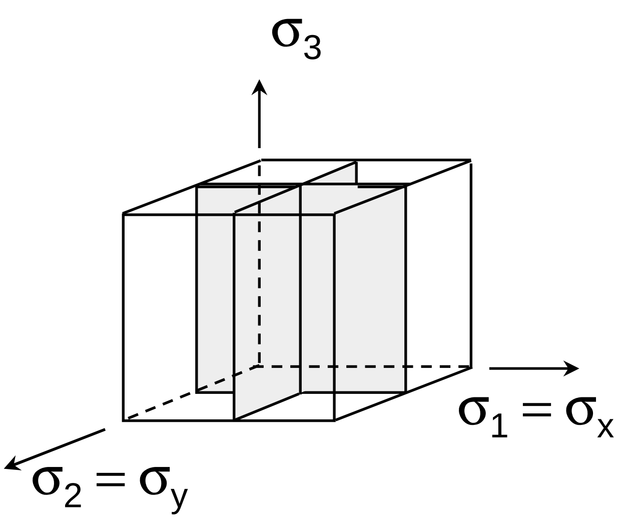
$$\varepsilon_2 = \varepsilon_3 = -\nu\varepsilon_1$$

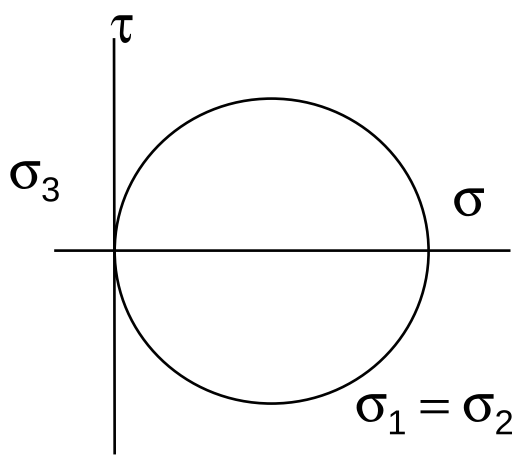
Torsion

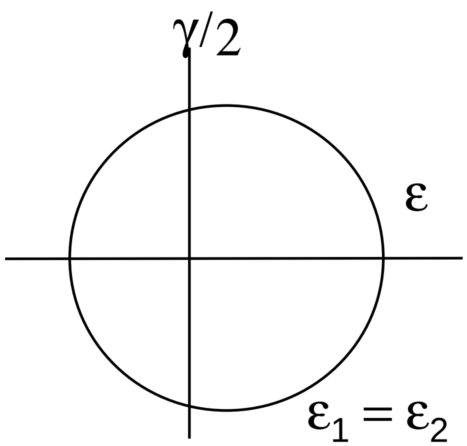


$\sigma_1 = \tau_{xy}$

Biaxial Tension

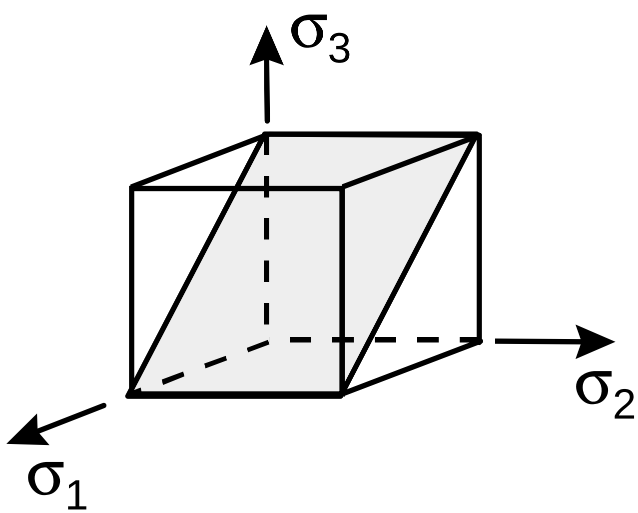


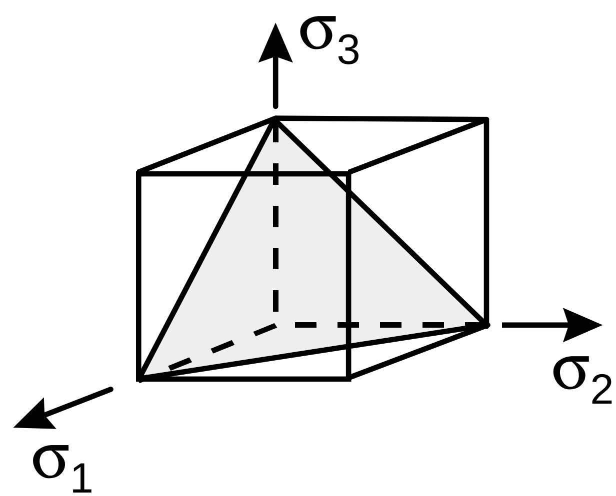




$$\varepsilon_3 = -\frac{2\nu}{1-\nu}\varepsilon$$

Shear Stresses





Maximum shear stress

Octahedral shear stress

$$\tau_{13} = \frac{|\sigma_1 - \sigma_3|}{2}$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2}$$

Mises: $\bar{\sigma} = \frac{3}{\sqrt{2}}\tau_{oct}$

$$\tau_{oct} = \frac{3}{2\sqrt{2}}\tau_{13} = 0.94\,\tau_{13}$$

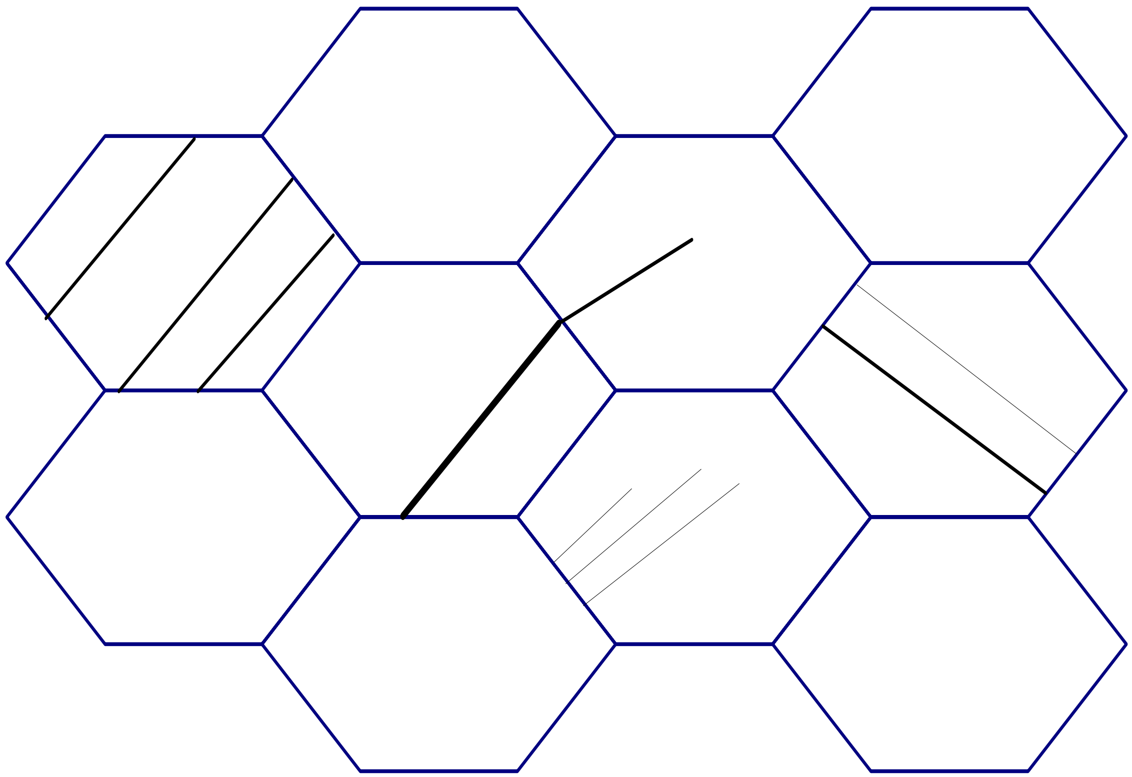
State of Stress Summary

- Stresses acting on a plane
- Principal stress
- Maximum shear stress
- Octahedral shear stress

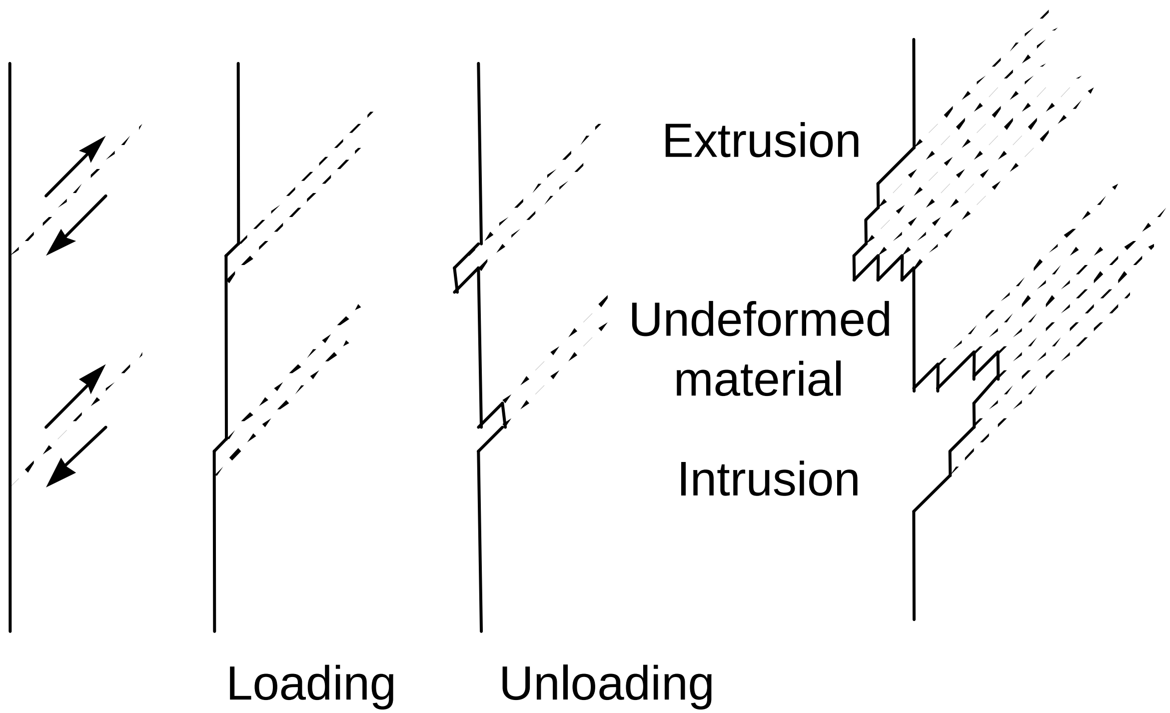
Fatigue Mechanisms

- Crack nucleation
- Fracture modes
- Crack growth
- State of stress effects

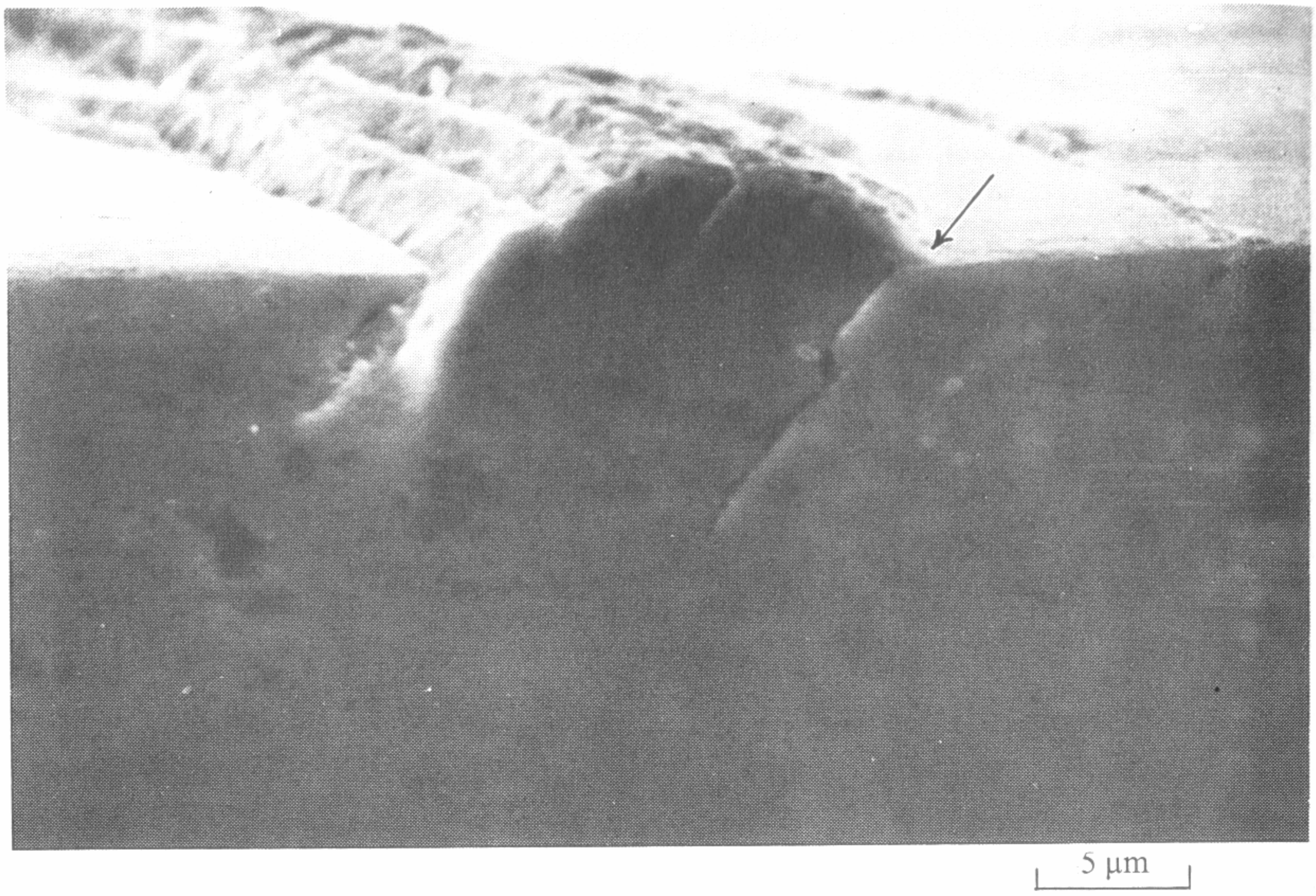
Crack Nucleation



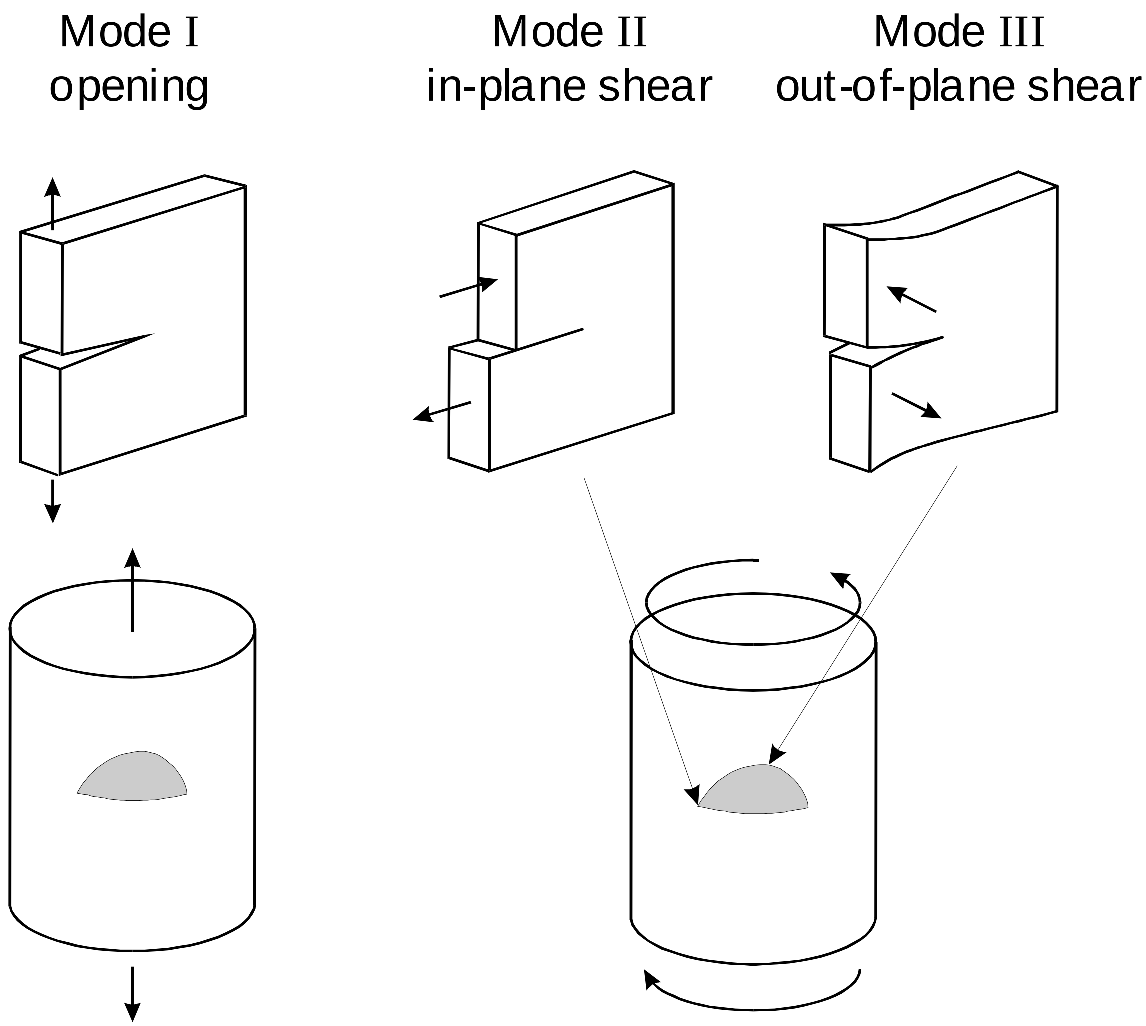
Slip Bands


Extrusion
Undeformed
material
Intrusion
Loading
Unloading

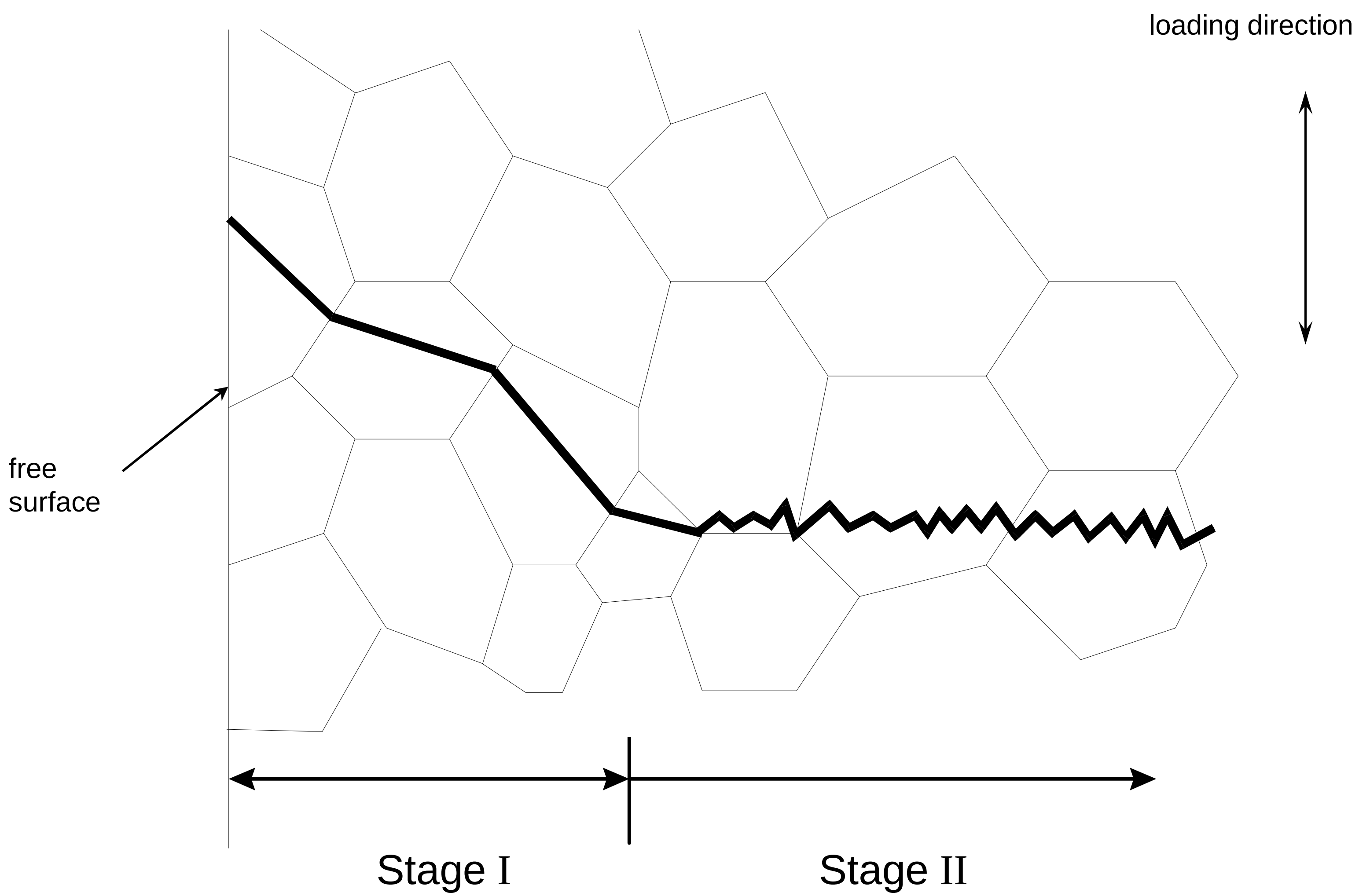
Slip Bands



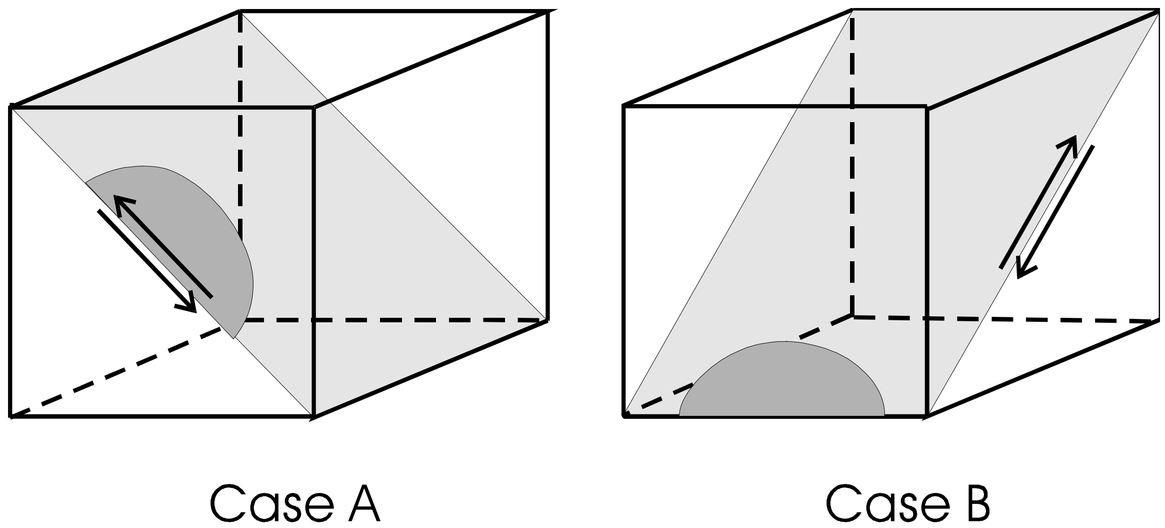
Mode I, Mode II, and Mode III



Stage I and Stage II



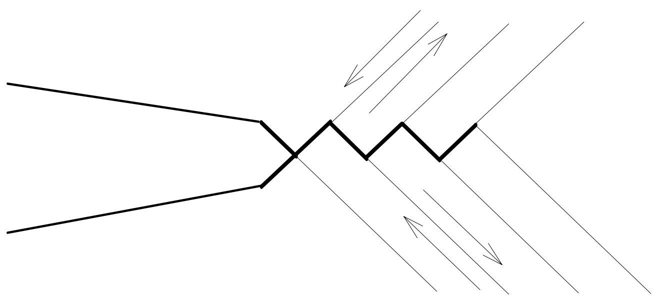
Case A and Case B

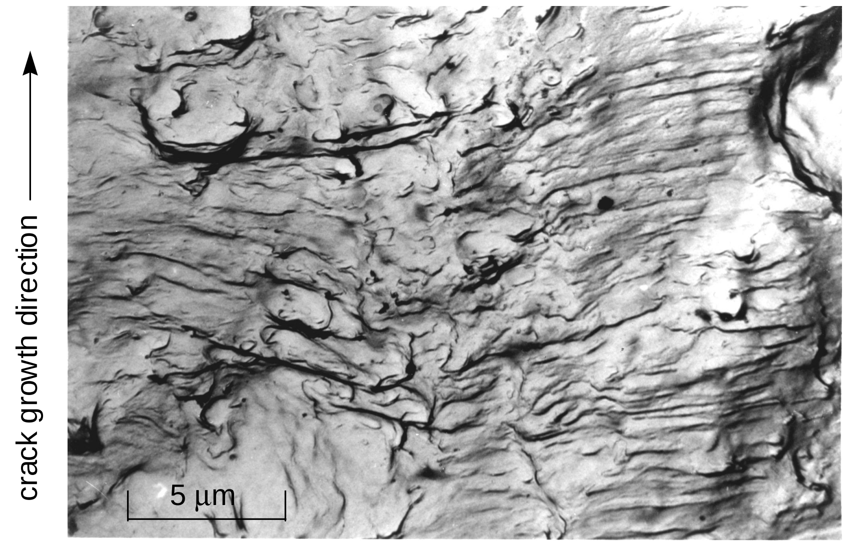


Growth along the surface

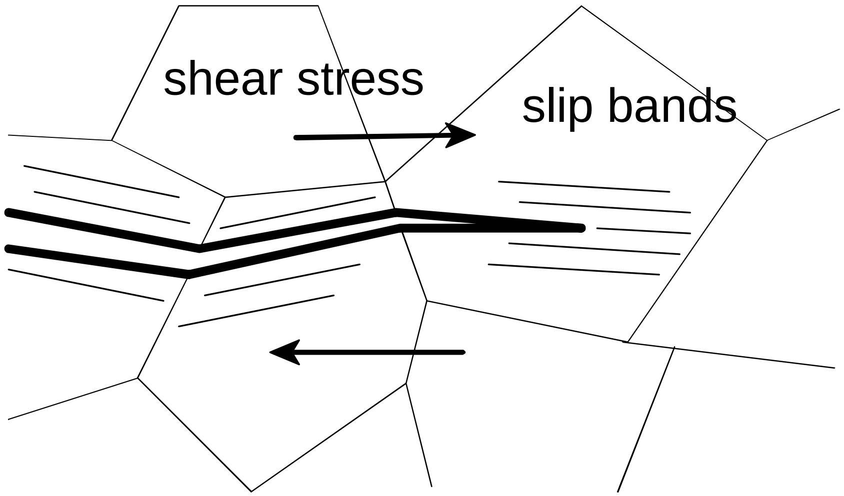
Growth into the surface

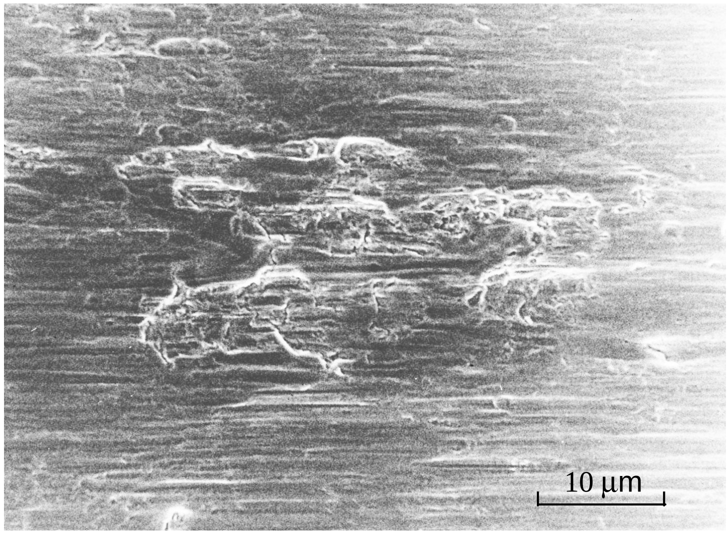
Mode I Growth





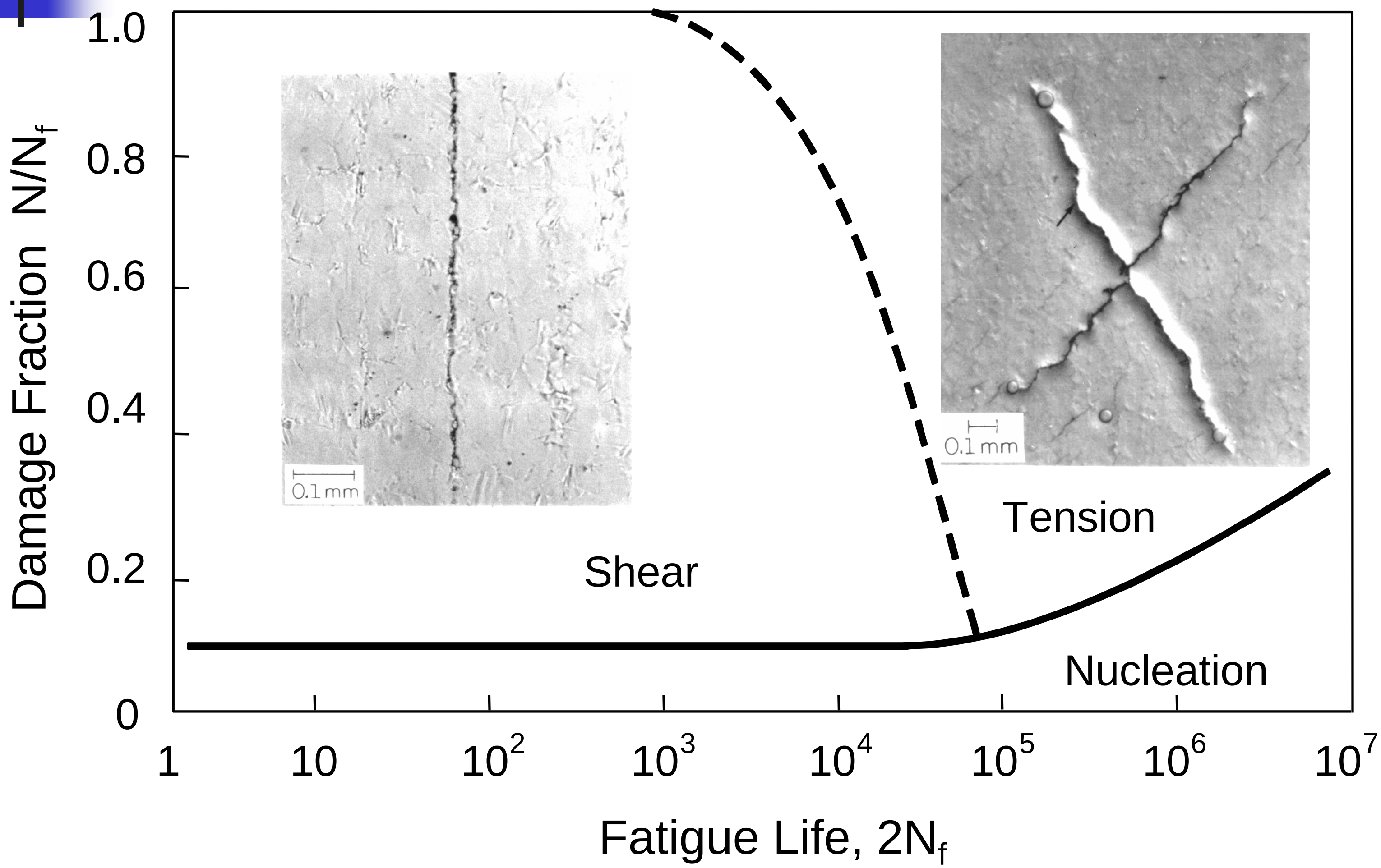
Mode II Growth



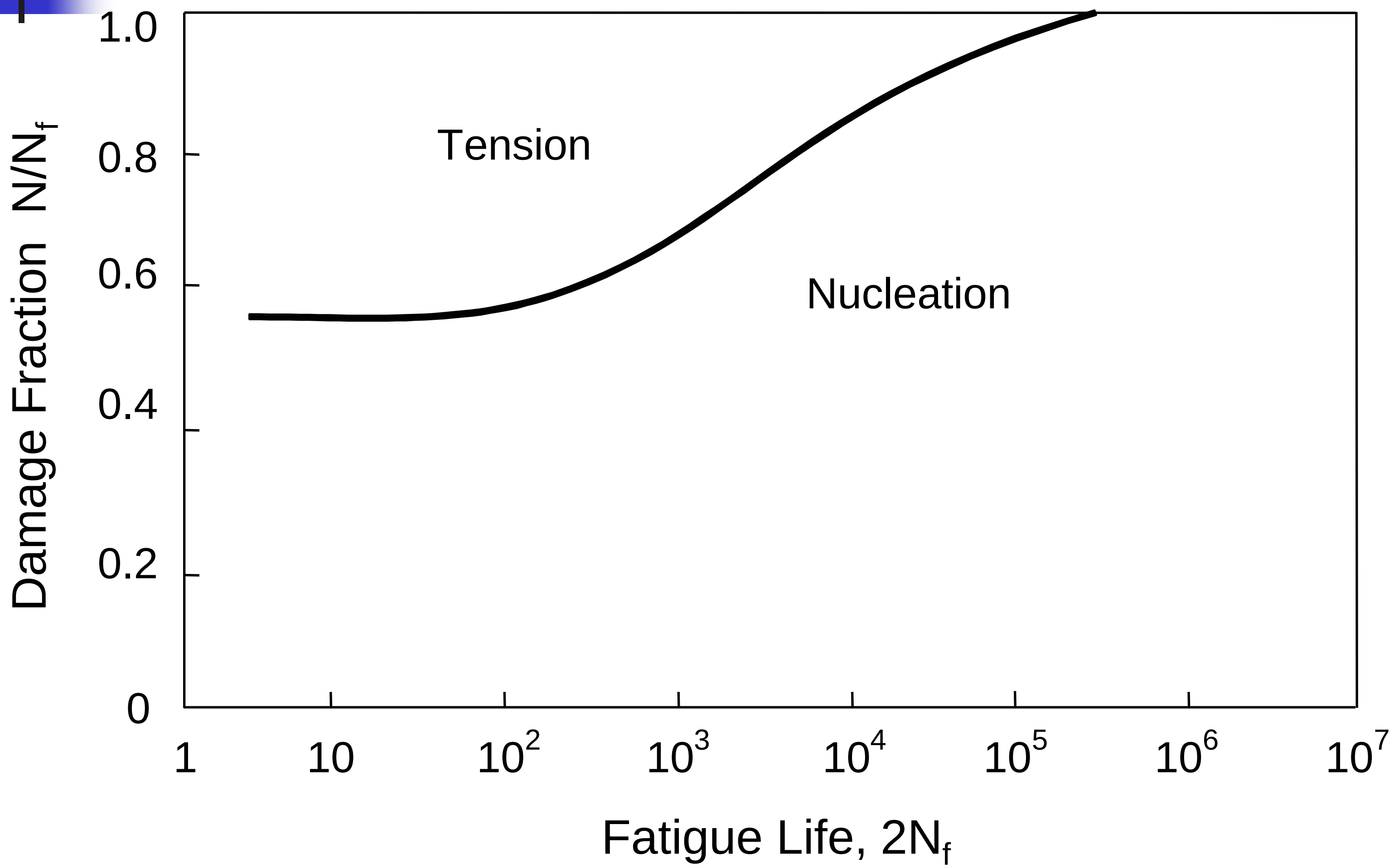


crack growth direction

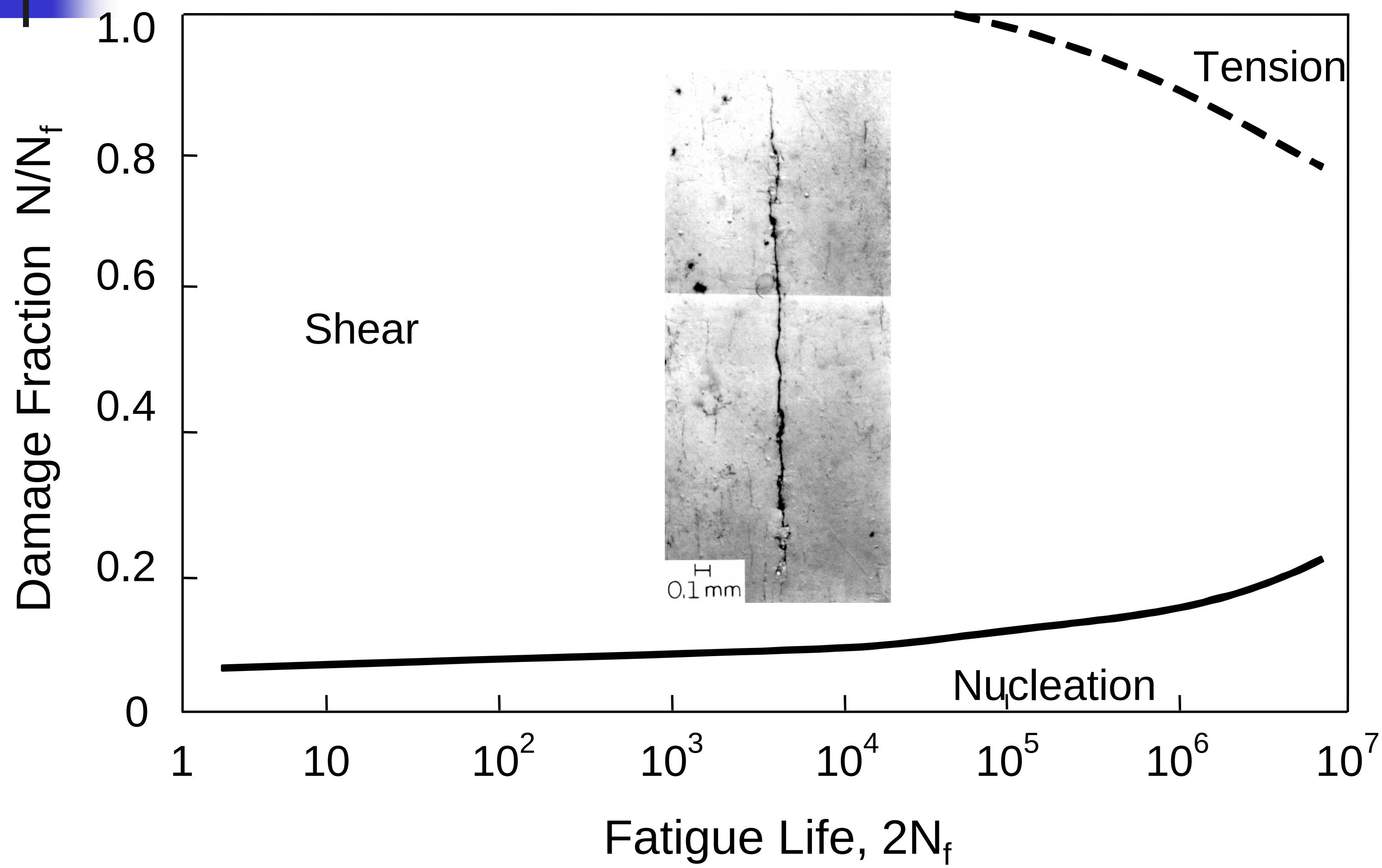
304 Stainless Steel - Torsion



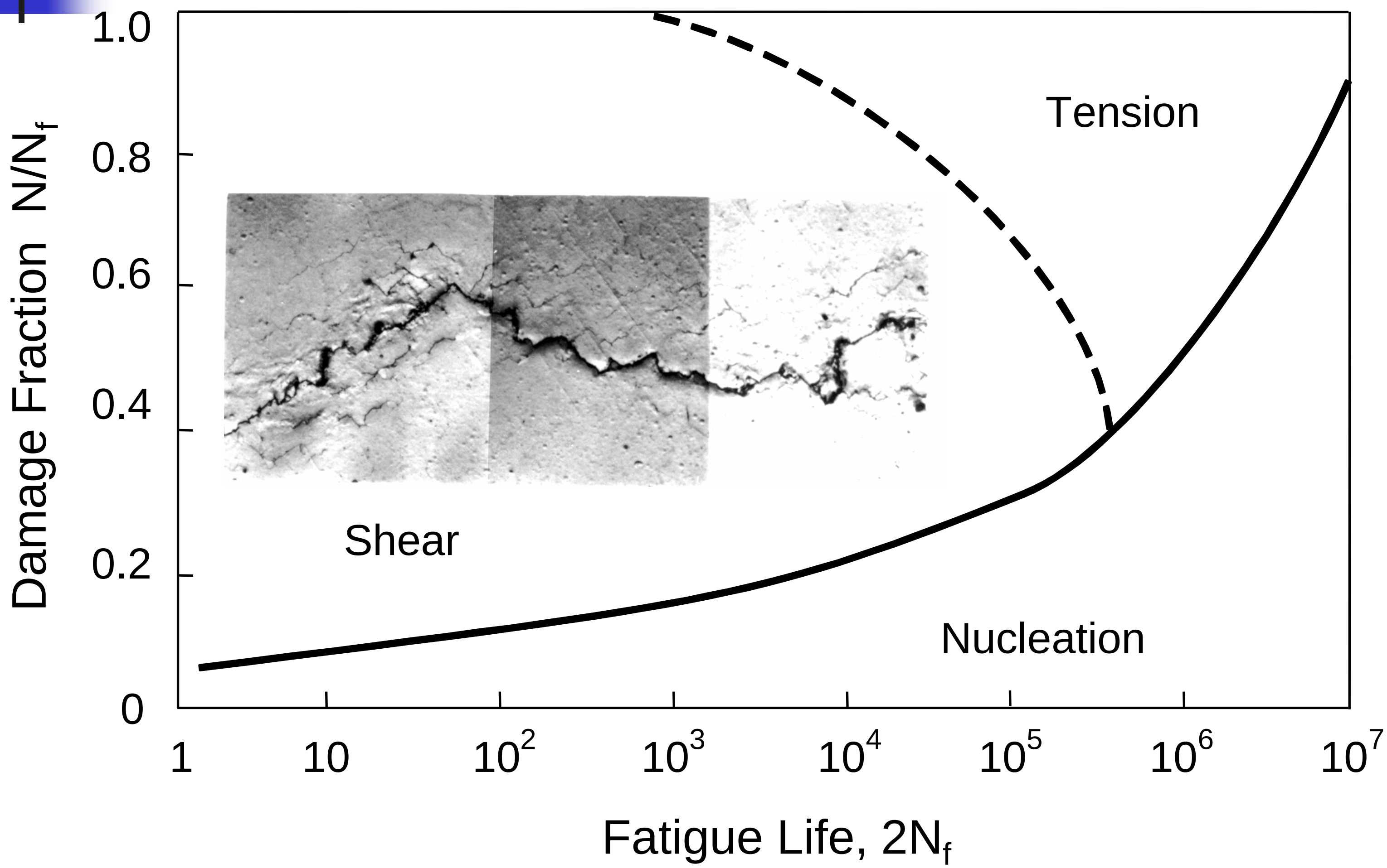
304 Stainless Steel - Tension



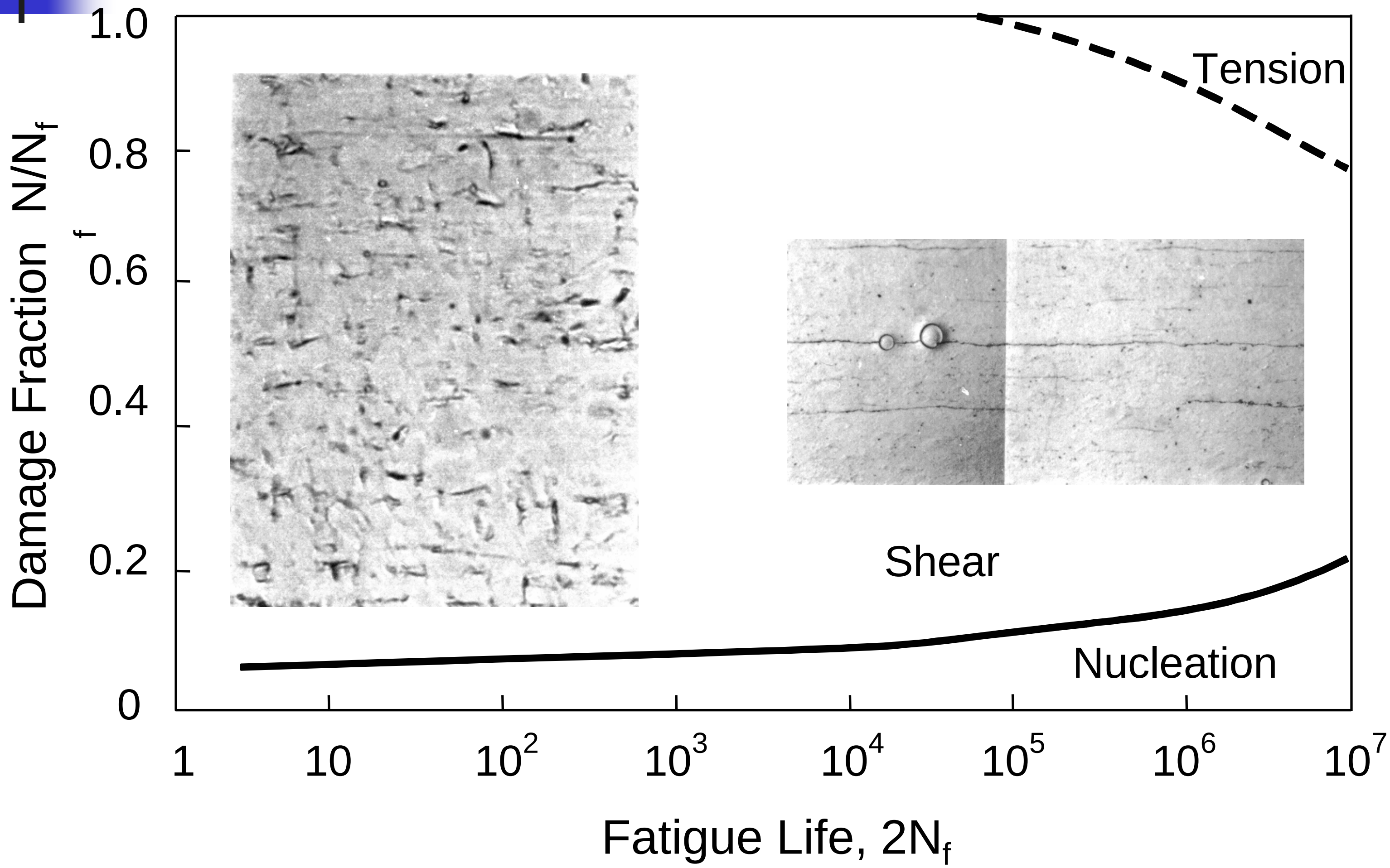
Inconel 718 - Torsion



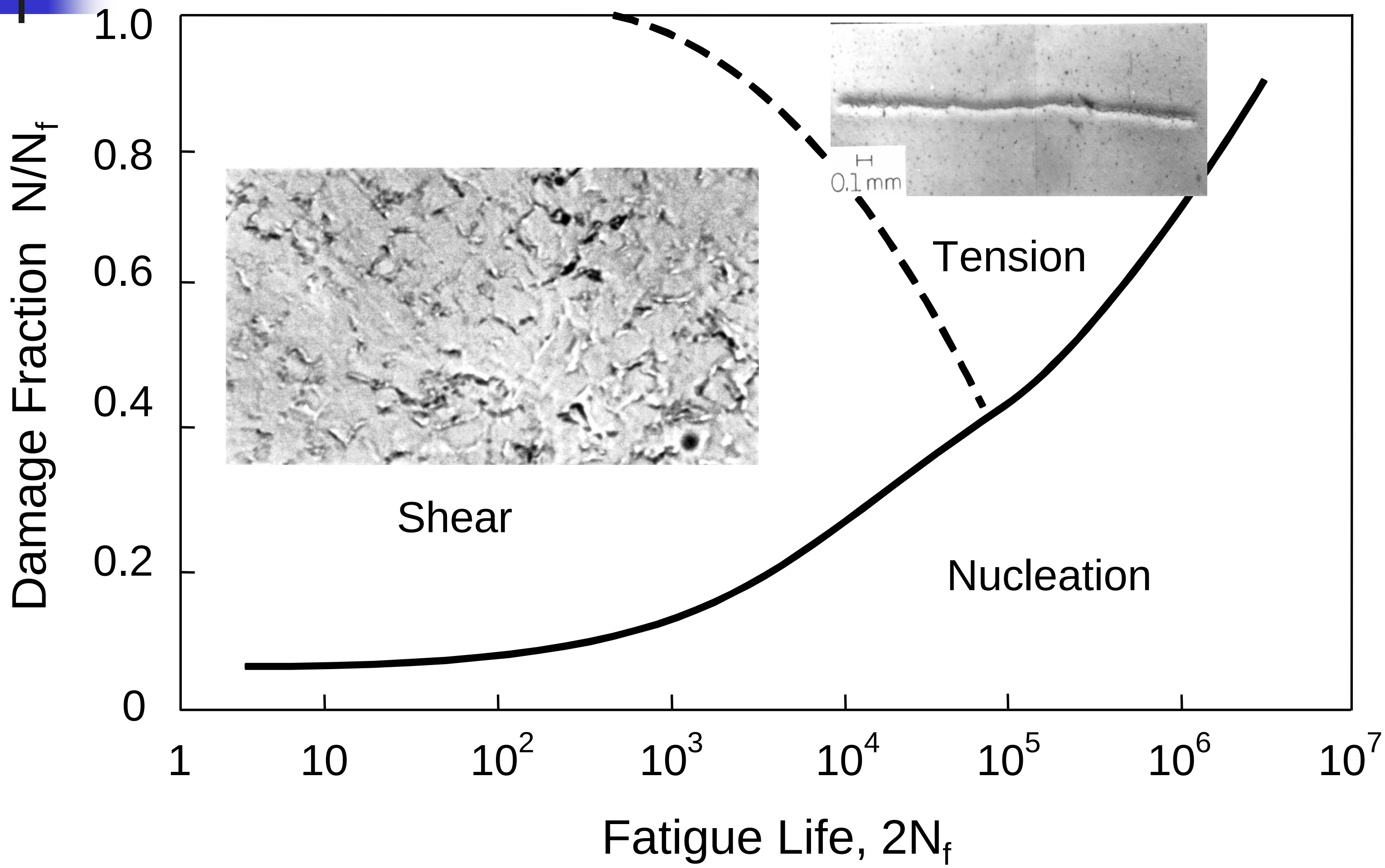
Inconel 718 - Tension



1045 Steel - Torsion



1045 Steel - Tension



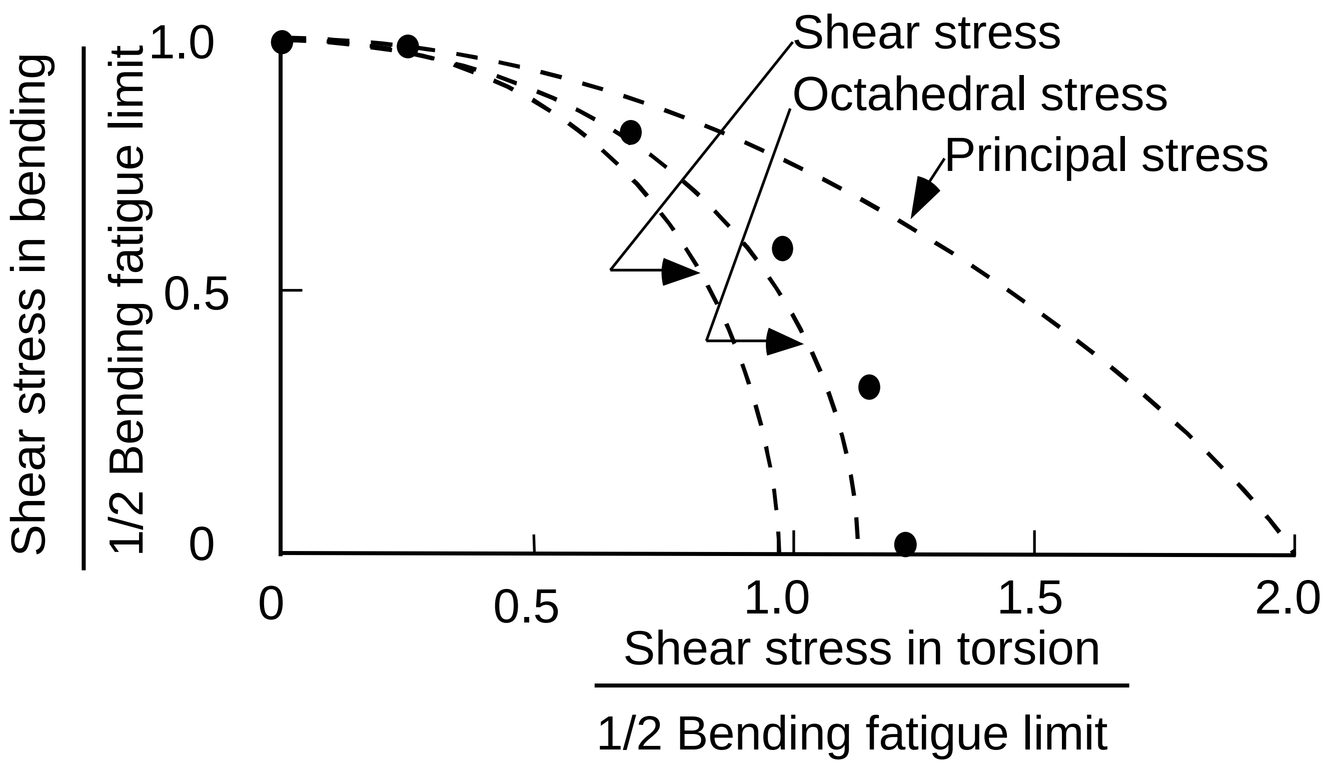
Fatigue Mechanisms Summary

- Fatigue cracks nucleate in shear
- Fatigue cracks grow in either shear or tension depending on material and state of stress

Stress Based Models

- Sines
- Findley
- Dang Van

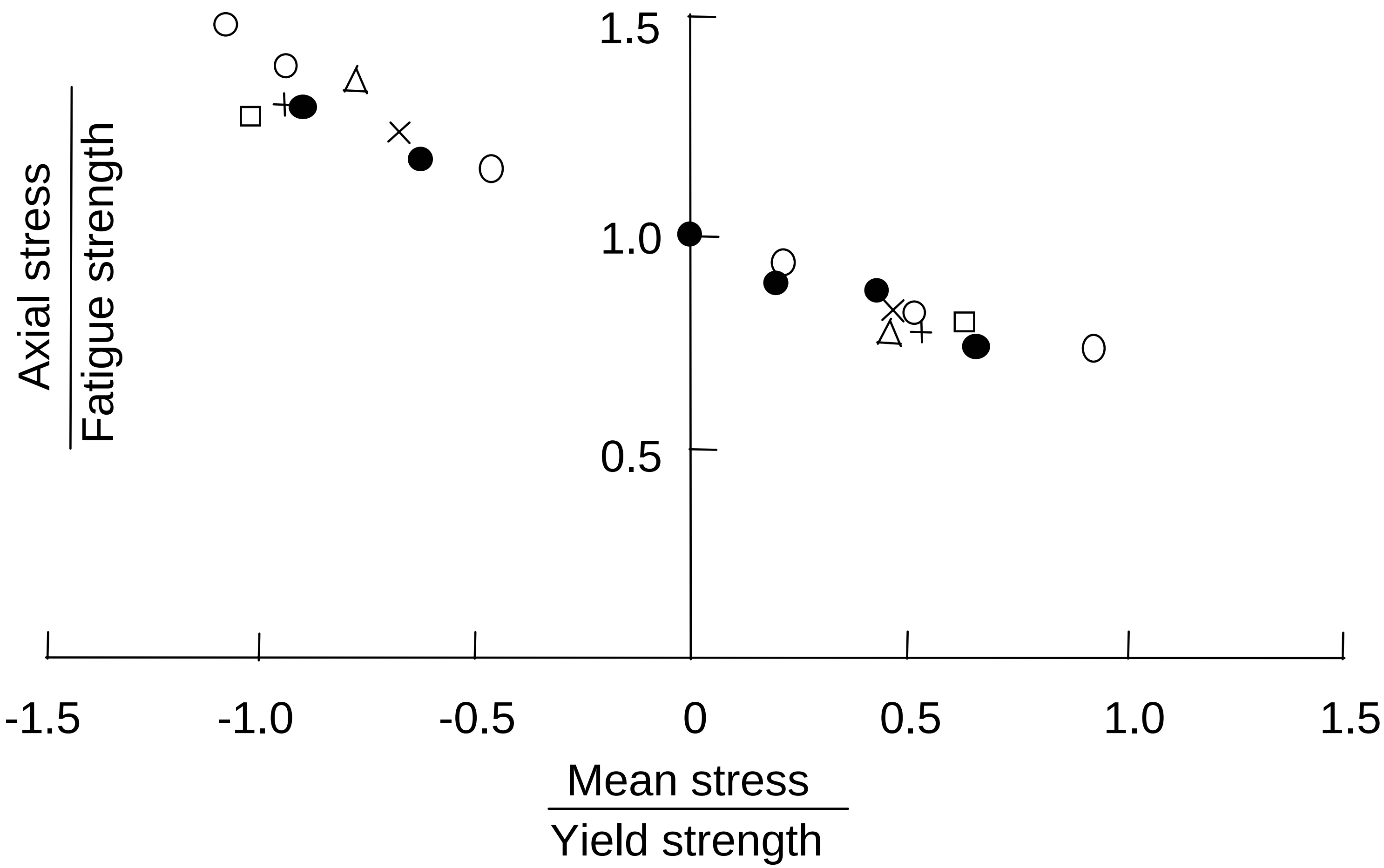
Bending Torsion Correlation



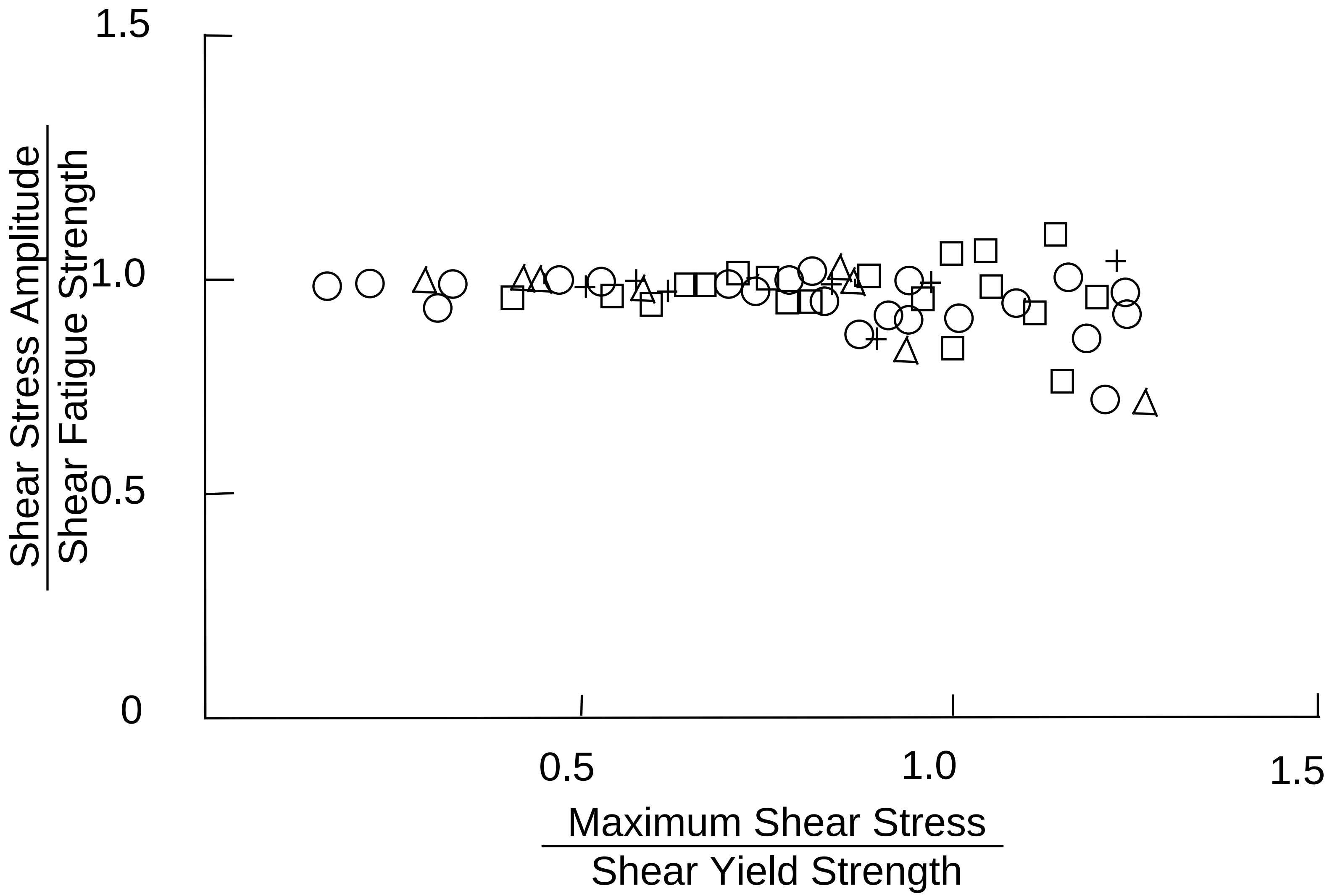
Test Results

- Cyclic tension with static tension
- Cyclic torsion with static torsion
- Cyclic tension with static torsion
- Cyclic torsion with static tension

Cyclic Tension with Static Tension

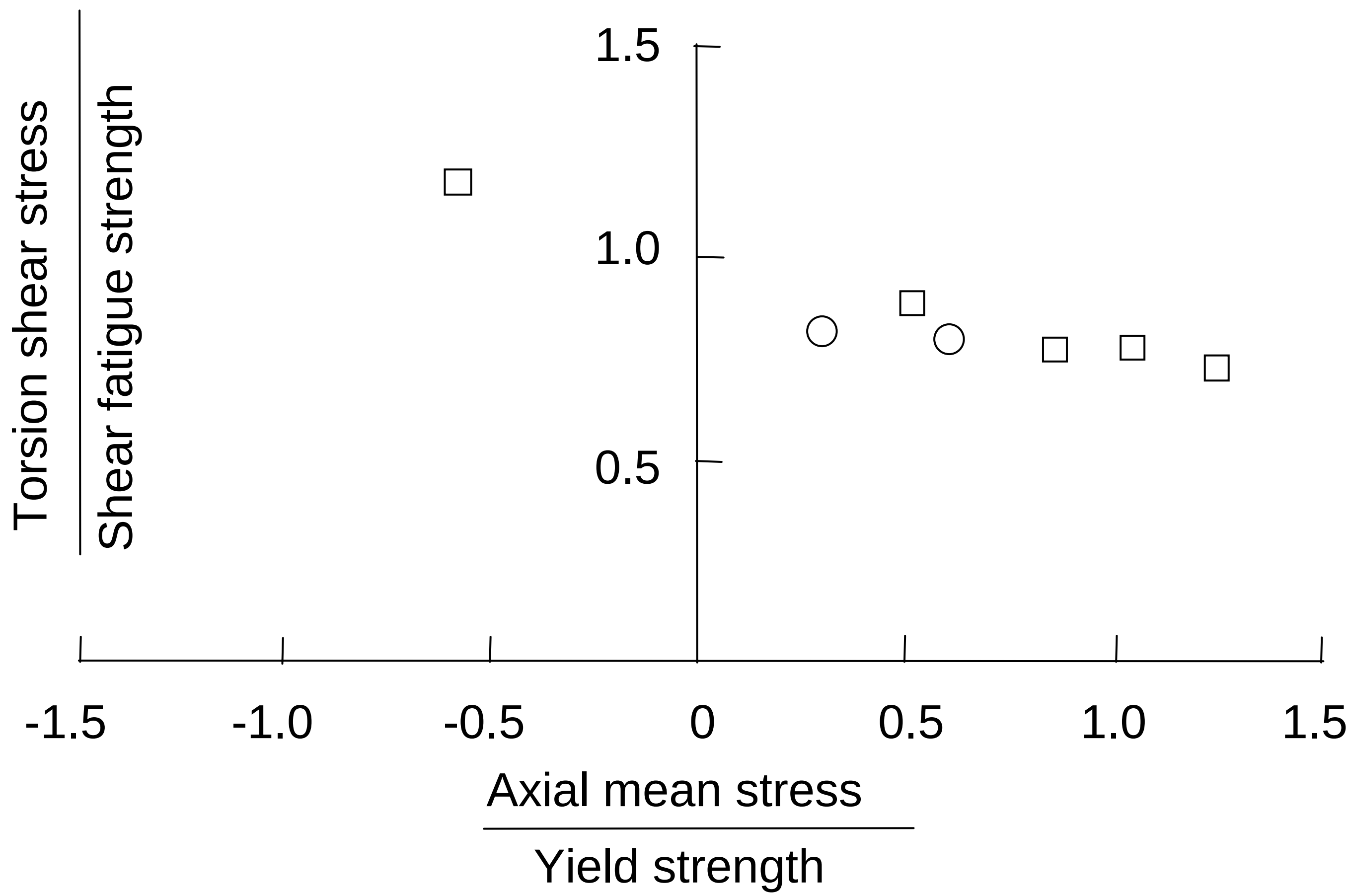


Cyclic Torsion with Static Torsion



Cyclic Tension with Static Torsion

Cyclic Torsion with Static Tension



Conclusions

- Tension mean stress affects both tension and torsion
- Torsion mean stress does not affect tension or torsion

Sines

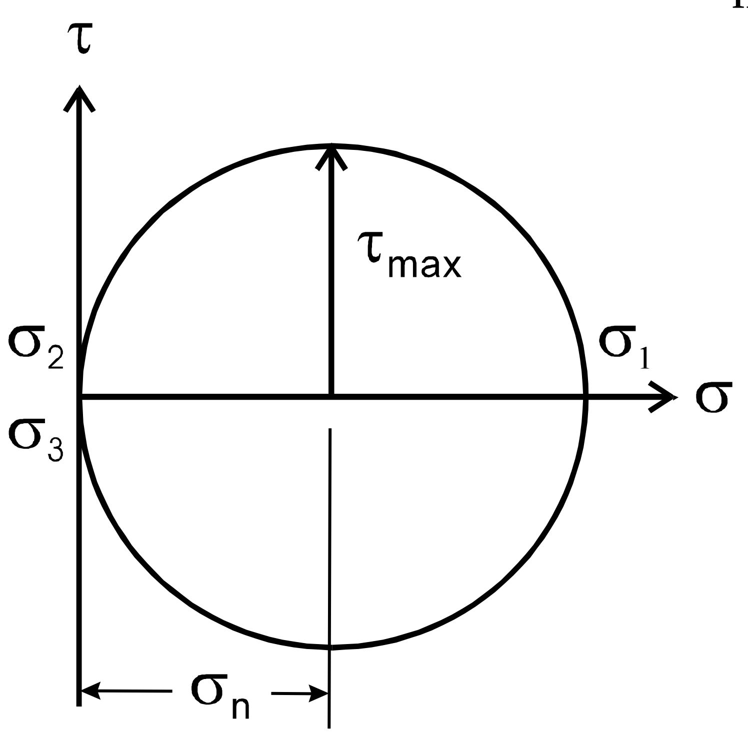
$$\frac{\Delta\tau_{oct}}{2} = \alpha(3\sigma_h) = \beta$$

$$\frac{1}{6}\sqrt{(\Delta\sigma_x - \Delta\sigma_y)^2 + (\Delta\sigma_x - \Delta\sigma_z)^2 + (\Delta\sigma_y - \Delta\sigma_z)^2 + 6(\Delta\tau_{xy}^2 + \Delta\tau_{xz}^2 + \Delta\tau_{yz}^2)} +$$

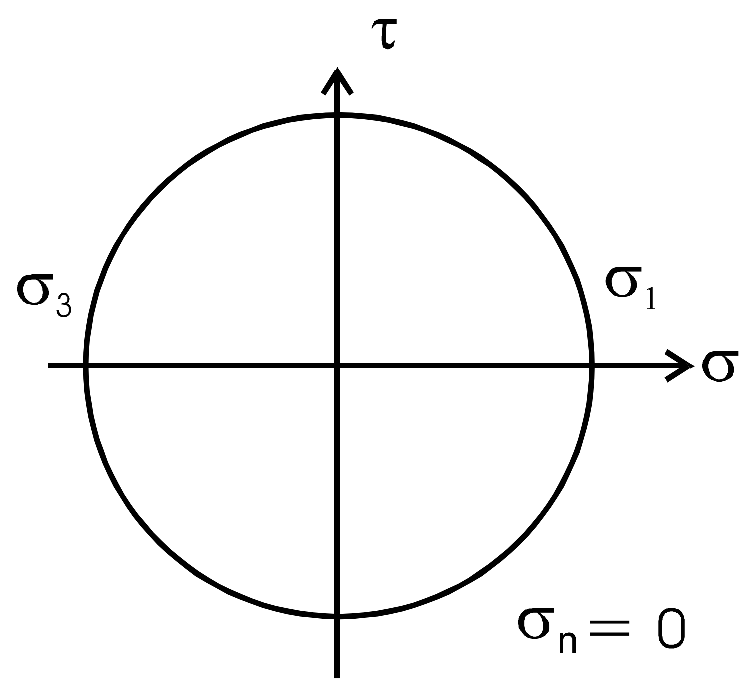
$$\alpha(\sigma_x^{mean} + \sigma_y^{mean} + \sigma_z^{mean}) = \beta$$

Findley

$$\left(\frac{\Delta\tau}{2} + k\sigma_n\right)_{max} = f$$

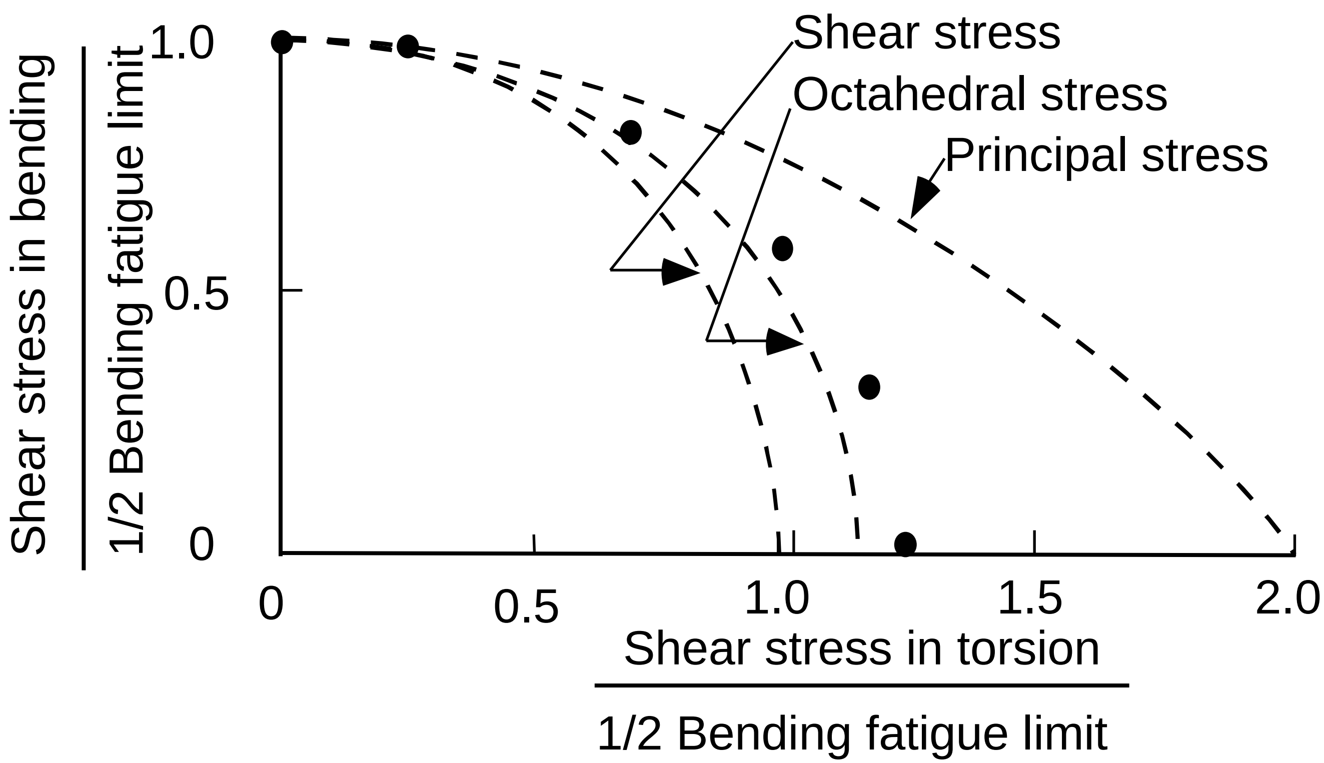


tension



torsion

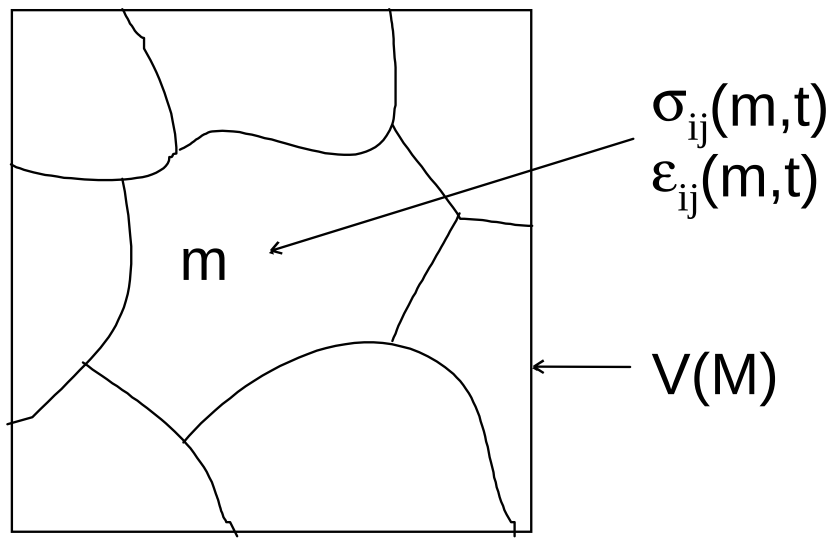
Bending Torsion Correlation



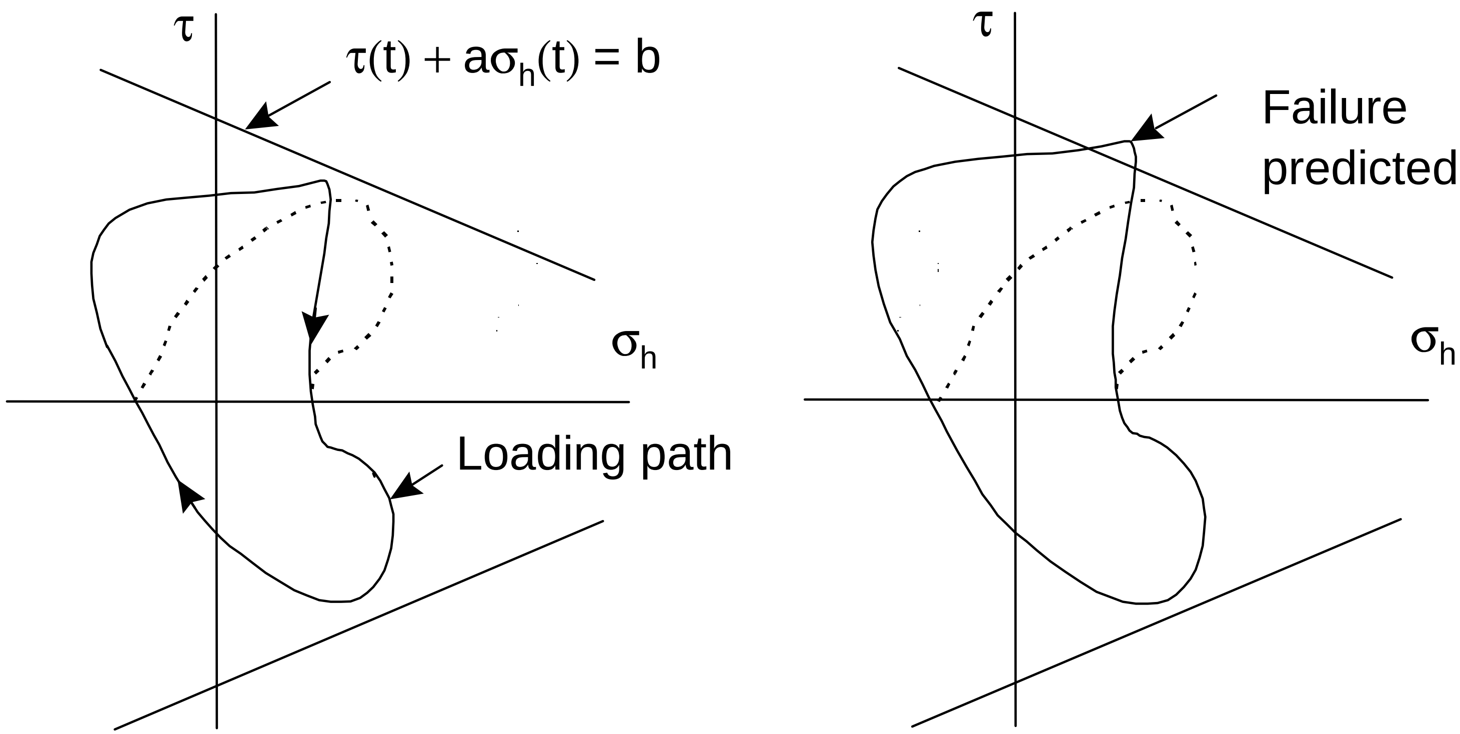
Dang Van

$$\tau(t) + a\sigma_h(t) = b$$

$\Sigma_{ij}(M,t)$ $E_{ij}(M,t)$



Dang Van (continued)



Stress Based Models Summary

Sines: $$\frac{\Delta\tau_{oct}}{2} = \alpha(3\sigma_h) = \beta$$

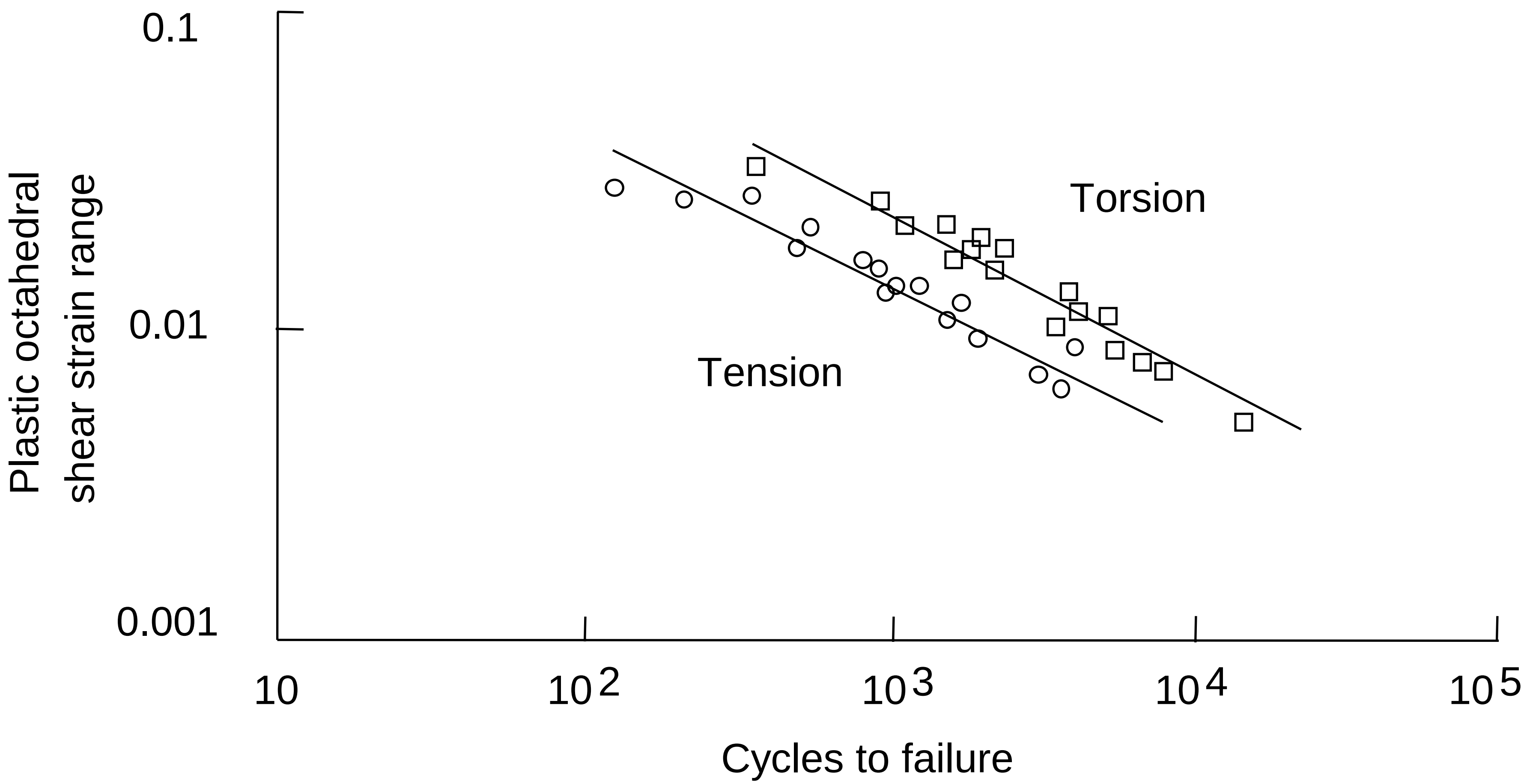
Findley: $$\left(\frac{\Delta\tau}{2} + k\sigma_n\right)_{max} = f$$

Dang Van: $$\tau(t) + a\sigma_h(t) = b$$

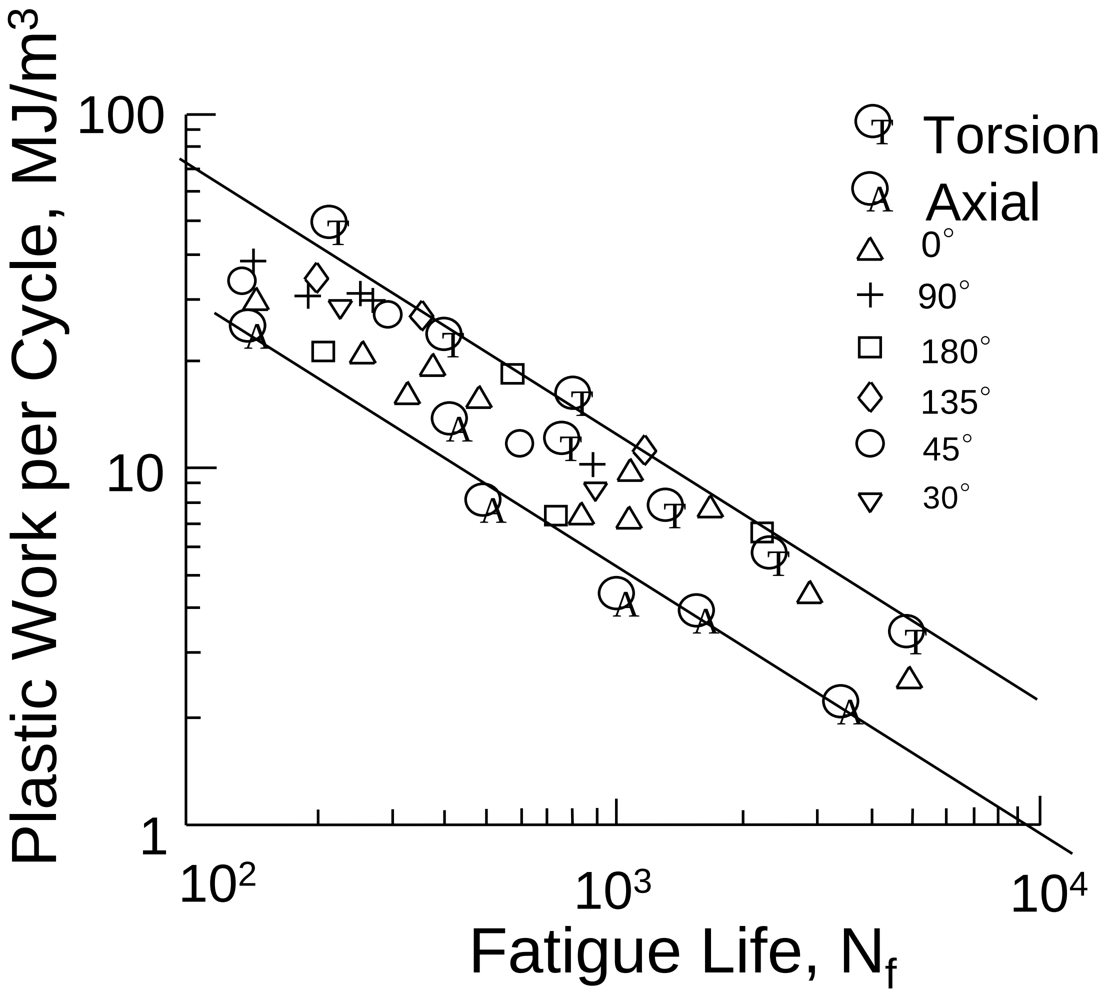
Strain Based Models

- Plastic Work
- Brown and Miller
- Fatemi and Socie
- Smith Watson and Topper
- Liu

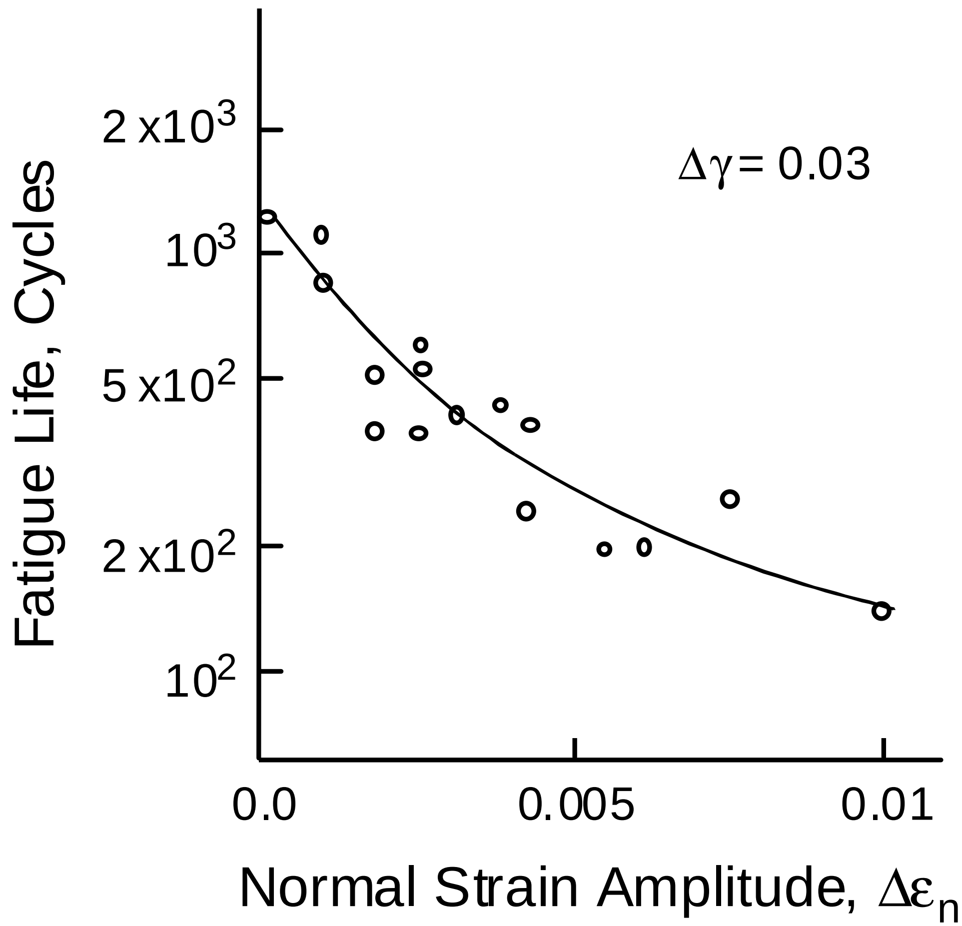
Octahedral Shear Strain



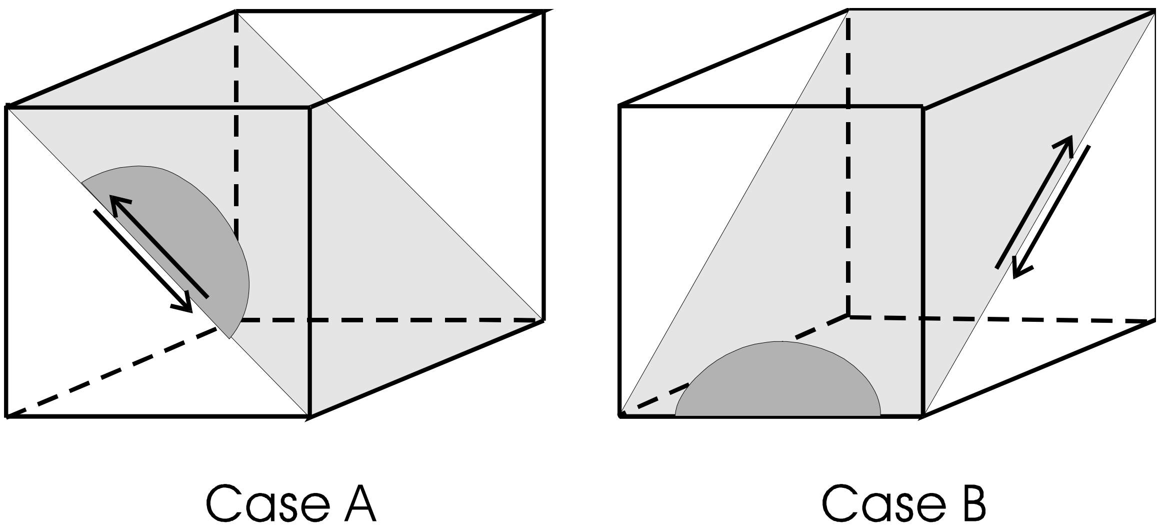
Plastic Work



Brown and Miller



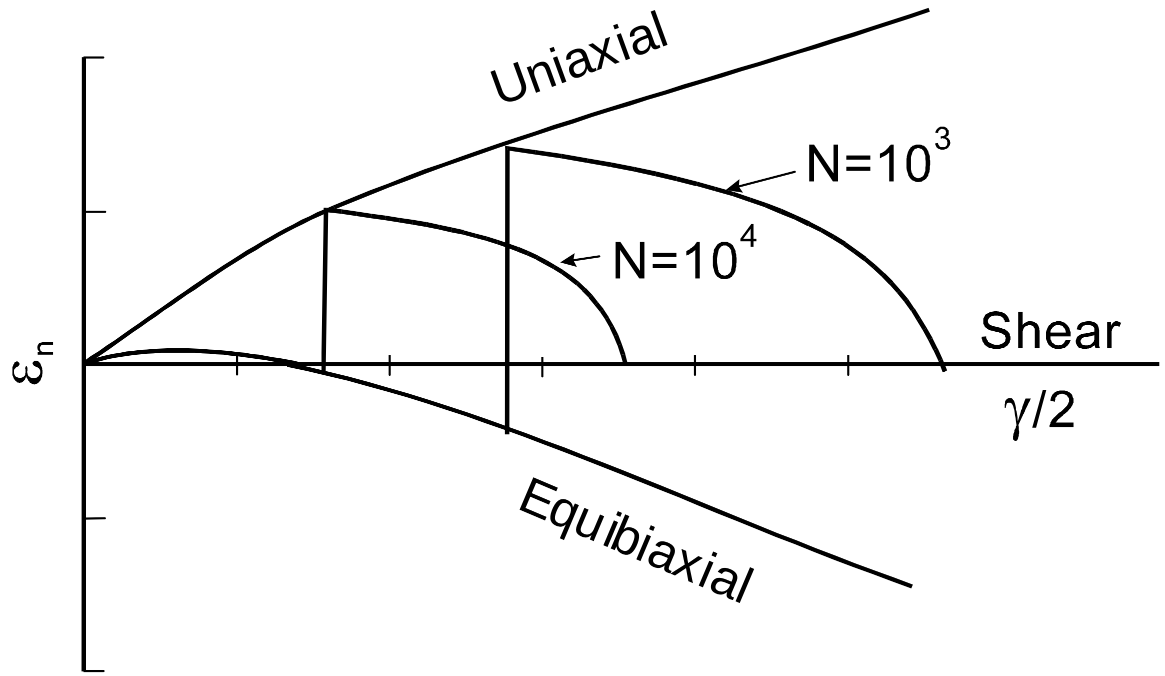
Case A and B



Growth along the surface

Growth into the surface

Brown and Miller (continued)

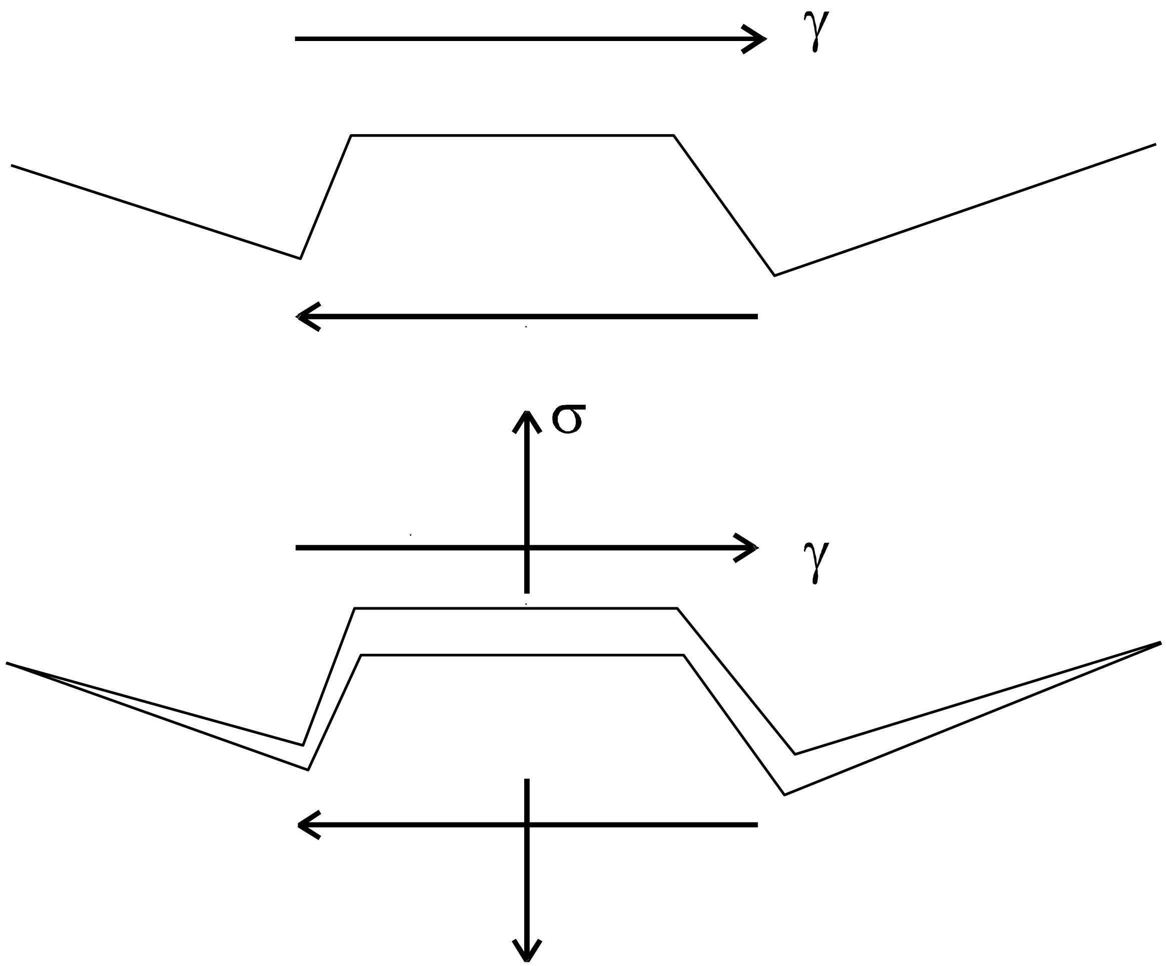


Brown and Miller (continued)

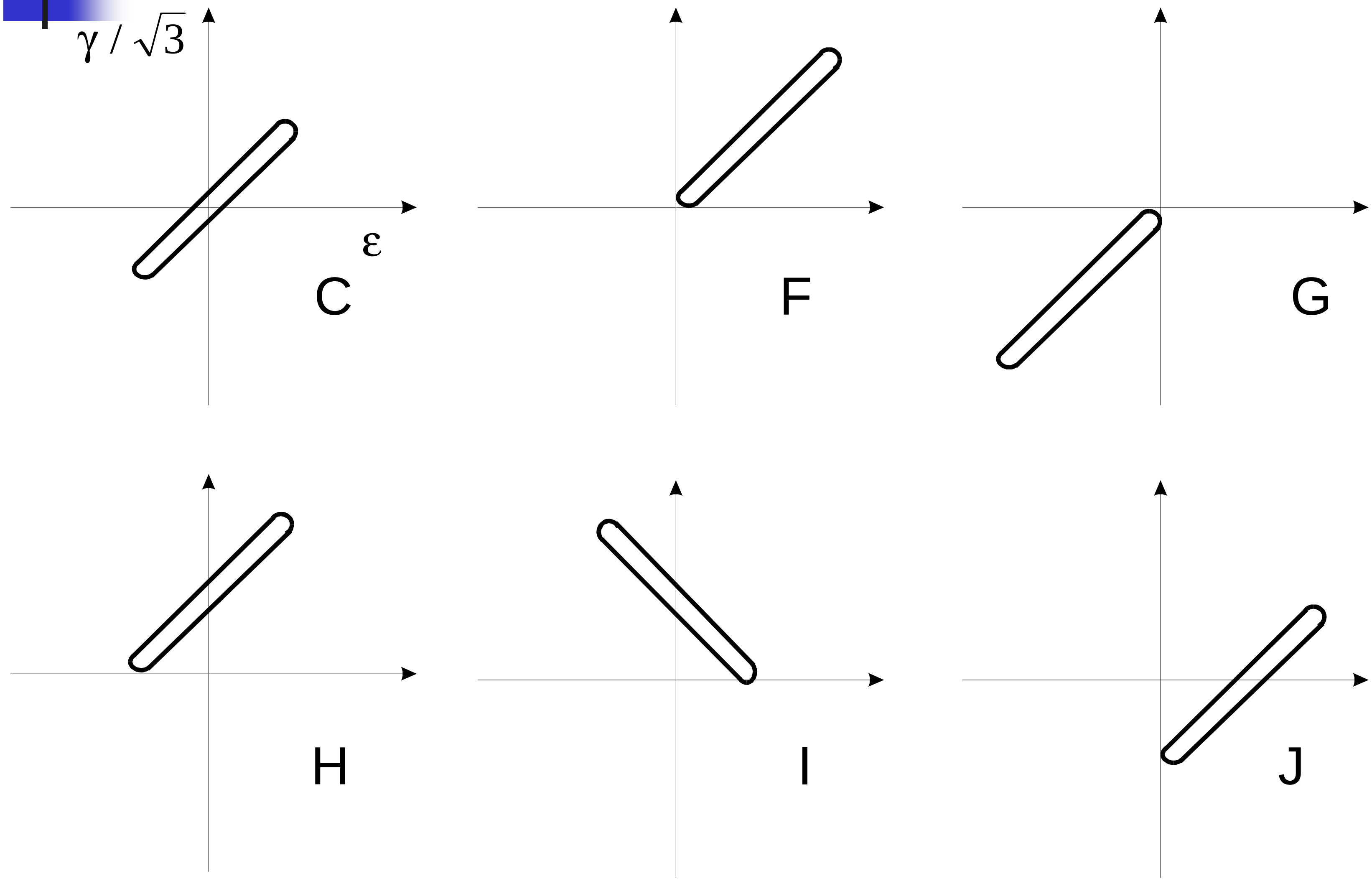
$$\Delta\hat{\gamma} = \left(\Delta\gamma_{max}^{\alpha} + S\Delta\varepsilon_{n}^{\alpha}\right)^{\frac{1}{\alpha}}$$

$$\frac{\Delta\gamma_{max}}{2} + S\,\Delta\varepsilon_{n} = A\,\frac{\sigma_{f}^{'} - 2\sigma_{n,mean}}{E}\left(2N_{f}\right)^{b} + B\varepsilon_{f}^{'}\left(2N_{f}\right)^{c}$$

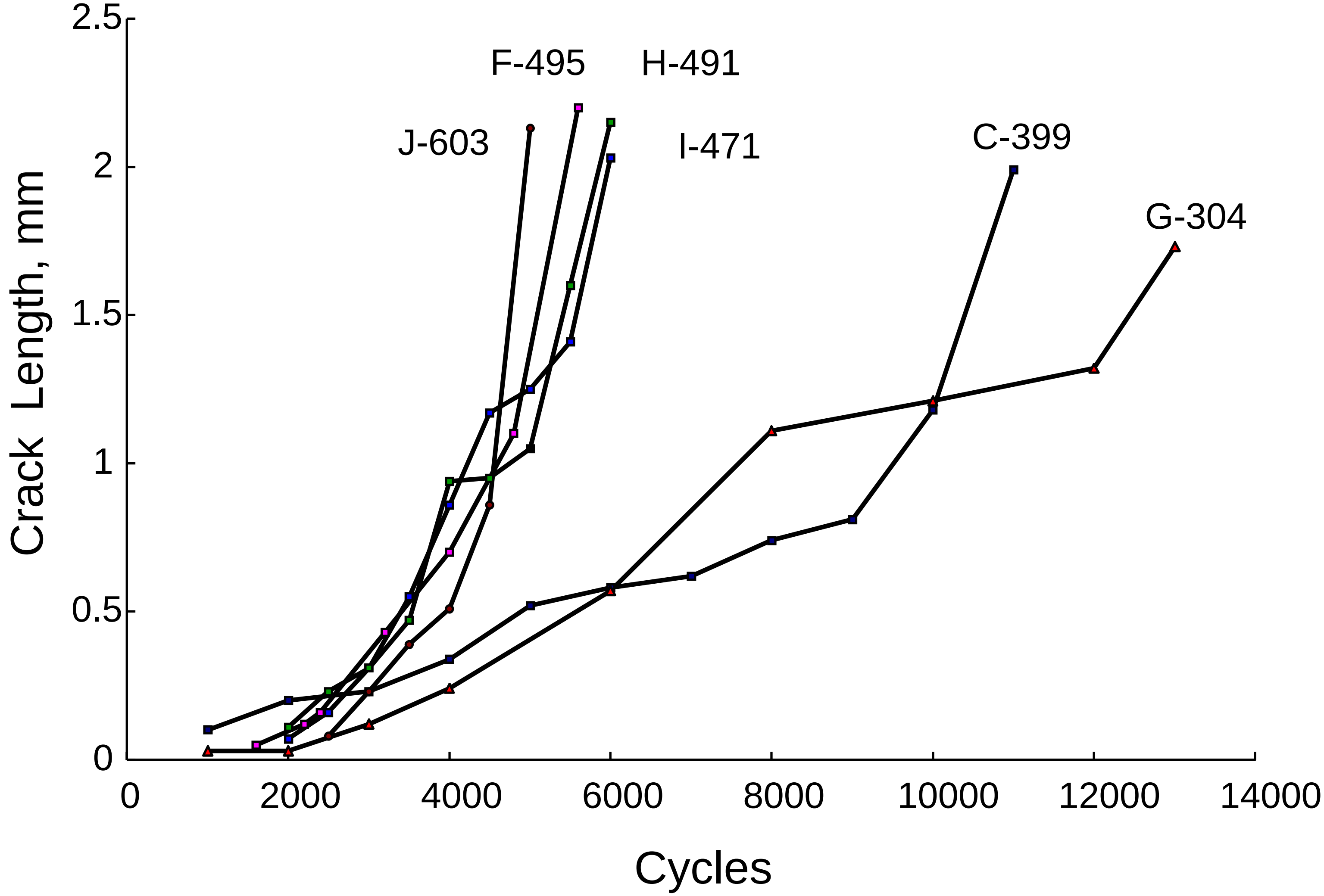
Fatemi and Socie



Loading Histories



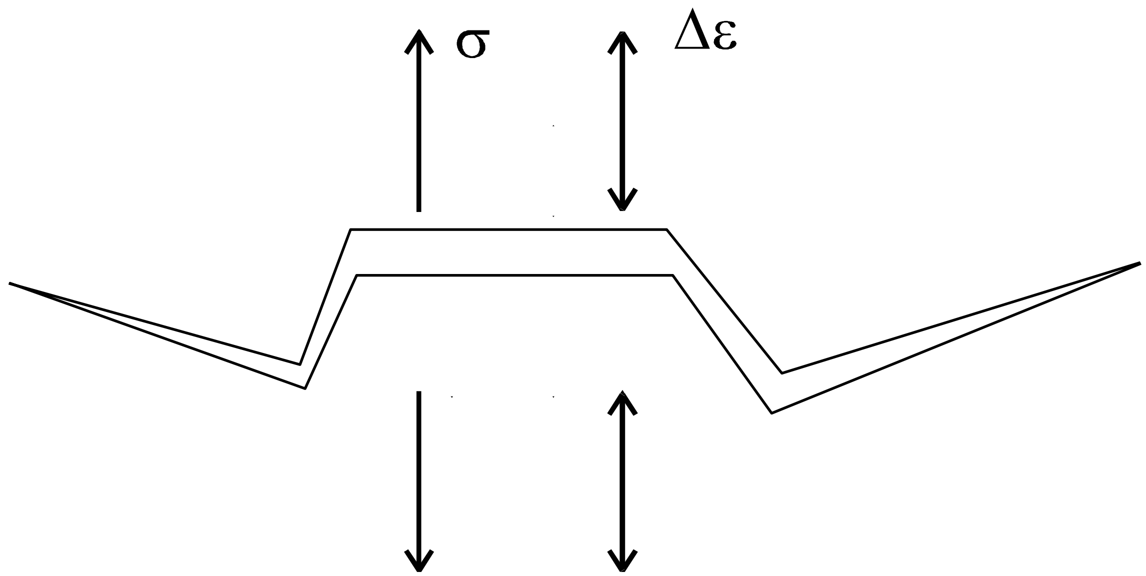
Crack Length Observations



Fatemi and Socie

$$\frac{\Delta\gamma}{2}\left(1+k\frac{\sigma_{n,max}}{\sigma_y}\right)=\frac{\tau_f^{'}}{G}(2N_f)^{bo}+\gamma_f^{'}(2N_f)^{co}$$

Smith Watson Topper



SWT

$$\sigma_n \frac{\Delta\varepsilon_1}{2} = \frac{\sigma_f'^{\,2}}{E}(2N_f)^{2b} + \sigma_f'\varepsilon_f'(2N_f)^{b+c}$$

Liu

Virtual strain energy for both mode I and mode II cracking

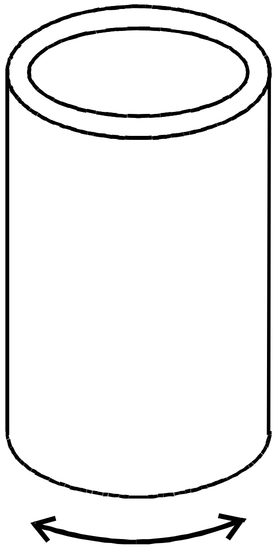
$$\Delta W_I = (\Delta\sigma_n \, \Delta\varepsilon_n)_{max} + (\Delta\tau \, \Delta\gamma)$$

$$\Delta W_I = 4\sigma_f^{'}\varepsilon_f^{'}(2N_f)^{b+c} + \frac{4\sigma_f^{'2}}{E}(2N_f)^{2b}$$

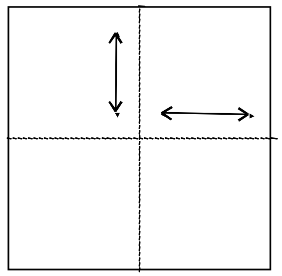
$$\Delta W_{II} = (\Delta\sigma_n \, \Delta\varepsilon_n) + (\Delta\tau \, \Delta\gamma)_{max}$$

$$\Delta W_{II} = 4\tau_f^{'}\gamma_f^{'}(2N_f)^{bo+co} + \frac{4\tau_f^{'2}}{G}(2N_f)^{2bo}$$

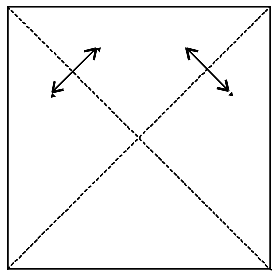
Cyclic Torsion



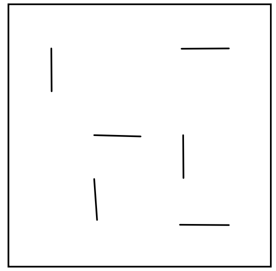
Cyclic Torsion



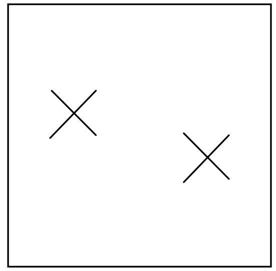
Cyclic Shear Strain



Cyclic Tensile Strain

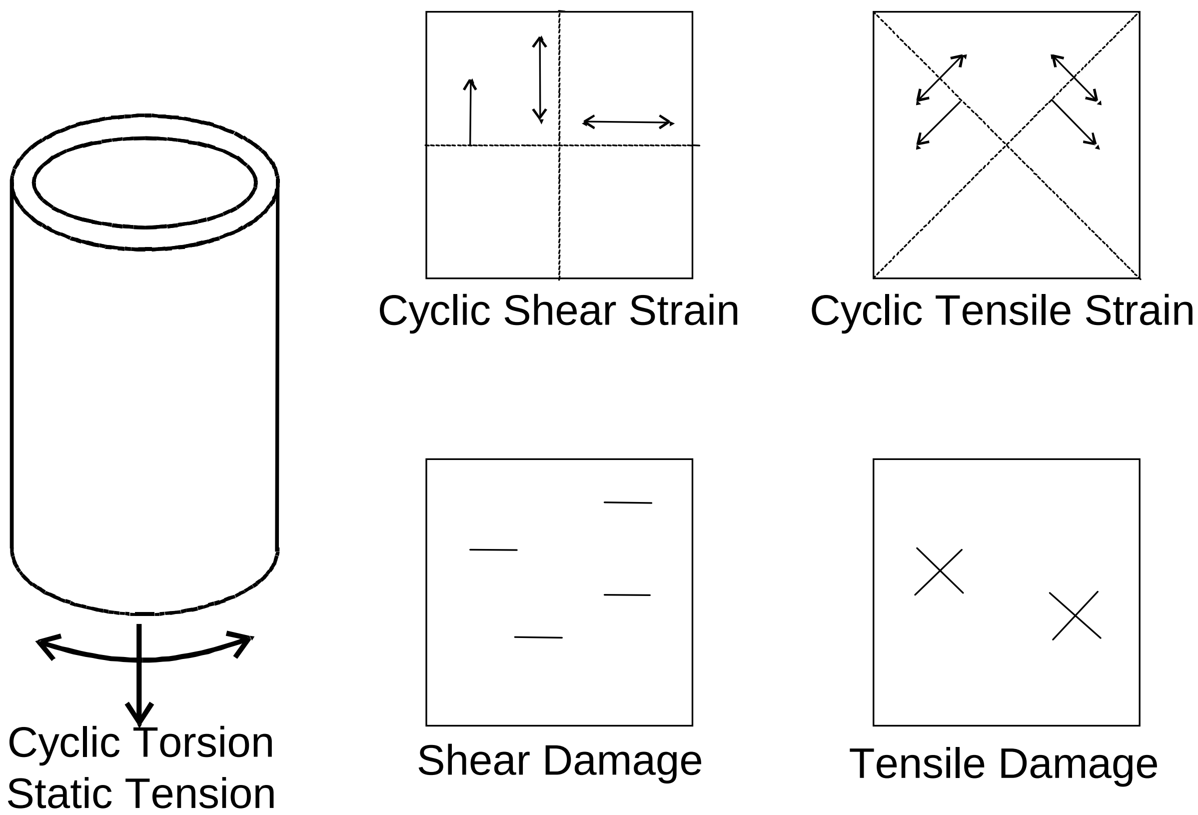


Shear Damage

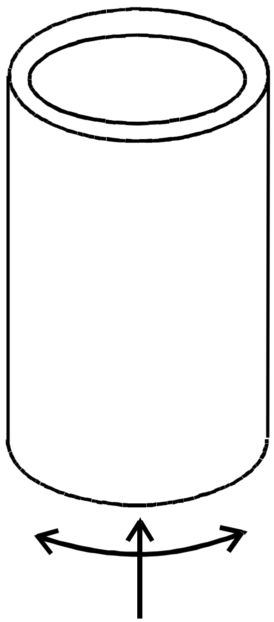


Tensile Damage

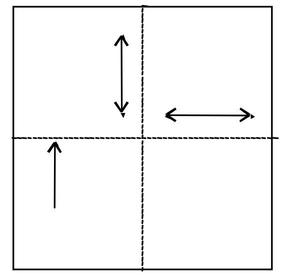
Cyclic Torsion with Static Tension



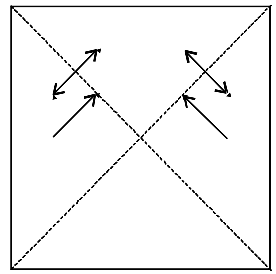
Cyclic Torsion with Compression



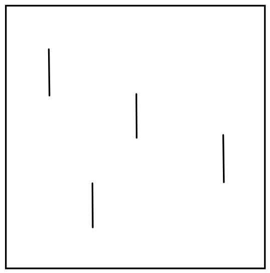
Cyclic Torsion
Static Compression



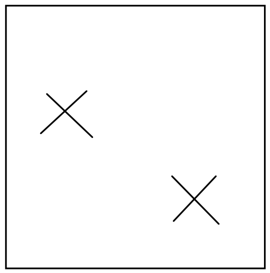
Cyclic Shear Strain



Cyclic Tensile Strain

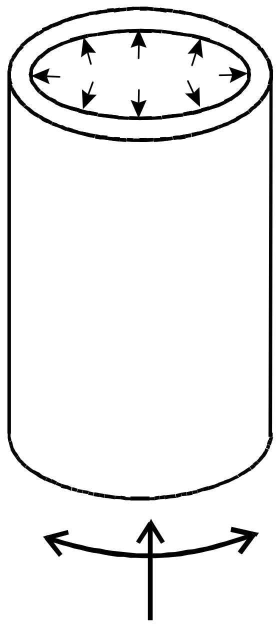


Shear Damage

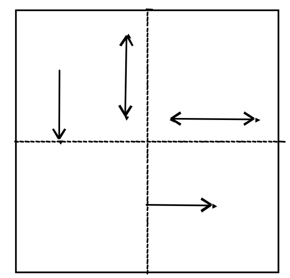


Tensile Damage

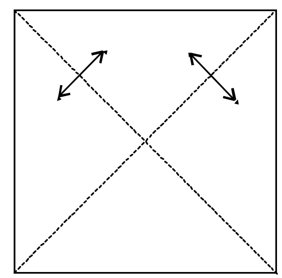
Cyclic Torsion with Tension and Compression



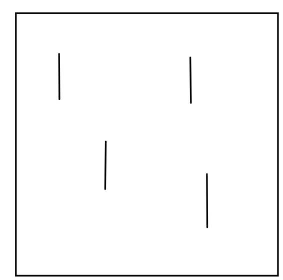
Cyclic Torsion
Static Compression
Hoop Tension



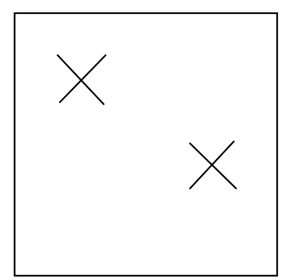
Cyclic Shear Strain



Cyclic Tensile Strain



Shear Damage



Tensile Damage

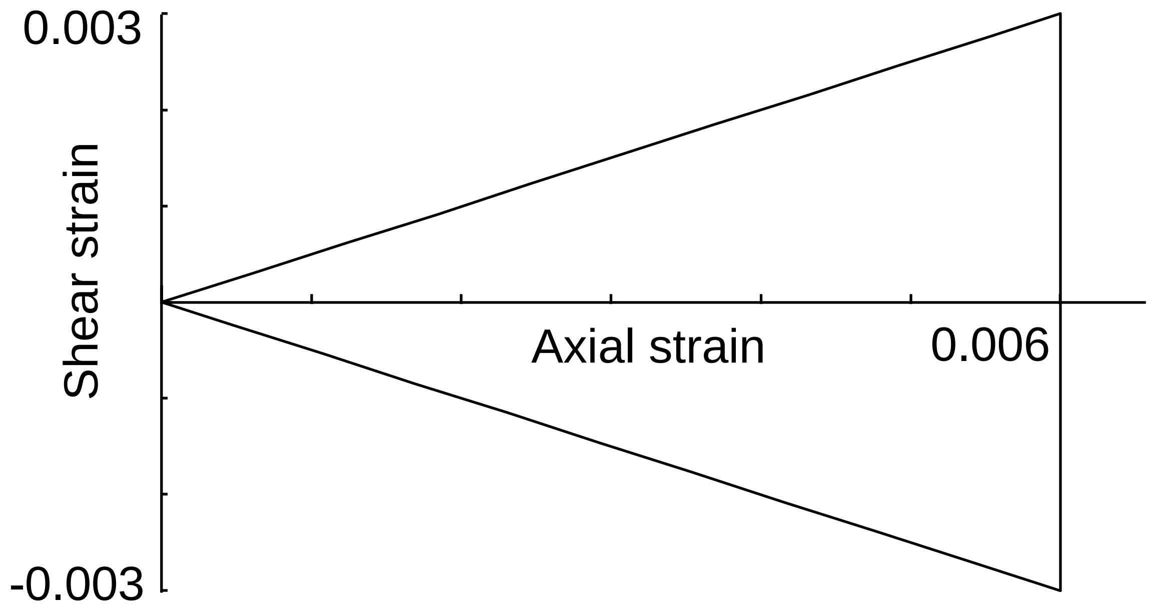
Test Results

Load Case	$\Delta\gamma/2$	σ_{hoop} MPa	σ_{axial} MPa	N_f
Torsion	0.0054	0	0	45,200
with tension	0.0054	0	450	10,300
with compression	0.0054	0	-500	50,000
with tension and compression	0.0054	450	-500	11,200

Conclusions

- All critical plane models correctly predict these results
- Hydrostatic stress models can not predict these results

Loading History



Model Comparison

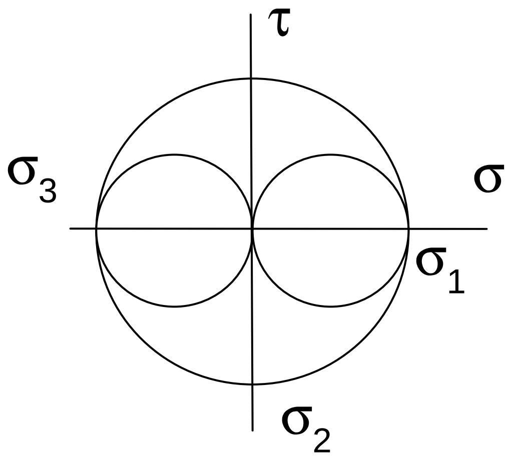
Summary of calculated fatigue lives

Model	Equation	Life
Epsilon	6.5	14,060
Garud	6.7	5,210
Ellyin	6.17	4,450
Brown-Miller	6.22	3,980
SWT	6.24	9,930
Liu I	6.41	4,280
Liu II	6.42	5,420
Chu	6.37	3,040
Gamma		26,775
Fatemi-Socie	6.23	10,350
Glinka	6.39	33,220

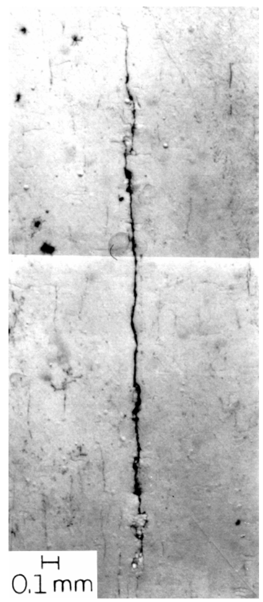
Strain Based Models Summary

- Two separate models are needed, one for tensile growth and one for shear growth
- Cyclic plasticity governs stress and strain ranges
- Mean stress effects are a result of crack closure on the critical plane

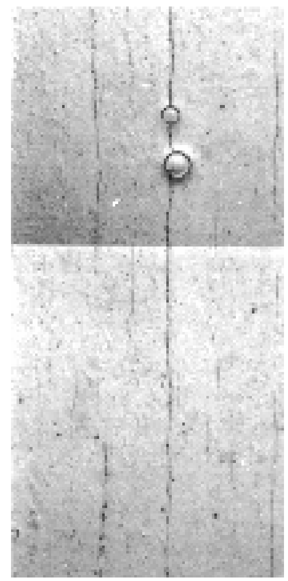
Separate Tensile and Shear Models



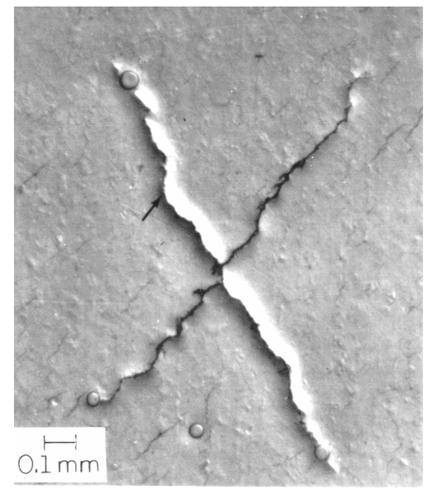
$\sigma_1 = \tau_{xy}$



Inconel



1045 steel



stainless steel

Cyclic Plasticity

$\Delta\varepsilon$

$\Delta\gamma$

$\Delta\varepsilon^p$

$\Delta\gamma^p$

$\Delta\varepsilon\Delta\sigma$

$\Delta\gamma\Delta\tau$

$\Delta\varepsilon^p\Delta\sigma$

$\Delta\gamma^p\Delta\tau$

Mean Stresses

$$\Delta\varepsilon_{eq} = \frac{\sigma'_f - \sigma_{mean}}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c$$

$$\frac{\Delta\gamma_{max}}{2} + S\Delta\varepsilon_n = (1.3 + 0.7S)\frac{\sigma'_f - 2\sigma_n}{E}(2N_f)^b + (1.5 + 0.5S)\varepsilon'_f(2N_f)^c$$

$$\frac{\Delta\gamma}{2}\left(1 + k\frac{\sigma_{n,max}}{\sigma_y}\right) = \frac{\tau'_f}{G}(2N_f)^{bo} + \gamma'_f(2N_f)^{co}$$

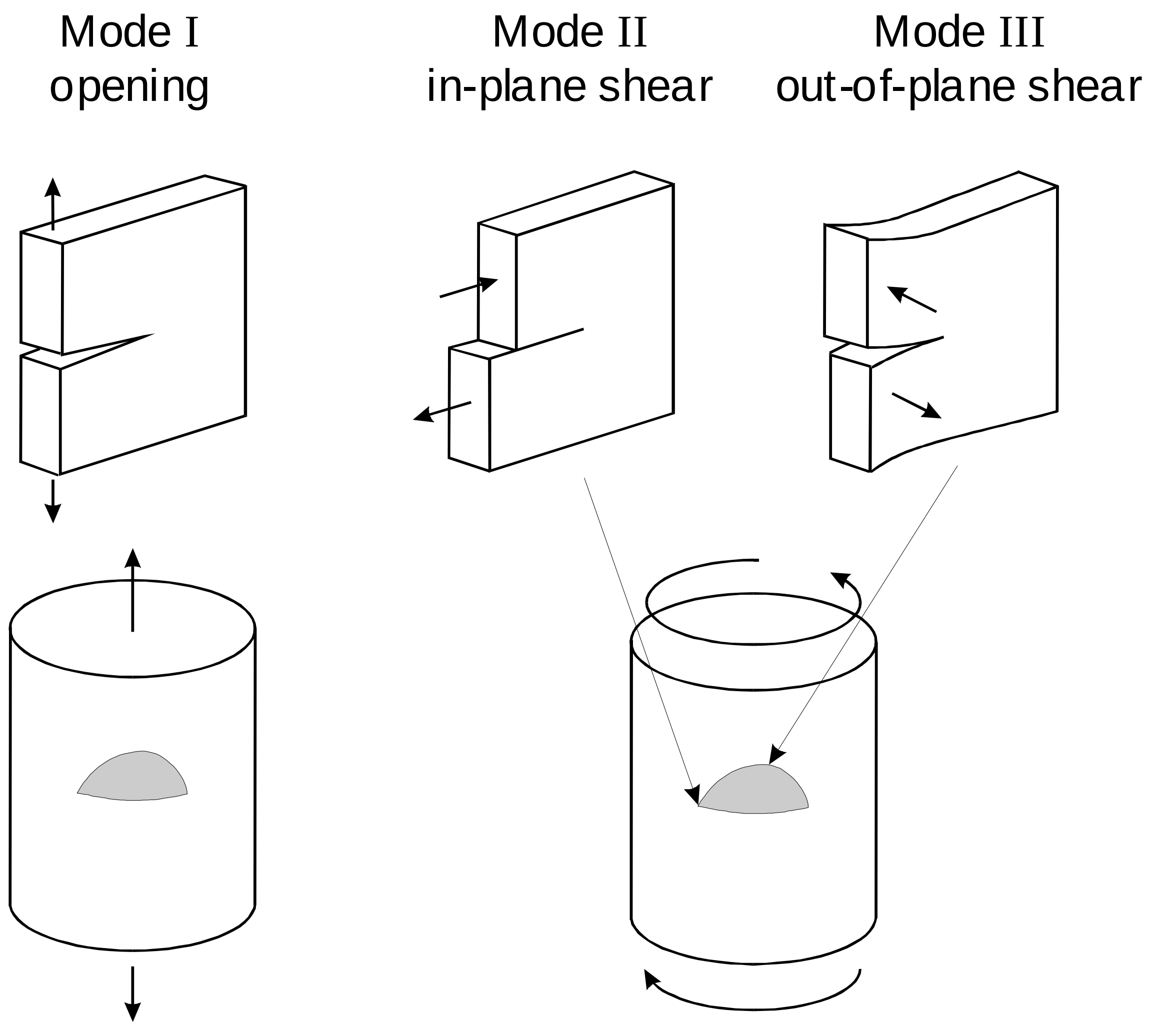
$$\sigma_n\frac{\Delta\varepsilon_1}{2} = \frac{\sigma_f'^{\,2}}{E}(2N_f)^{2b} + \sigma'_f\varepsilon'_f(2N_f)^{b+c}$$

$$\Delta W_I = \left[(\Delta\sigma_n\Delta\varepsilon_n)_{max} + (\Delta\tau\Delta\gamma)\right]\left(\frac{2}{1-R}\right)$$

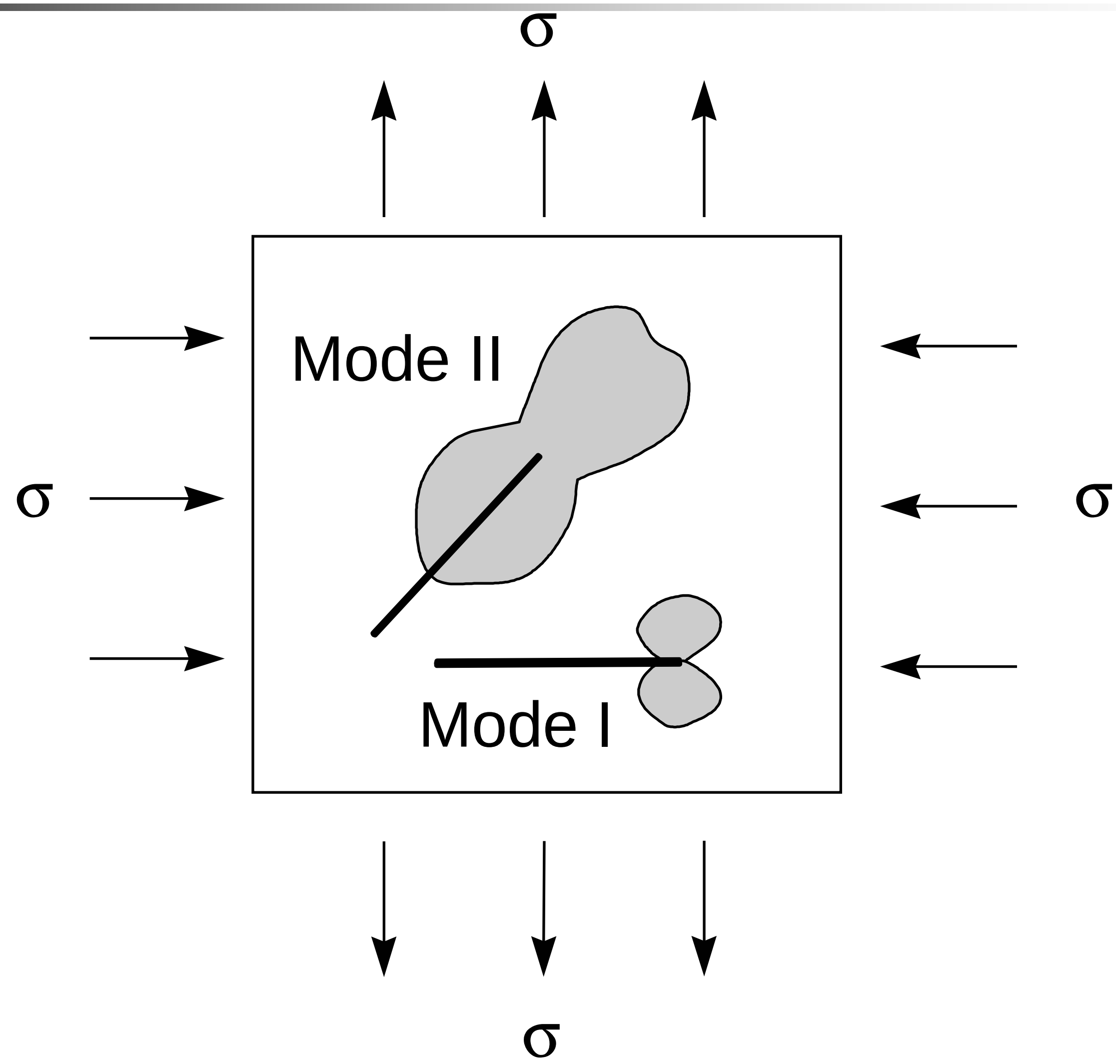
Fracture Mechanics Models

- Mode I growth
- Torsion
- Mode II growth
- Mode III growth

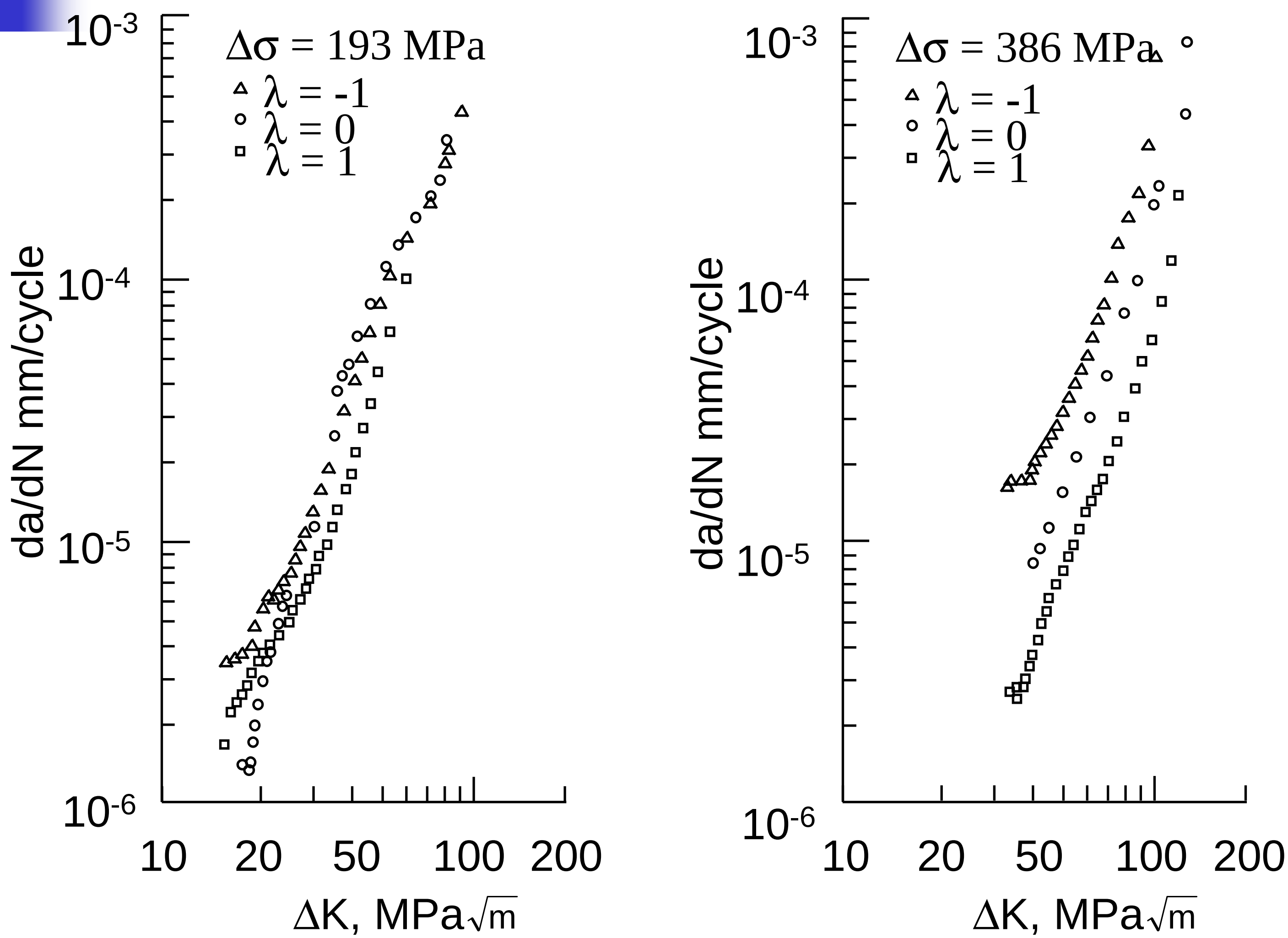
Mode I, Mode II, and Mode III



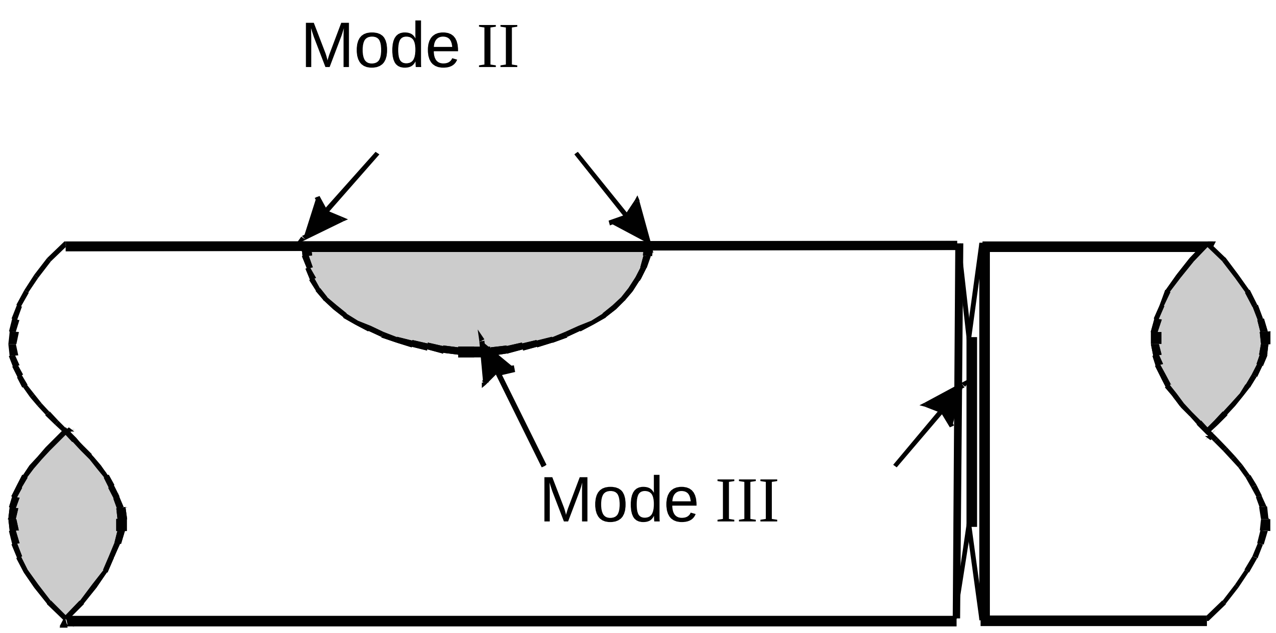
Mode I and Mode II Surface Cracks



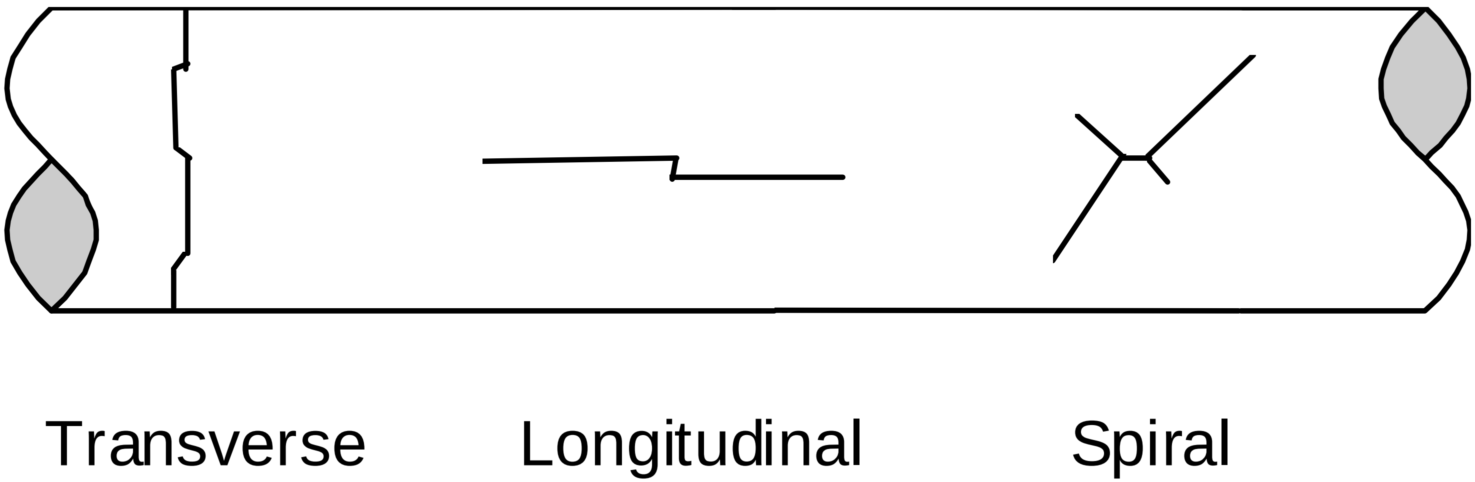
Biaxial Mode I Growth



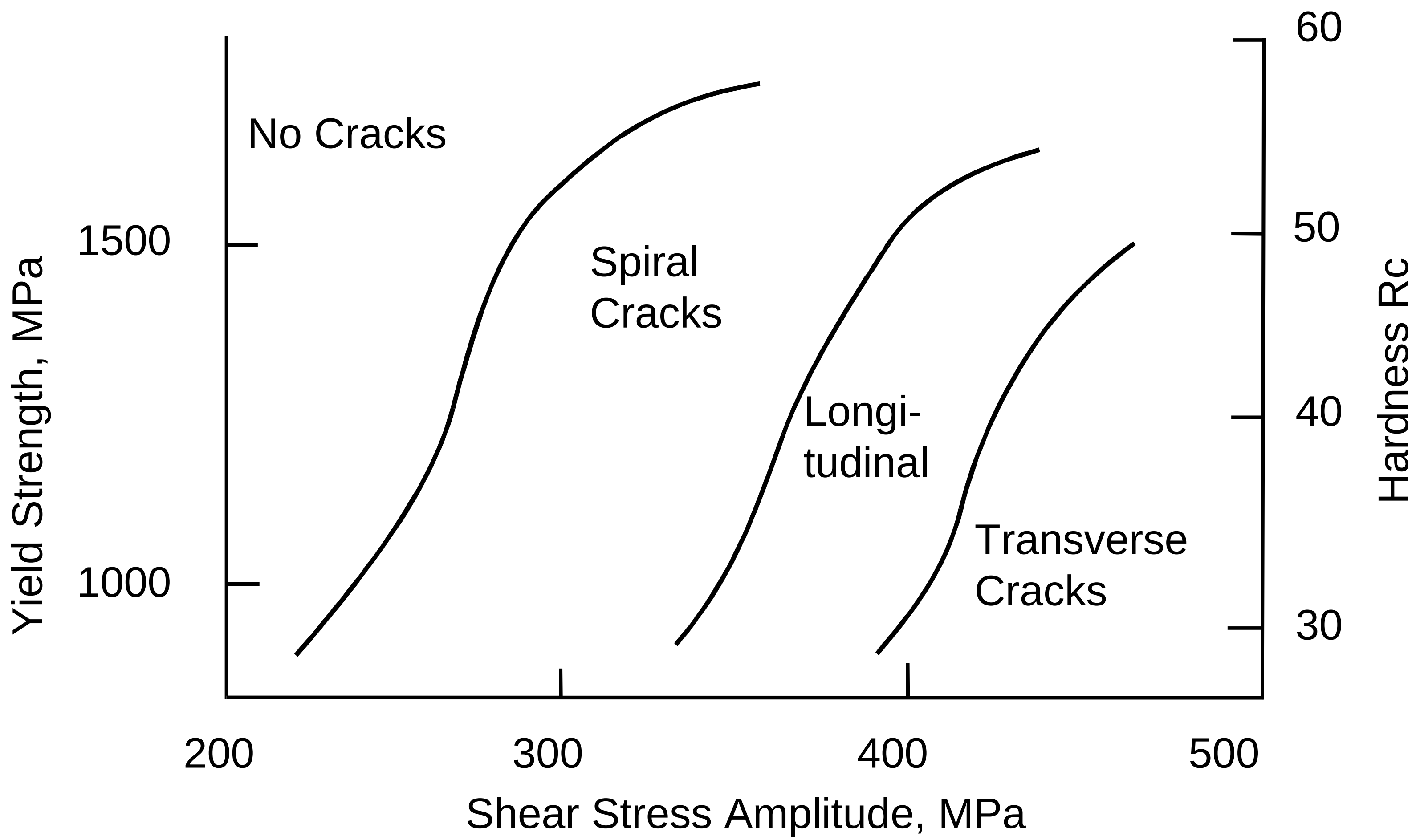
Surface Cracks in Torsion



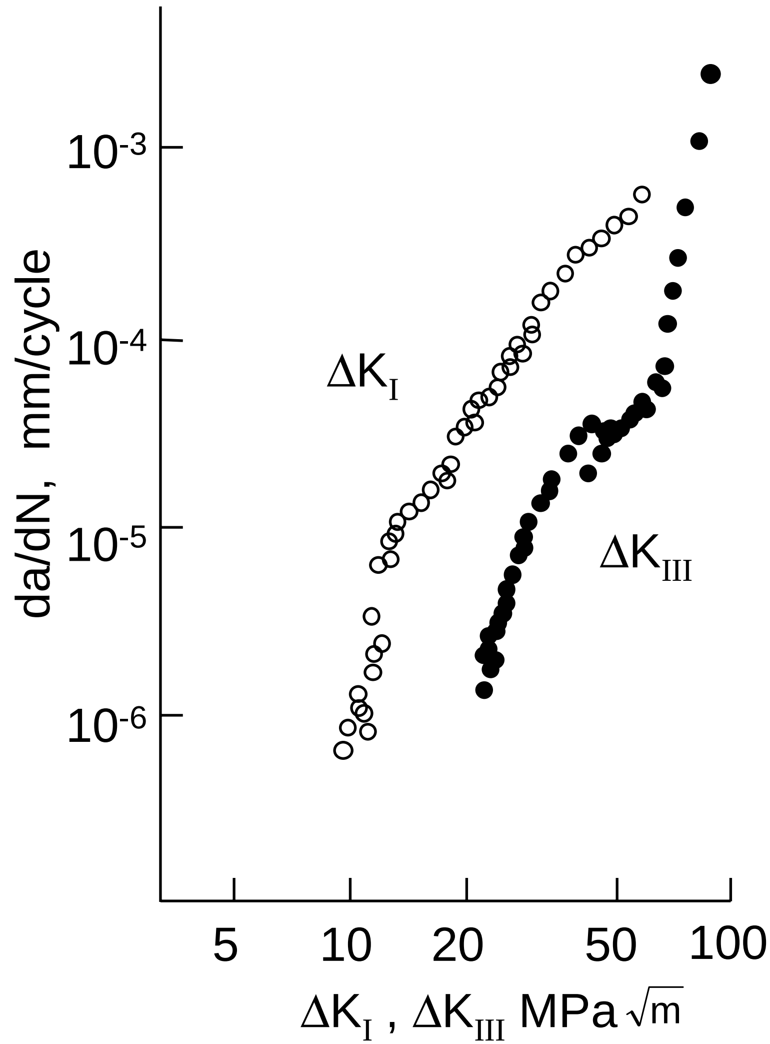
Failure Modes in Torsion



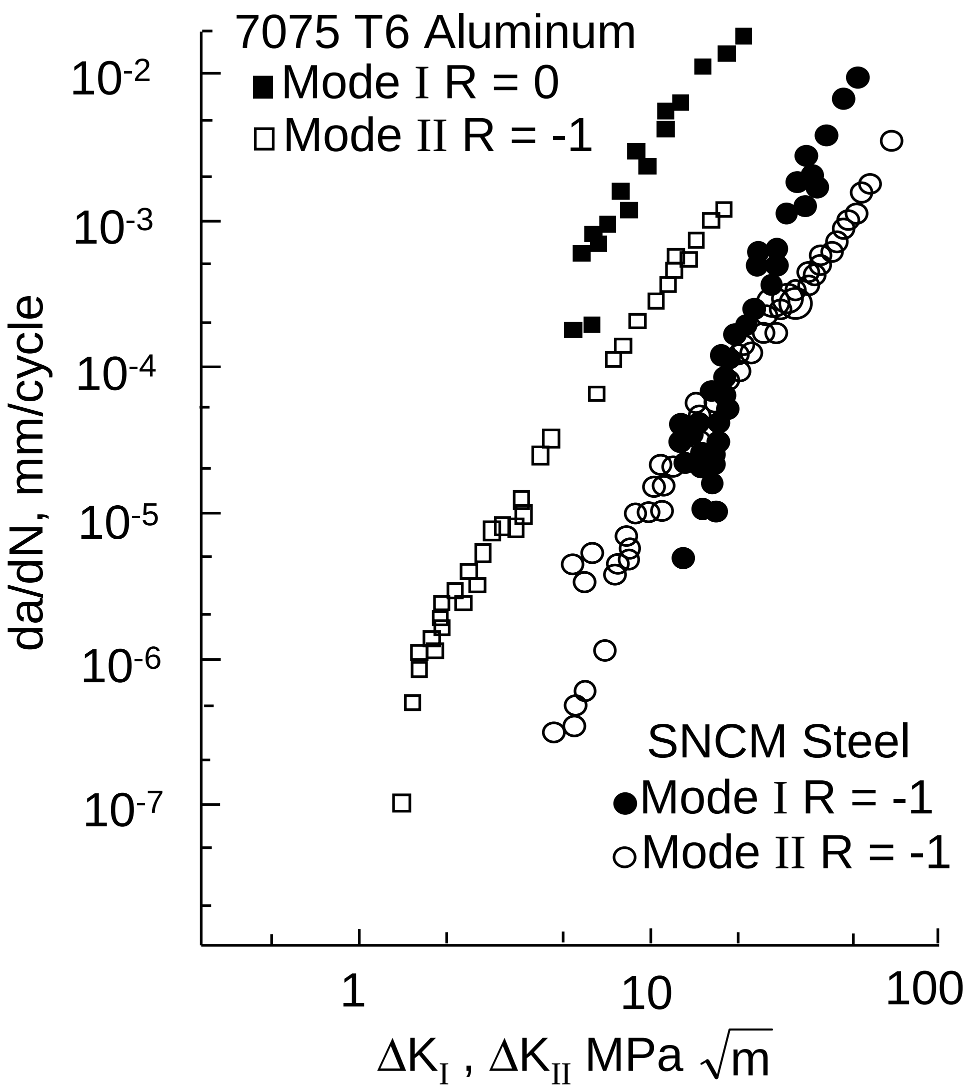
Fracture Mechanism Map



Mode I and Mode III Growth



Mode I and Mode II Growth



Fracture Mechanics Models

$$\frac{da}{dN} = C\left(\Delta K_{eq}\right)^m$$

$$\Delta K_{eq} = \left[\Delta K_I^4 + 8\Delta K_{II}^4 + 8\Delta K_{III}^4/(1-\nu)\right]^{0.25}$$

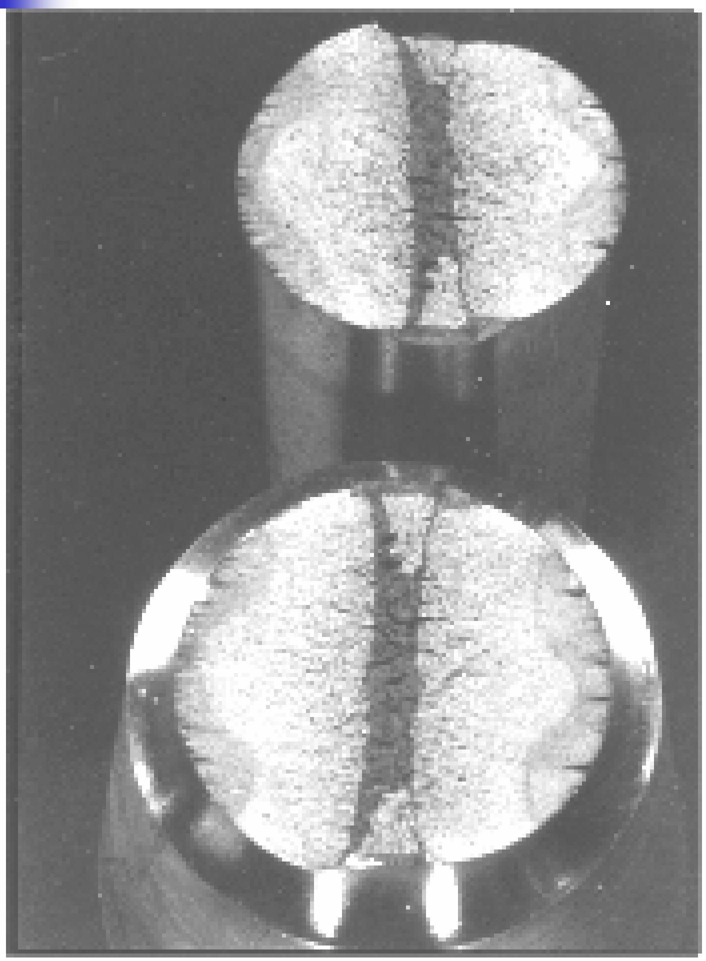
$$\Delta K_{eq} = \left[\Delta K_I^2 + \Delta K_{II}^2 + (1+\nu)\Delta K_{III}^2\right]^{0.5}$$

$$\Delta K_{eq} = \left[\Delta K_I^2 + \Delta K_I \, \Delta K_{II} + \Delta K_{II}^2\right]^{0.5}$$

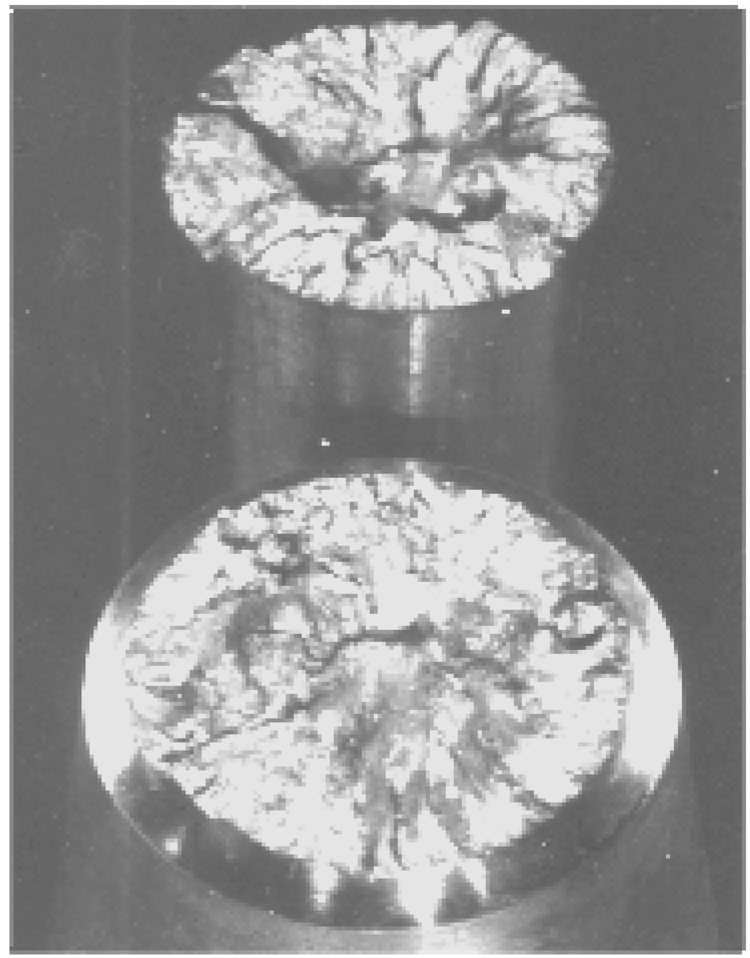
$$\Delta K_{eq}(\varepsilon) = \left[\left(F_{II}\frac{E}{2(1+\nu)}\Delta\gamma\right)^2 + \left(F_I E \Delta\varepsilon\right)^2\right]^{0.5}\sqrt{\pi a}$$

$$\Delta K_{eq}(\varepsilon) = FG\Delta\gamma\left(1 + k\frac{\sigma_{n,max}}{\sigma_{ys}}\right)\sqrt{\pi a}$$

Fracture Surfaces

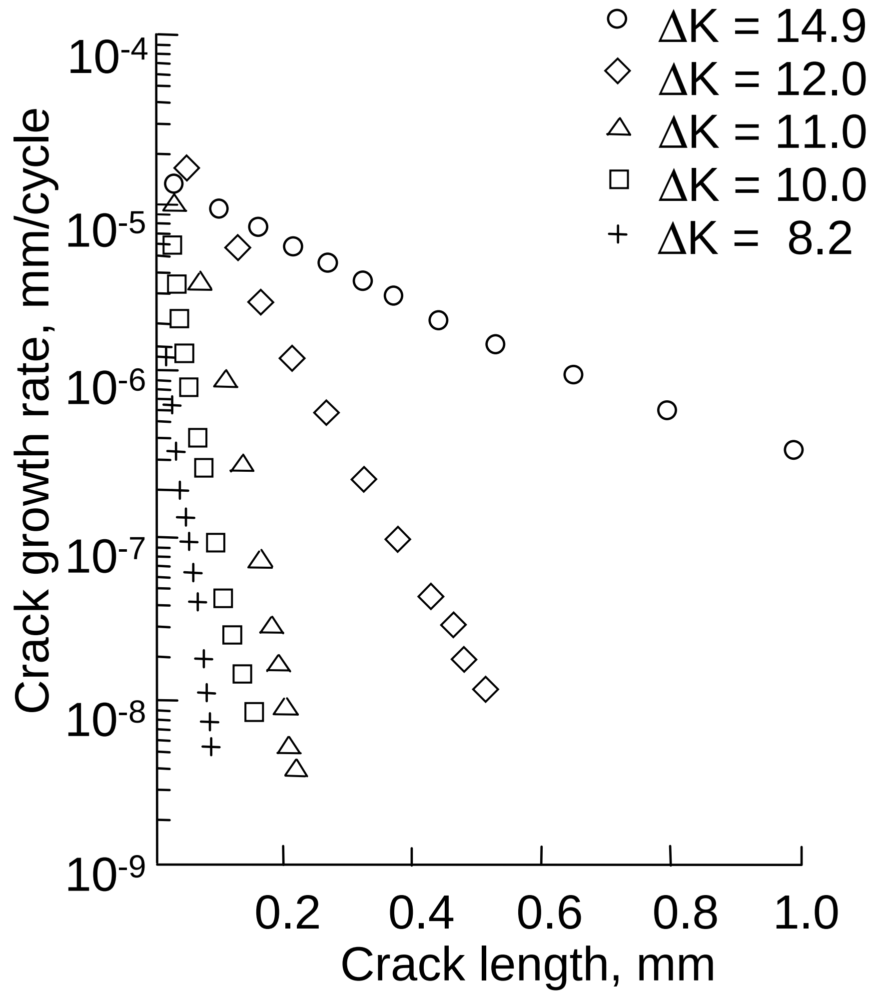


Bending



Torsion

Mode III Growth



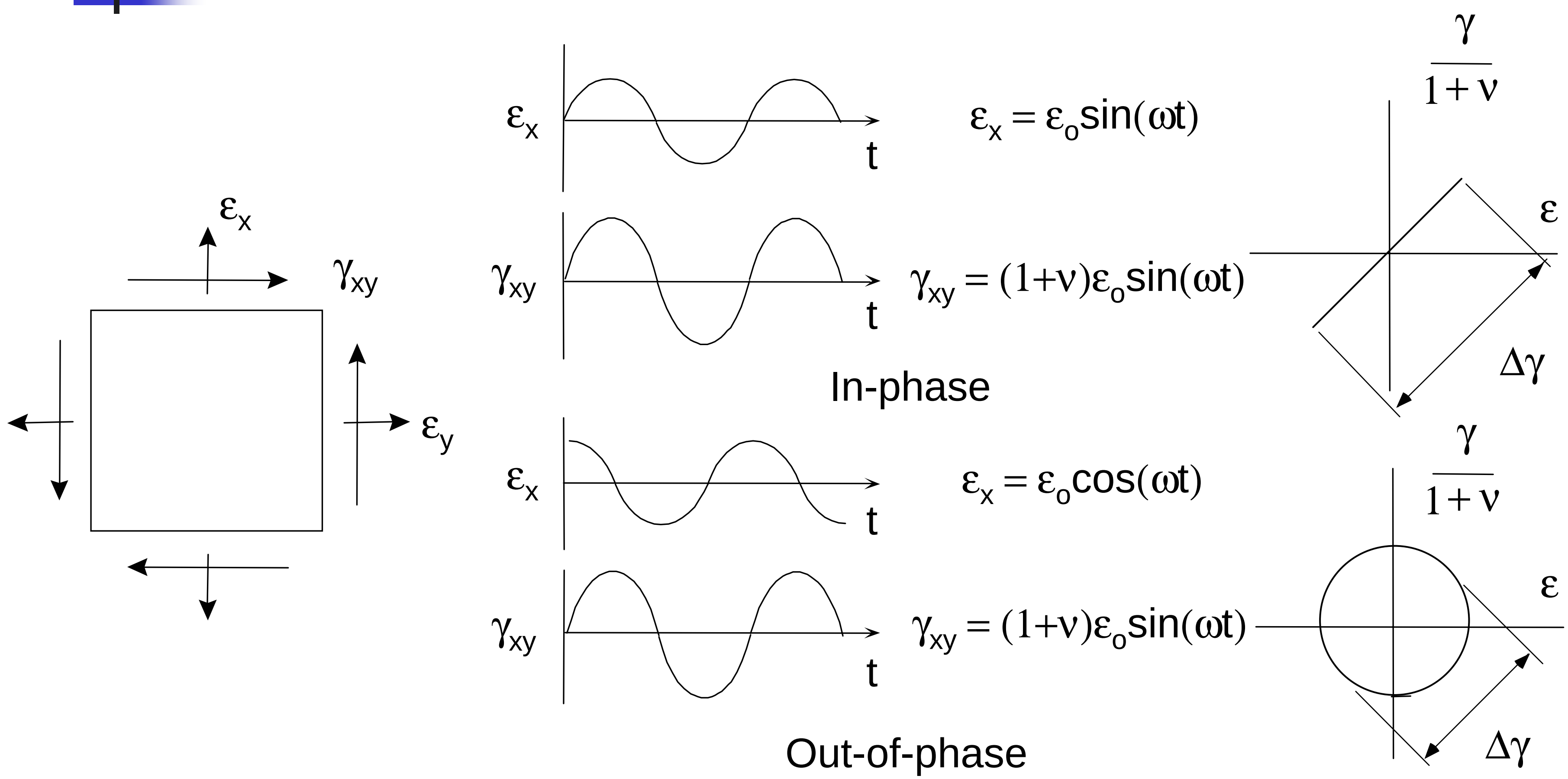
Fracture Mechanics Models Summary

- Multiaxial loading has little effect in Mode I
- Crack closure makes Mode II and Mode III calculations difficult

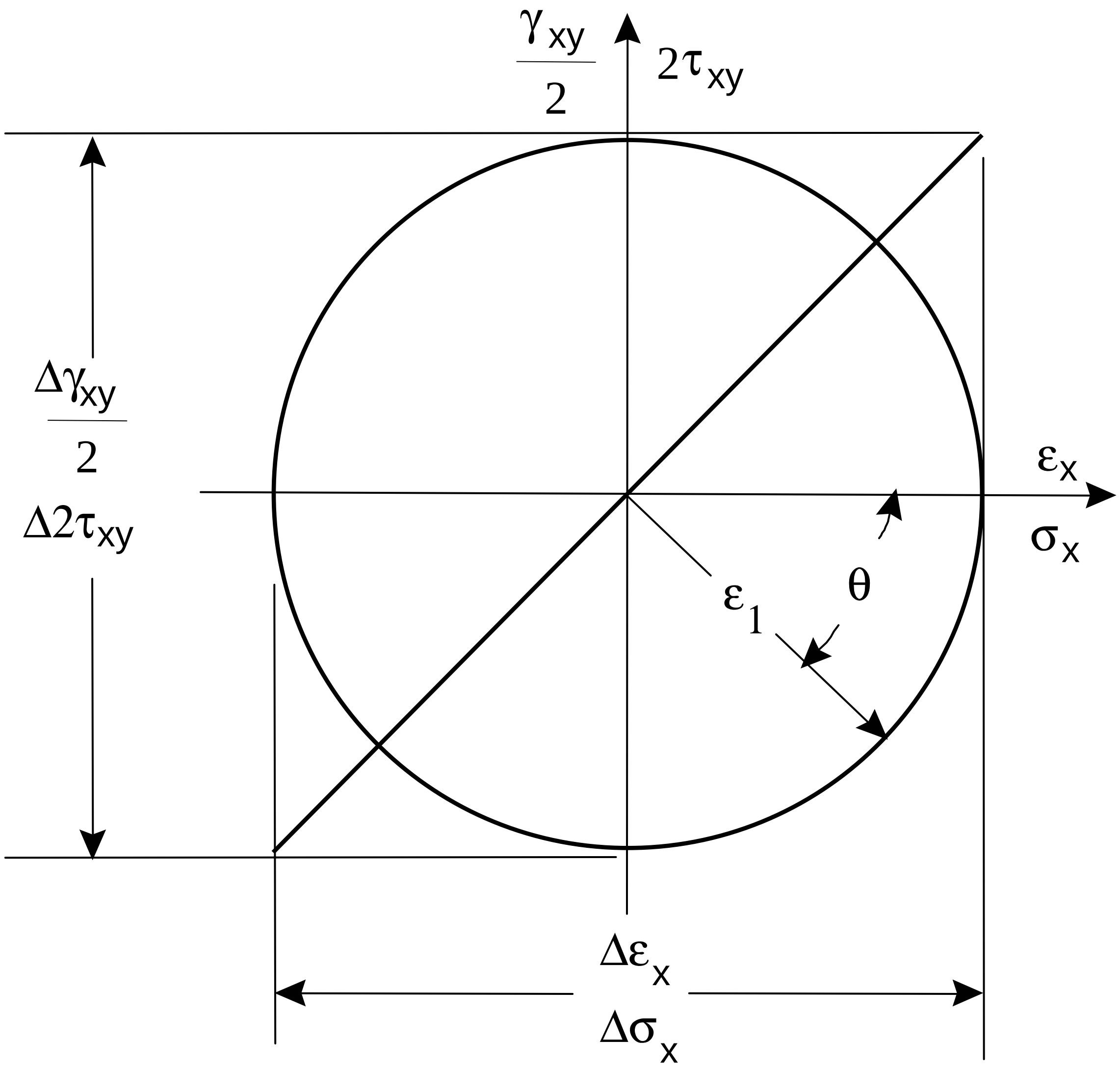
Nonproportional Loading

- In and Out-of-phase loading
- Nonproportional cyclic hardening
- Variable amplitude

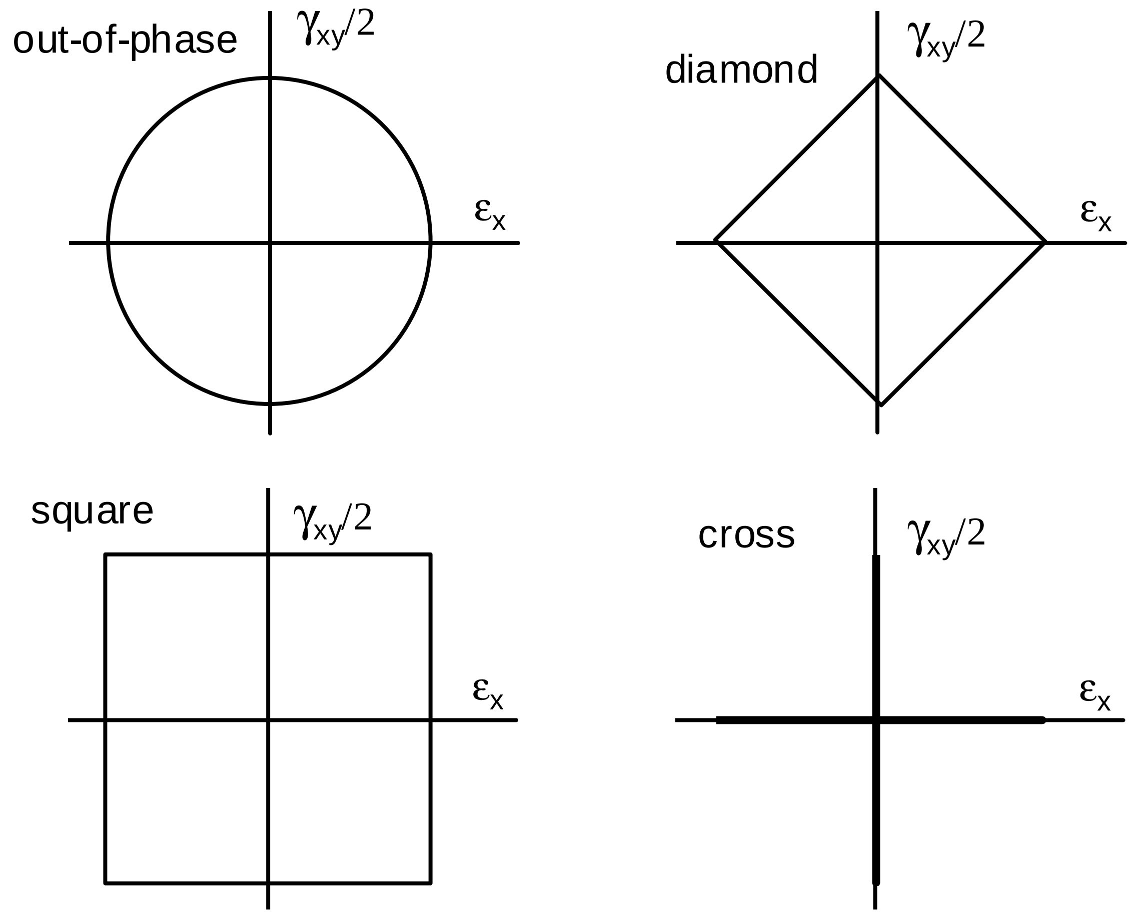
In and Out-of-Phase Loading



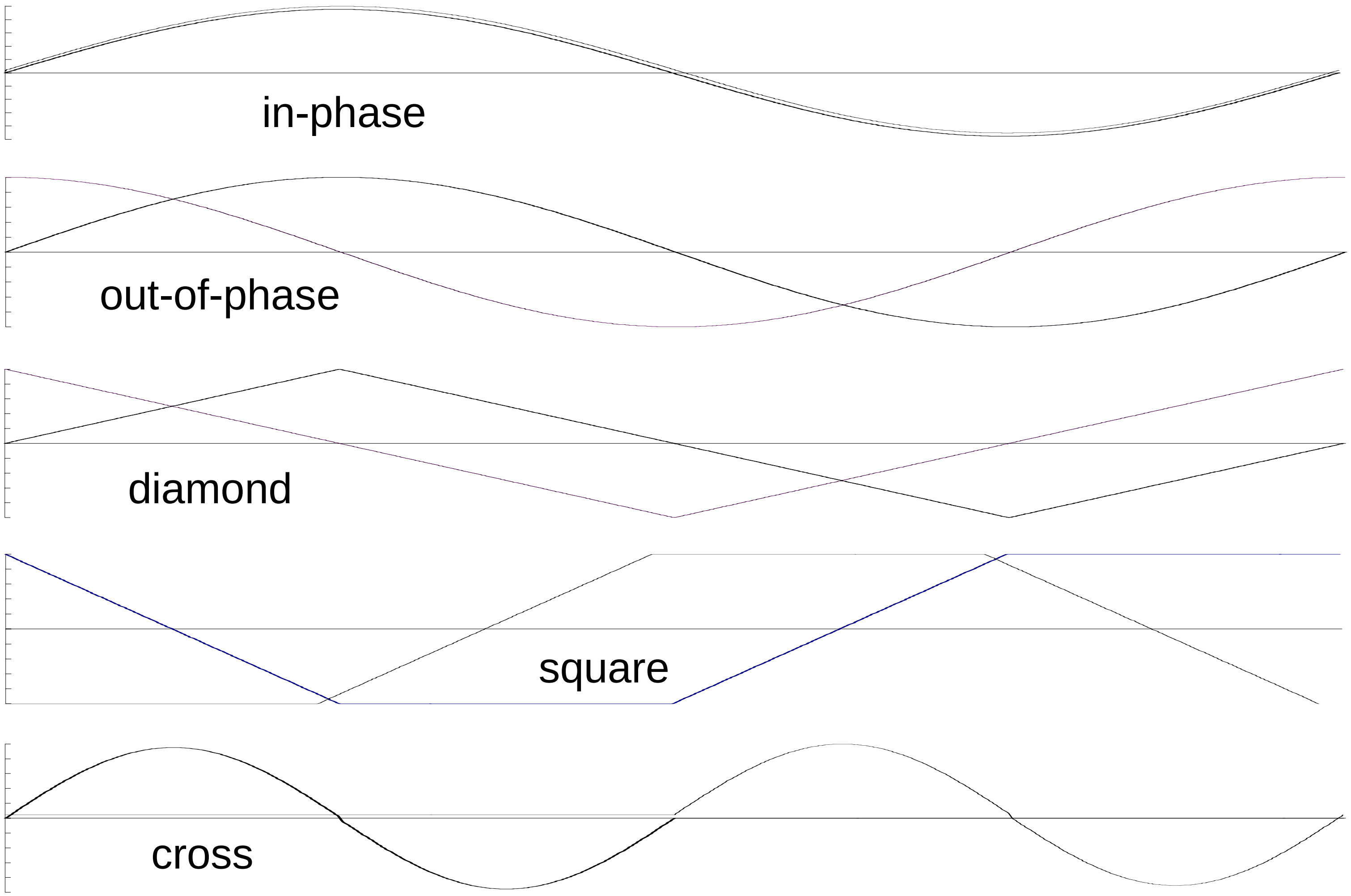
In-Phase and Out-of-Phase



Loading Histories

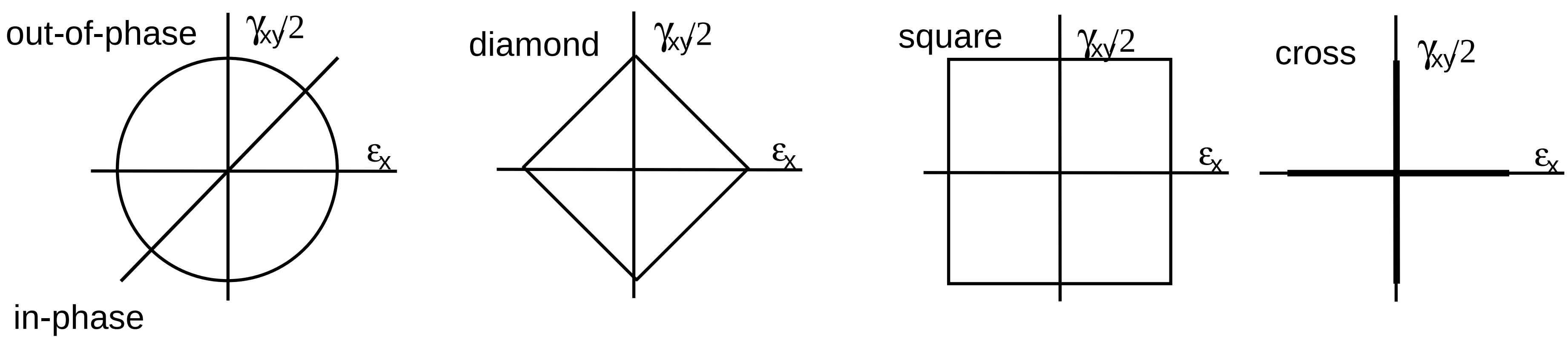


Loading Histories

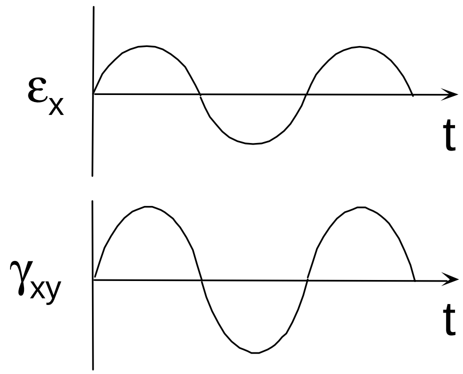


Findley Model Results

	$\Delta\tau/2$ MPa	$\sigma_{n,max}$ MPa	$\Delta\tau/2 + 0.3\,\sigma_{n,max}$	N/N_{ip}
in-phase	353	250	428	1.0
90° out-of-phase	250	500	400	2.0
diamond	250	500	400	2.0
square	353	603	534	0.11
cross - tension cycle	250	250	325	16
cross - torsion cycle	250	0	250	216



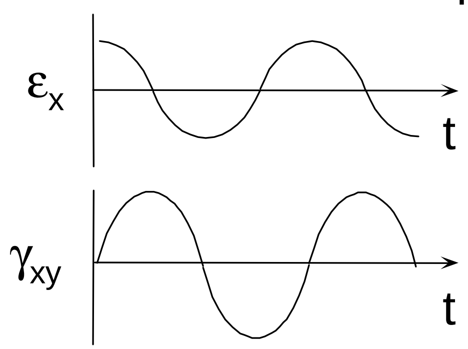
Nonproportional Hardening



$$\varepsilon_x = \varepsilon_o \sin(\omega t)$$

$$\gamma_{xy} = (1+\nu)\varepsilon_o \sin(\omega t)$$

In-phase

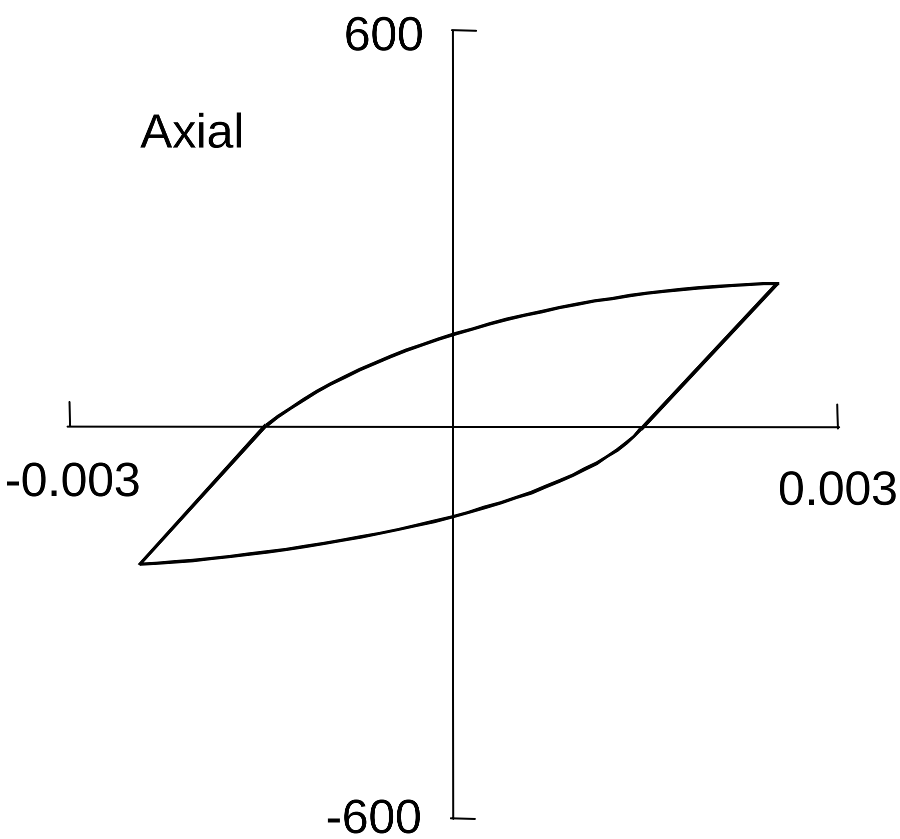


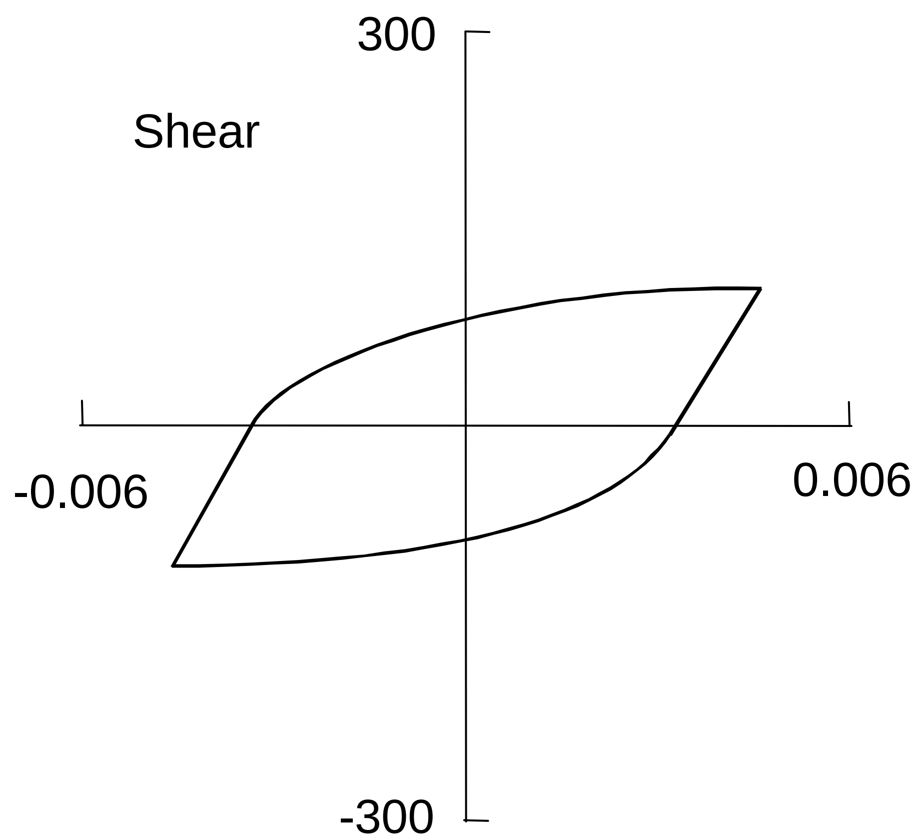
$$\varepsilon_x = \varepsilon_o \cos(\omega t)$$

$$\gamma_{xy} = (1+\nu)\varepsilon_o \sin(\omega t)$$

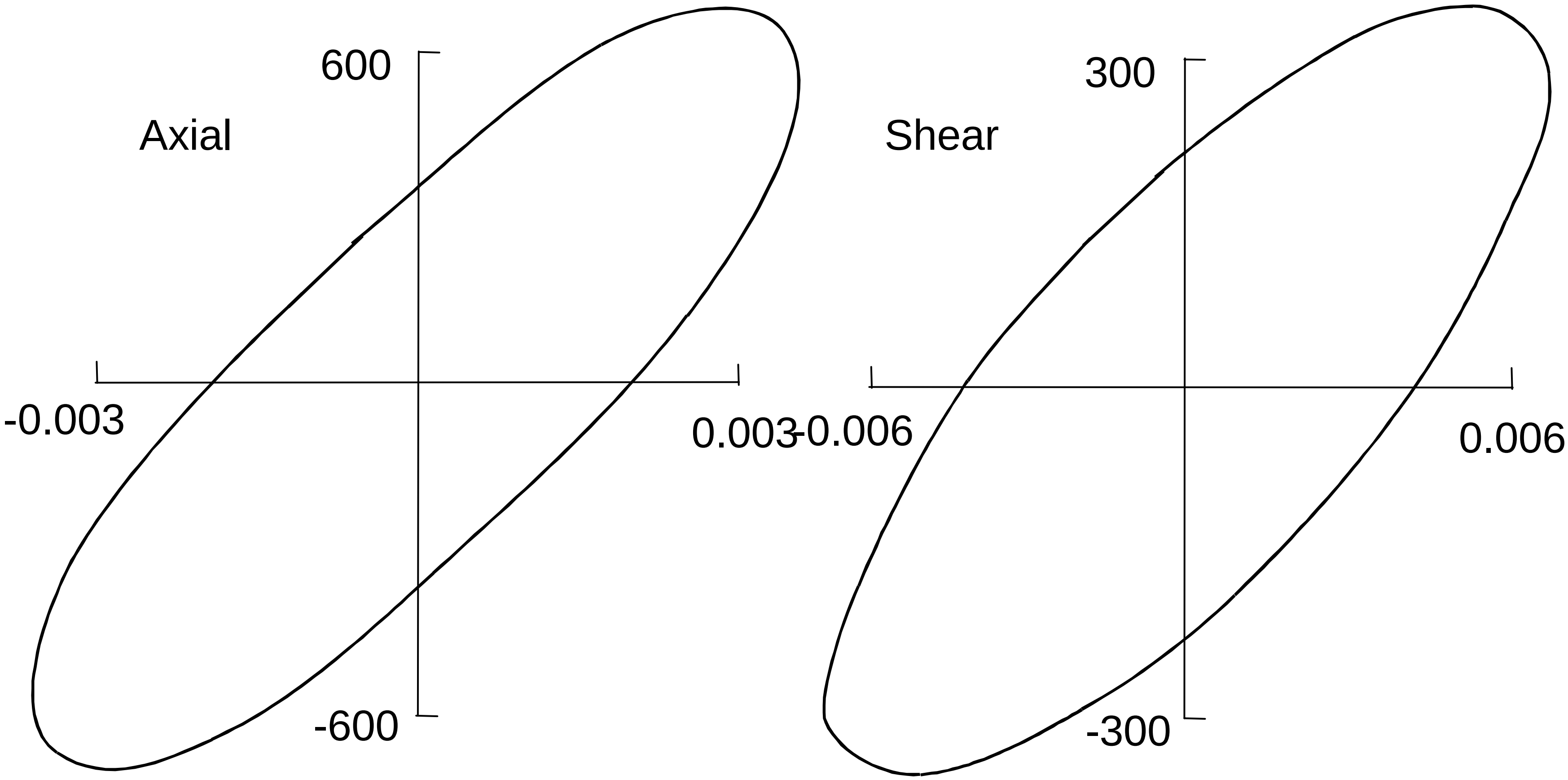
Out-of-phase

In-Phase

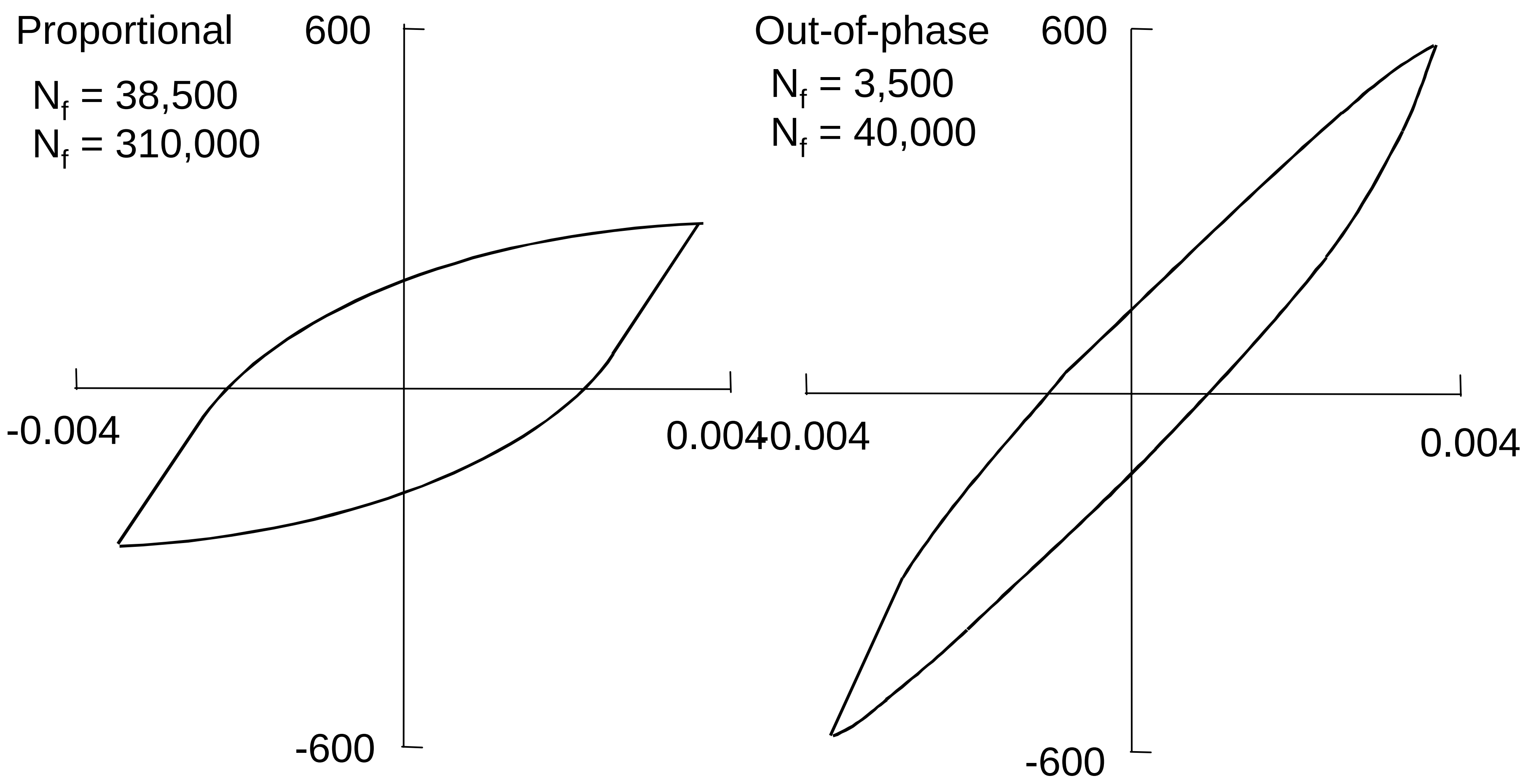




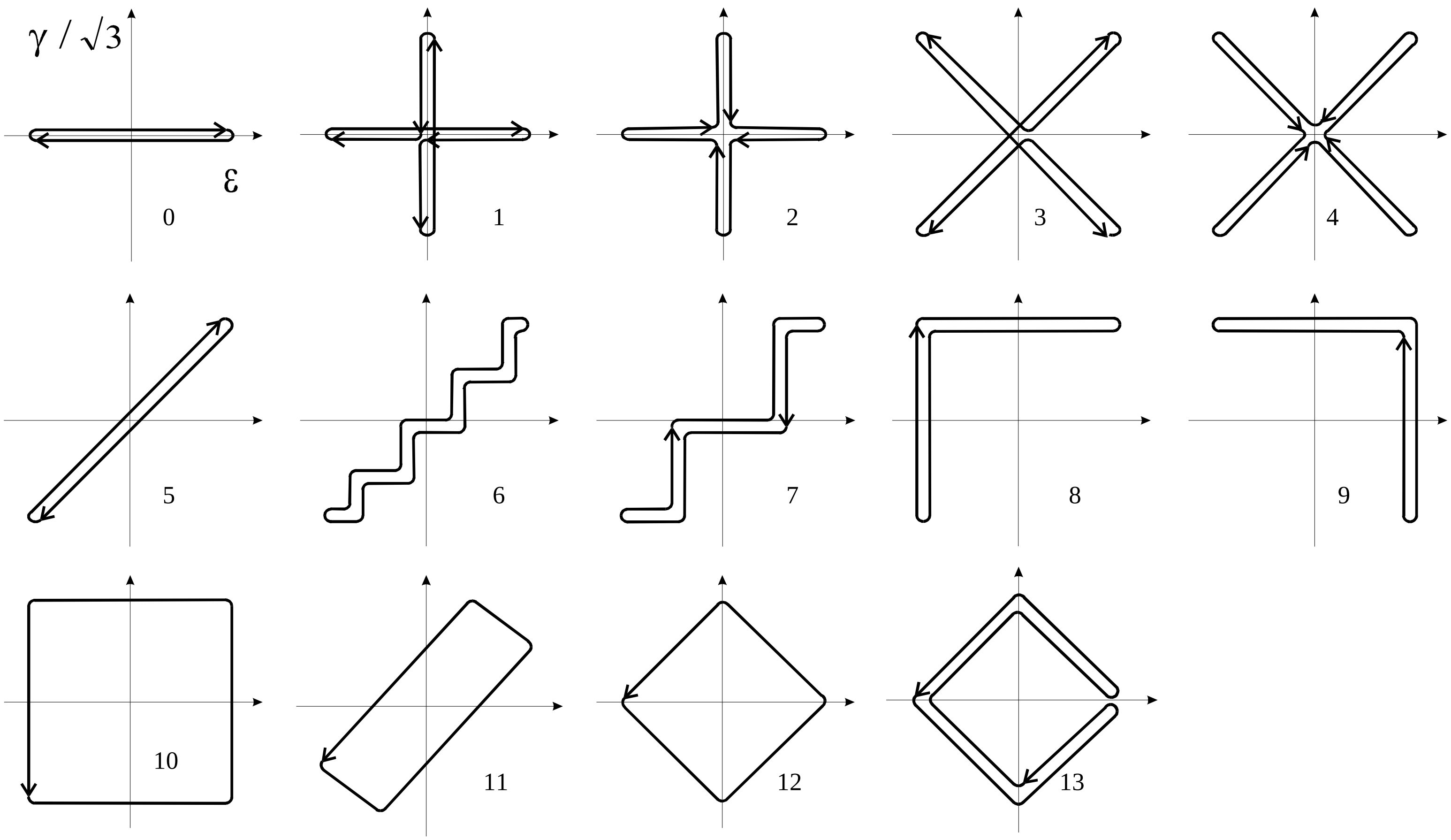
90° Out-of-Phase



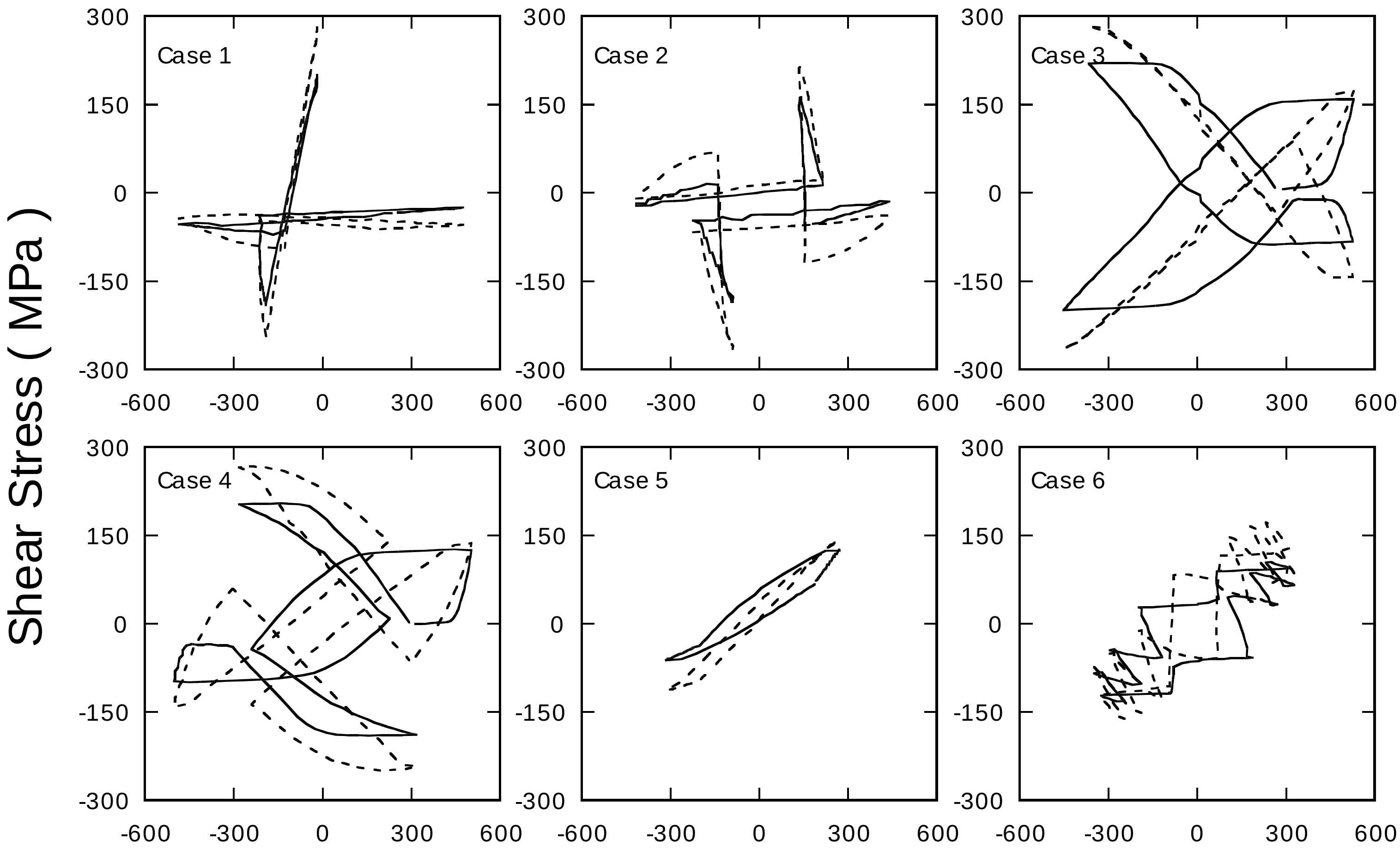
Critical Plane



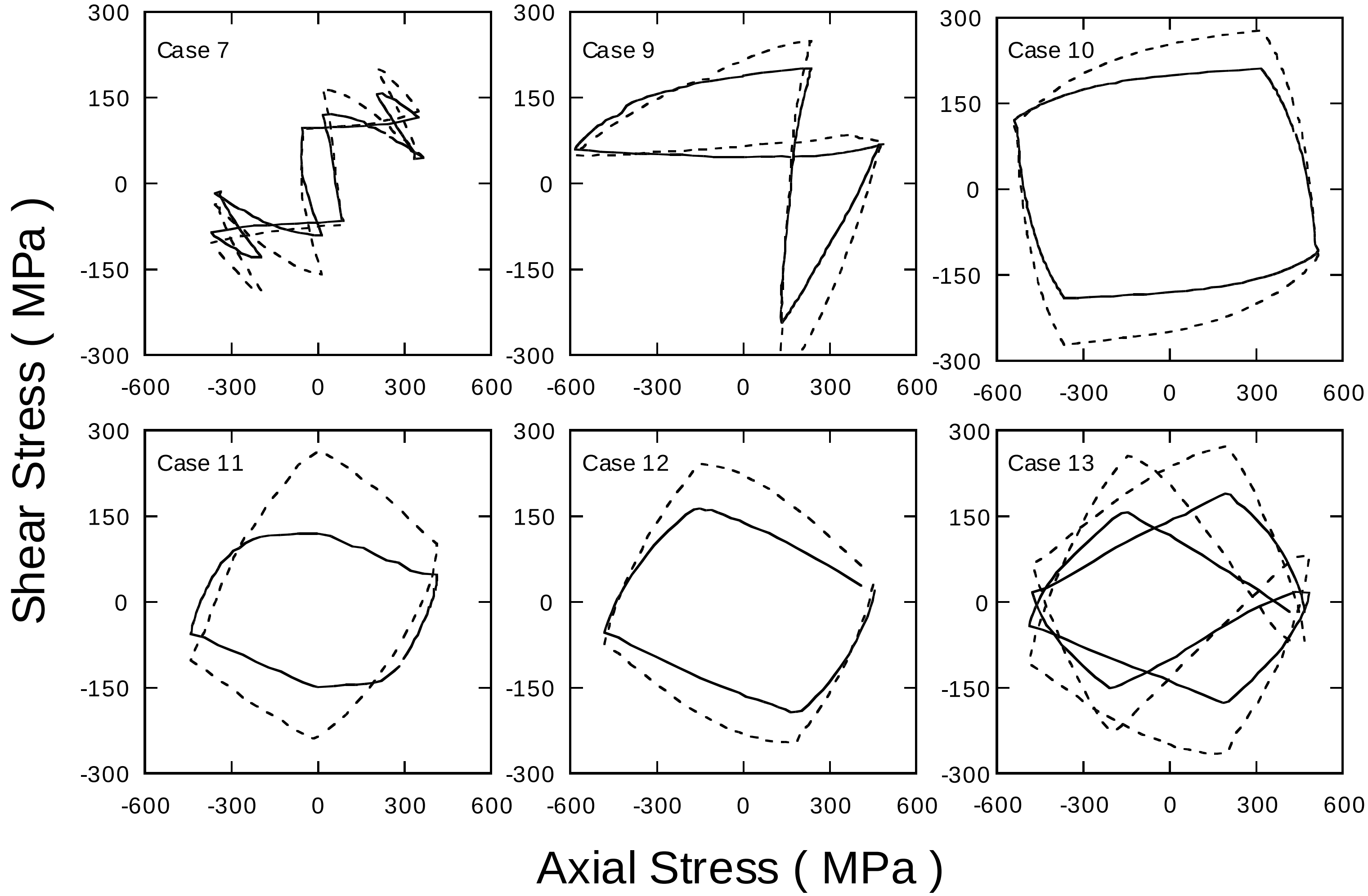
Loading Histories



Stress-Strain Response

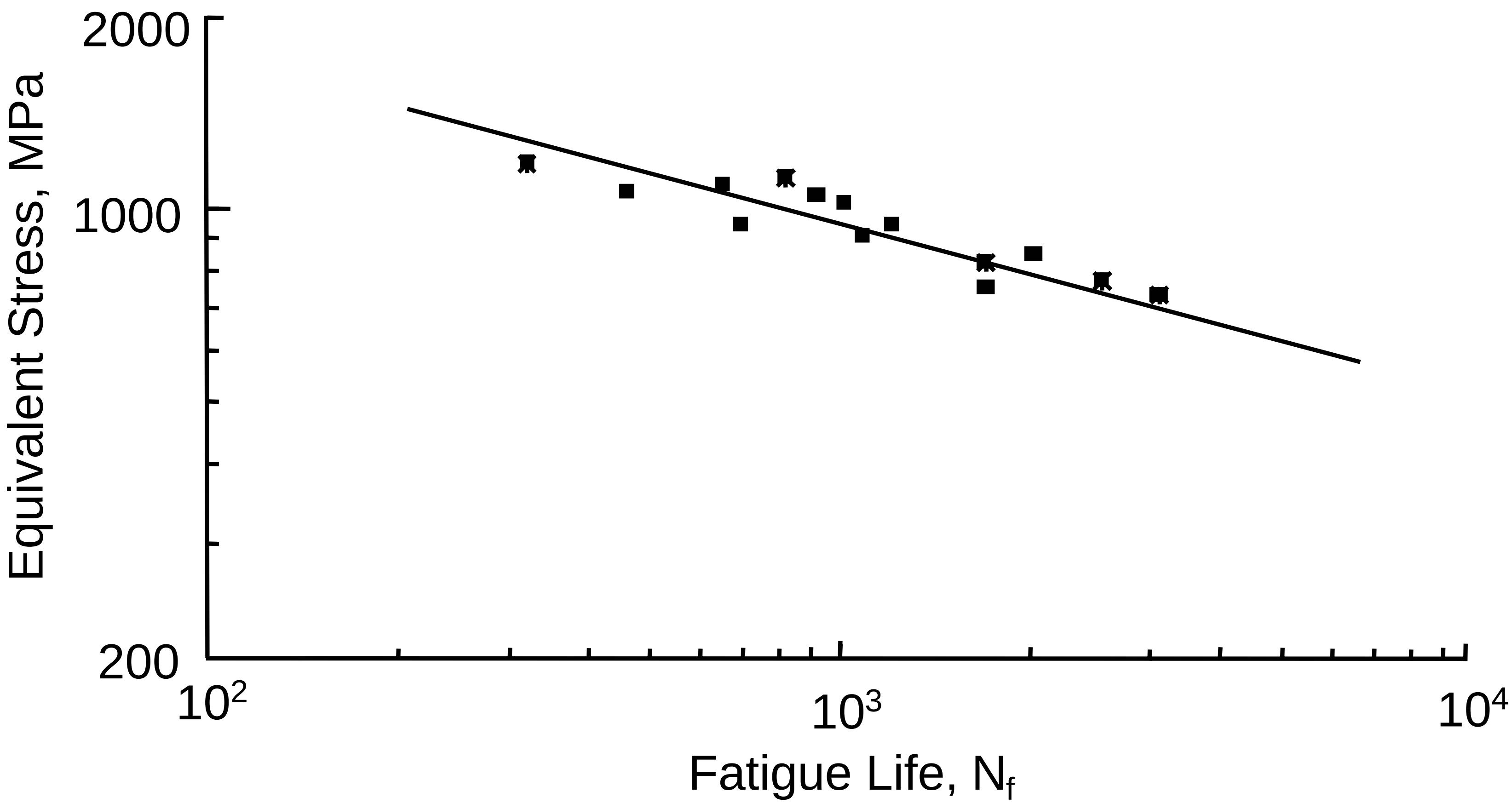


Stress-Strain Response (continued)



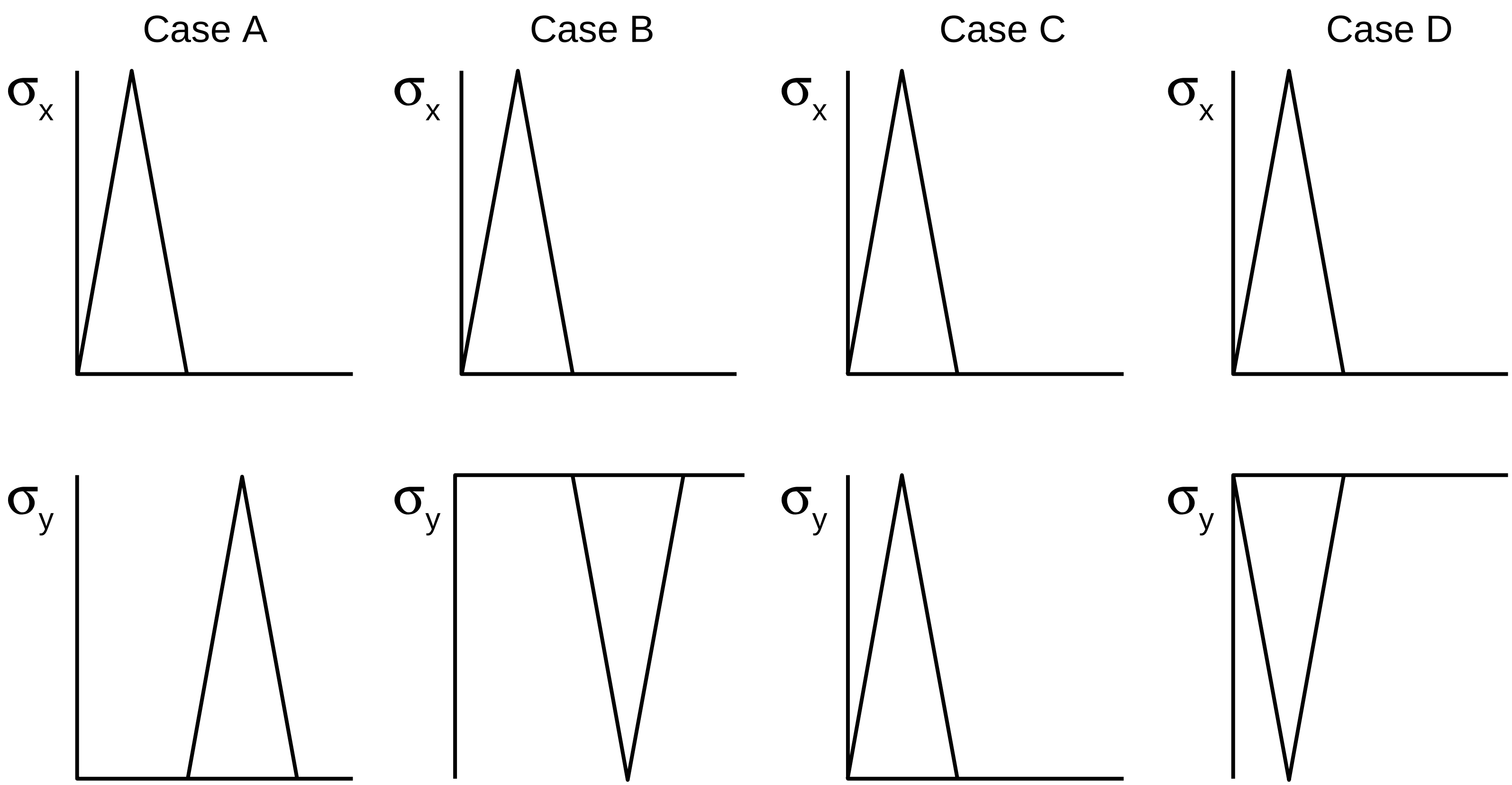
Maximum Stress

All tests have the same strain ranges

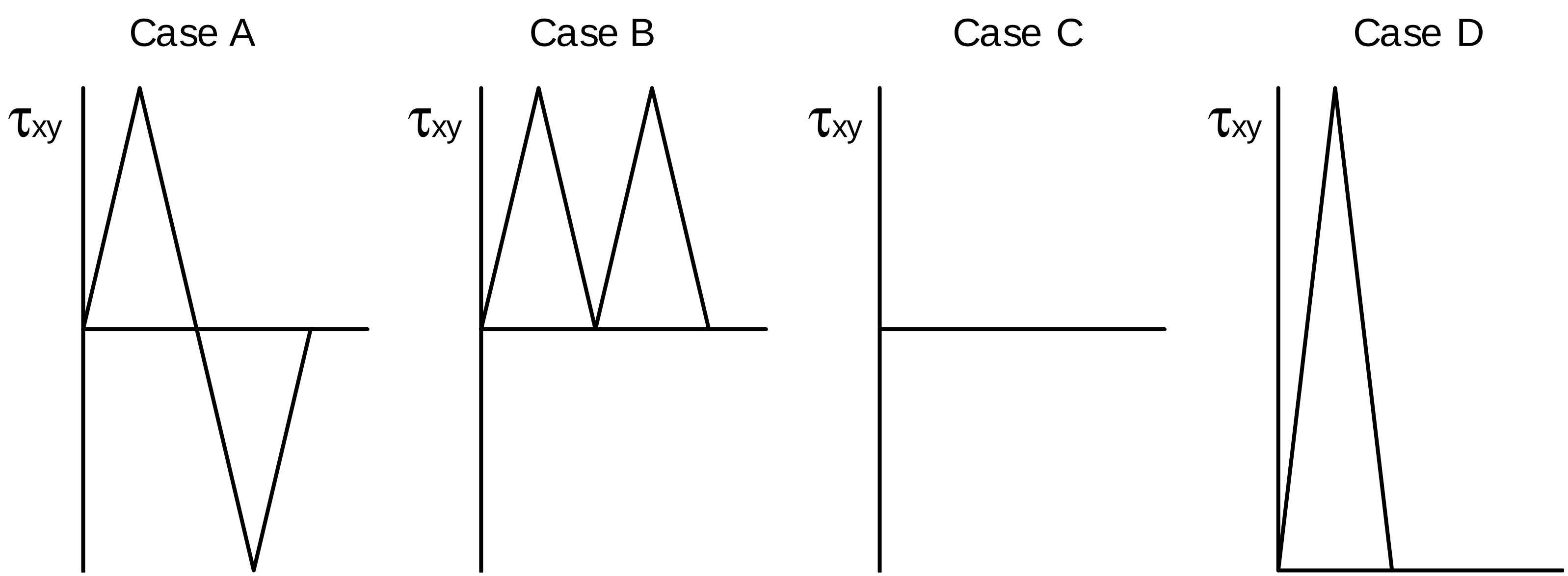


Nonproportional hardening results in lower fatigue lives

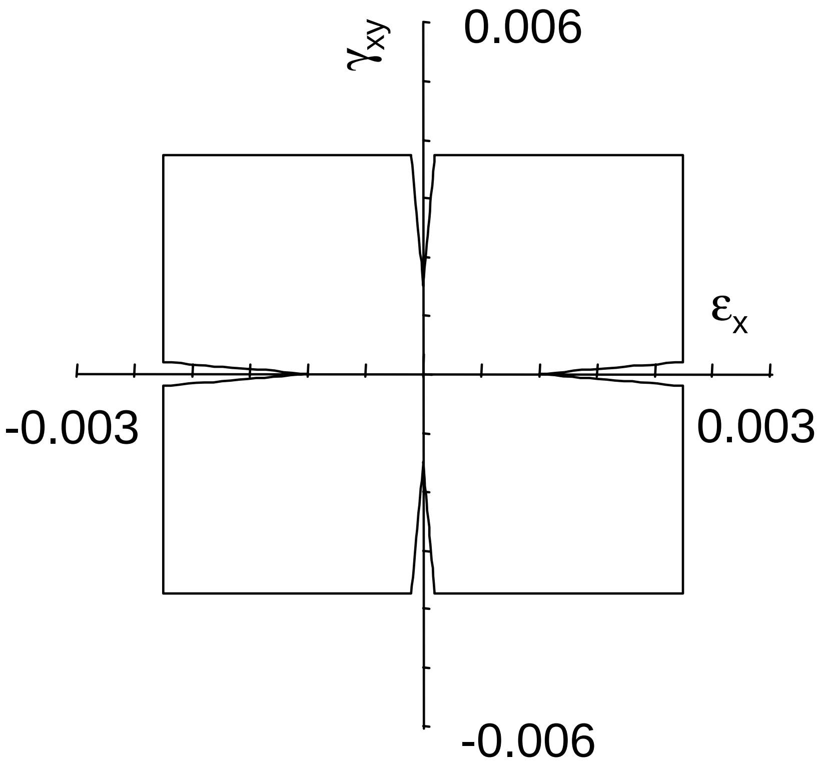
Nonproportional Example

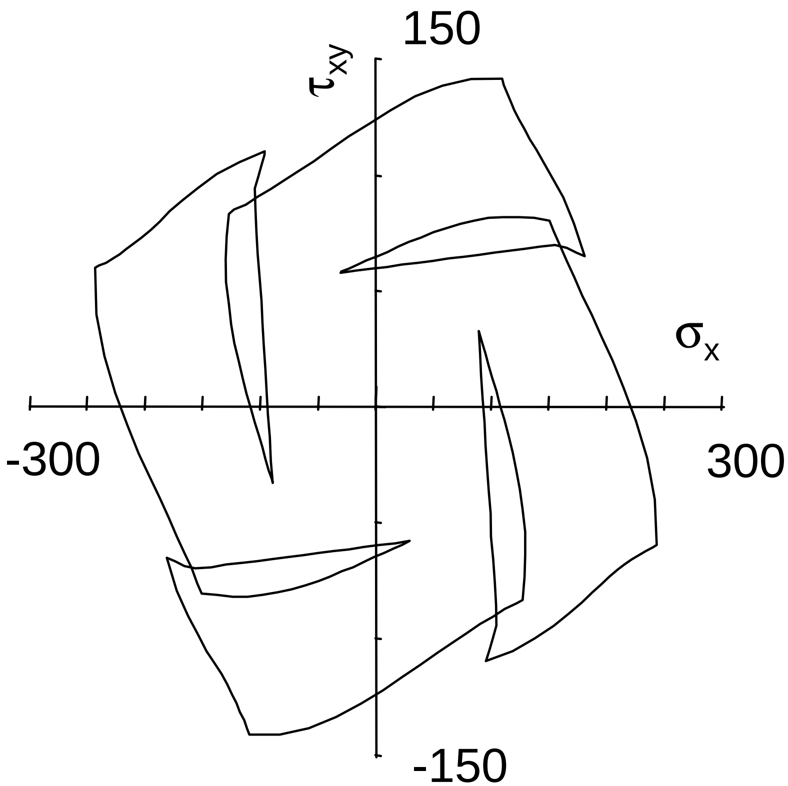


Shear Stresses

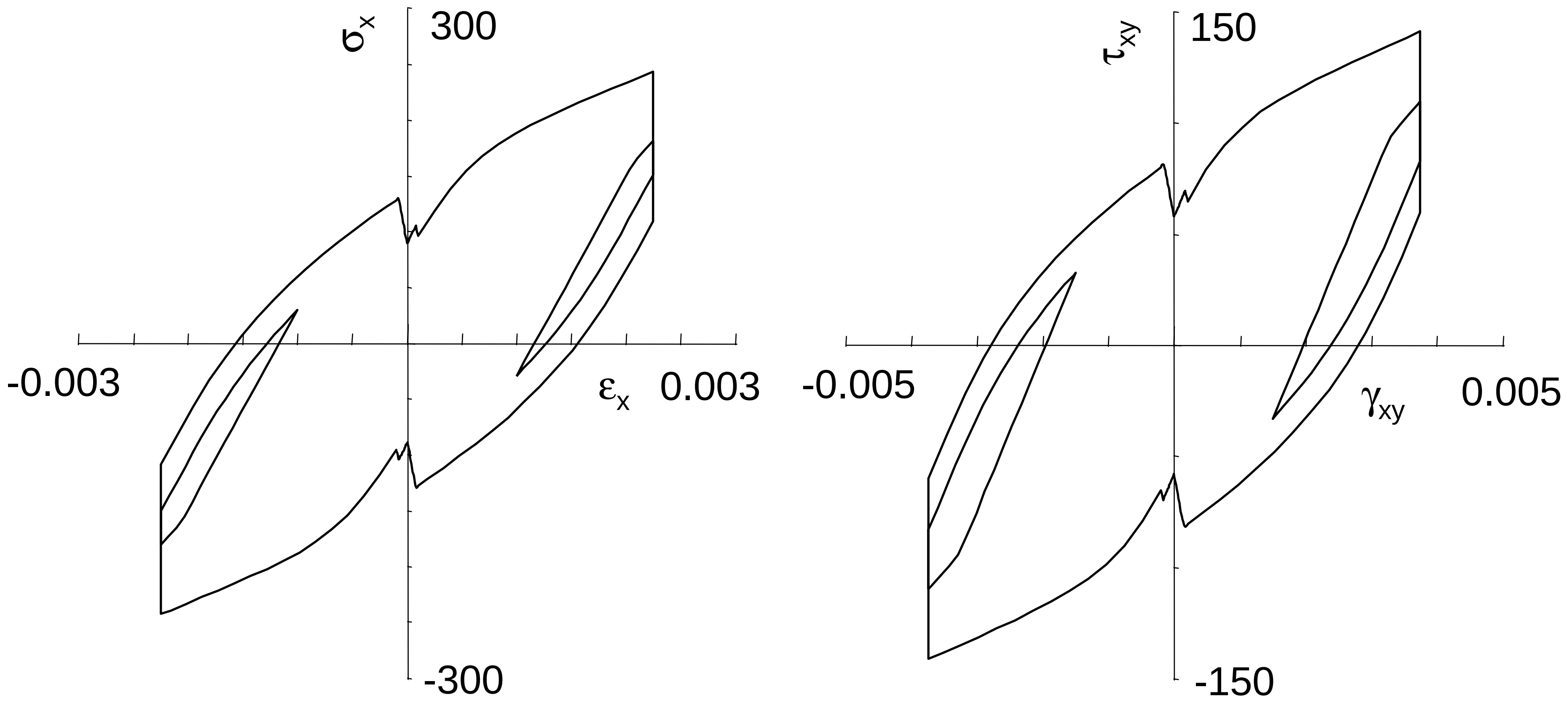


Simple Variable Amplitude History

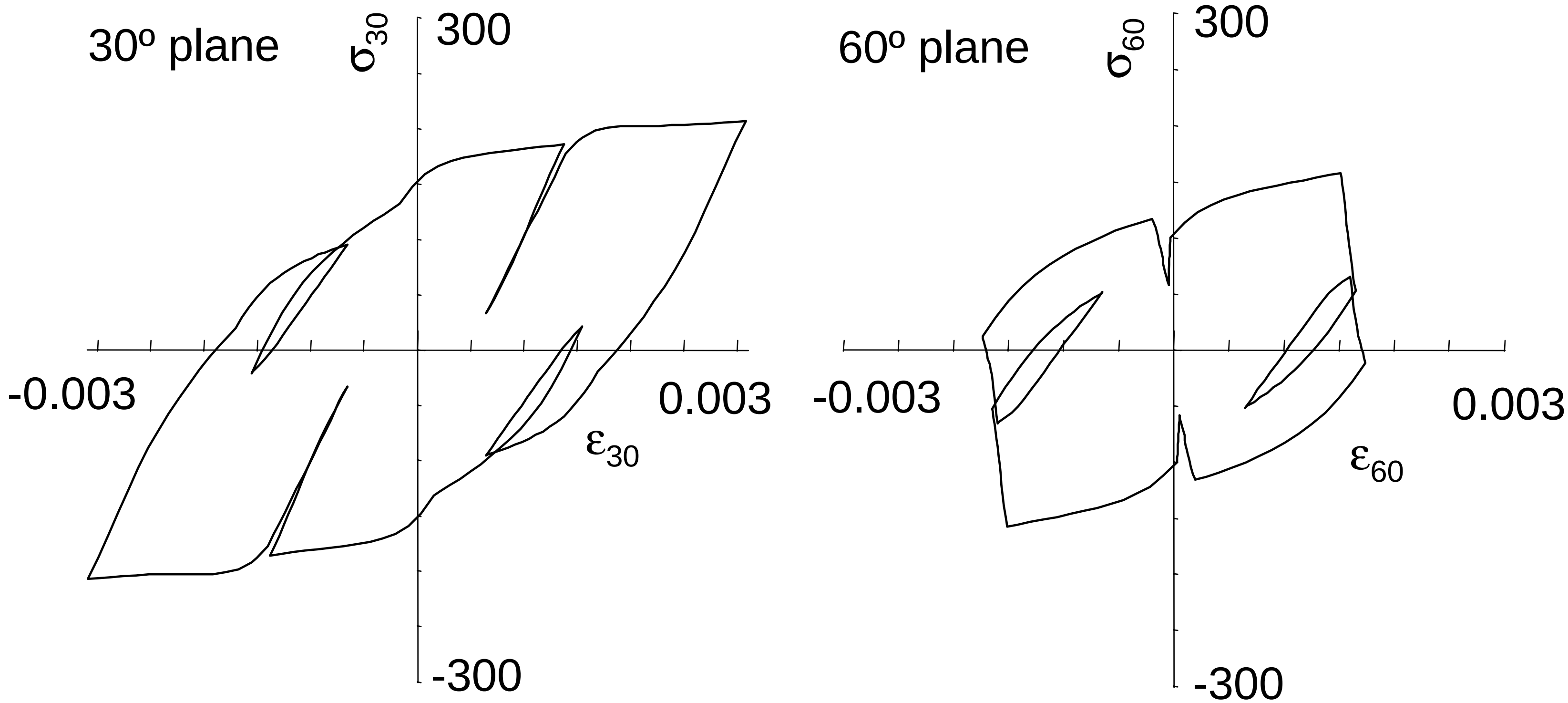




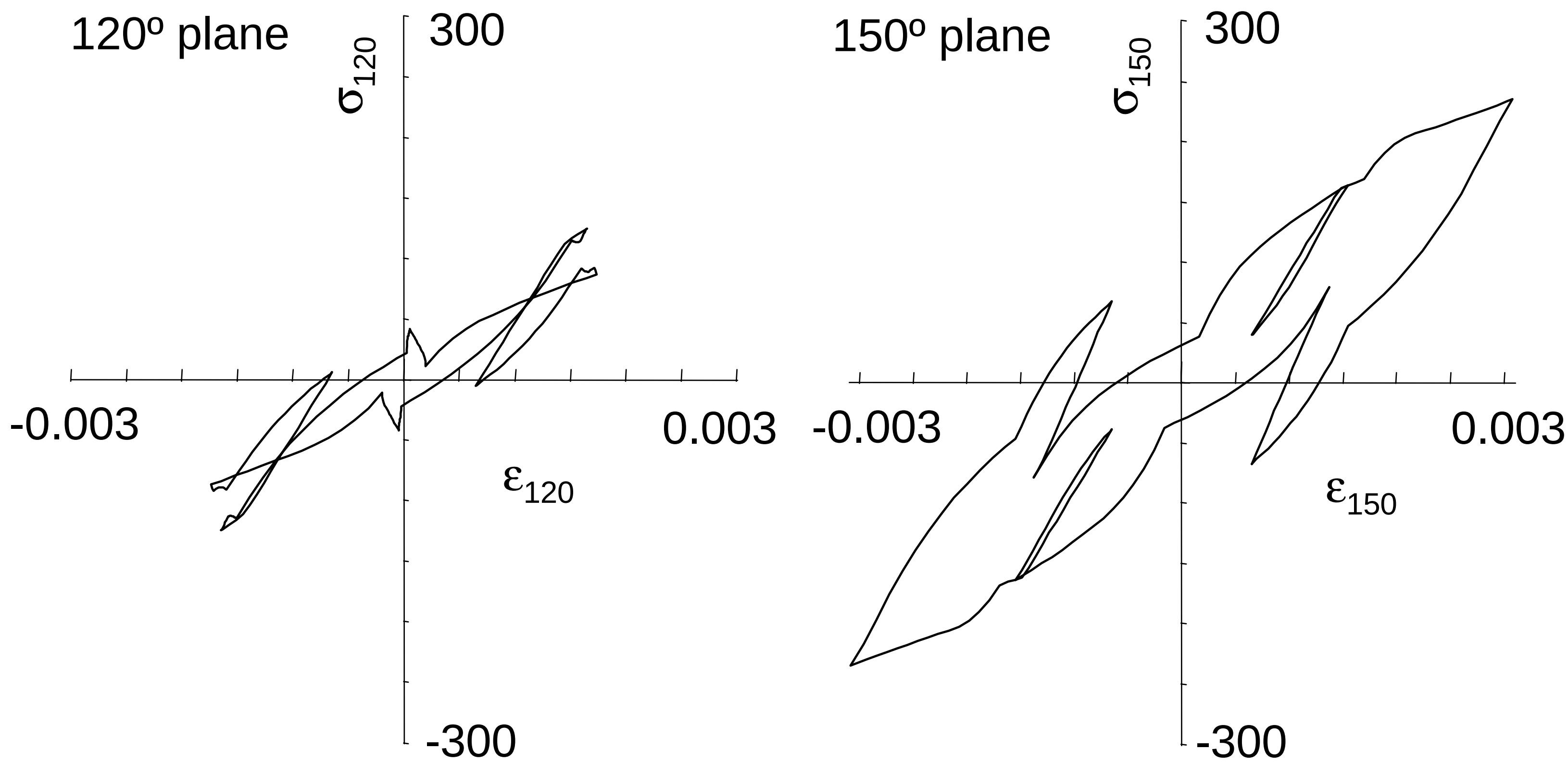
Stress-Strain on 0° Plane



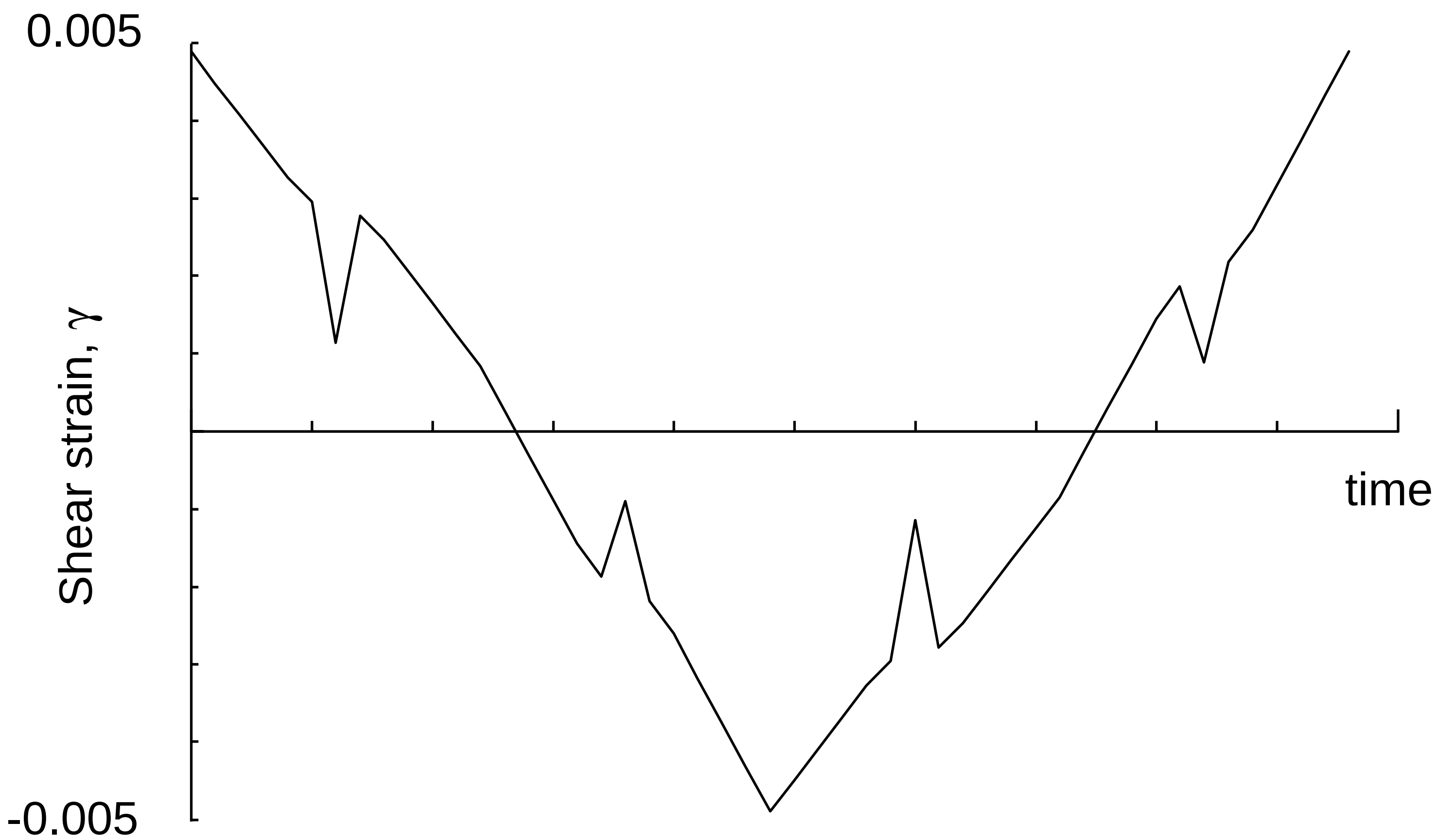
Stress-Strain on 30° and 60° Planes



Stress-Strain on 120° and 150° Planes



Shear Strain History on Critical Plane



Fatigue Calculations

Load or strain history

Cyclic plasticity model

Stress and strain tensor

Search for critical plane

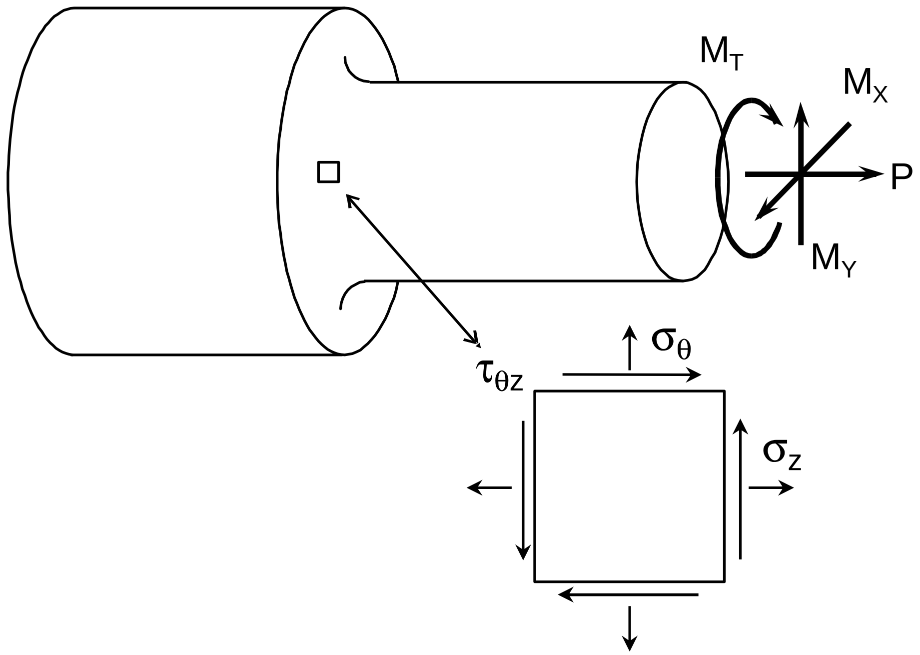
Nonproportional Loading Summary

- Nonproportional cyclic hardening increases stress levels
- Critical plane models are used to assess fatigue damage

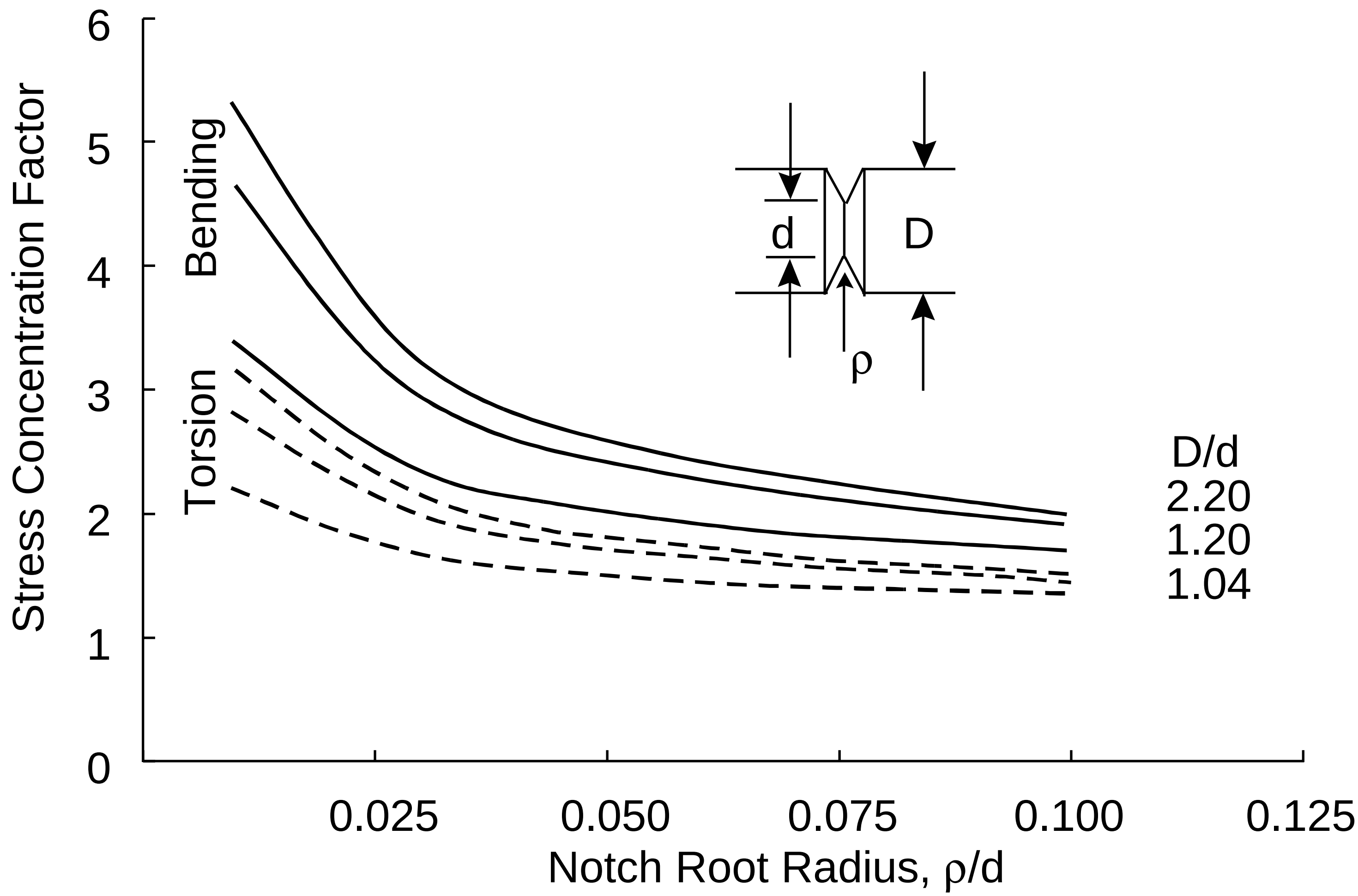
Notches

- Stress and strain concentrations
- Nonproportional loading and stressing
- Fatigue notch factors
- Cracks at notches

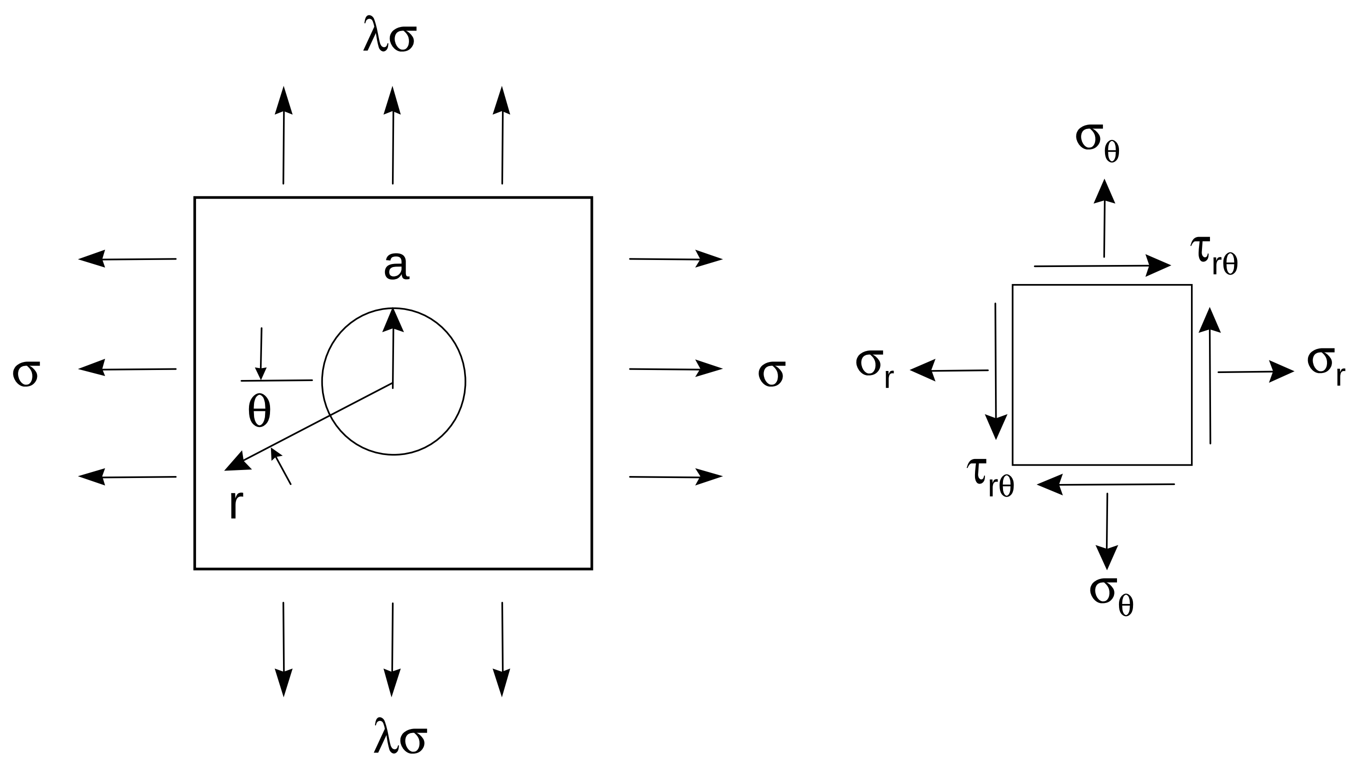
Notched Shaft Loading



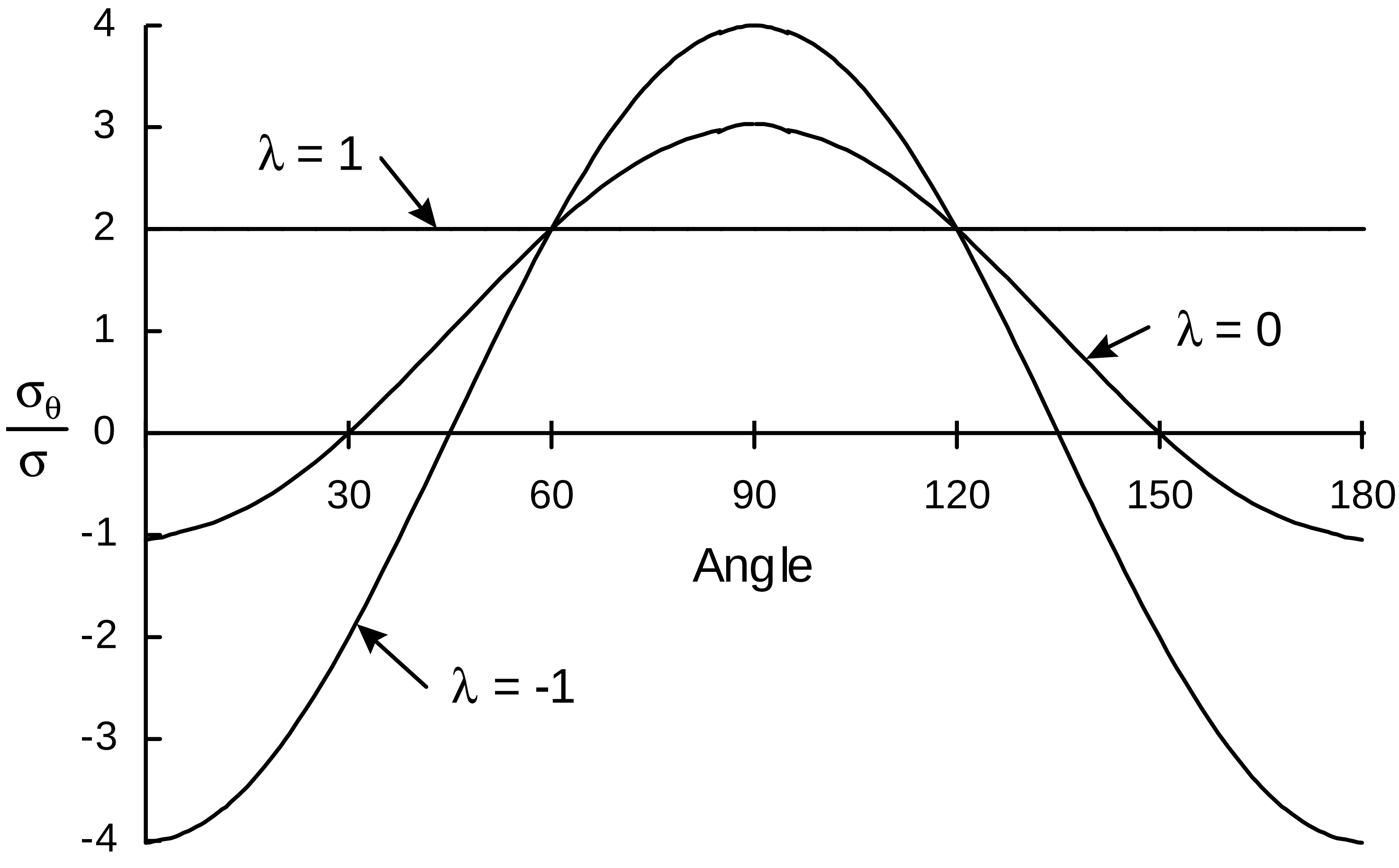
Stress Concentration Factors



Hole in a Plate

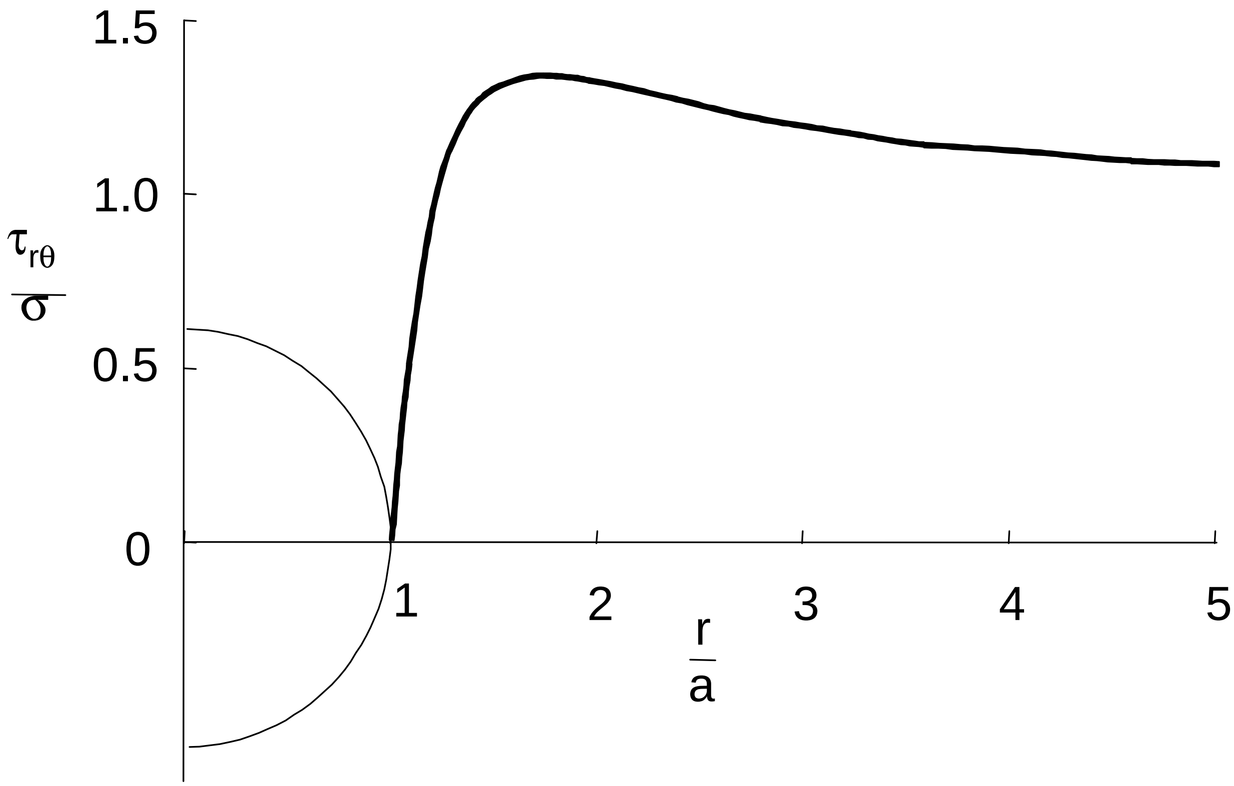


Stresses at the Hole

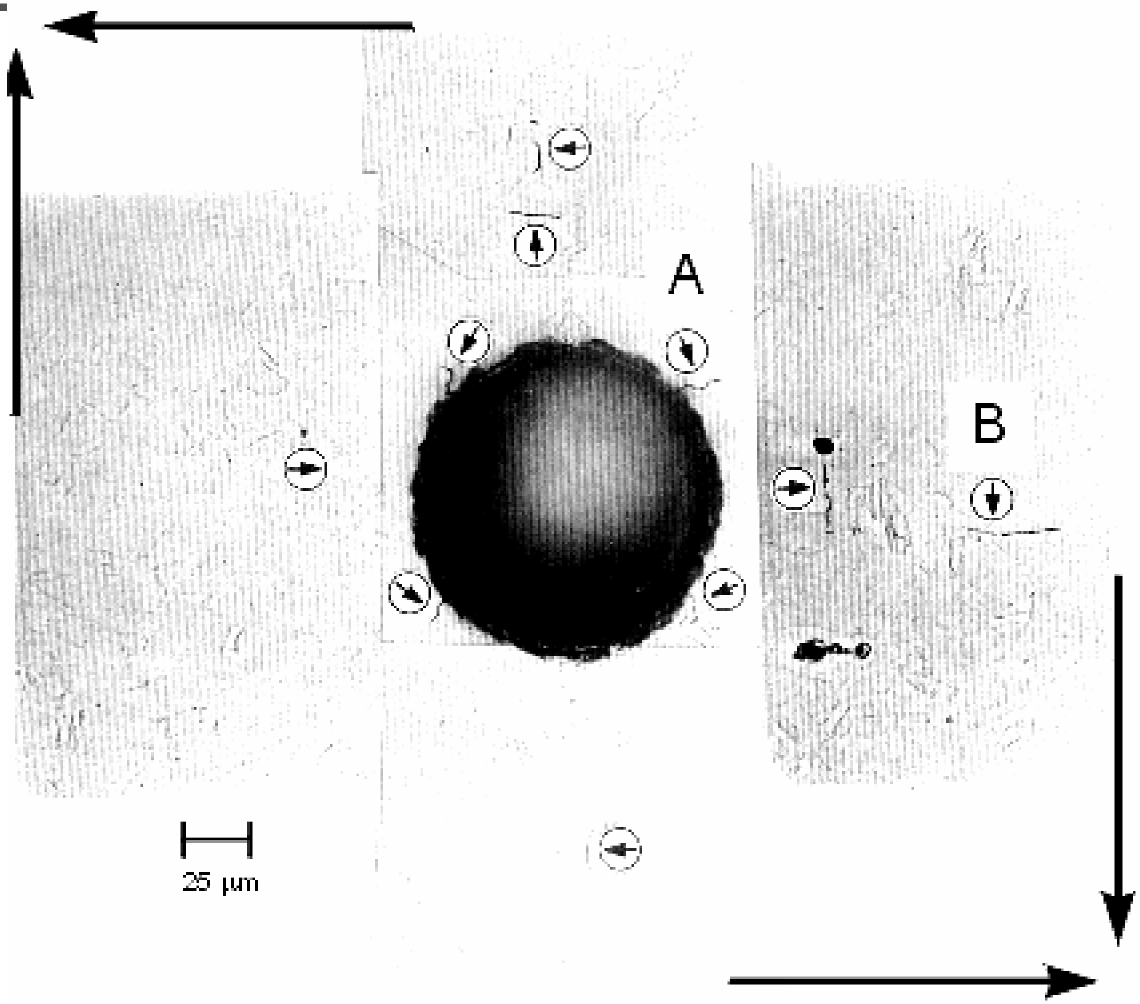


Stress concentration factor depends on type of loading

Shear Stresses during Torsion



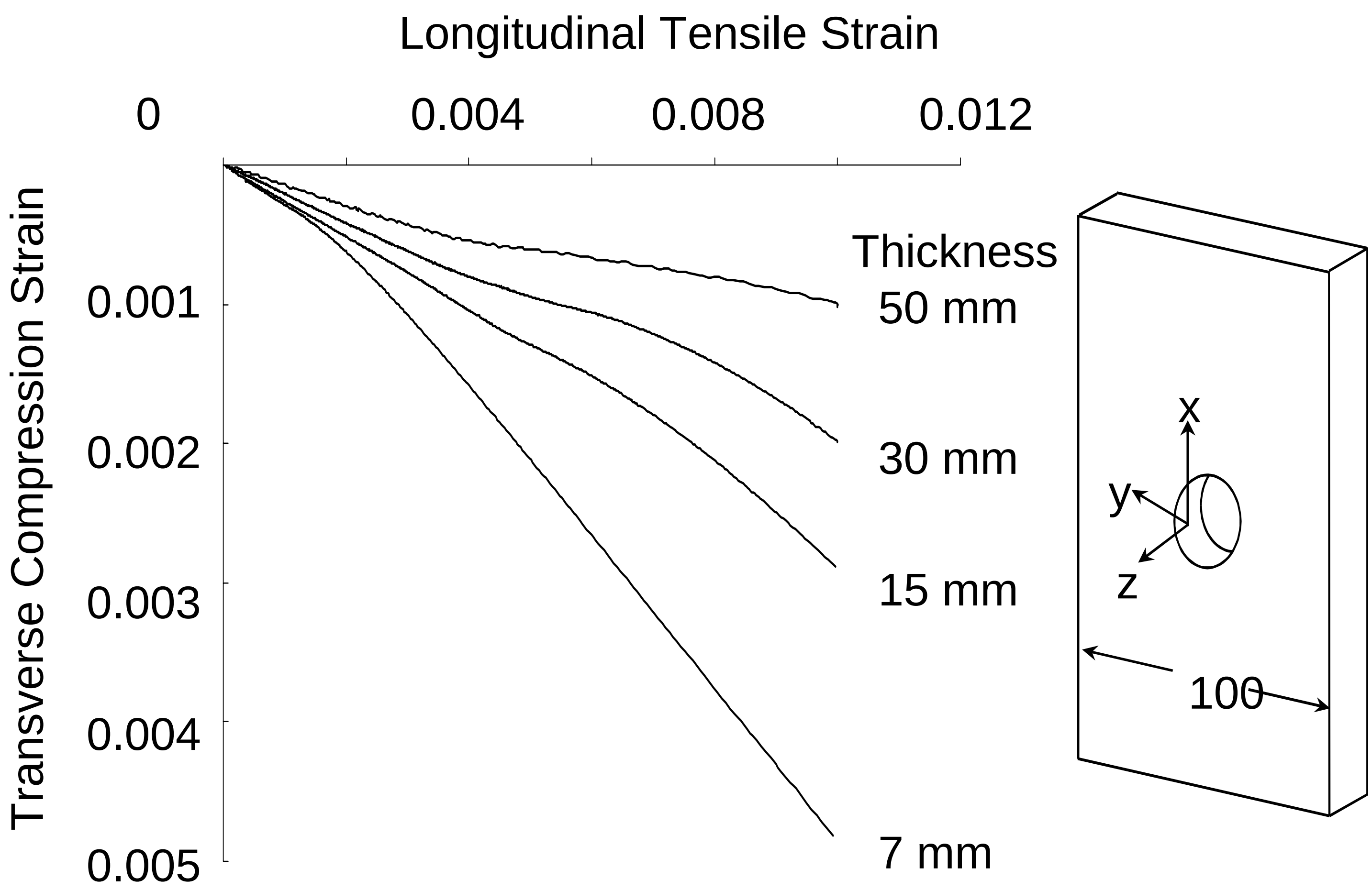
Torsion Experiments



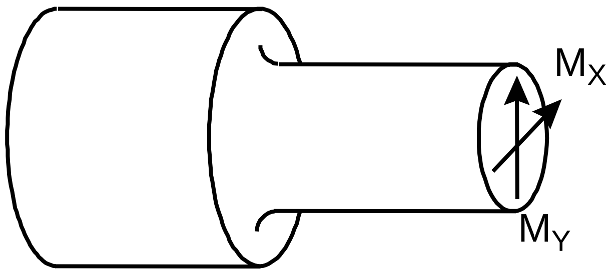
Multiaxial Loading

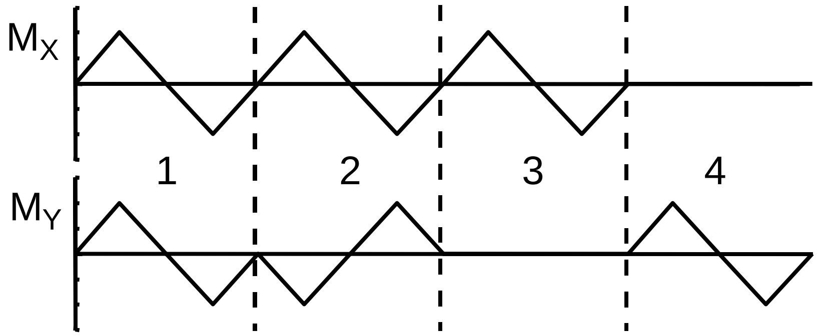
- Uniaxial loading that produces multiaxial stresses at notches
- Multiaxial loading that produces uniaxial stresses at notches
- Multiaxial loading that produces multiaxial stresses at notches

Thickness Effects

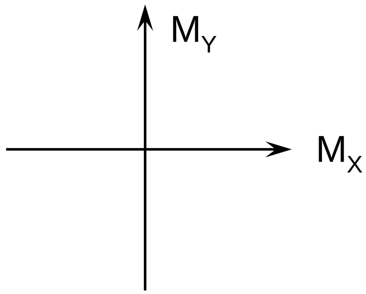


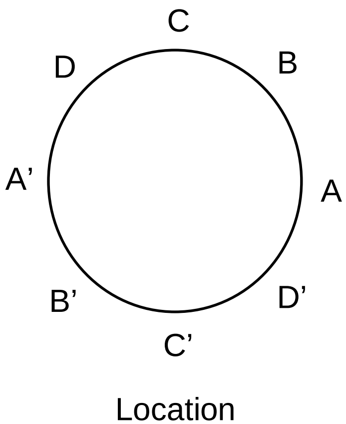
Applied Bending Moments

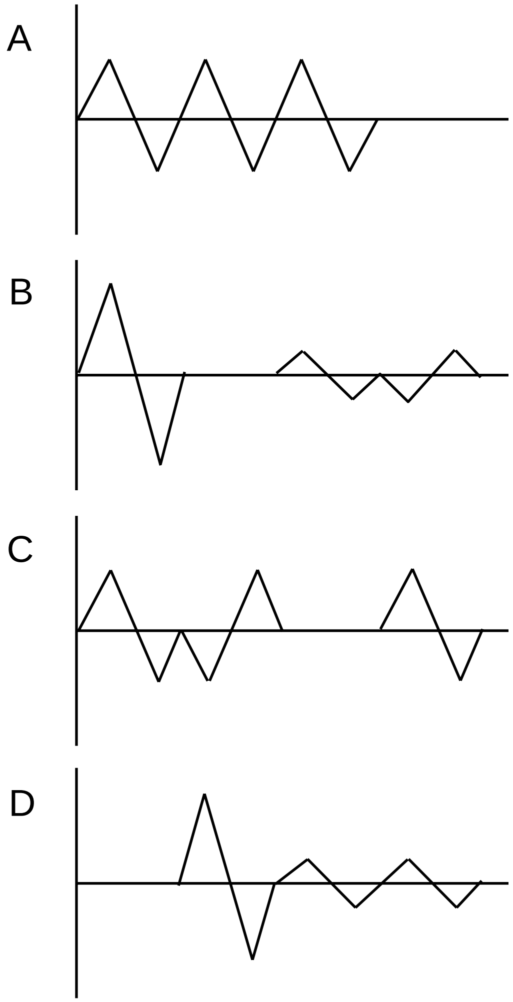




Bending Moments on the Shaft







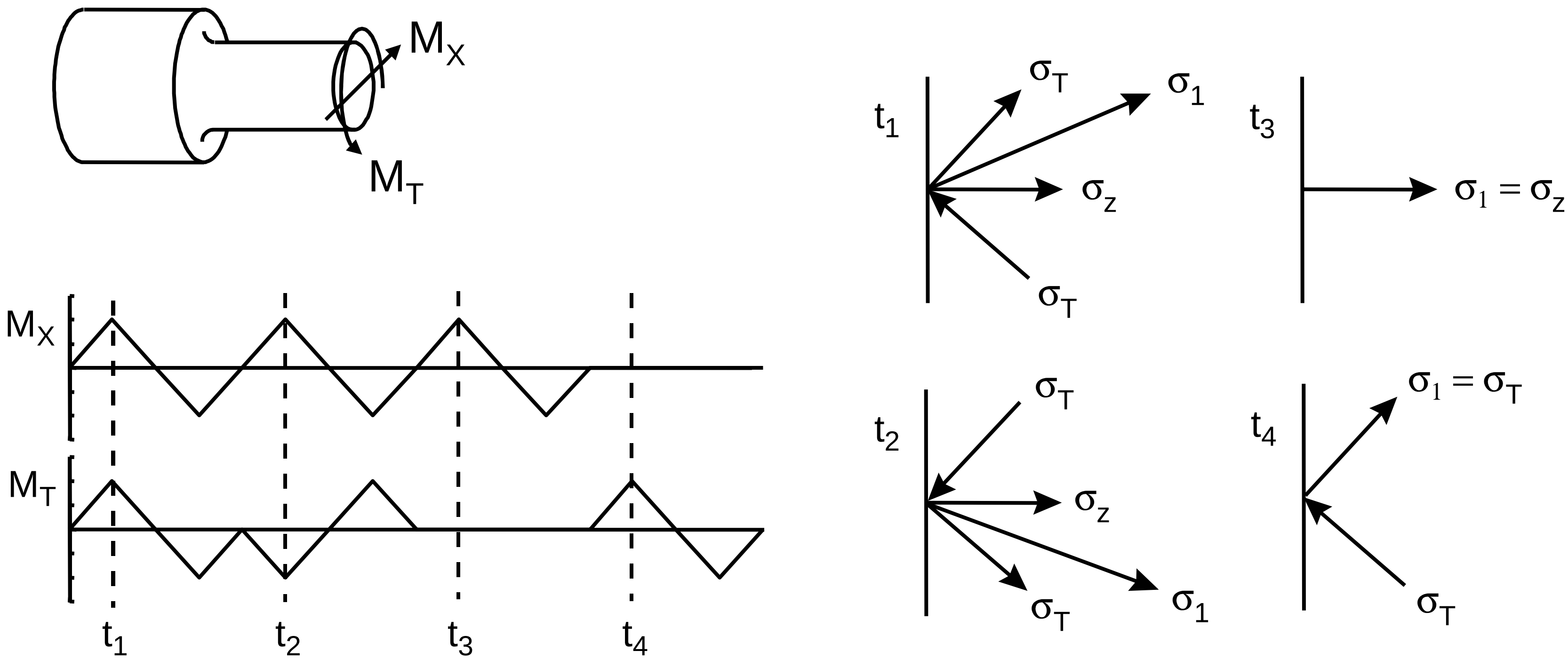
Bending Moments

ΔM	A	B	C	D
2.82		1		1
2.00	3		2	
1.41		2		1
1.00			2	
0.71				2

$$\Delta\overline{M} = \sqrt[5]{\sum \Delta M^5}$$

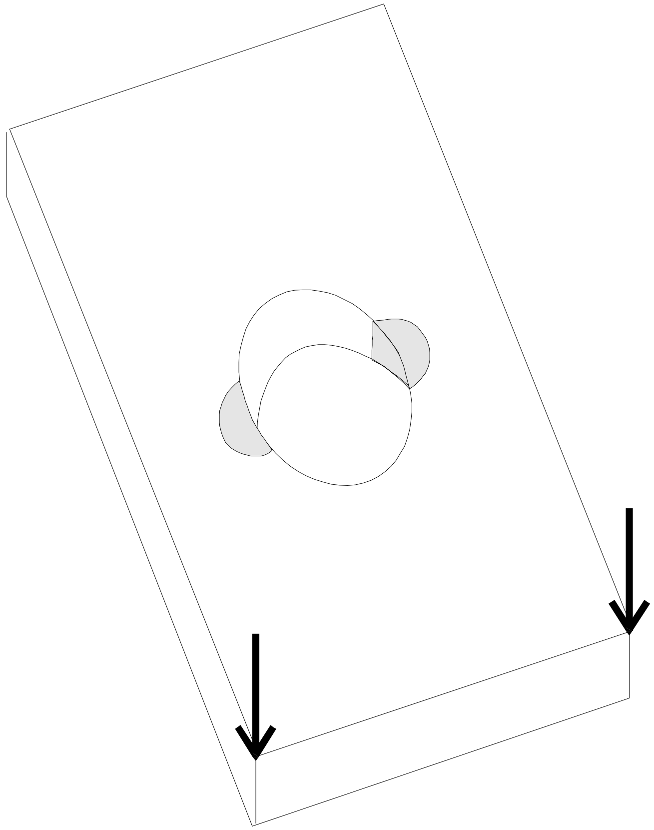
	A	B	C	D
$\Delta\overline{M}$	2.49	2.85	2.31	2.84

Torsion Loading

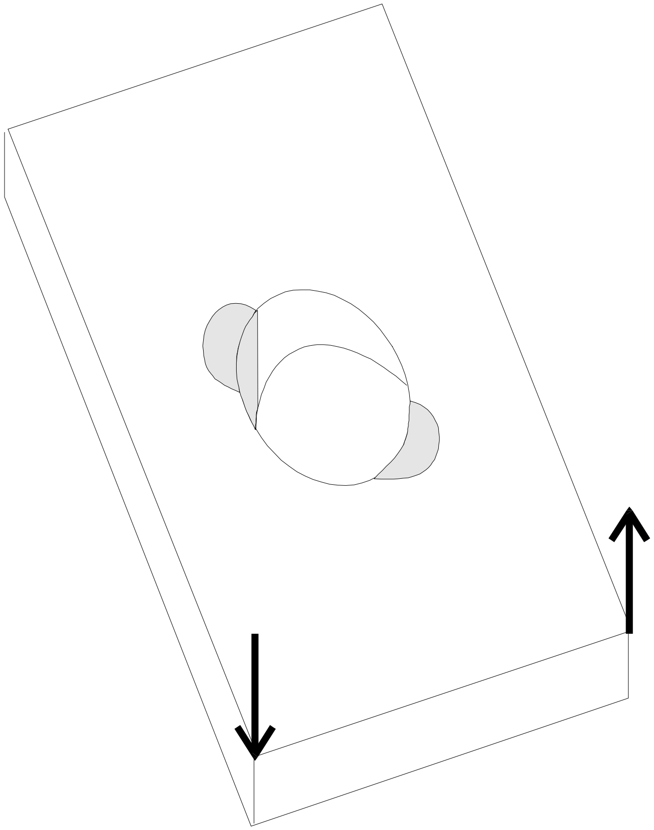


Out-of-phase shear loading is needed to produce nonproportional stressing

Plate and Shell Structures

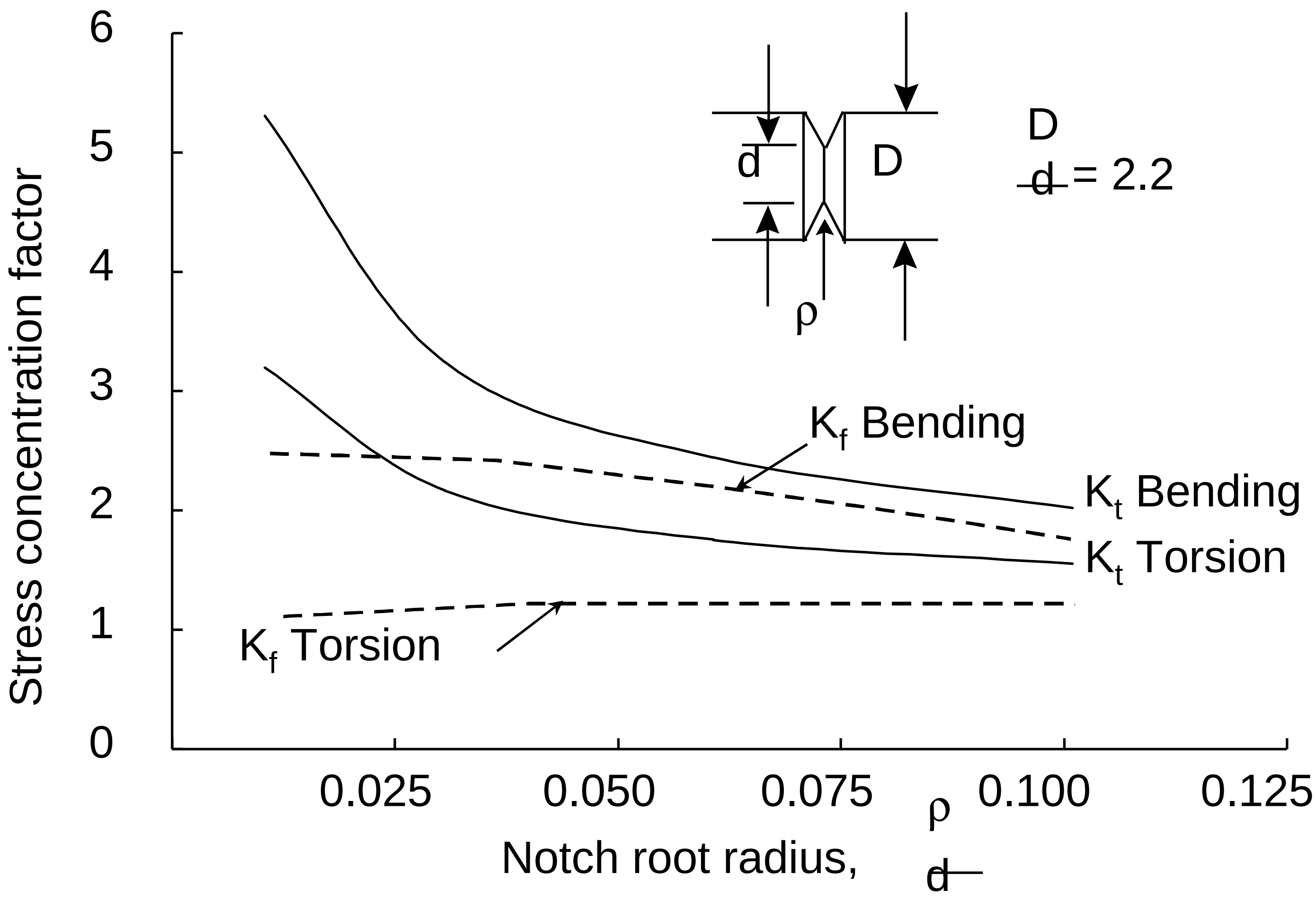


$K_t = 3$

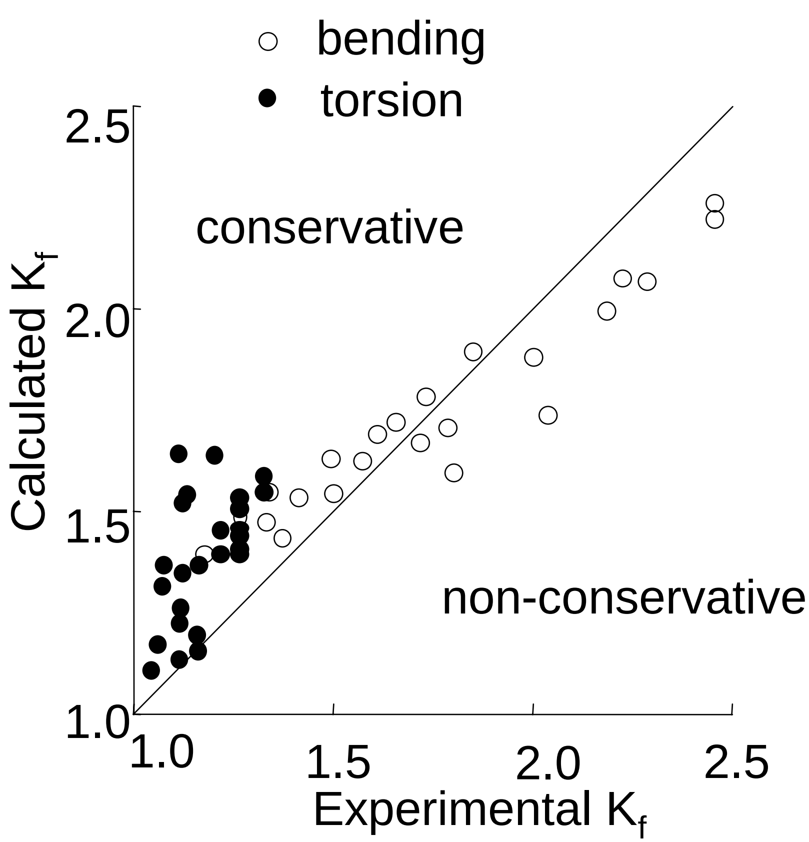


$K_t = 4$

Fatigue Notch Factors



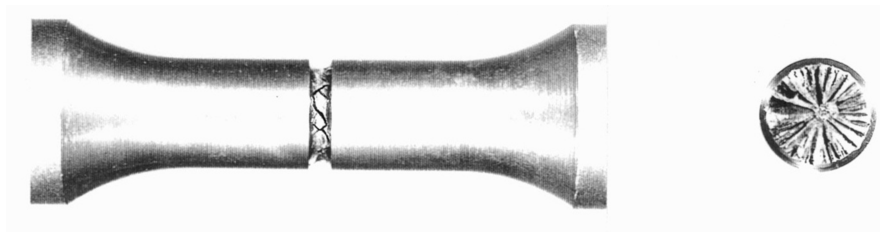
Fatigue Notch Factors (continued)



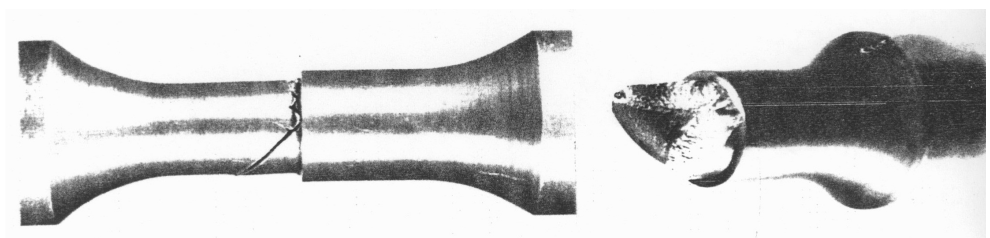
Peterson's Equation

$$K_f = 1 + \frac{K_T - 1}{1 + \frac{a}{r}}$$

Fracture Surfaces in Torsion

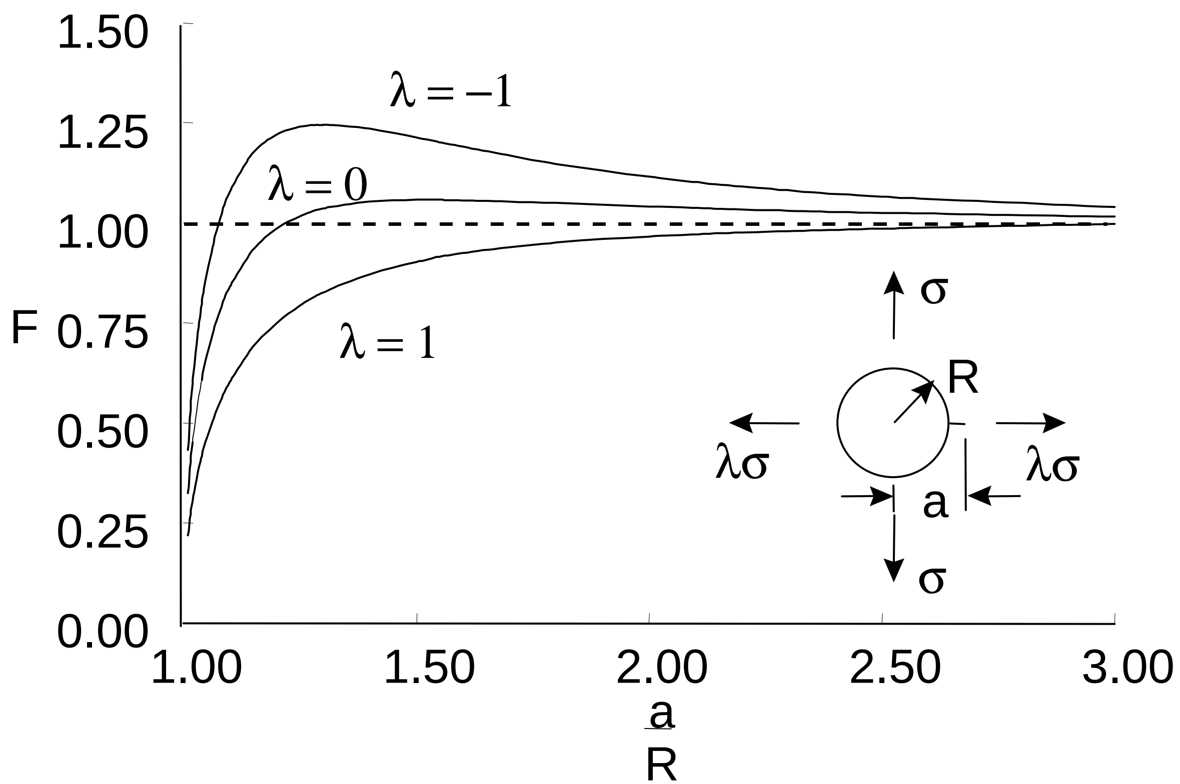


Circumferencial Notch



Shoulder Fillet

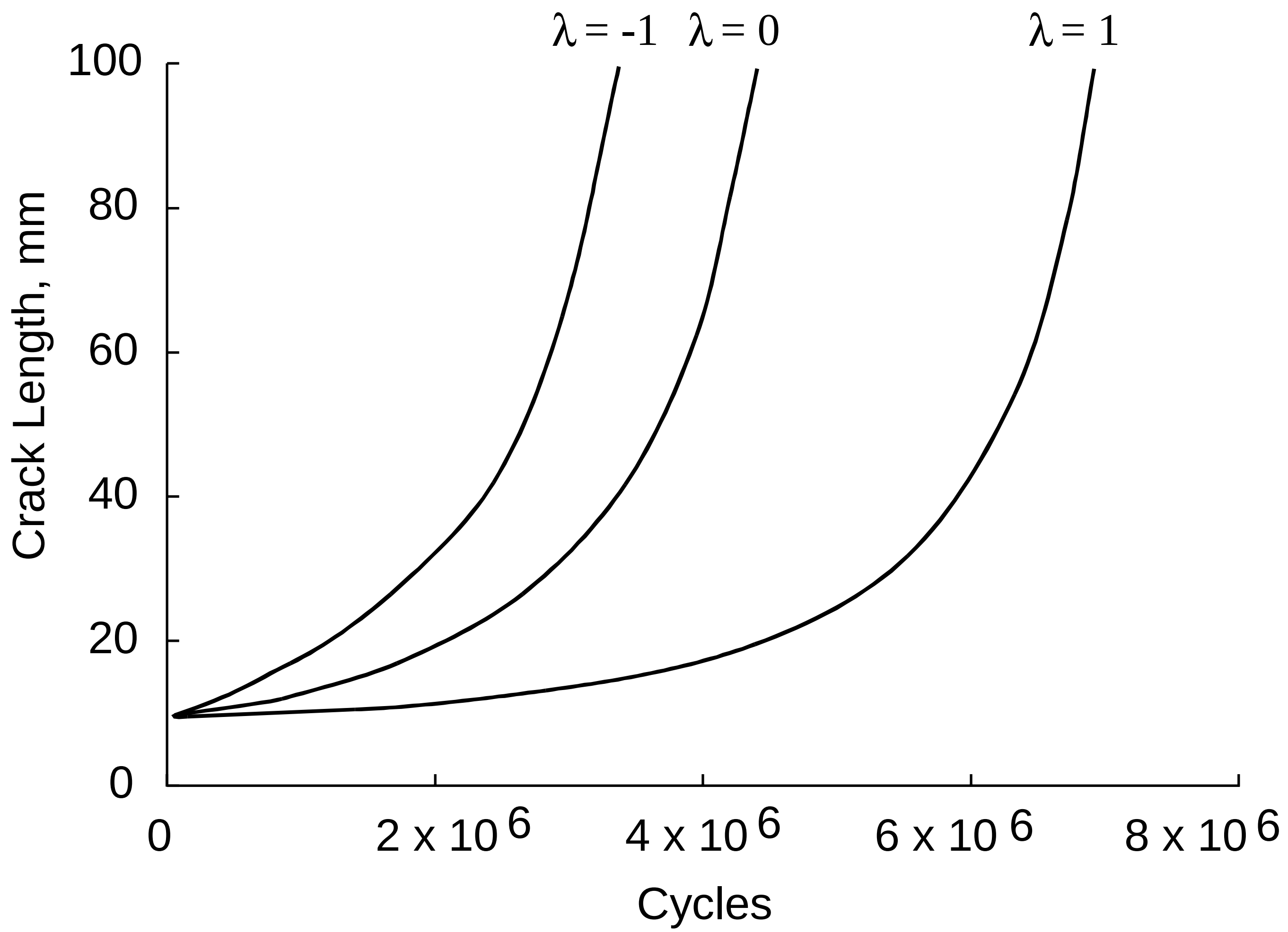
Stress Intensity Factors



$$\frac{da}{dN} = C\left(\Delta K_{eq}\right)^m$$

$$K_I = F\,\Delta\sigma\sqrt{\pi a}$$

Crack Growth From a Hole



Notches Summary

- Uniaxial loading can produce multiaxial stresses at notches
- Multiaxial loading can produce uniaxial stresses at notches
- Multiaxial stresses are not very important in thin plate and shell structures
- Multiaxial stresses are not very important in crack growth

Final Summary

- Fatigue is a planar process involving the growth of cracks on many size scales
- Critical plane models provide reasonable estimates of fatigue damage

Multiaxial Fatigue

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