



# Sequence Effects in Fatigue

---

**Professor Darrell F. Socie**  
**University of Illinois at Urbana-Champaign**

© 2002 Darrell Socie, All Rights Reserved

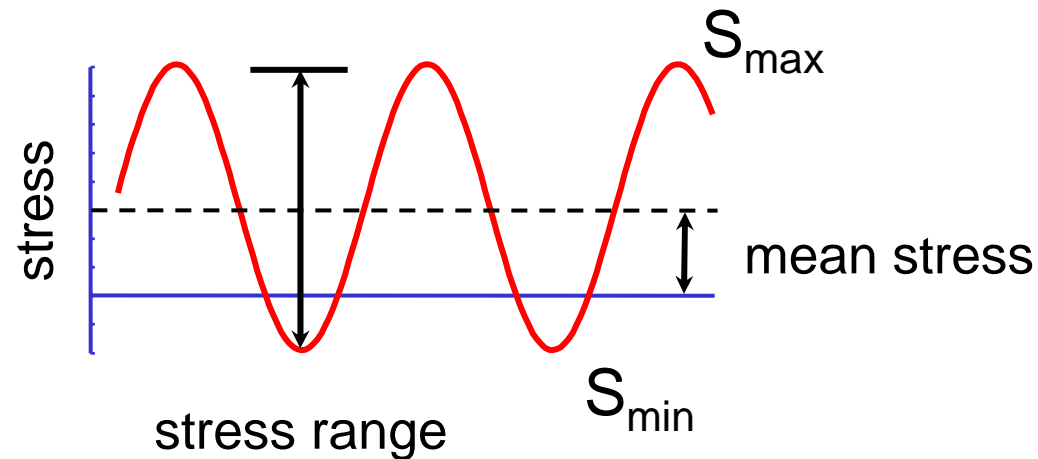


# Outline

---

1. Mean Stress
2. Nonlinear Damage Models
3. Crack Growth Interactions
4. Crack Tip Plasticity Models
5. Crack Closure Models

# Mean Stresses



$$S_{\text{mean}} = \frac{S_{\max} + S_{\min}}{2}$$

$$R = \frac{S_{\min}}{S_{\max}}$$



# General Observations

---

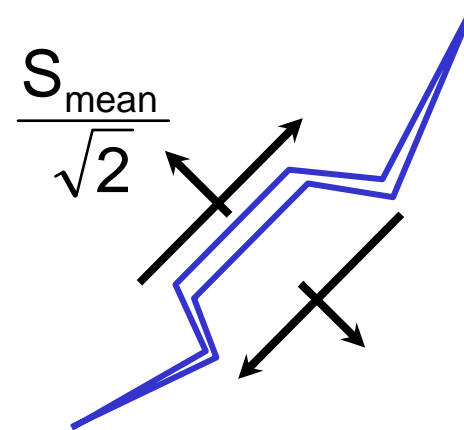
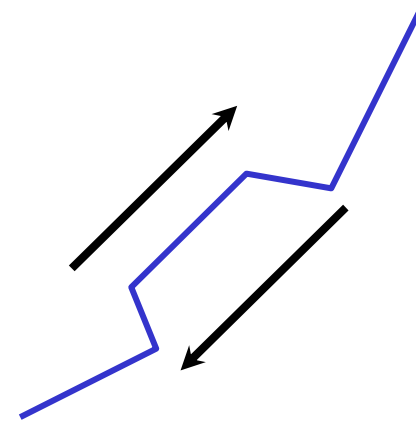
- Tensile mean stresses reduce the fatigue life or decrease the allowable stress range
- Compressive mean stresses increase the fatigue life or increase the allowable stress range

# Mechanism



Fatigue damage is a shear process

Tensile mean stresses open microcracks and make sliding easier



# Goodman 1890

*Mechanics Applied to Engineering*  
John Goodman, 1890

“.. whether the assumptions of the theory are justifiable or not .... We adopt it simply because it is the easiest to use, and for all practical purposes, represents Wöhlers data.

$$S_{\text{ultimate}} = S_{\text{min}} + 2 \Delta S$$

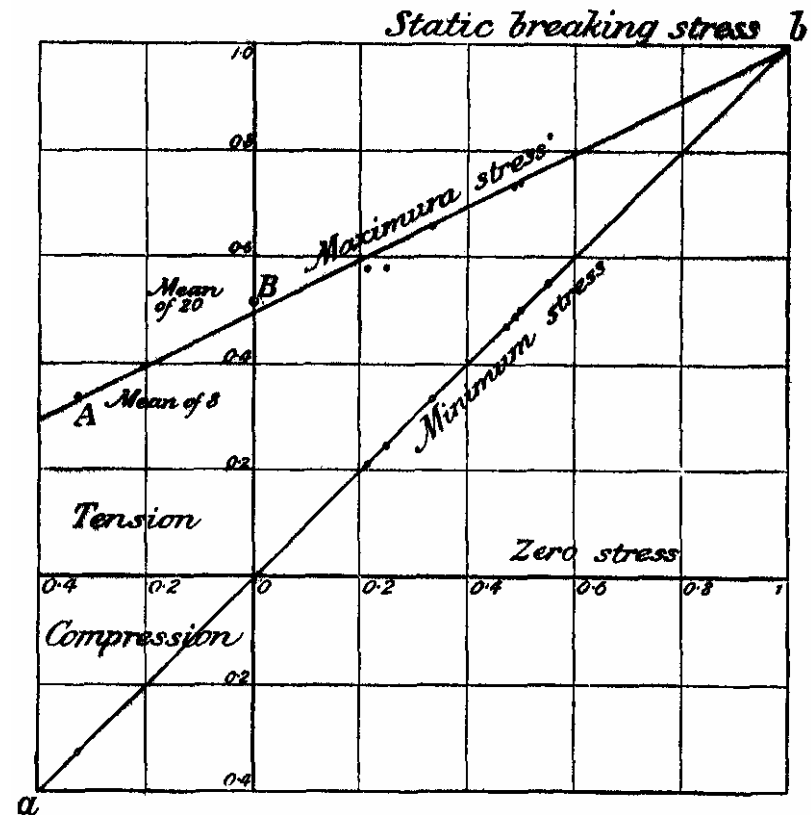
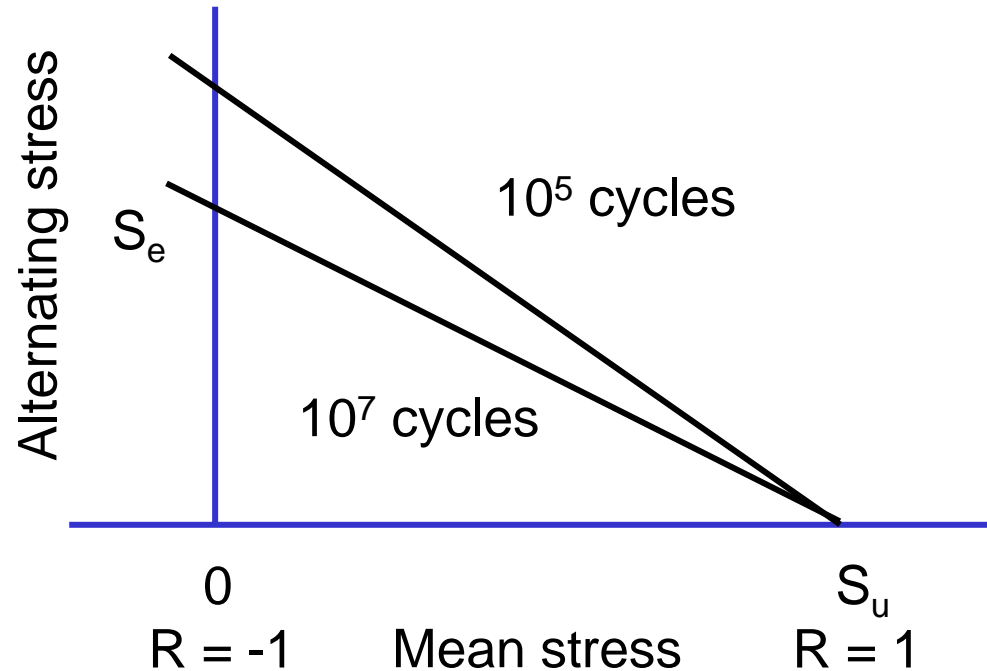


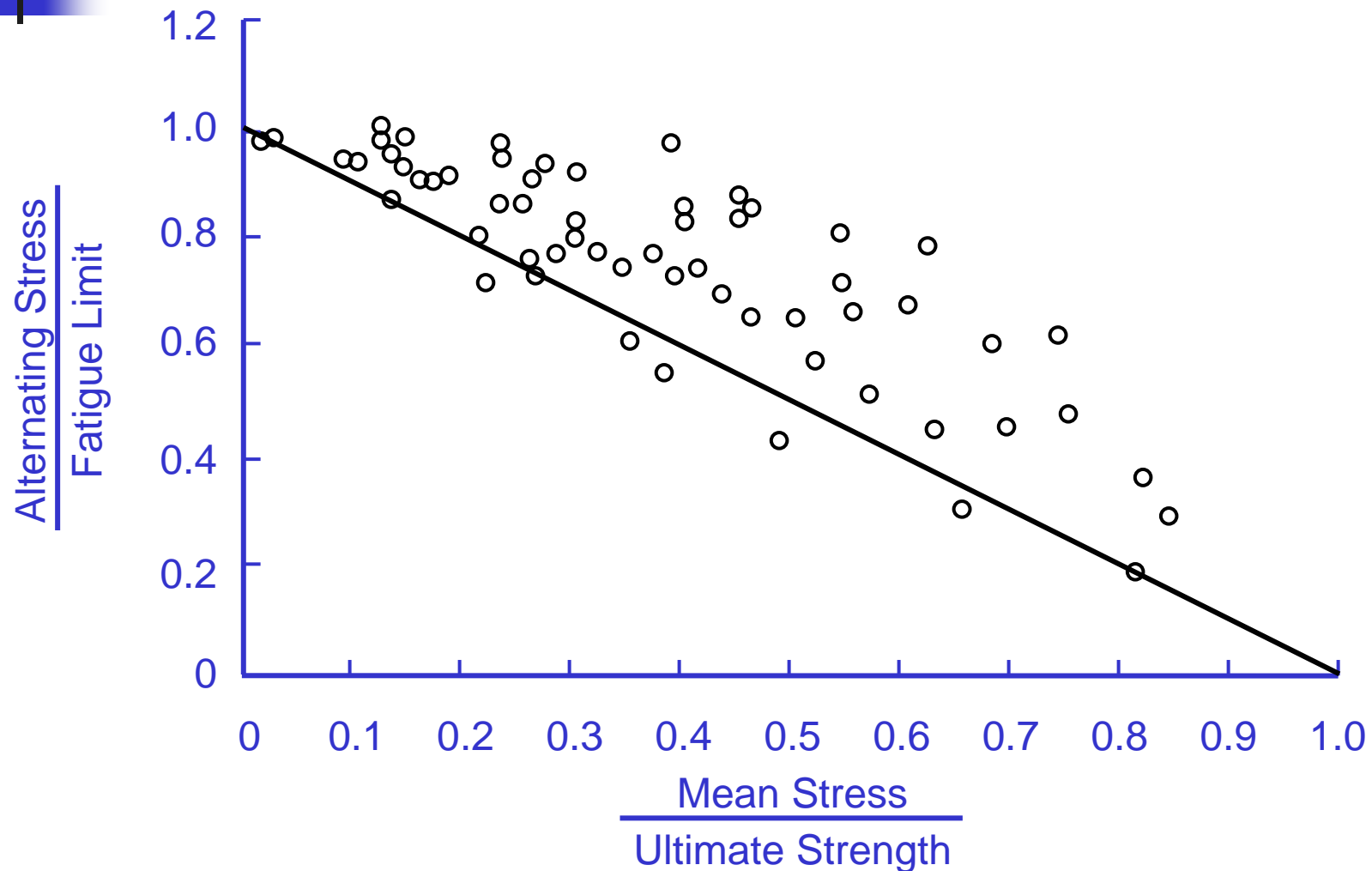
FIG. 517.

# Goodman Diagram



$$\frac{\Delta S}{2} = \left( \frac{\Delta S}{2} \right)_{R=-1} \left( 1 - \frac{S_{\text{mean}}}{S_{\text{ultimate}}} \right)$$

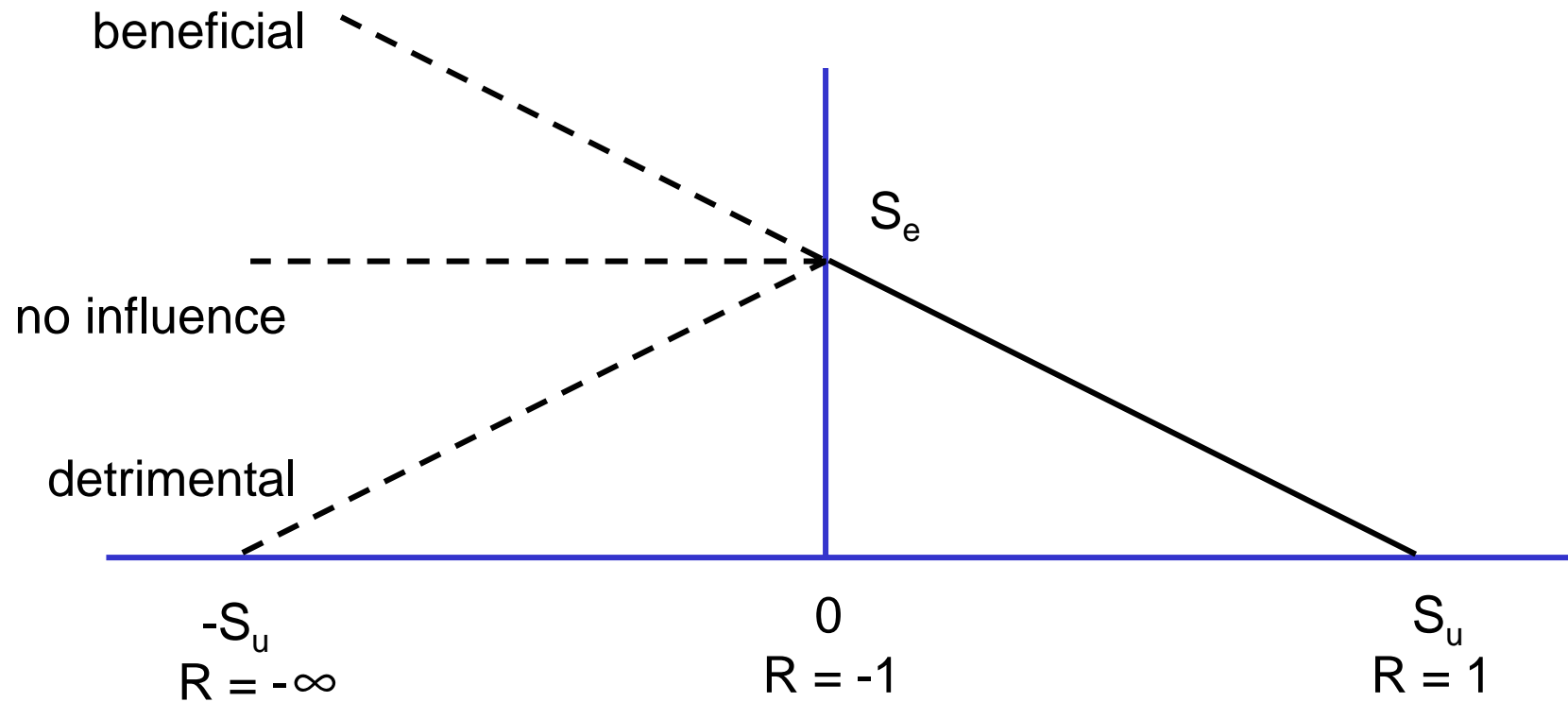
## Test Data ( 1941 )



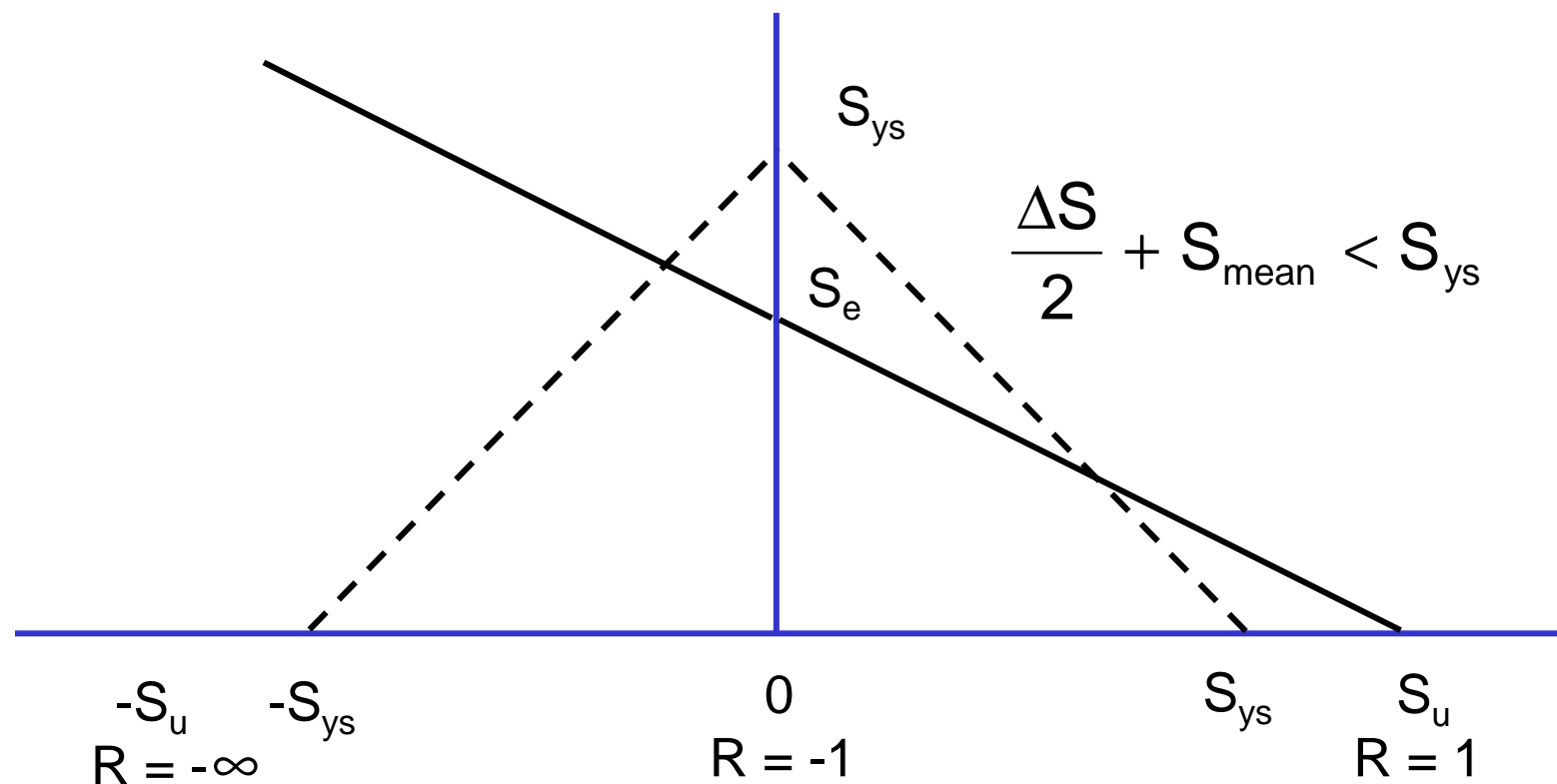
J.O. Smith, The Effect of Range of Stress on the Fatigue Strength of Metals,  
Engineering Experiment Station Bulletin 334, University of Illinois, 1941



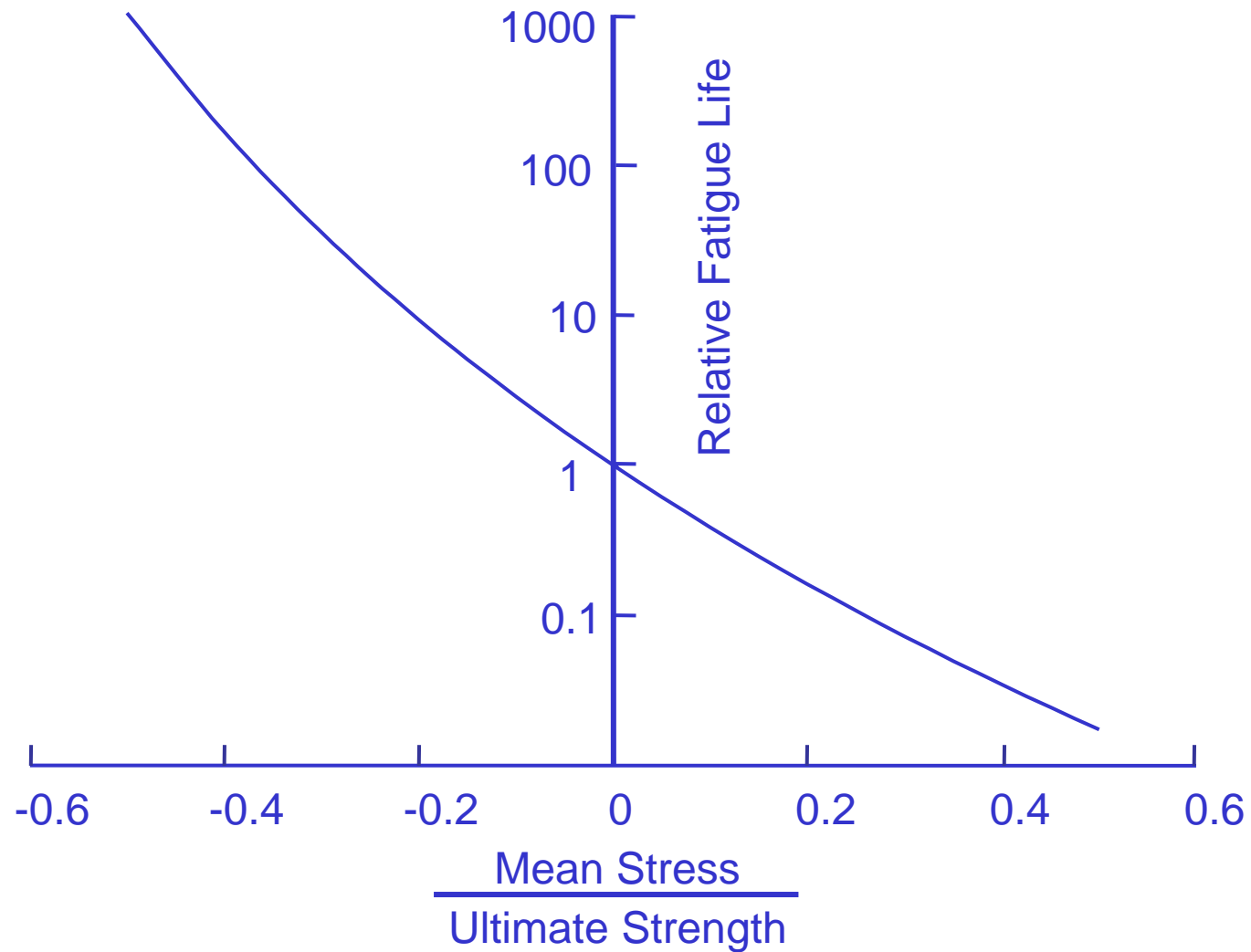
# Compression



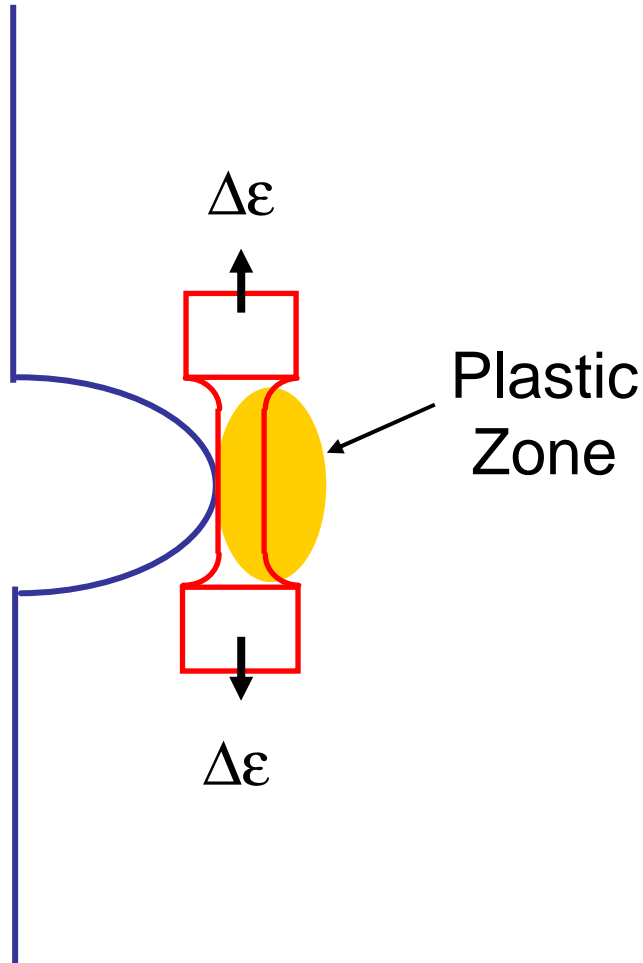
# Modified Goodman ( no yielding )



# Mean Stress Influence on Life



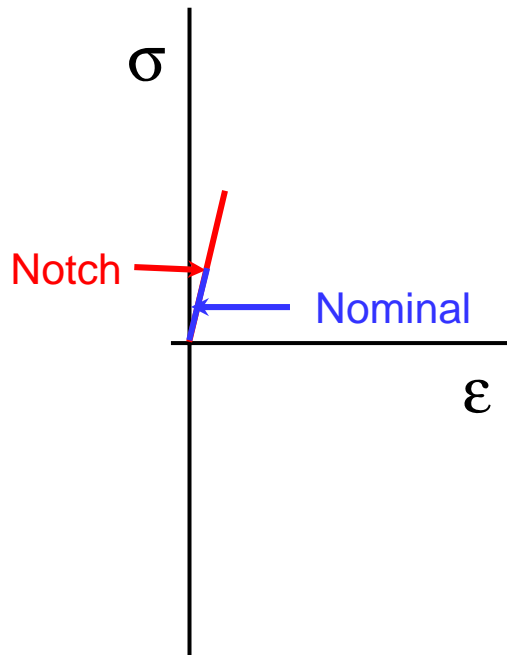
# Stress Concentrations



The elastic material surrounding the plastic zone around a stress concentration forces the material to deform in strain control

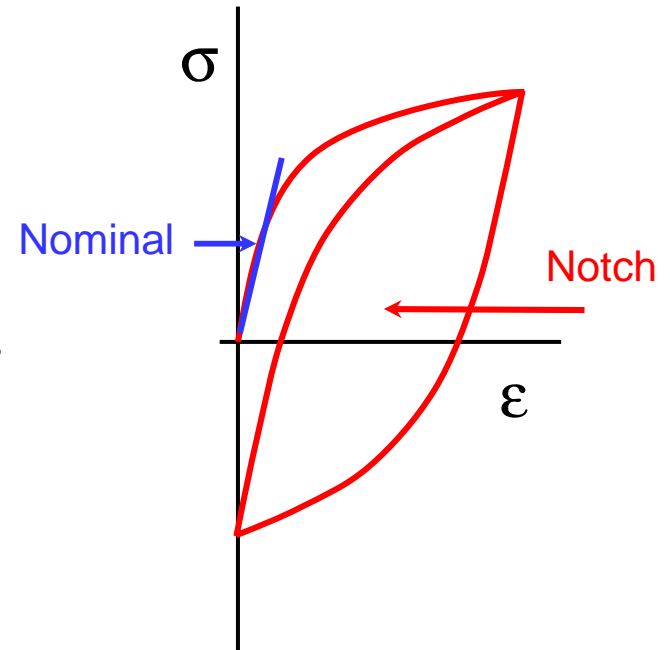
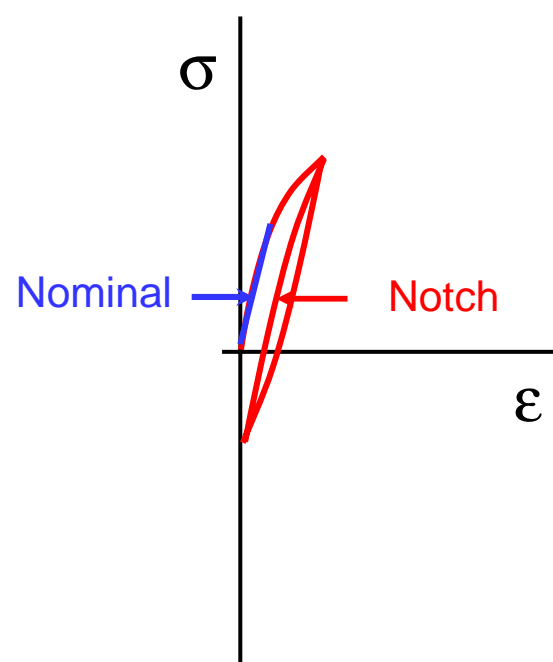
# Mean Stresses at Notches

elastic



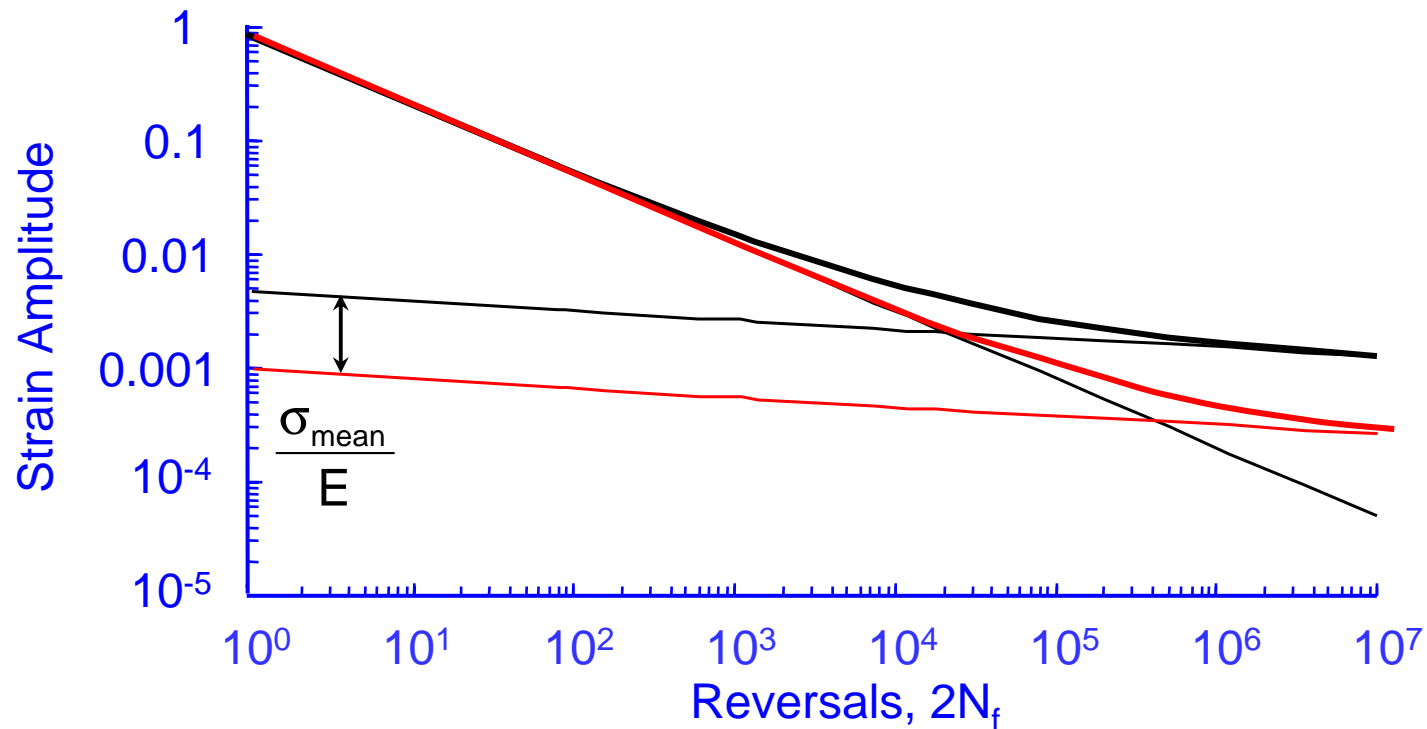
Nominal mean stress is less than notch mean stress

plastic



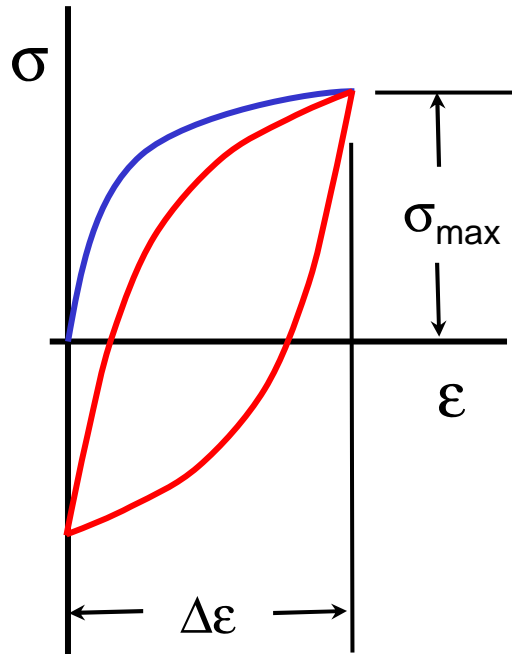
Nominal mean stress is greater than notch mean stress

# Morrow Mean Stress Correction



$$\frac{\Delta \epsilon}{2} = \frac{\sigma'_f - \sigma_{\text{mean}}}{E} (2N_f)^b + \epsilon'_f (2N_f)^c$$

# Smith Watson Topper



$$\sigma_{\max} \frac{\Delta\epsilon}{2} = \frac{\sigma_f'^2}{E} (2N_f)^{2b} + \sigma_f' \epsilon_f' (2N_f)^{b+c}$$

# Mean Stress Relaxation

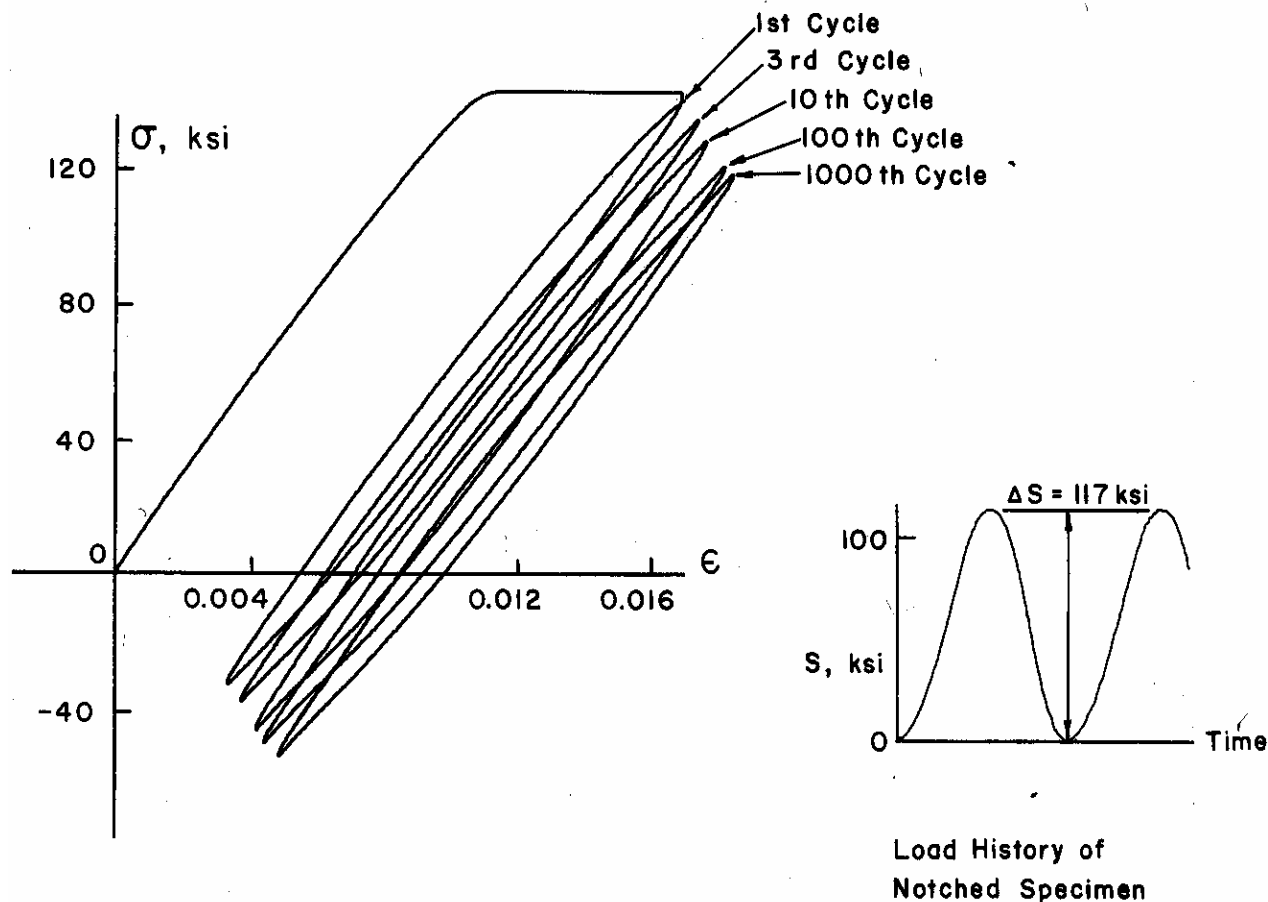
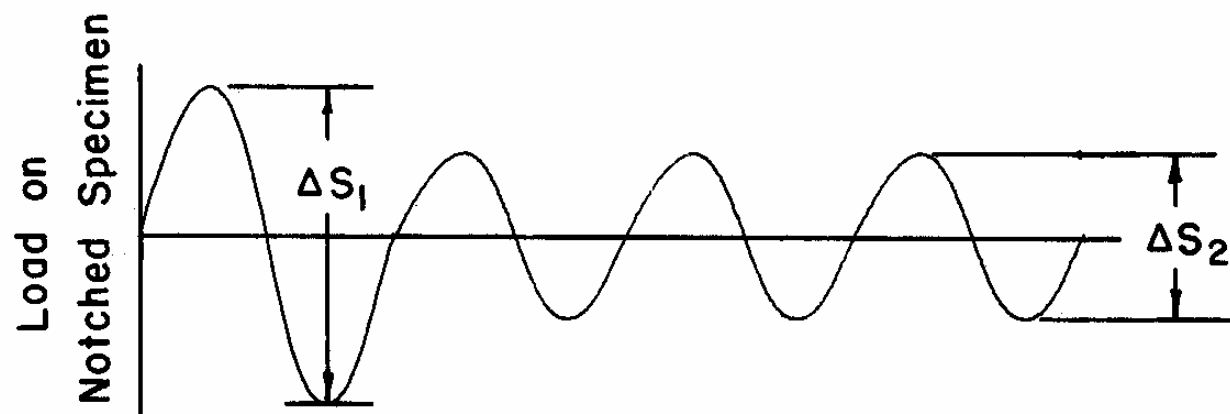


FIG. 7—Cyclic softening and relaxation of mean stress under Neuber control (Ti-8Al-1Mo-IV,  $K_t = 1.75$ ).

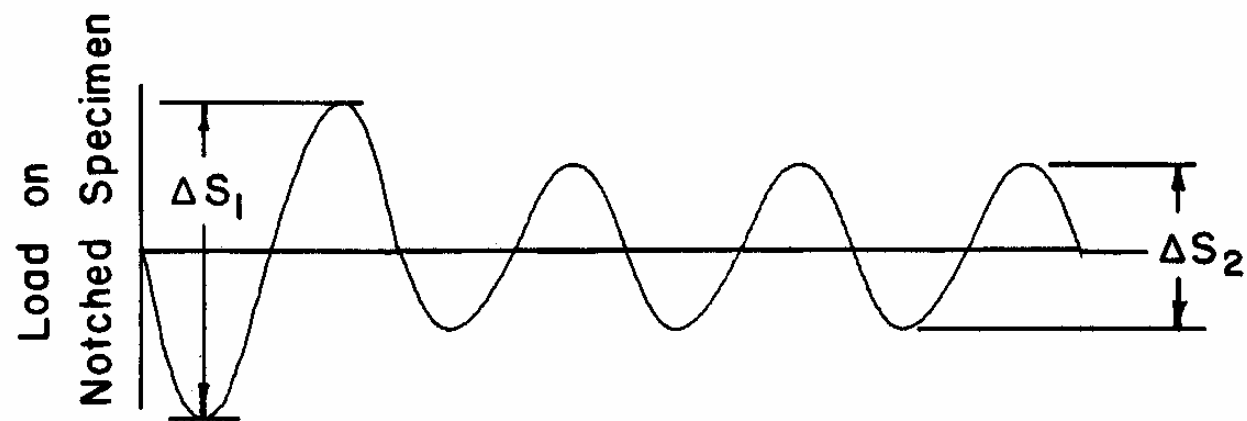
Stadnick and Morrow, "Techniques for Smooth Specimen Simulation of Fatigue Behavior of Notched Members" ASTM STP 515, 1972, 229-252



# Loading Histories

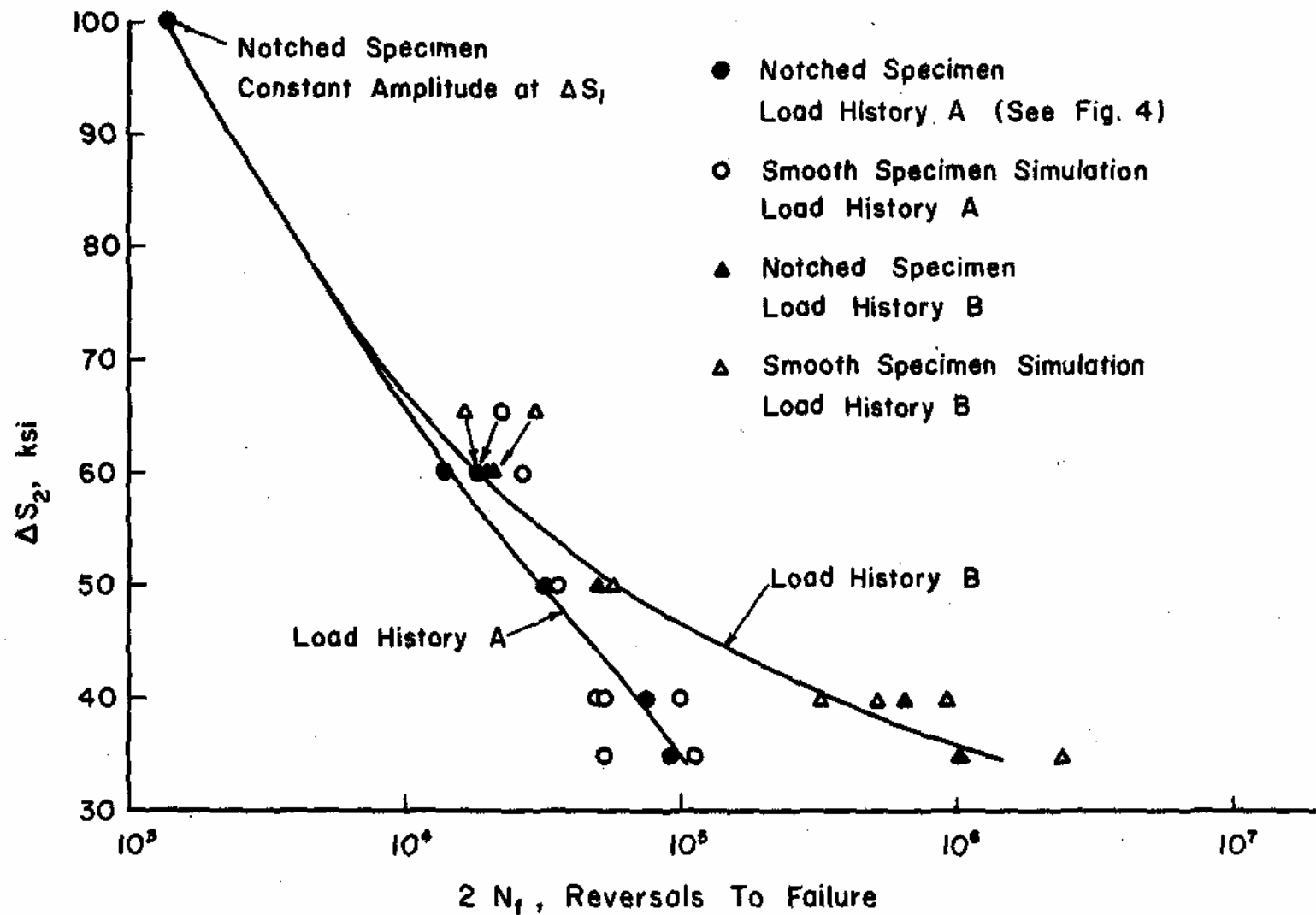


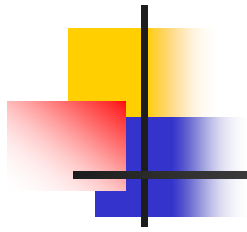
Load History A



Load History B

# Test Results

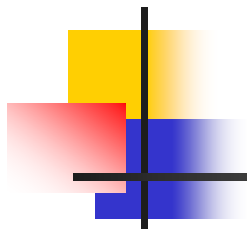




# Sources of Mean/Residual Stress

---

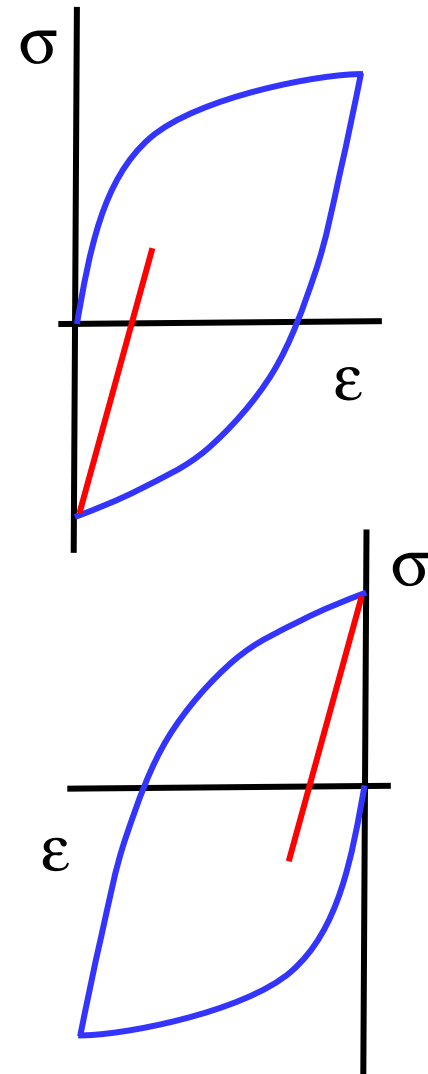
- Loading History
- Fabrication
- Shot Peening
- Heat Treating

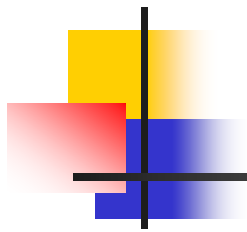


# Loading History

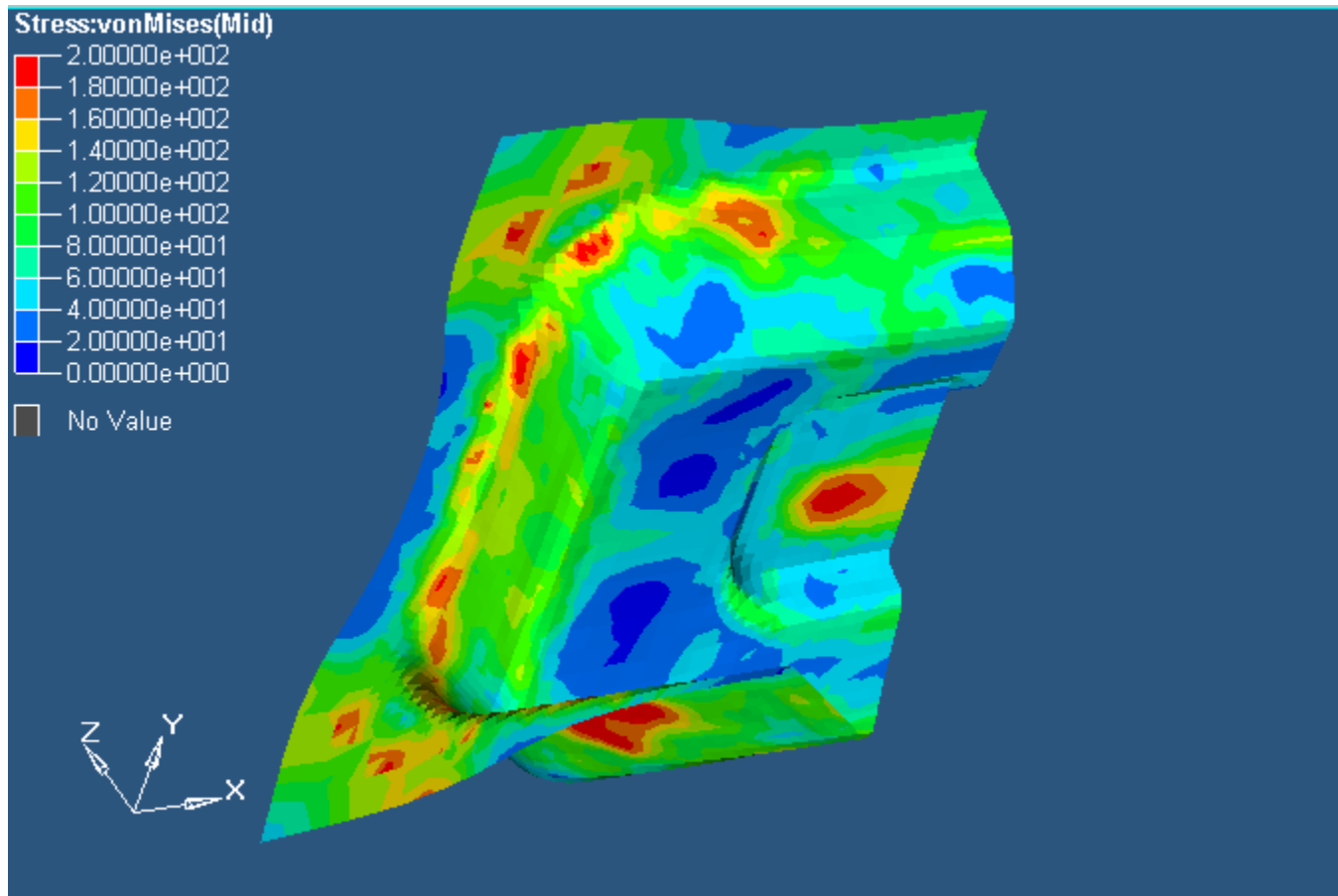
Tension overloads produce favorable compressive residual stress

Compressive overloads produce unfavorable tensile residual stress





# Fabrication

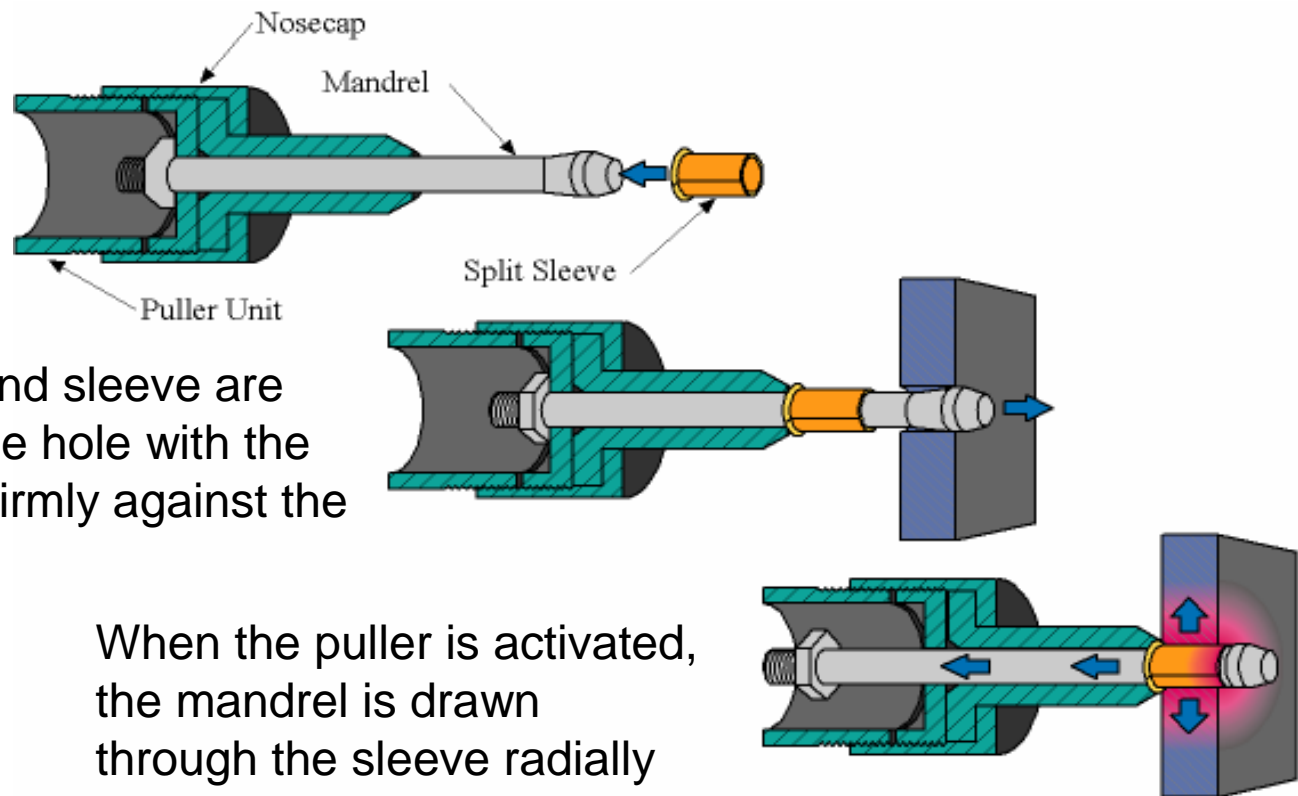


# Cold Expansion

## 1965 Basic Cx process conceptualized (Boeing)

The split sleeve is slipped onto the mandrel, which is attached to the hydraulic puller unit.

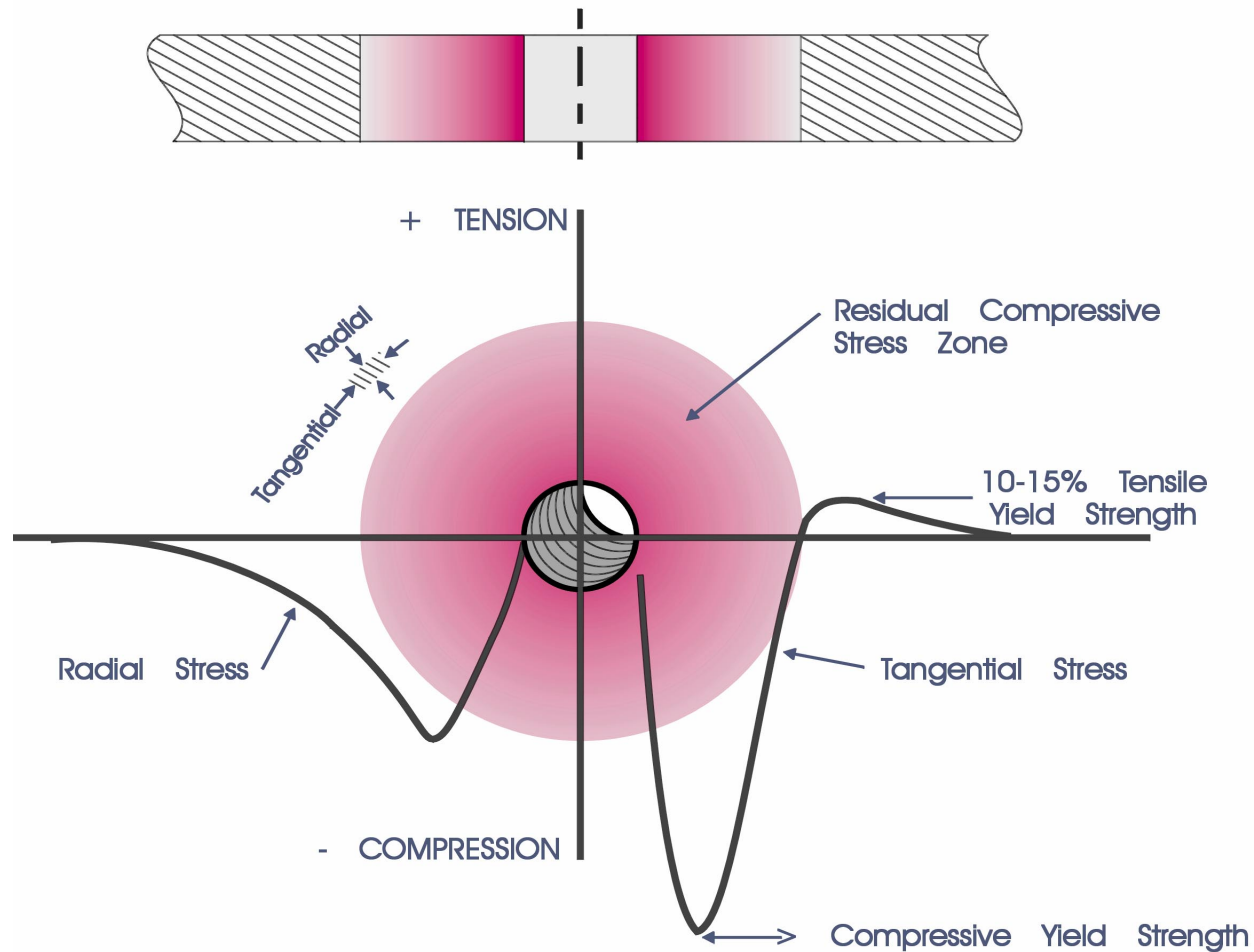
The mandrel and sleeve are inserted into the hole with the nosecap held firmly against the workpiece.



When the puller is activated, the mandrel is drawn through the sleeve radially expanding the hole.

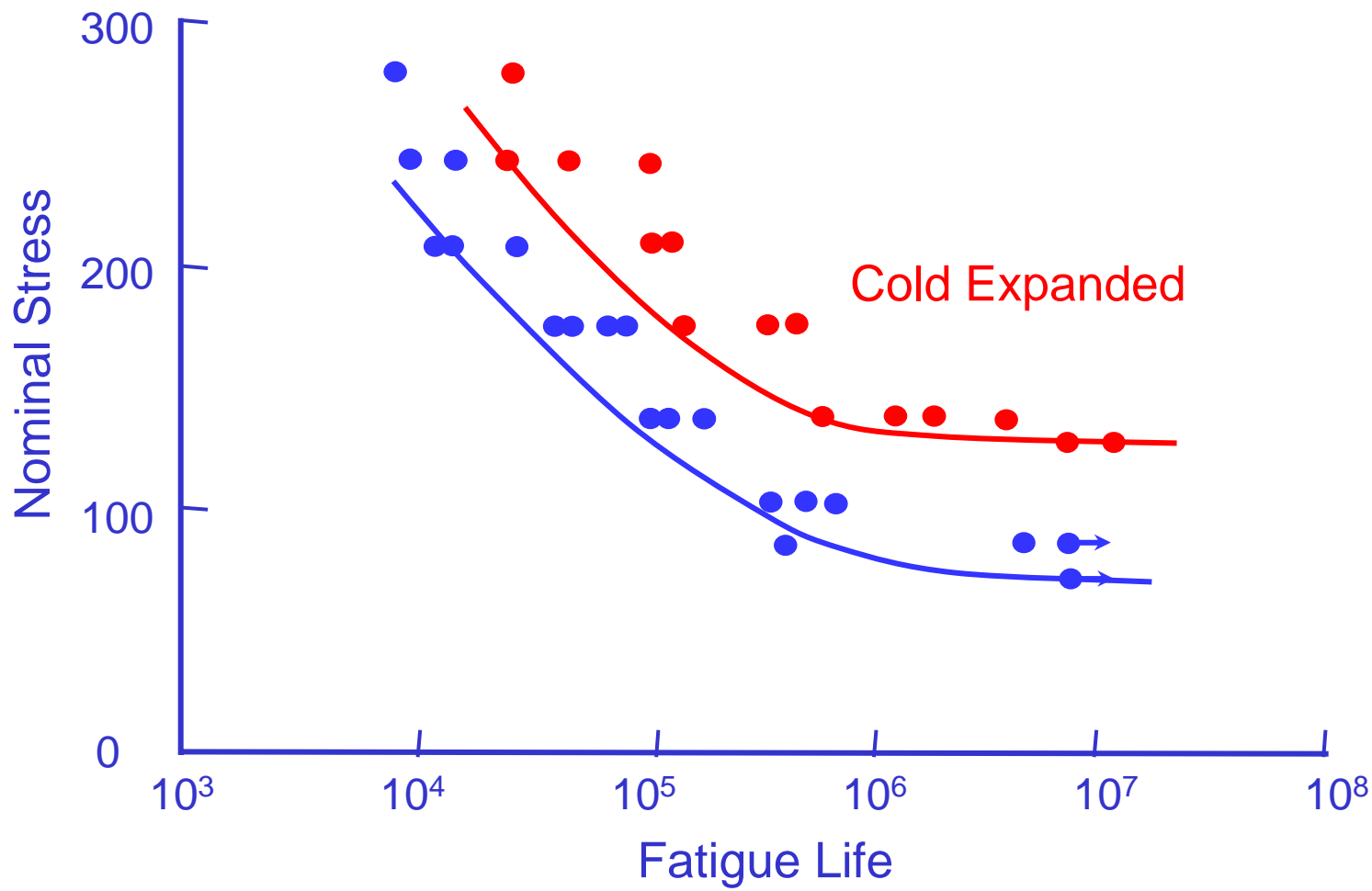
Courtesy of Fatigue Technology Inc.

# Theory of Cold Expansion



Courtesy of Fatigue Technology Inc.

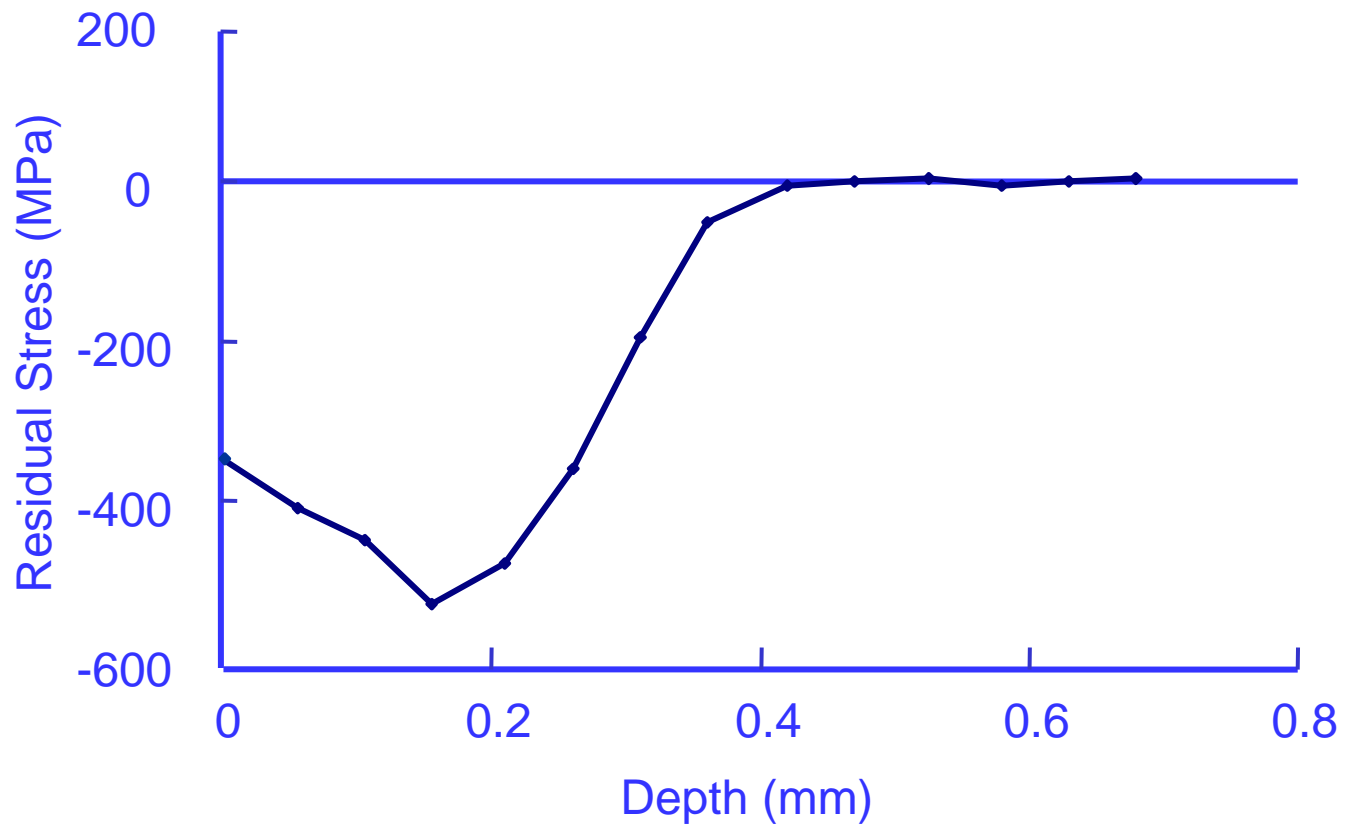
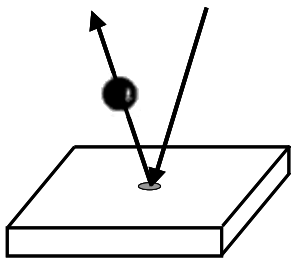
# Fatigue Life Improvement



Courtesy of Fatigue Technology Inc.

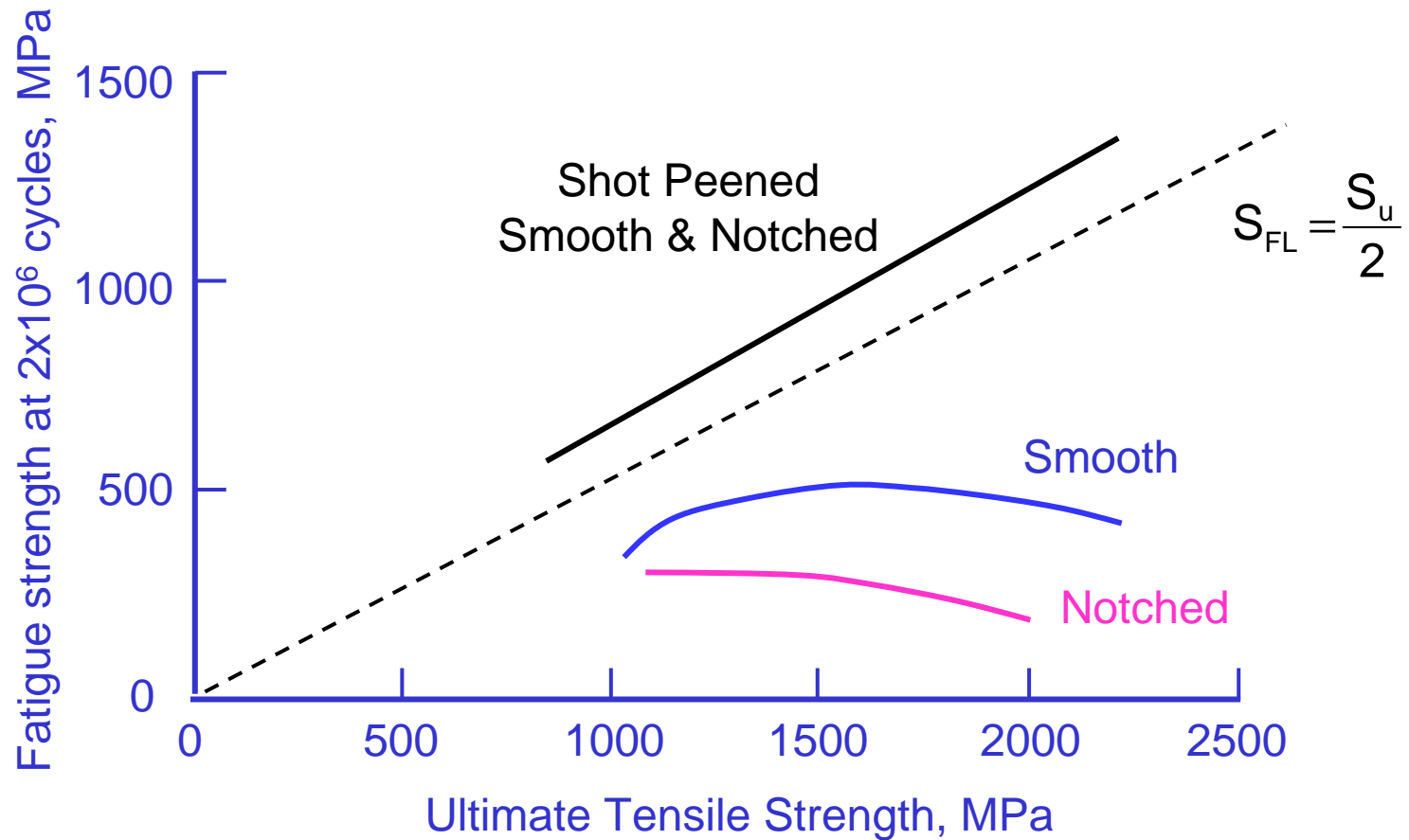


# Shot Peening

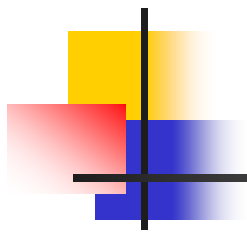


Residual stress in a shot peened leaf spring

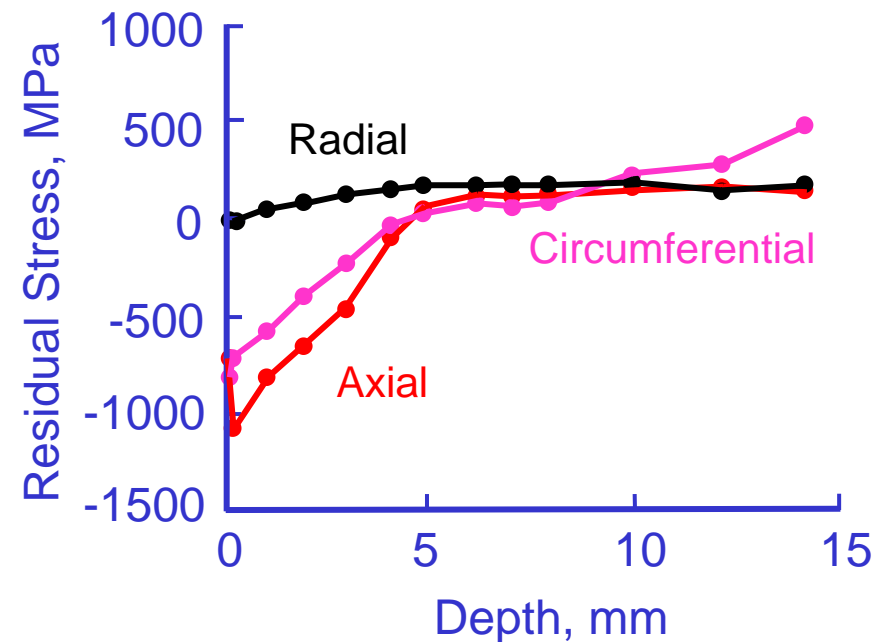
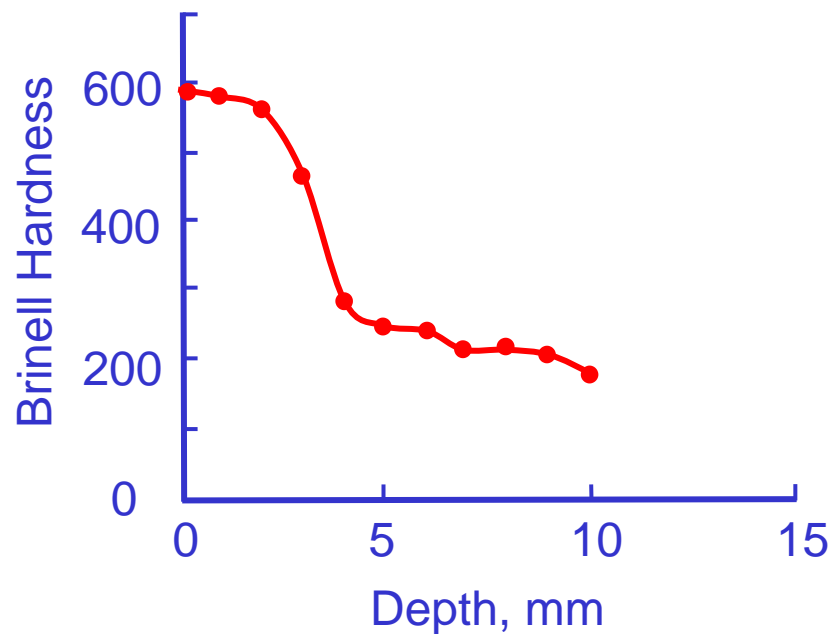
# Shot Peening Results



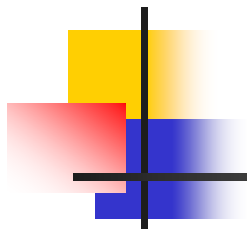
[www.metalimprovement.com](http://www.metalimprovement.com)



# Heat Treating



50 mm diameter induction hardened 1045 steel shaft



# Outline

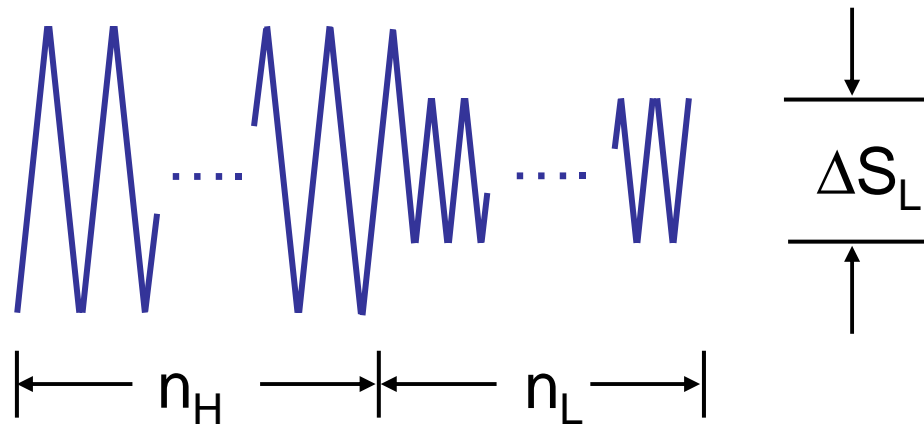
---

1. Mean Stress
2. Nonlinear Damage Models
3. Crack Growth Interactions
4. Crack Tip Plasticity Models
5. Crack Closure Models

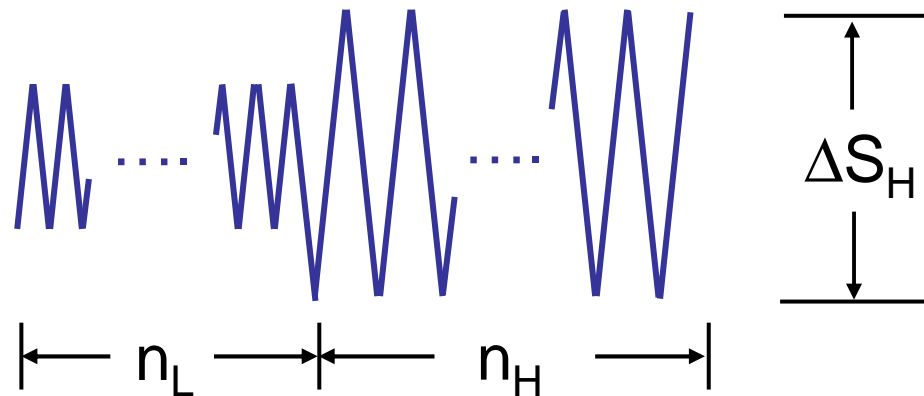


# Cumulative Damage

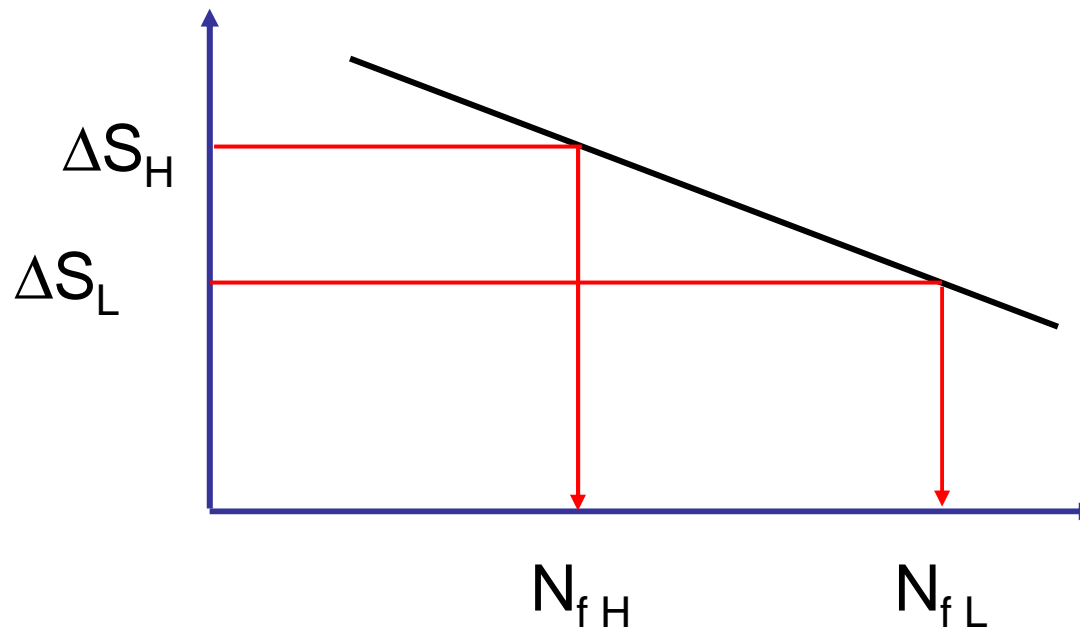
High - Low



Low - High



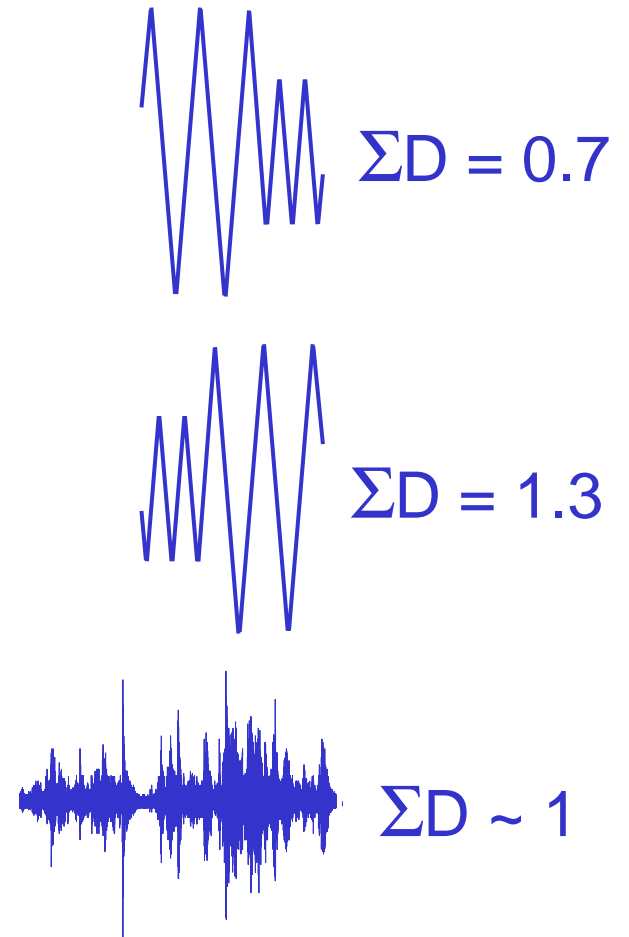
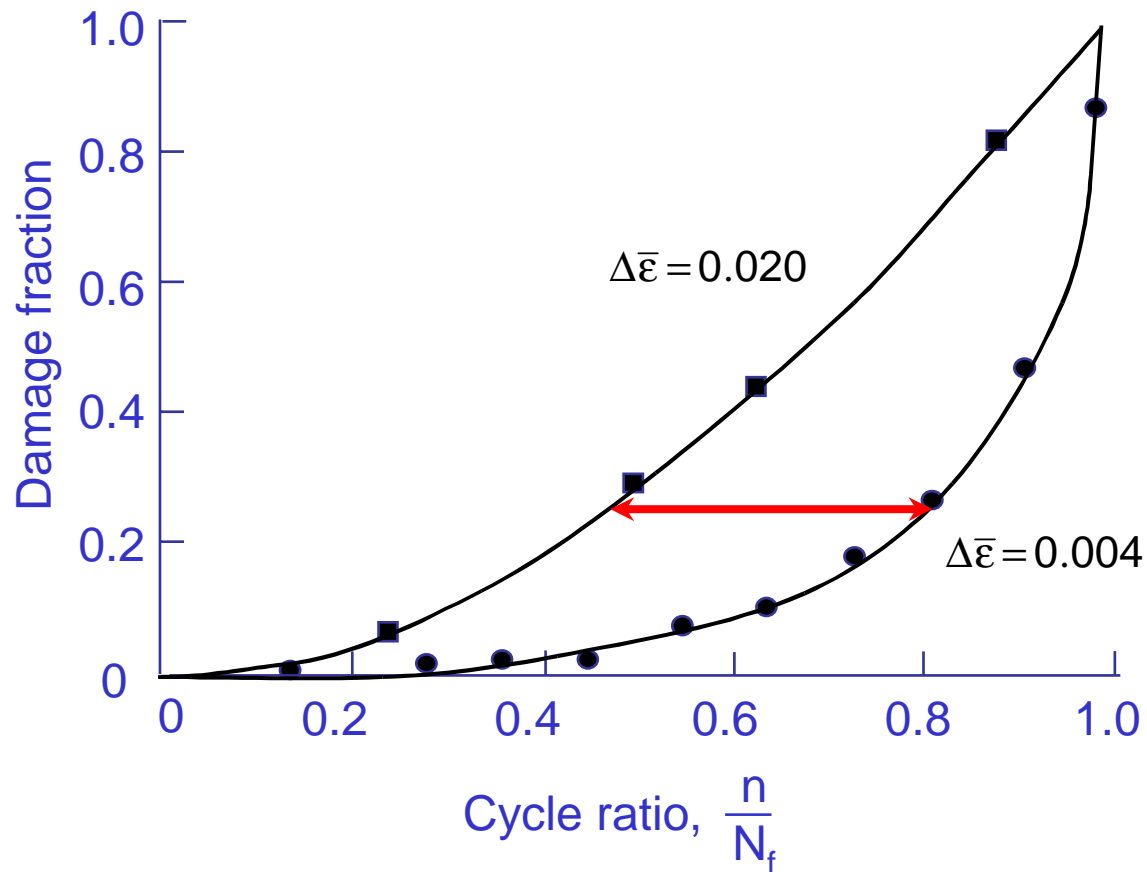
# Linear Damage



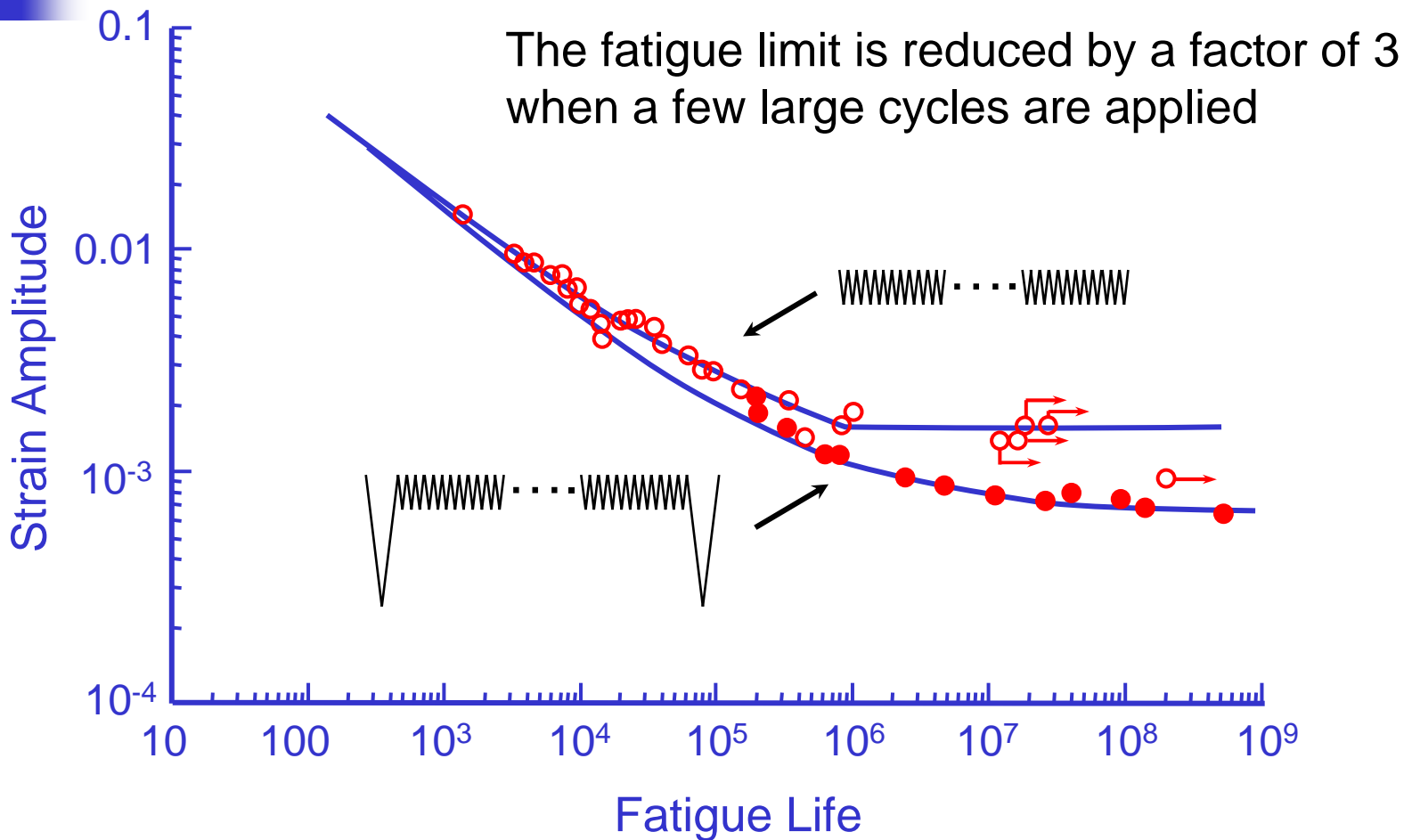
Miners Rule:

$$\text{Damage} = \sum \frac{n}{N_F} = \frac{n_H}{N_{fH}} + \frac{n_L}{N_{fL}}$$

# Nonlinear Damage

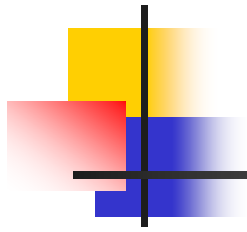


# Periodic Overload Results



Bonnen and Topper, "The Effects of Periodic Overloads on Biaxial Fatigue of Normalized SAE 1045 Steel"  
ASTM STP 1387, 2000, 213-231





## Relative Miner's Rule

---

$$\sum D = \sum \frac{n}{N_f} = 0.3$$

Set Miner's damage sum to a number less than 1

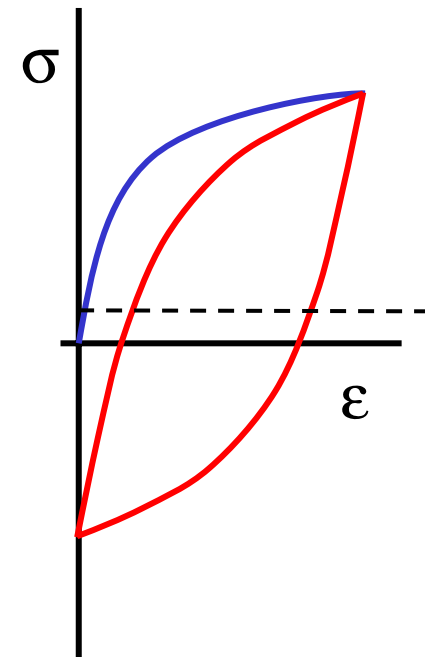
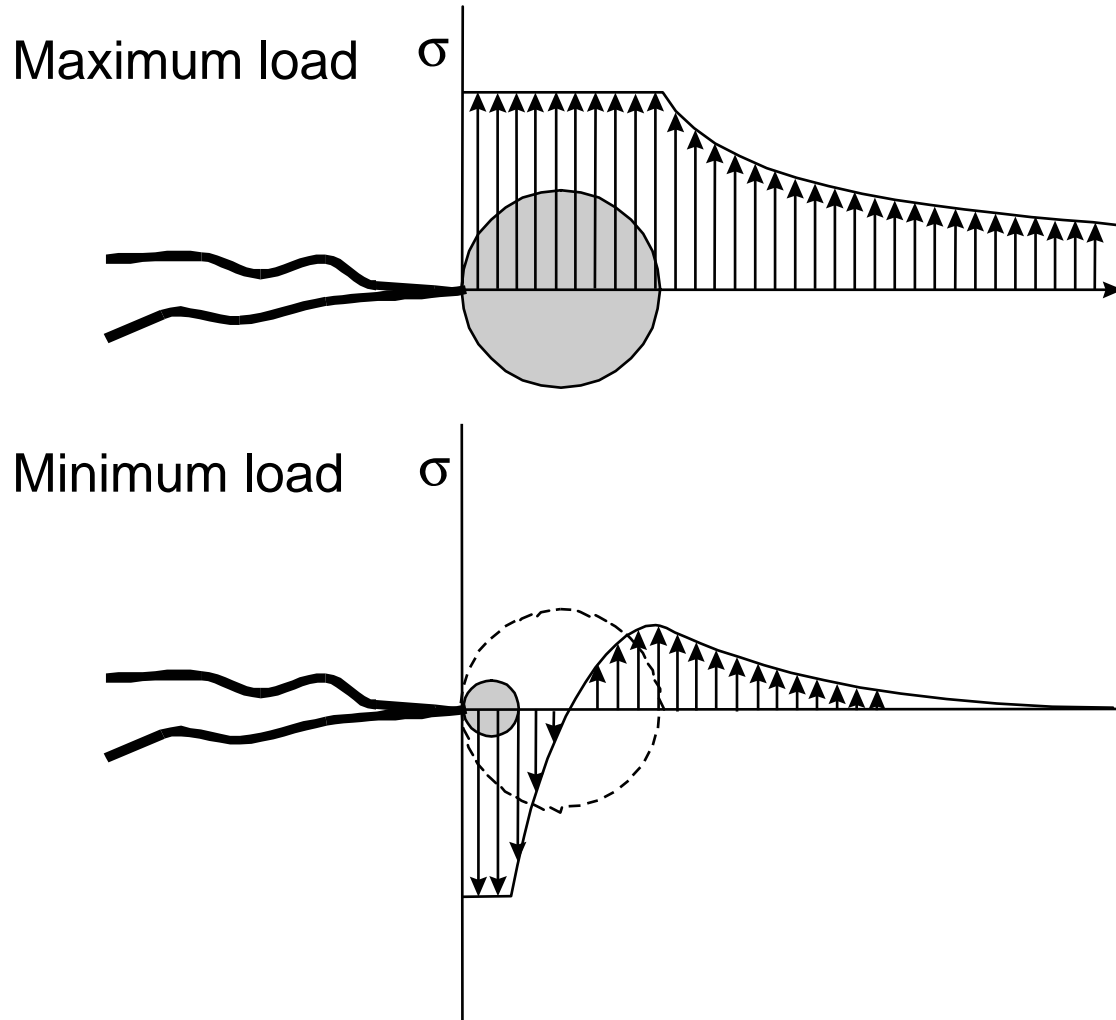


# Outline

---

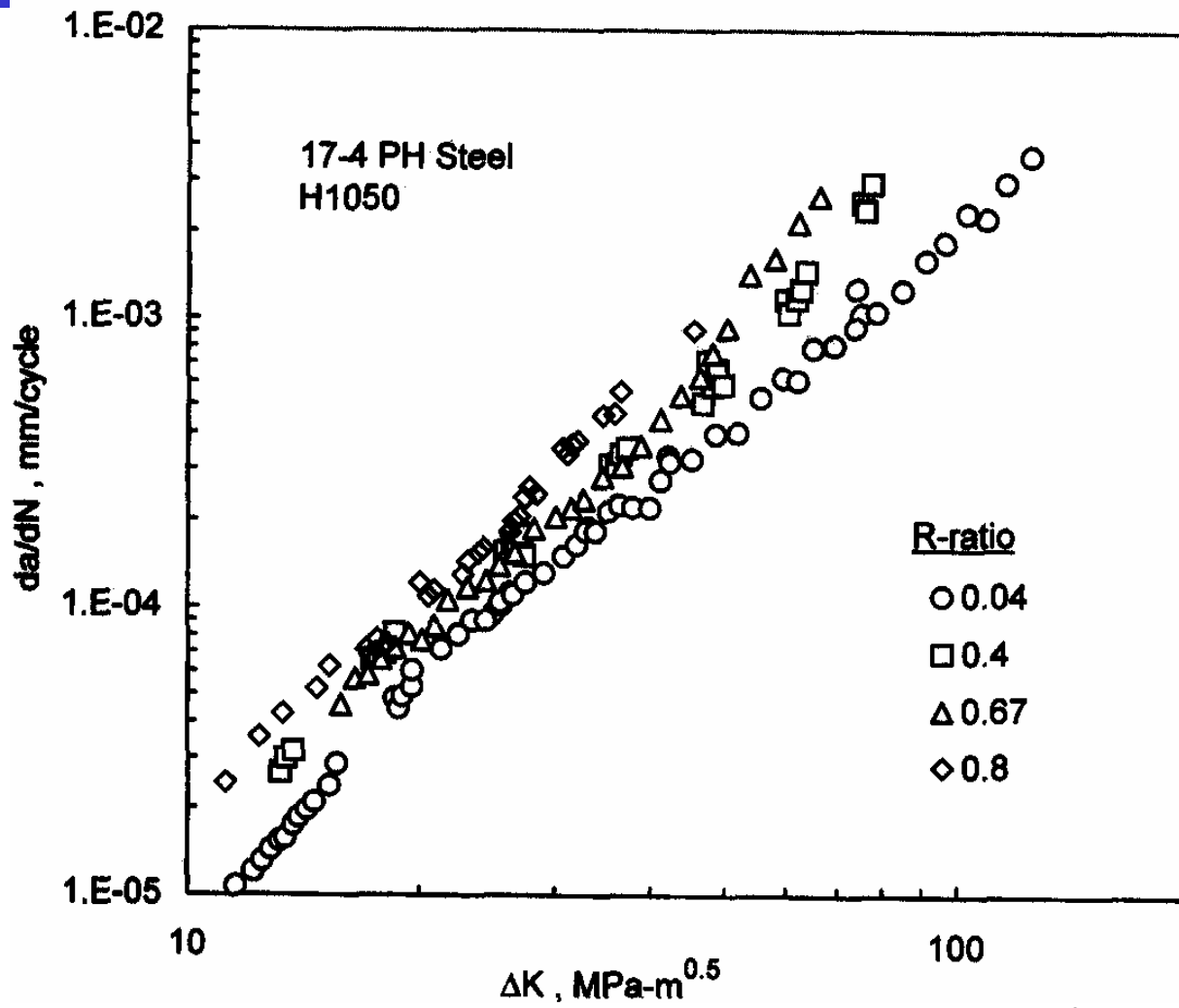
1. Mean Stress
2. Nonlinear Damage Models
3. Crack Growth Interactions
4. Crack Tip Plasticity Models
5. Crack Closure Models

# Crack Growth Physics



Mean stresses in plastic zone are small

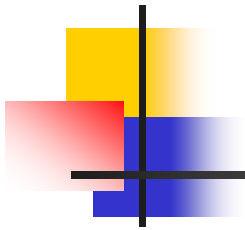
# Mean Stress Effects



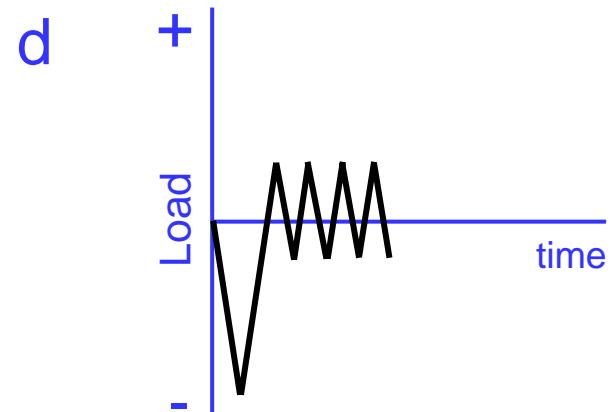
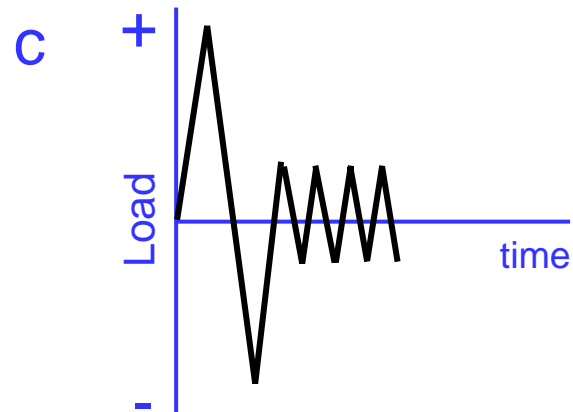
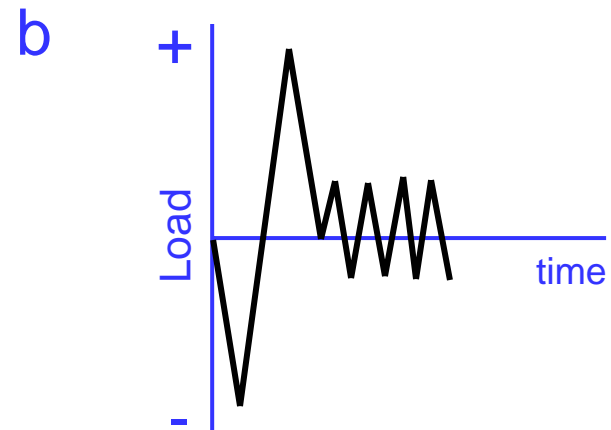
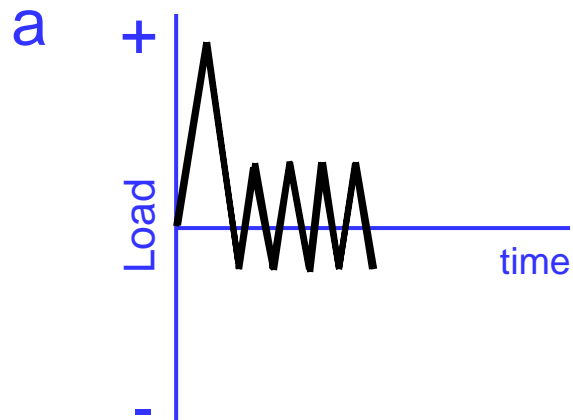
$$\frac{da}{dN} = \frac{C \Delta K^m}{(1-R)^\gamma}$$

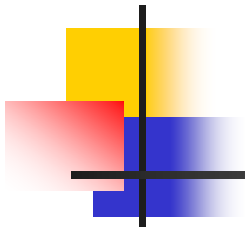
$$0 < \gamma < 0.5$$

From Dowling, Mechanical Behavior of Materials, 1999

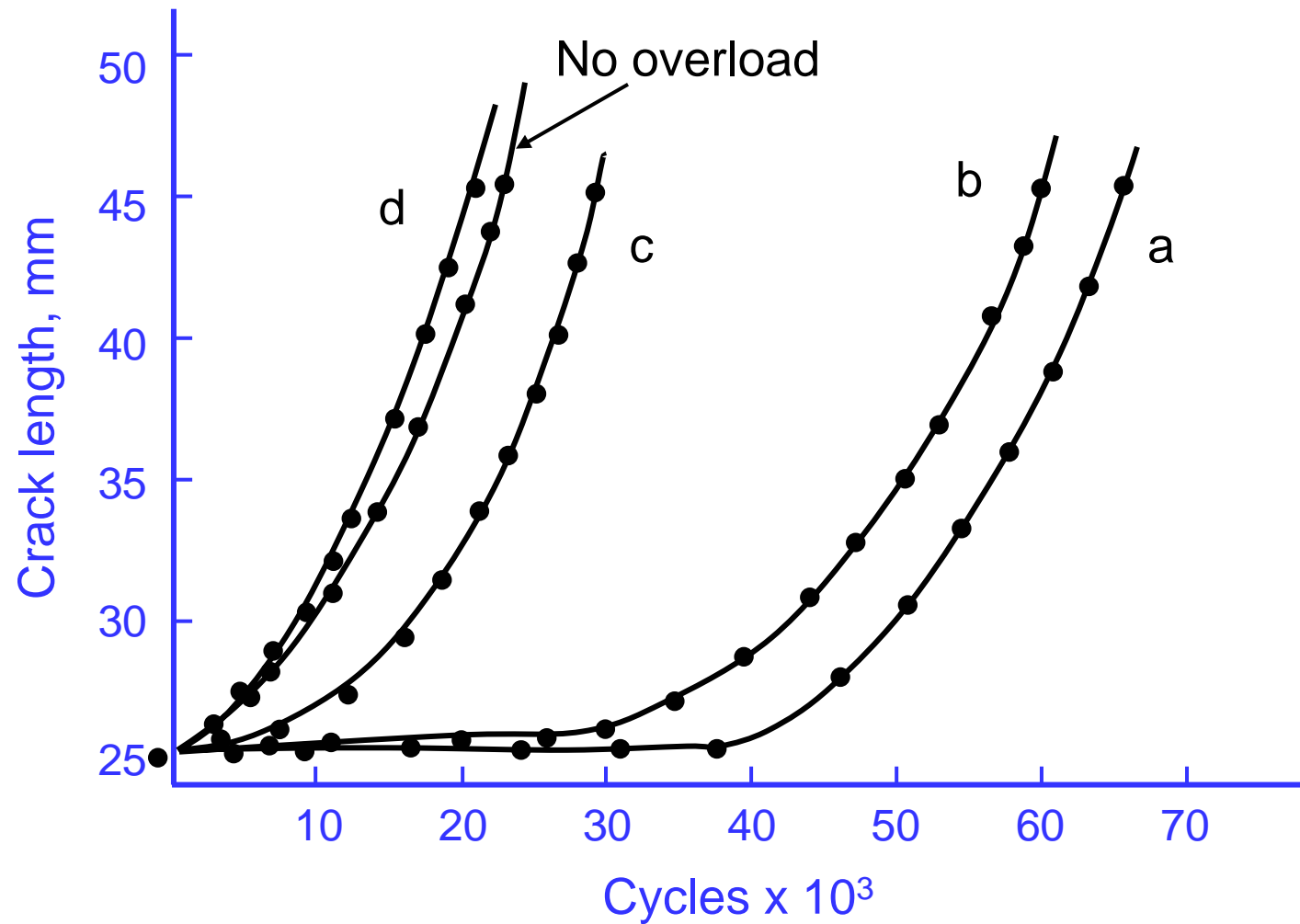


# Sequences

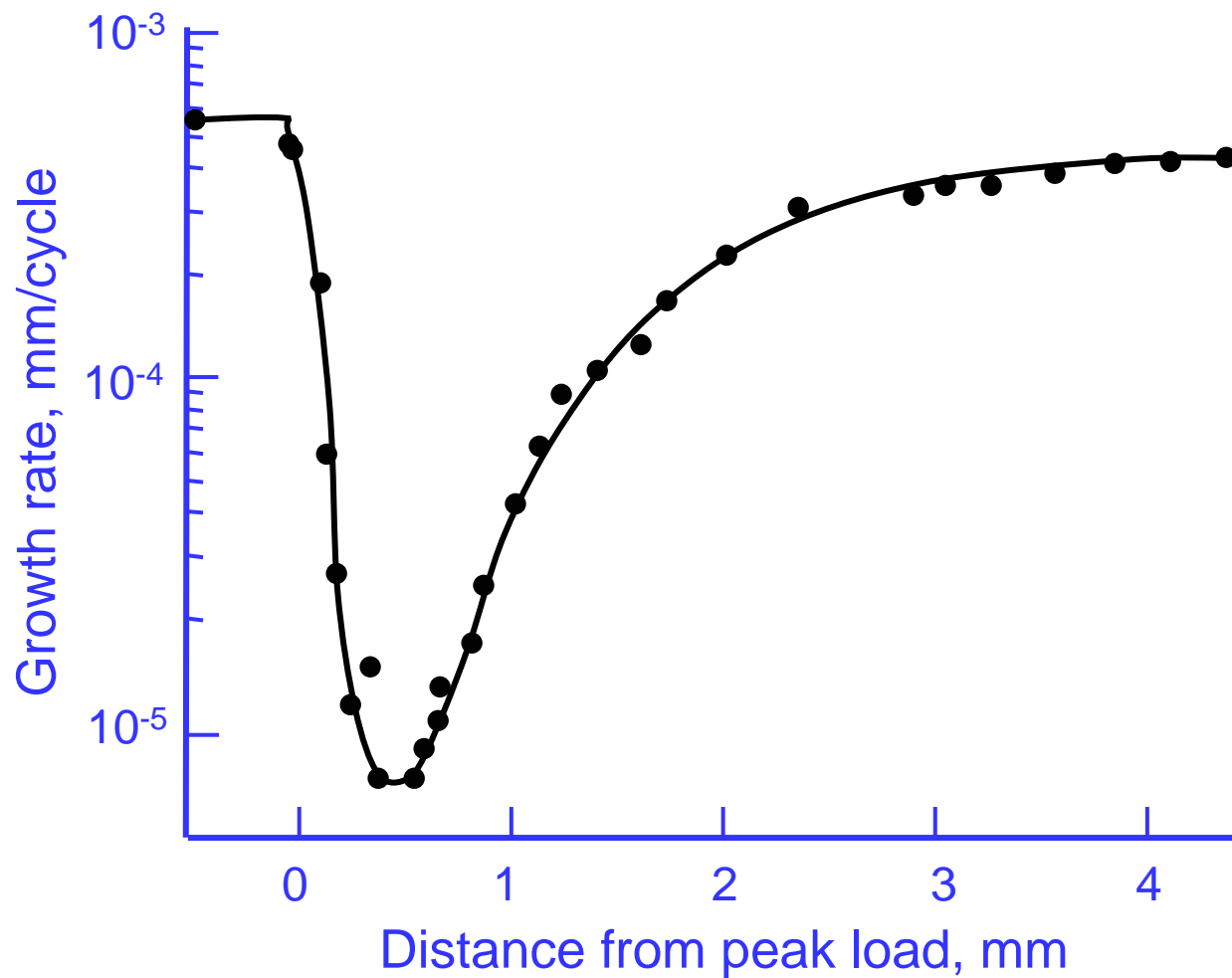




# Crack Growth



# Crack Growth Rate





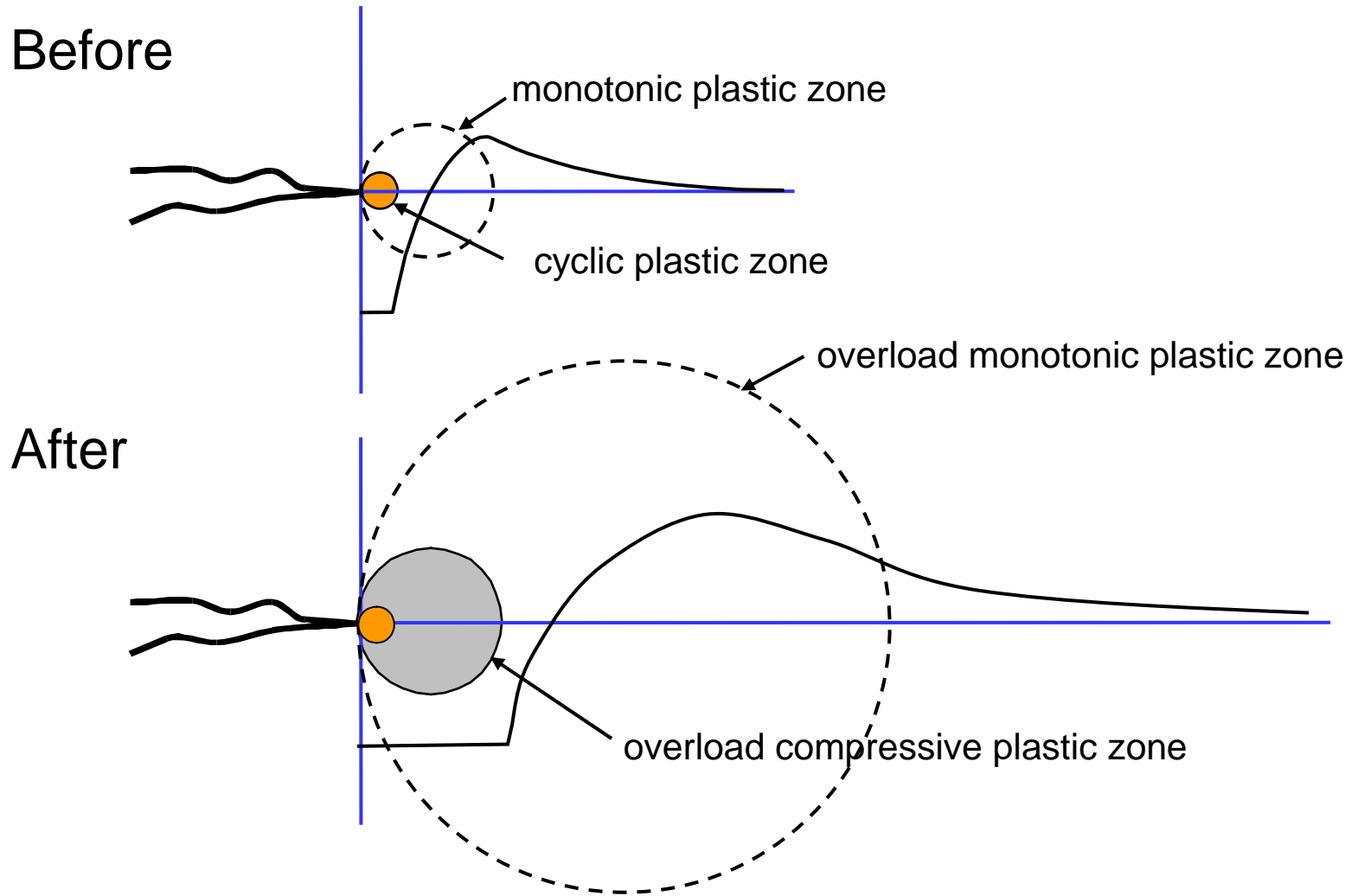
# Outline

---

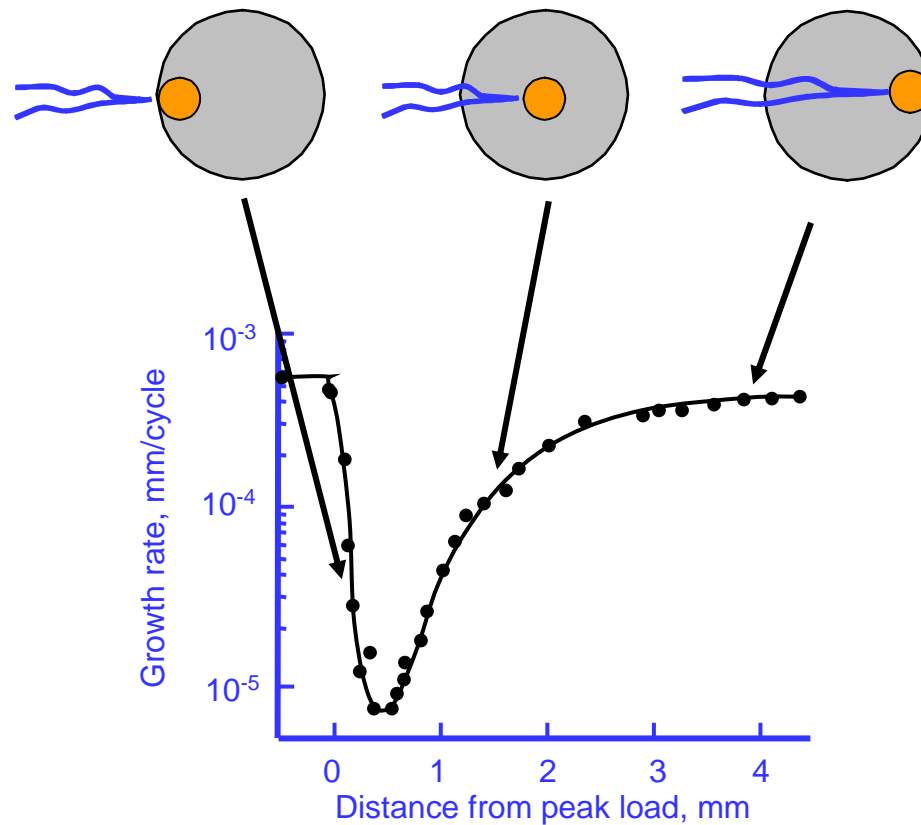
1. Mean Stress
2. Nonlinear Damage Models
3. Crack Growth Interactions
4. Crack Tip Plasticity Models
5. Crack Closure Models



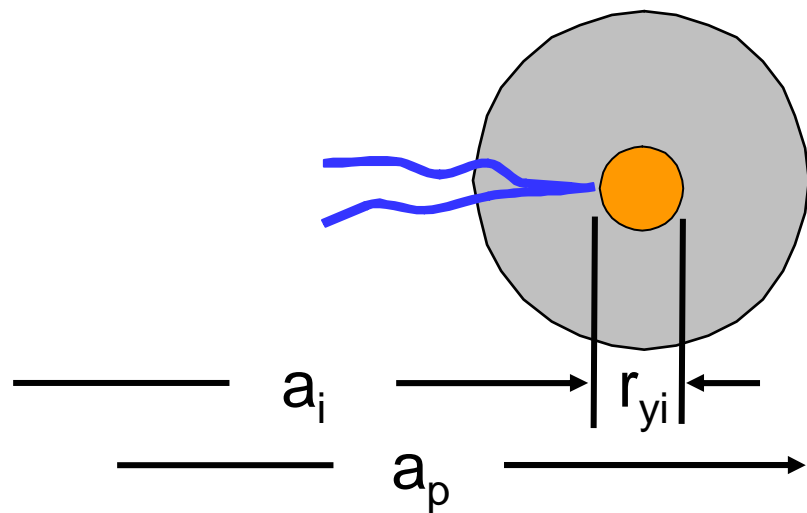
# Stress Fields After Overload



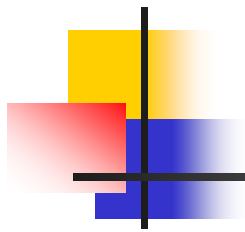
# Crack Growth Retardation



# Wheeler Model



$$\frac{da}{dN} = \left( \frac{r_{yi}}{a_p - a_i} \right)^\gamma \left( \frac{da}{dN} \right)_{CA}$$



# Outline

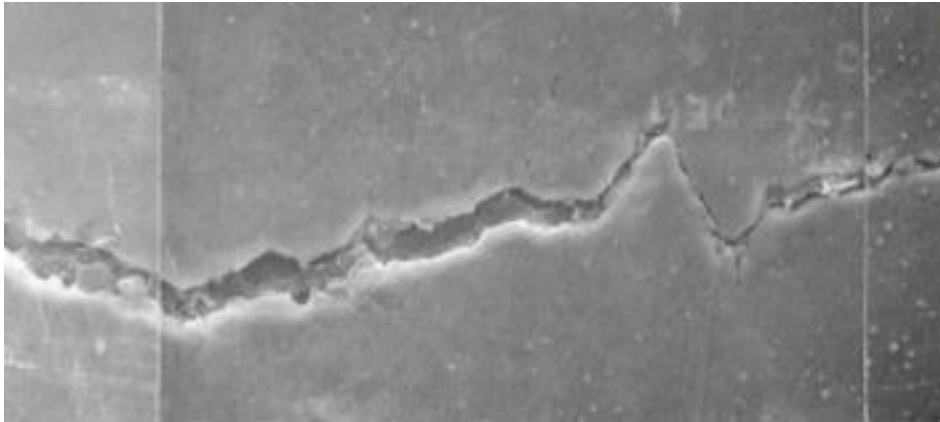
---

1. Mean Stress
2. Nonlinear Damage Models
3. Crack Growth Interactions
4. Crack Tip Plasticity Models
5. Crack Closure Models

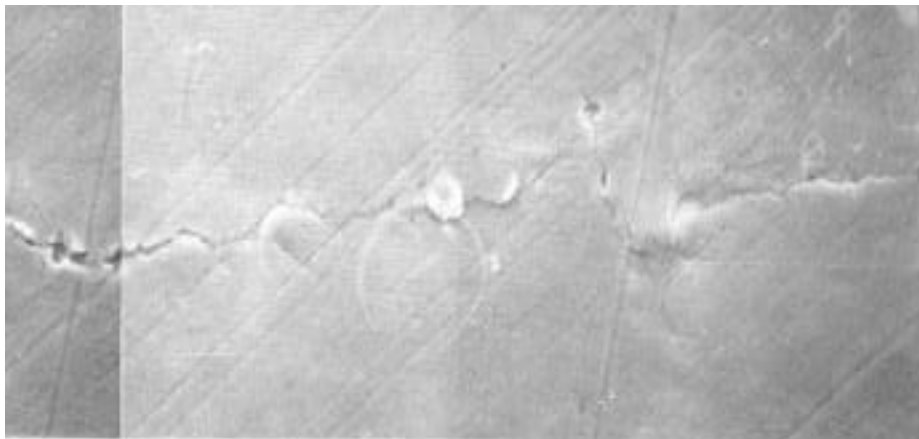


# Compression

---



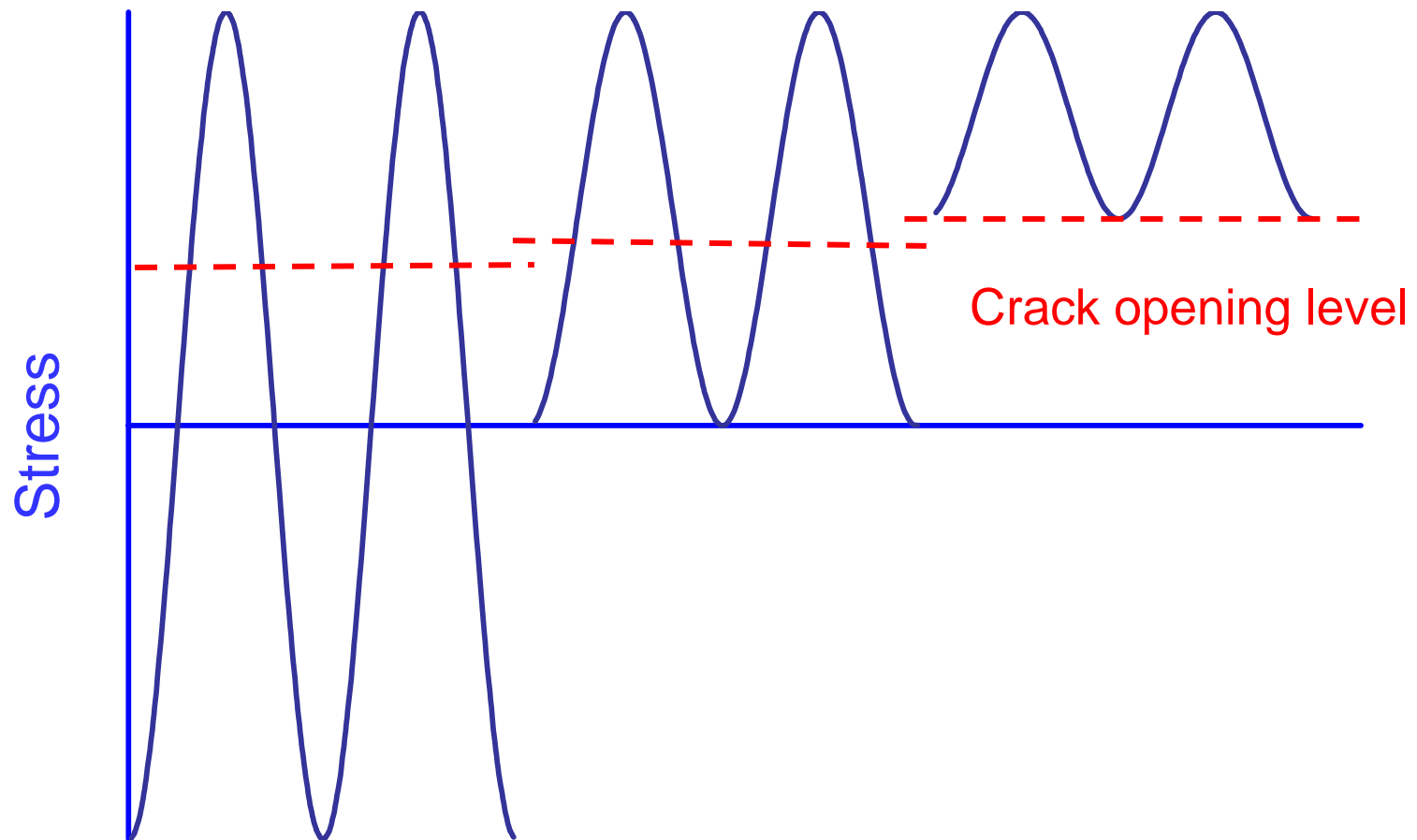
Crack open



Crack closed



# Compressive Stresses

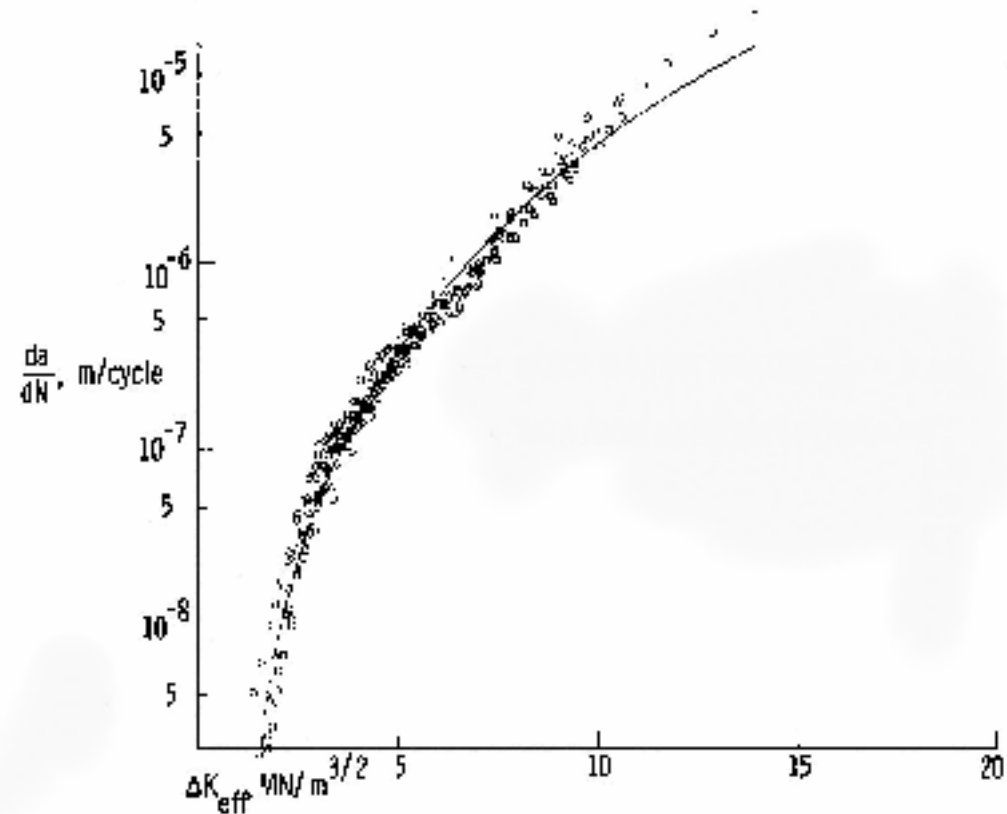


Compressive stresses are not very damaging in crack growth

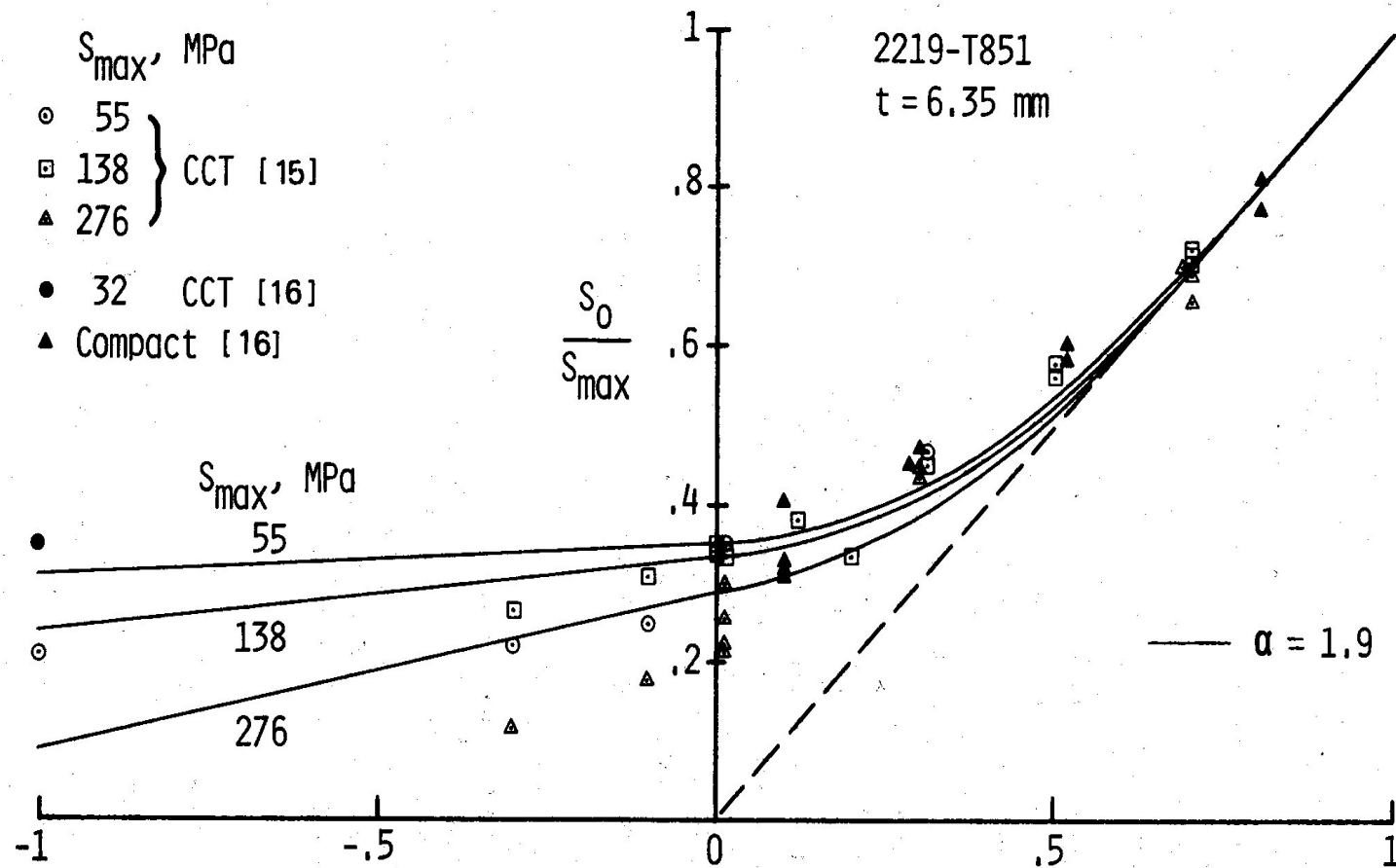
# Crack Closure - Elber

$$\Delta K_{\text{eff}} = (0.5 + 0.4R) \Delta K$$

$$\frac{da}{dN} = C(\Delta K_{\text{eff}})^m$$



# Maximum Stress Dependence







# Newman Model

$$\Delta K_{\text{eff}} = \left[ \frac{\left( 1 - \frac{S_o}{S_{\max}} \right)}{1 - R} \right] \Delta K$$

Newman

$$\frac{S_o}{S_{\max}} = A_0 + A_1 R + A_2 R^2 + A_3 R^3$$

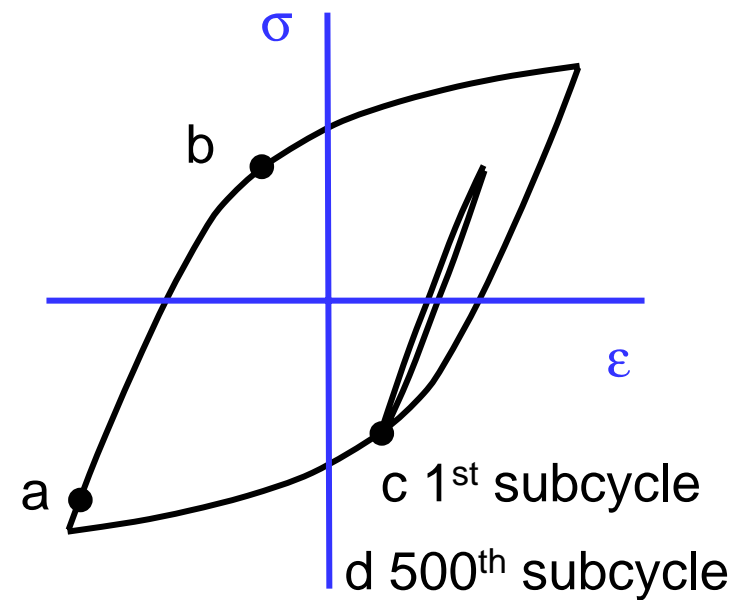
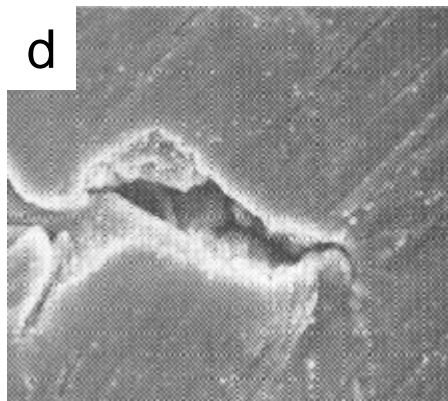
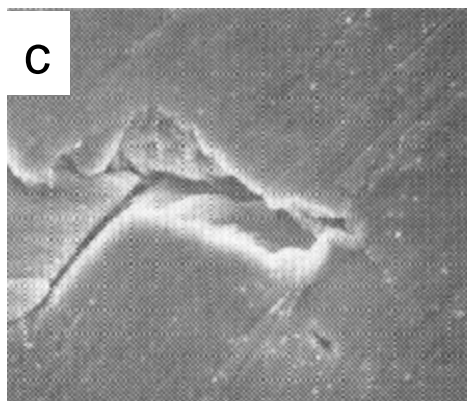
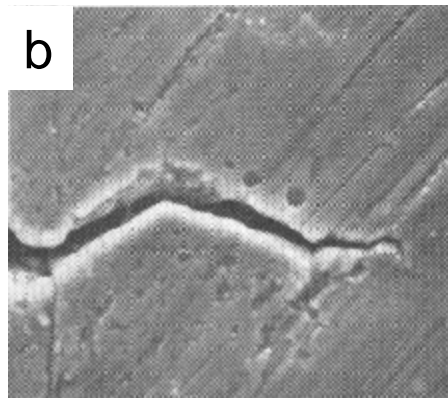
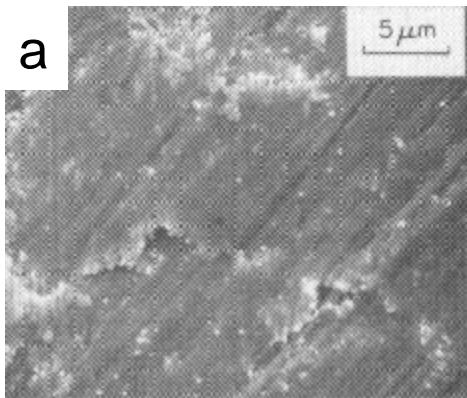
$$A_0 = (0.825 - 0.34 \alpha + 0.05 \alpha^2) \left[ \cos \left( \frac{\pi S_{\max}}{2 \sigma_0} \right) \right]^{\frac{1}{\alpha}}$$

$$A_1 = (0.415 - 0.071 \alpha) \frac{S_{\max}}{\sigma_0}$$

$$A_2 = 1 - A_0 - A_1 - A_3$$

$$A_3 = 2A_0 + A_1 - 1$$

# Closure Observations

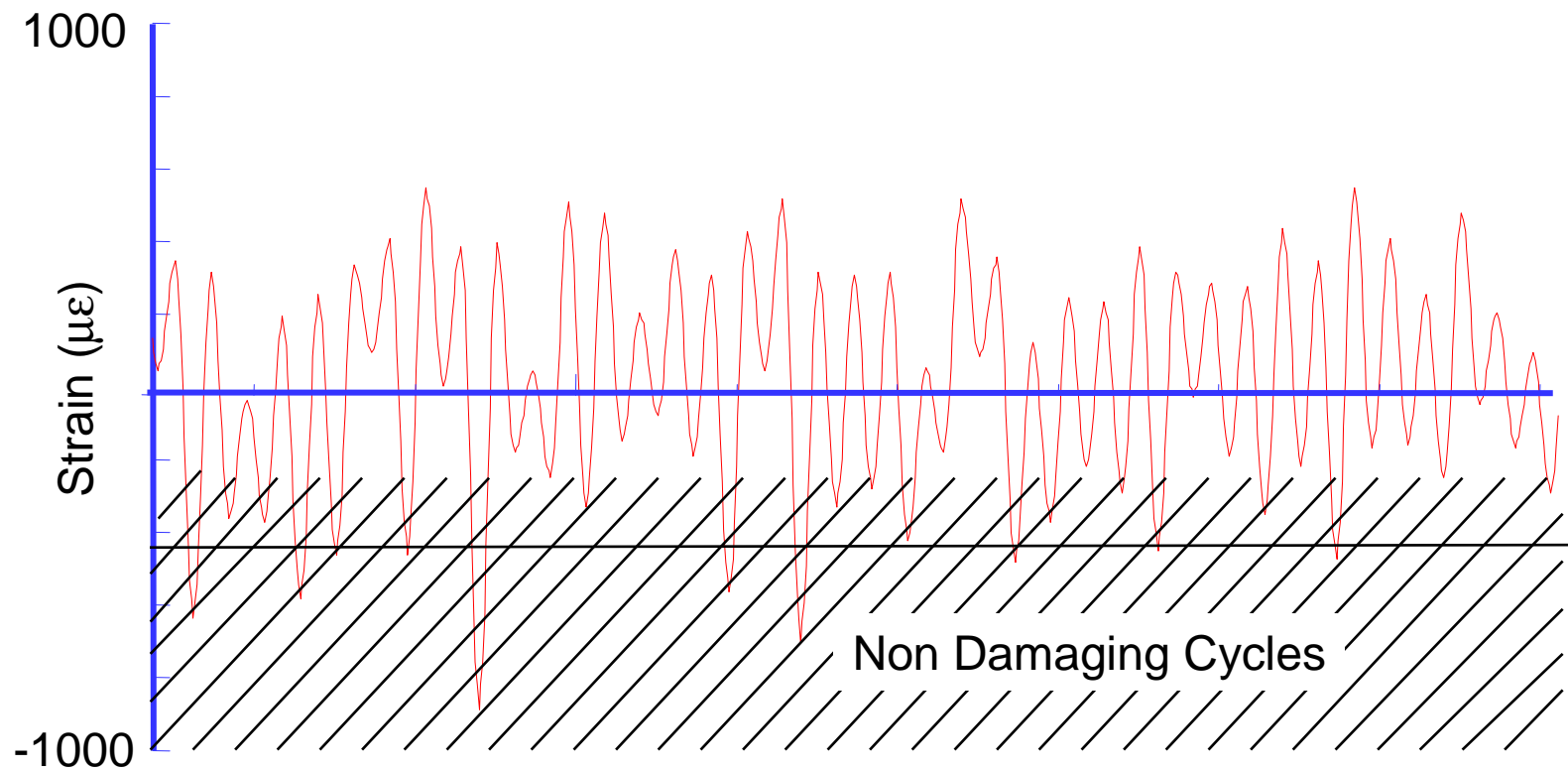


1026 steel

$$\Delta\epsilon_1/2 = 0.005$$

$$\Delta\epsilon_2/2 = 0.001$$

# Spectrum Loading



# Sequence Effects in Fatigue

