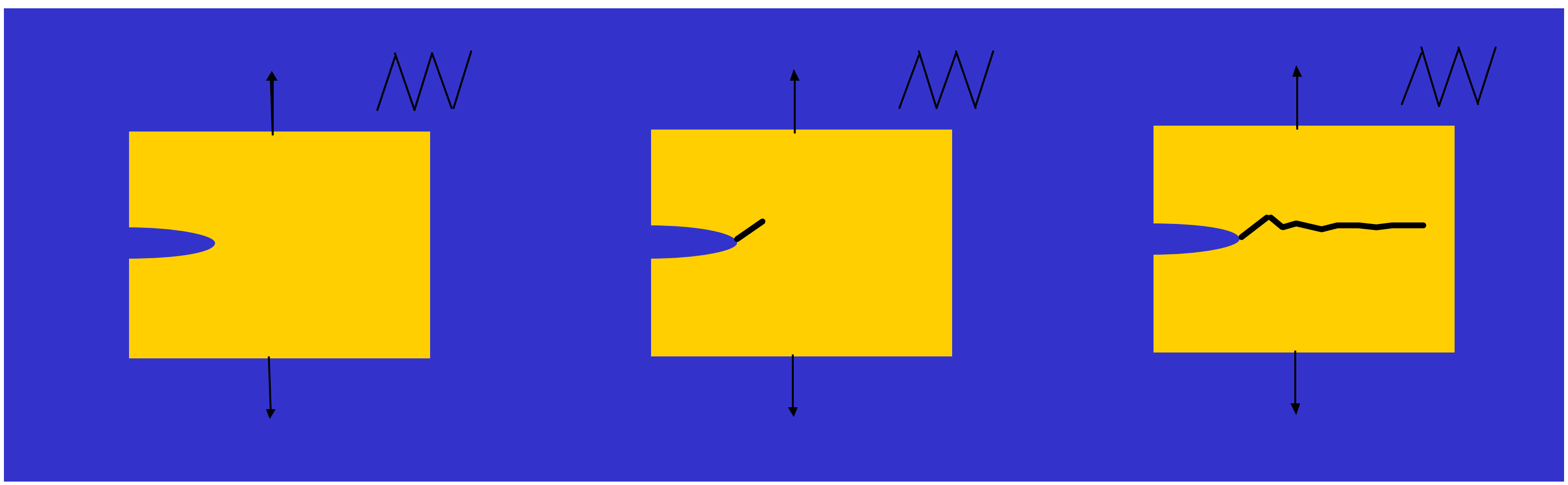
# III Fatigue Models



1. Will a crack nucleate?
2. Will it grow?
3. How fast will it grow?

# Outline

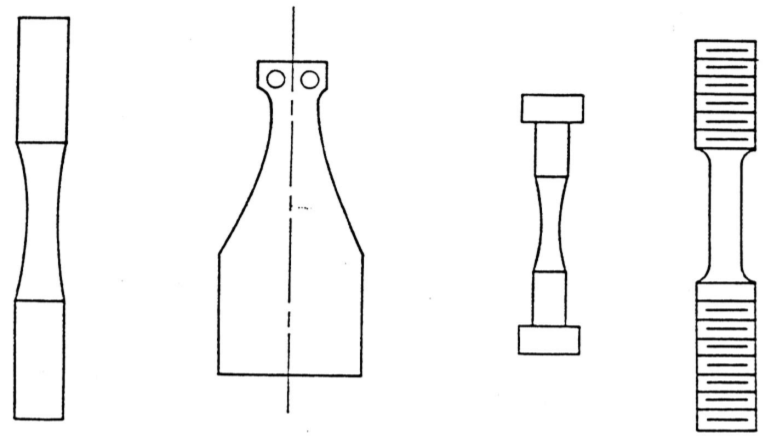
- Sources of knowledge
- Modeling
  - Crack nucleation
  - Non propagating cracks
  - Crack growth

# Source of knowledge

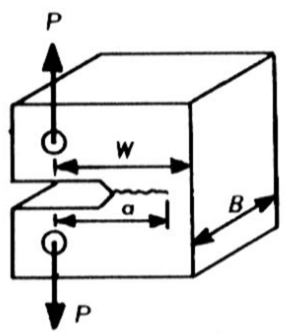
The intrinsic fatigue properties of the component's material are determined using small, smooth or cracked specimens.

# Laboratory fatigue test specimens

Smooth specimens



Crack growth specimen




Compact tension specimen (CT):

$$K_I = \frac{P}{B\,W^{\frac{1}{2}}} \cdot f\left(\frac{a}{W}\right)$$

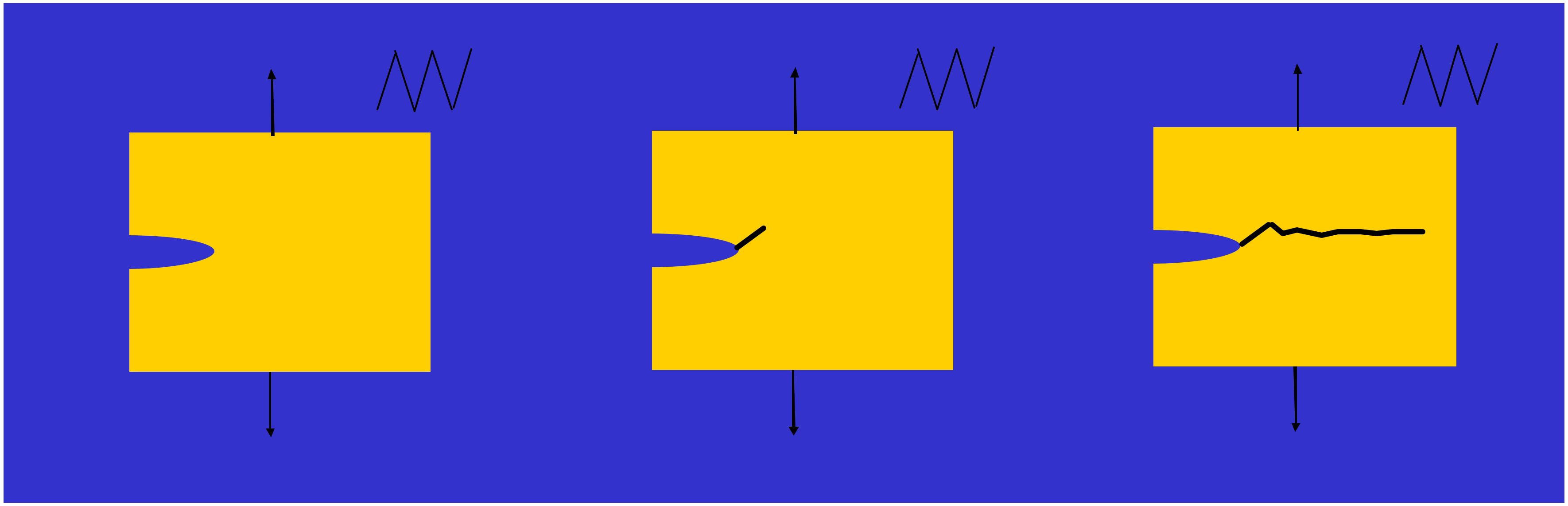
$$f\left(\frac{a}{W}\right) = \frac{\left(2+\frac{a}{W}\right)\left[0.886+4.64\left(\frac{a}{W}\right)-13.32\left(\frac{a}{W}\right)^2+14.72\left(\frac{a}{W}\right)^3-5.6\left(\frac{a}{W}\right)^4\right]}{\left(1-\frac{a}{W}\right)^{\frac{3}{2}}}$$

This information can be used to predict, to compute the answer to…….

# Outline

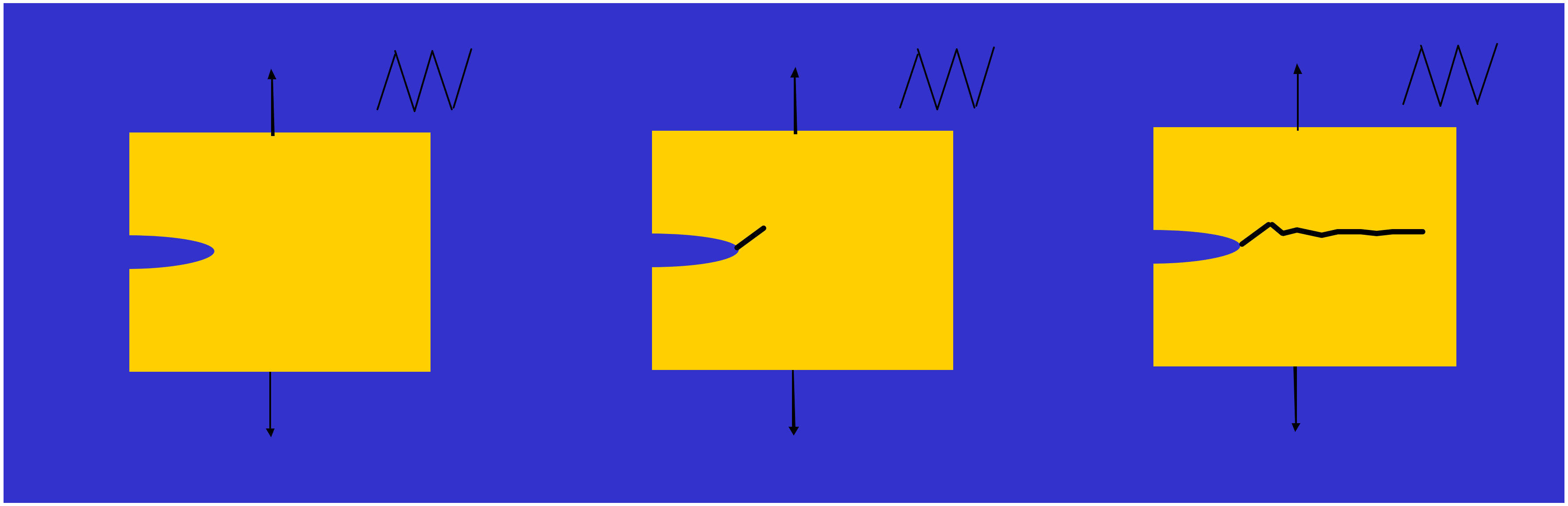
- Sources of knowledge
- Modeling
  - Crack nucleation
  - Non propagating cracks
  - Crack growth

# The three BIG fatigue questions



1. Will a crack nucleate? 2. Will it grow? 3. How fast will it grow?

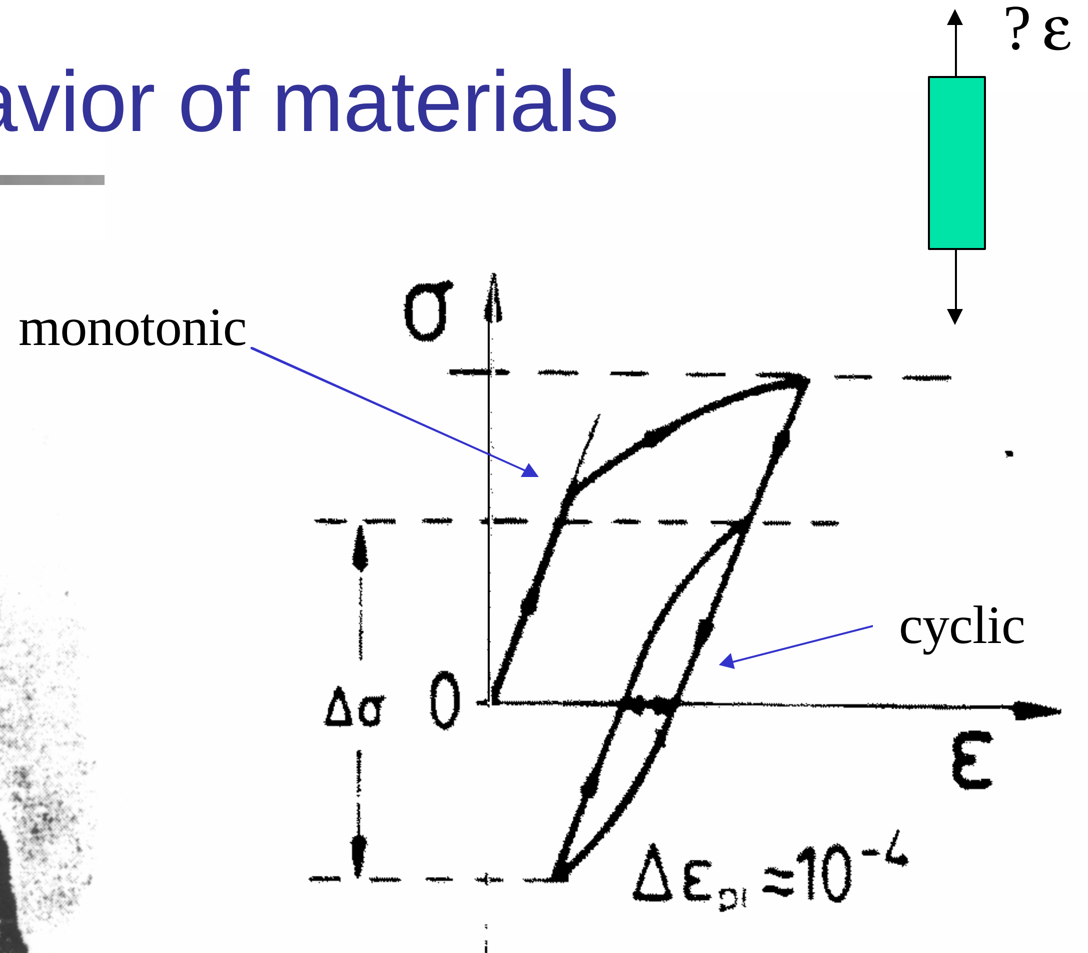
# Crack nucleation



1. **Will a crack nucleate?** 2. Will it grow? 3. How fast will it grow?

# Cyclic behavior of materials

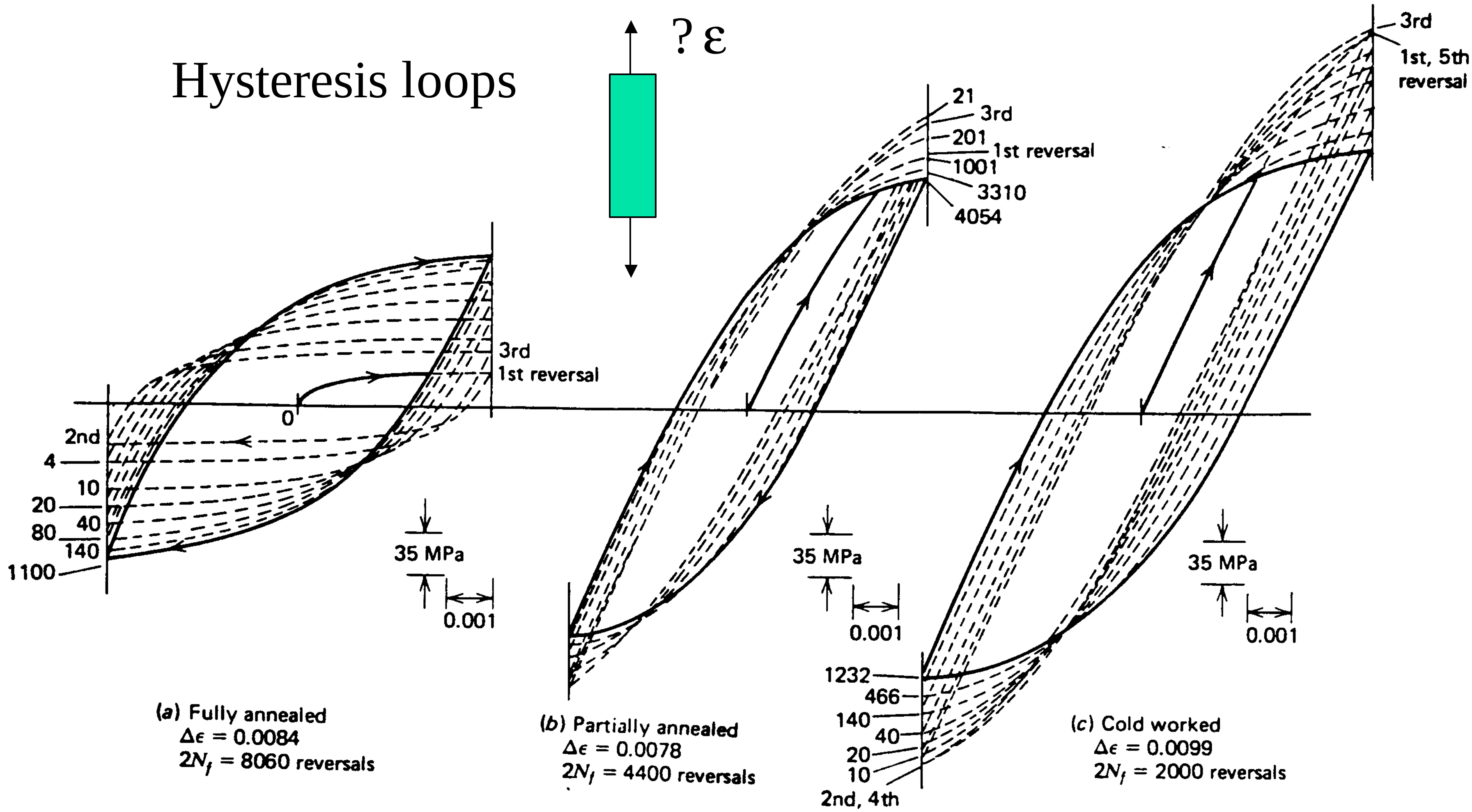
Fig. 1: Portrait of Johann Bauschinger, born on June 11, 1834, and died on November 25, 1893.



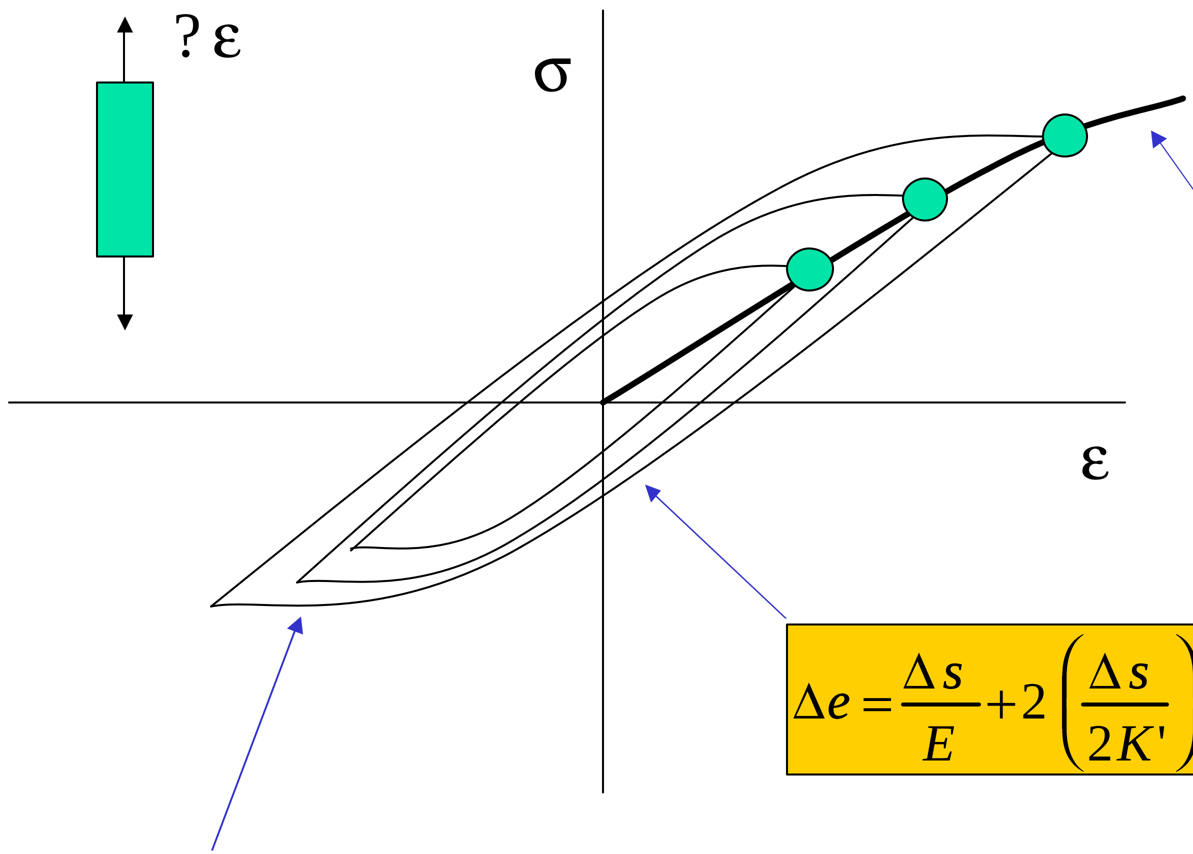

Bauschinger effect: monotonic and cyclic stress-strain different!

# Cyclic Deformation

Hysteresis loops



# MODEL: Cyclic stress-strain curve



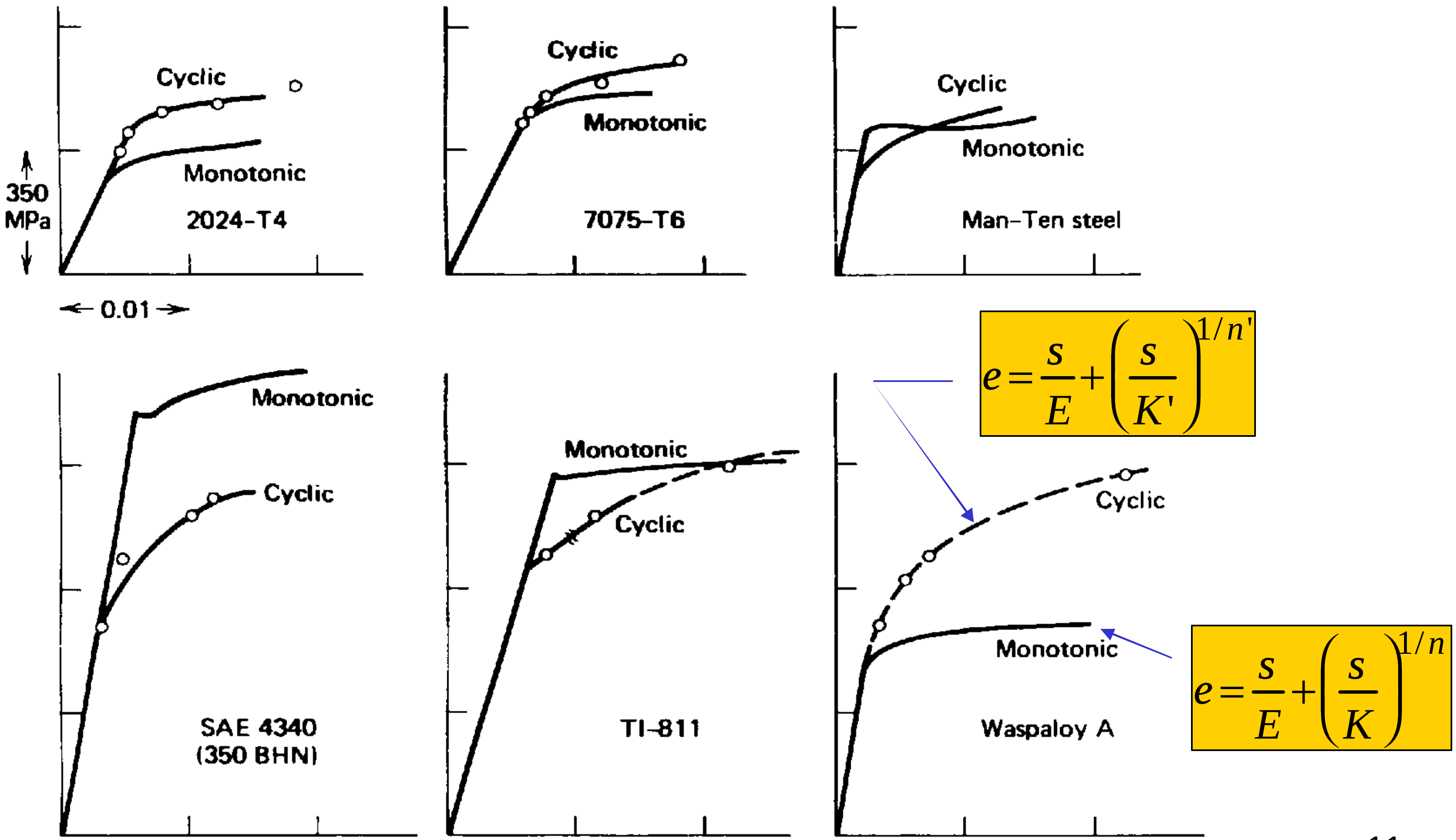

Cyclic stress-strain curve

$$e = \frac{s}{E} + \left(\frac{s}{K'}\right)^{1/n'}$$

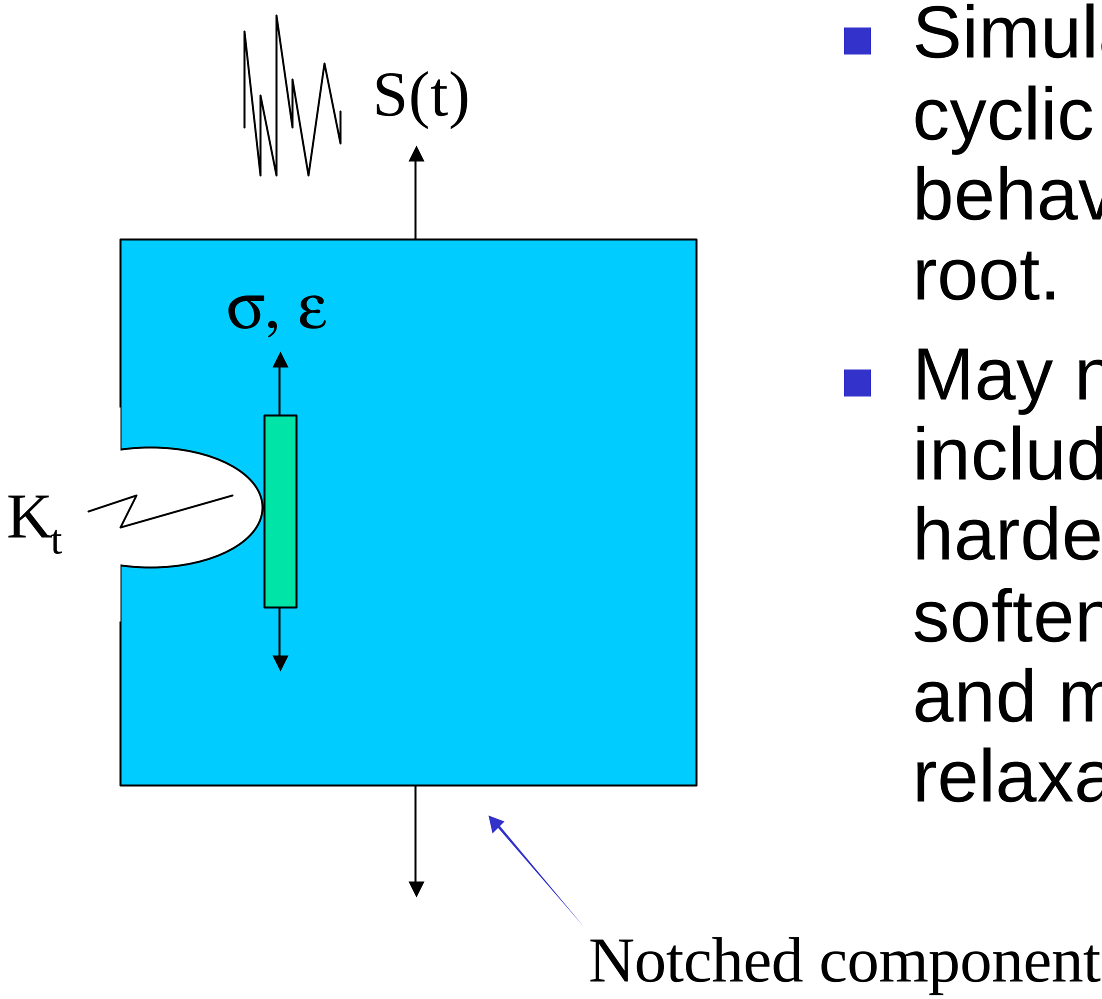
$$\Delta e = \frac{\Delta s}{E} + 2\left(\frac{\Delta s}{2K'}\right)^{1/n'}$$

Hysteresis loops for different levels of applied strain

# MODEL: Cyclic stress-strain curve

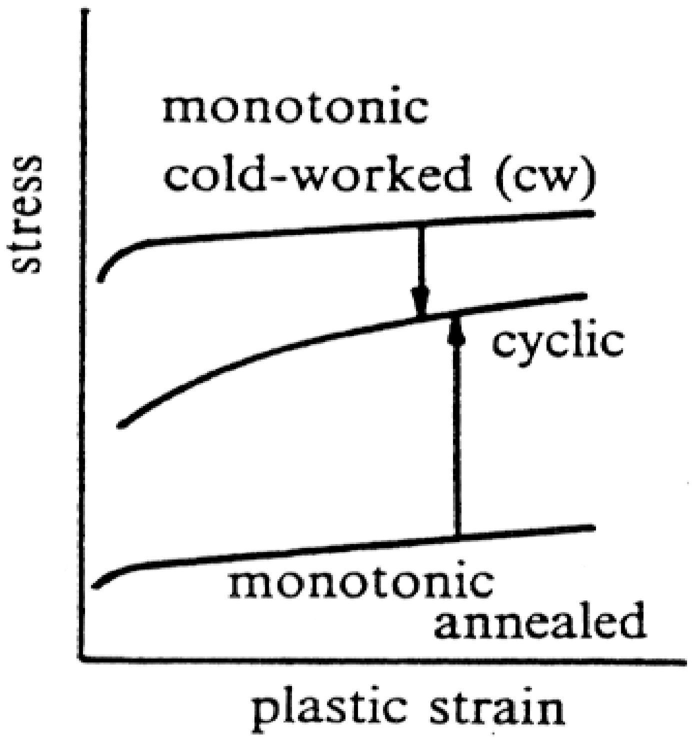


# MODEL USES

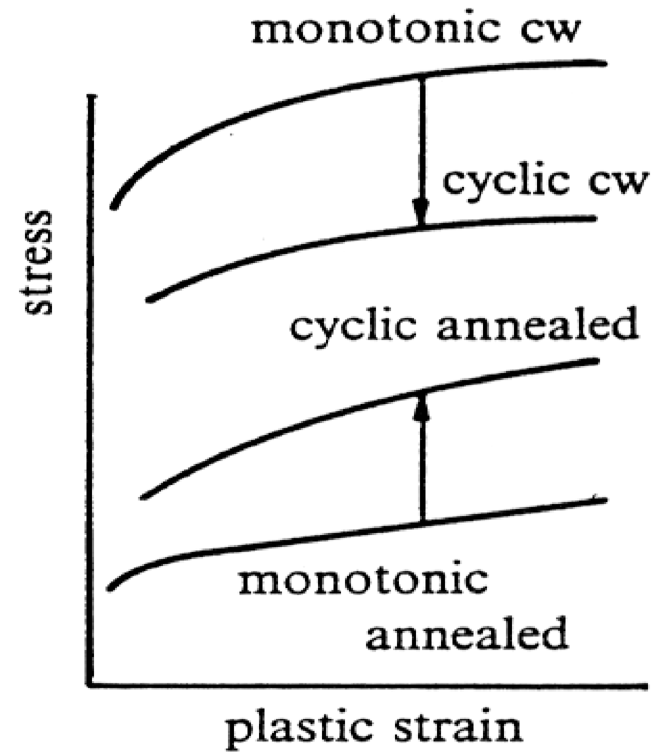



- Simulate local cyclic stress-strain behavior at a notch root.
- May need to include cyclic hardening and softening behavior and mean stress relaxation effects.

# Planar and wavy slip materials

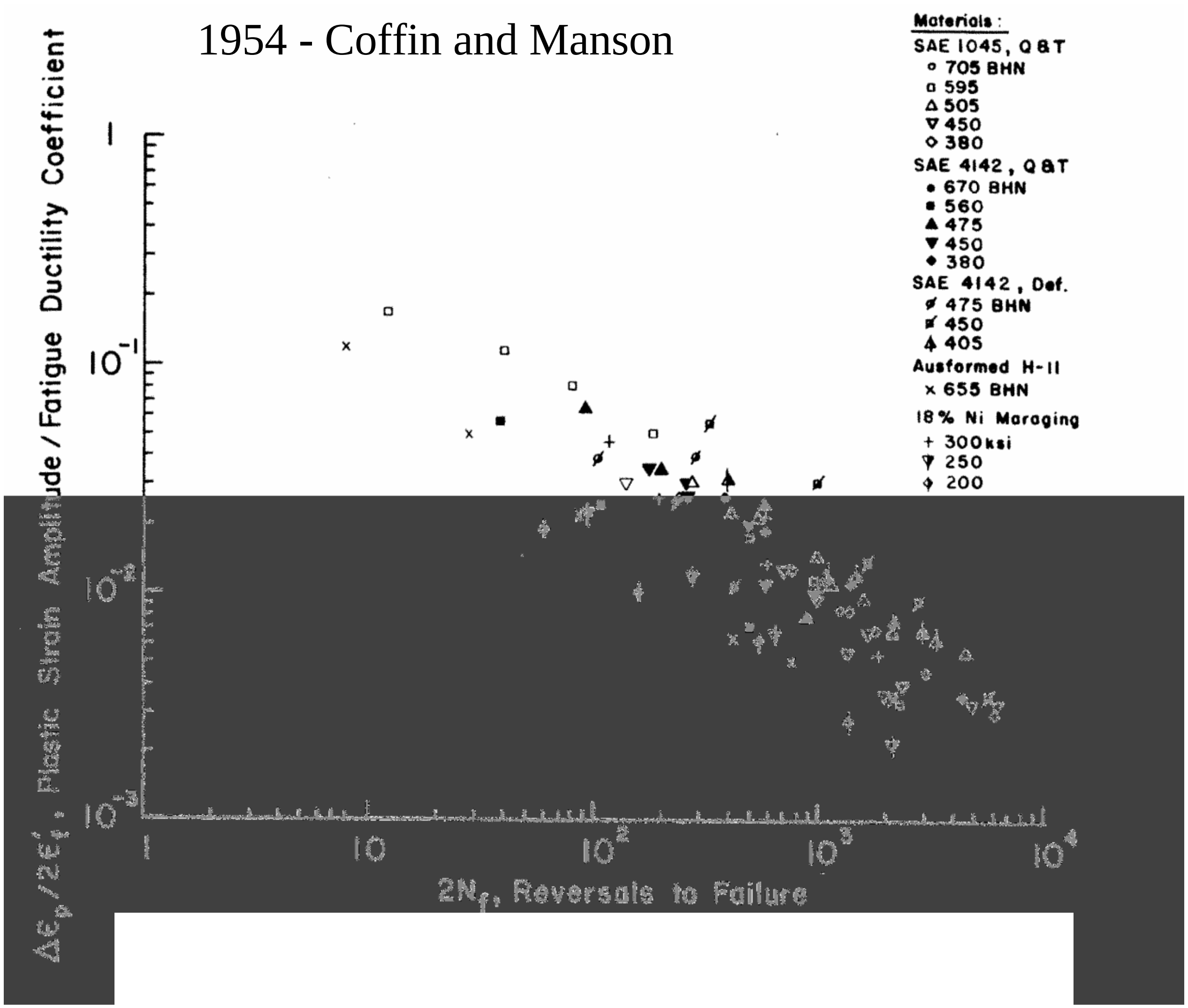



Wavy slip materials



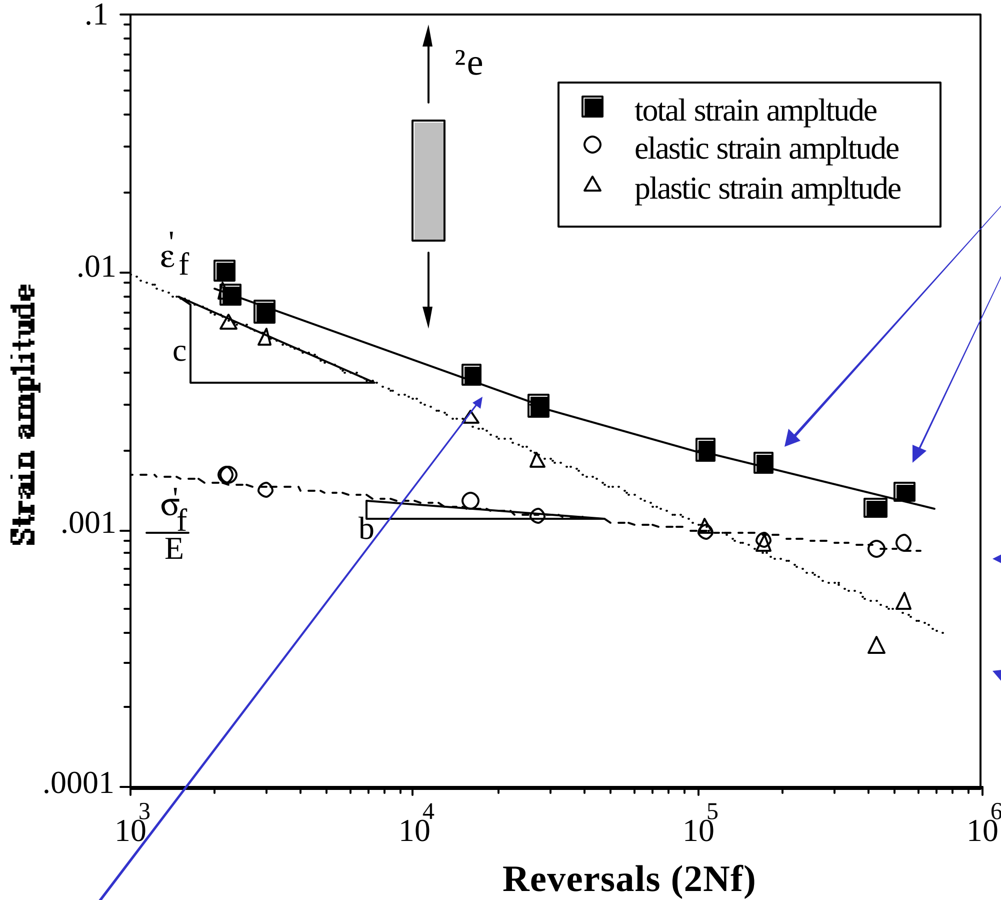

Planar slip materials

# Concept of fatigue damage




Plastic strains cause the accumulation of “fatigue damage.”

# MODEL: Strain-life curve



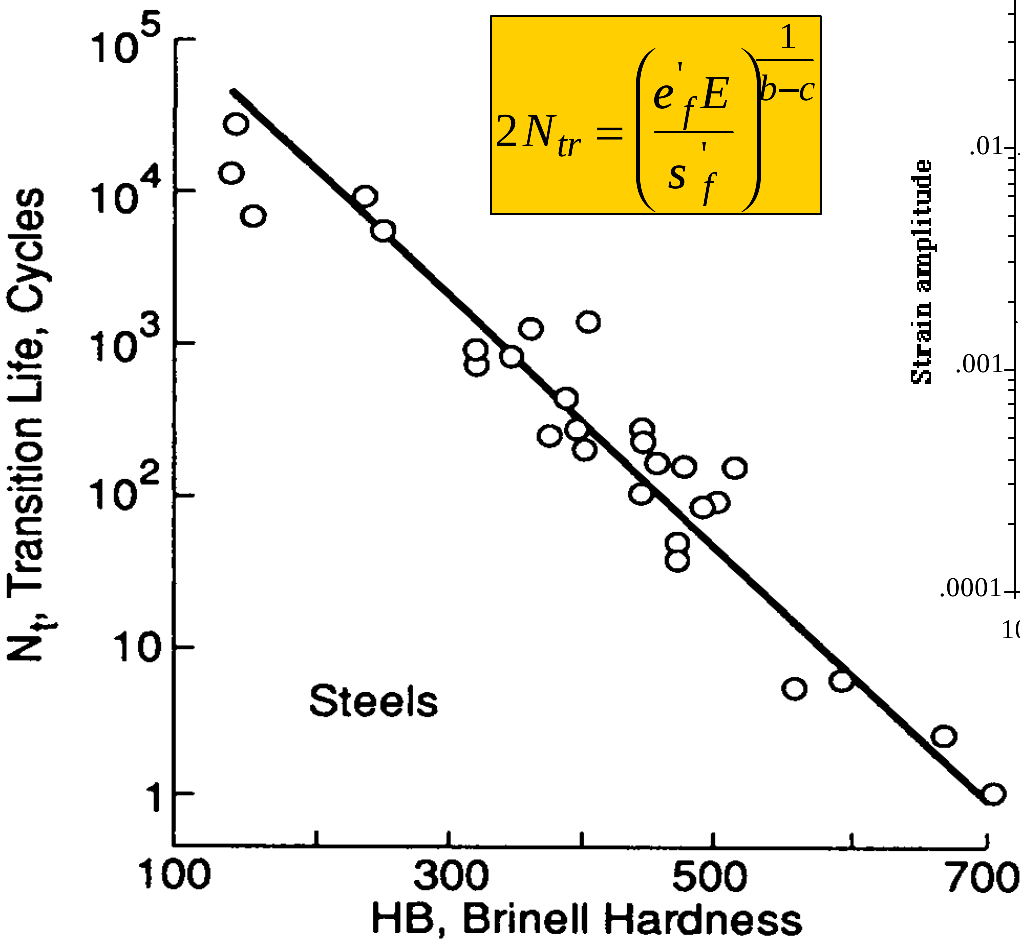

Fatigue test data from strain-controlled tests on smooth specimens.

Elastic component of strain, ? $\varepsilon_e$

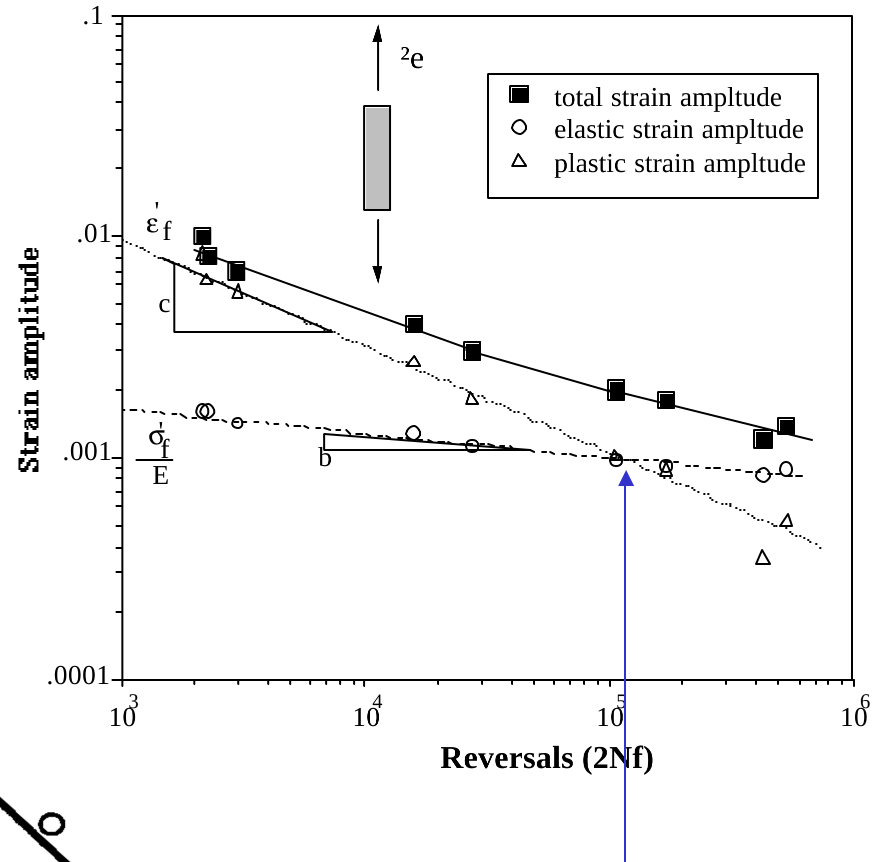
Plastic component of strain, ? $\varepsilon_p$

$$\Delta\varepsilon_t = \Delta\varepsilon_e + \Delta\varepsilon_p = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c$$

# Transition Fatigue Life

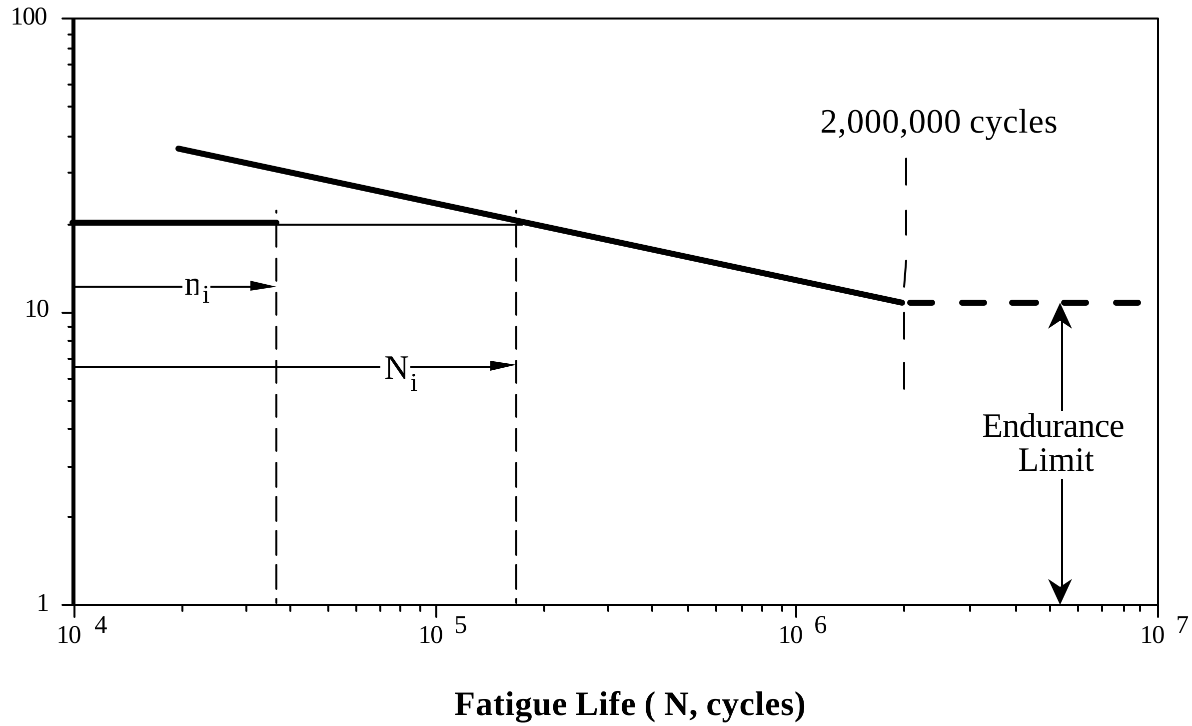



$$2N_{tr} = \left(\frac{e'_f E}{s'_f}\right)^{\frac{1}{b-c}}$$




Transition fatigue life, $N_{tr}$
Elastic strains = plastic strains

# MODEL: Linear cumulative damage



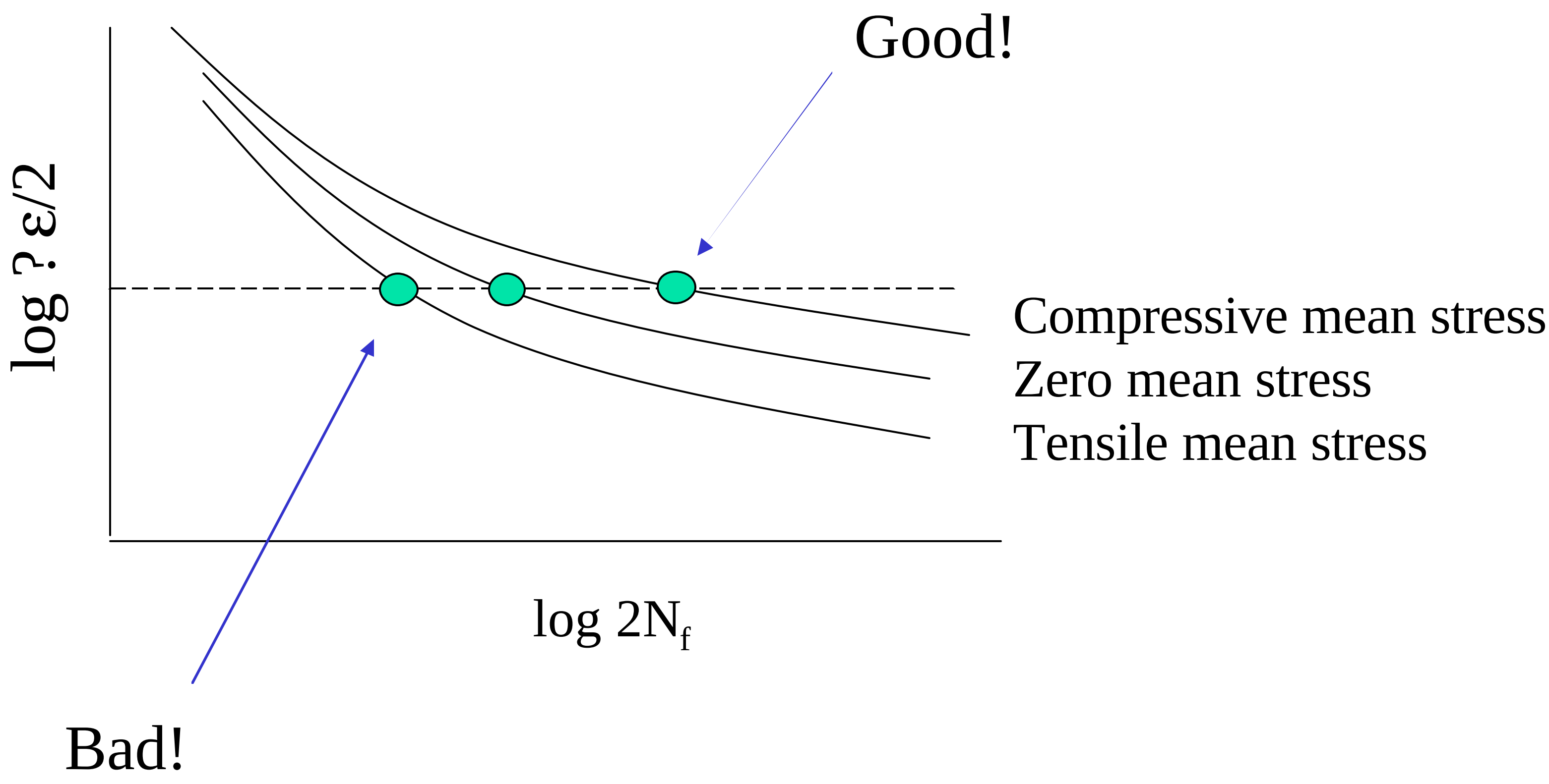

Miner's Rule of Cumulative Fatigue Damage is the best and simplest tool available to estimate the likelihood of fatigue failure under variable load histories. Miner's rule hypothesizes that fatigue failure will occur when the sum of the cycle ratios ($n_i/N_i$) is equal to or greater than 1

$$\sum_i \frac{n_i}{N_i} \geq 1.0$$

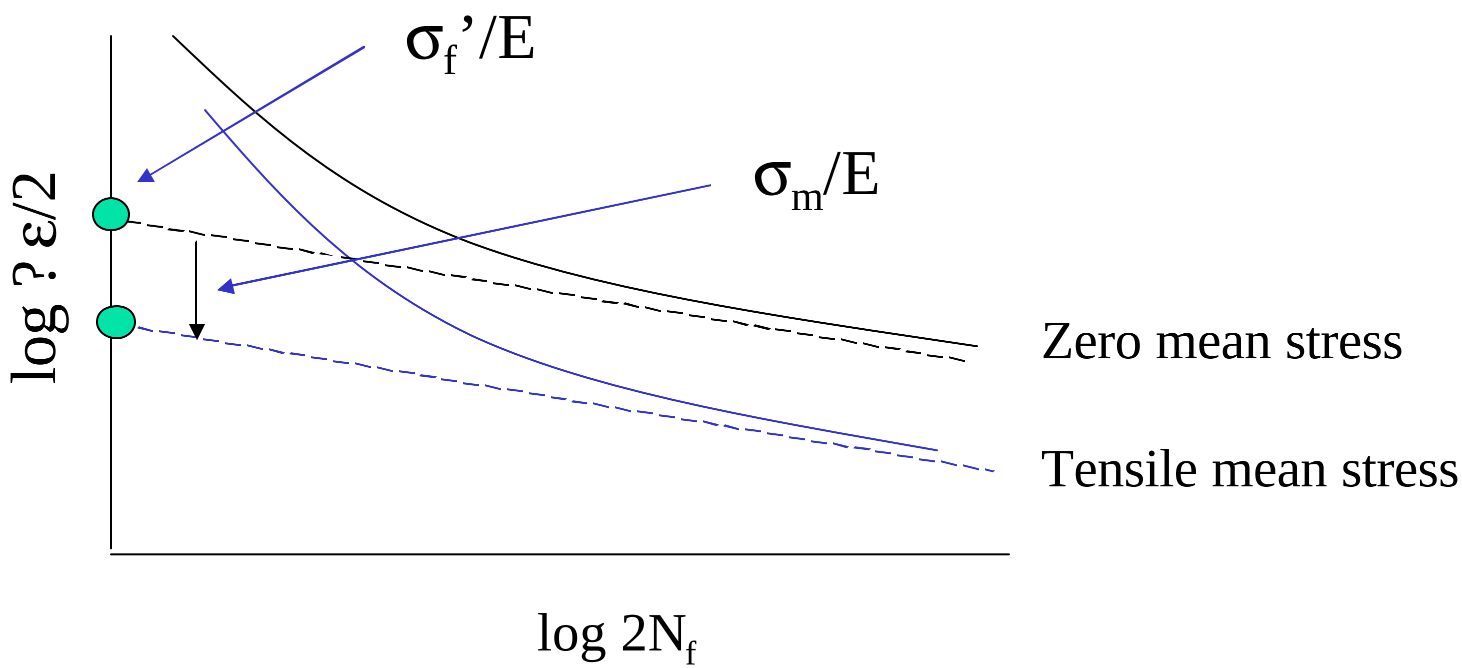
$n_i$ = Number of cycles experienced at a given stress or load level.
$N_i$ = Expected fatigue life at that stress or load level.

# Mean stress effects

Effects the growth of (small) fatigue cracks.



# MODEL: Morrow’s mean stress correction

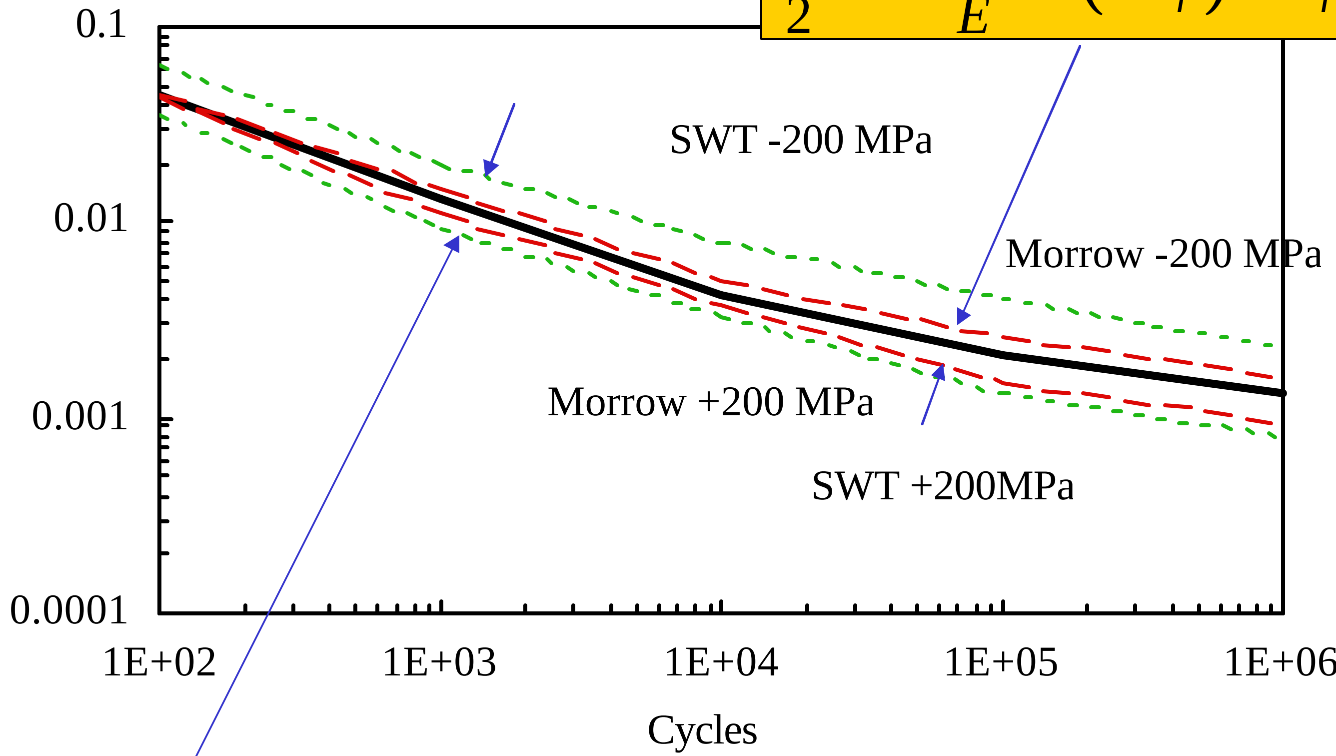



$$\frac{\Delta \varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$

# MODEL: Mean stress effects

Morrow

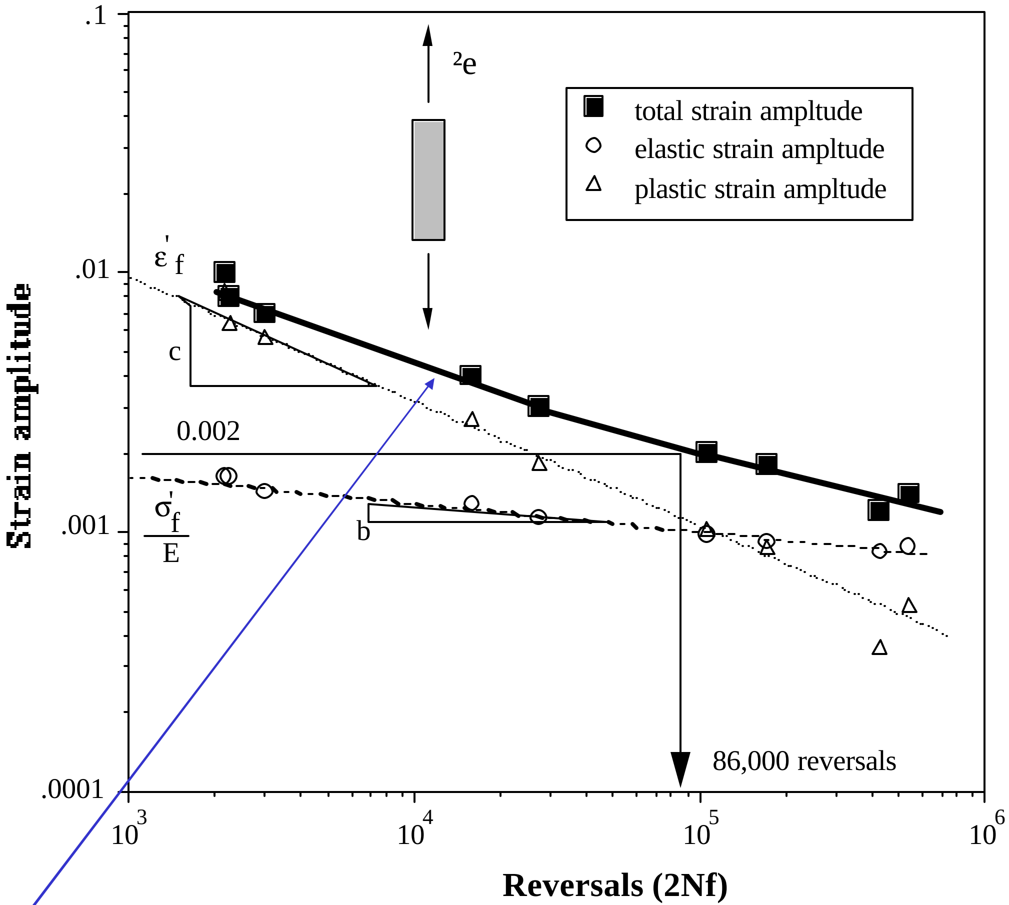
$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c$$




$$\frac{\sigma_{max}\Delta\varepsilon}{2} = \frac{\sigma'^{2}_f}{E}\left(2N_f\right)^{2b} + \sigma'_f\varepsilon'_f\left(2N_f\right)^{b+c}$$

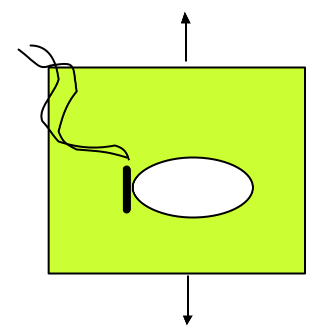
Smith-Watson-Topper

# MODEL USES




$$\Delta\varepsilon_t = \Delta\varepsilon_e + \Delta\varepsilon_p = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c$$

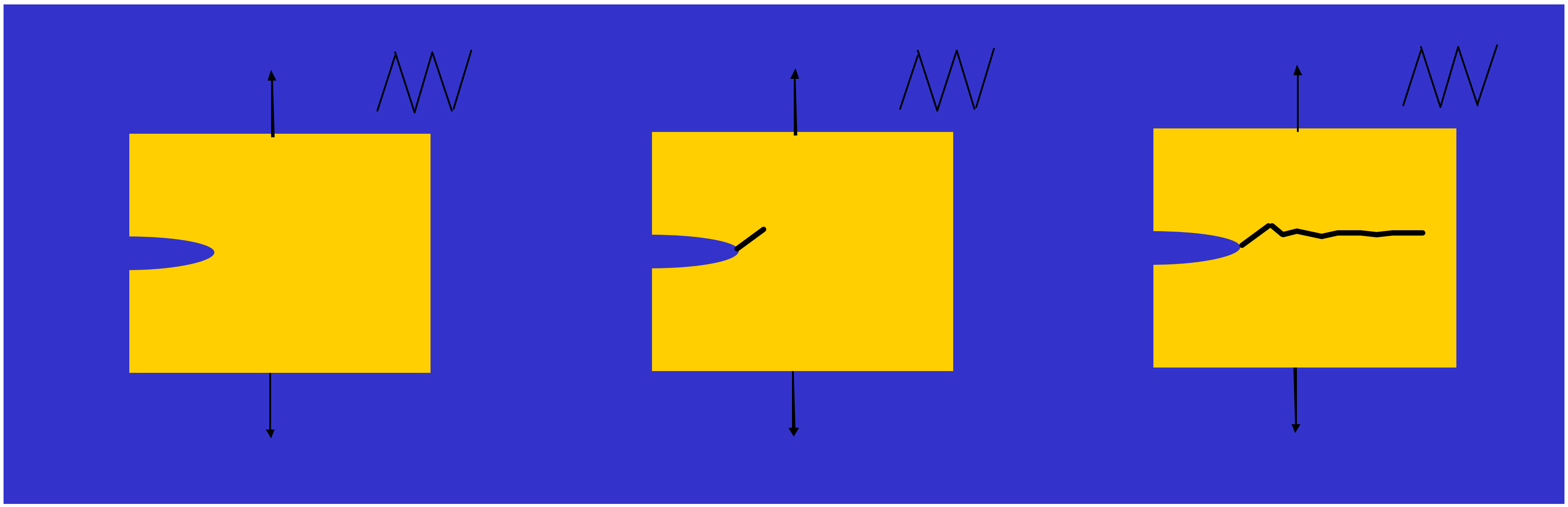
A strain gage mounted in a high stress region of a component indicates a cyclic strain of 0.002. The number of reversals to nucleate a crack ($2N_I$) is estimated to be 86,000 or 43,000 cycles.



# Outline

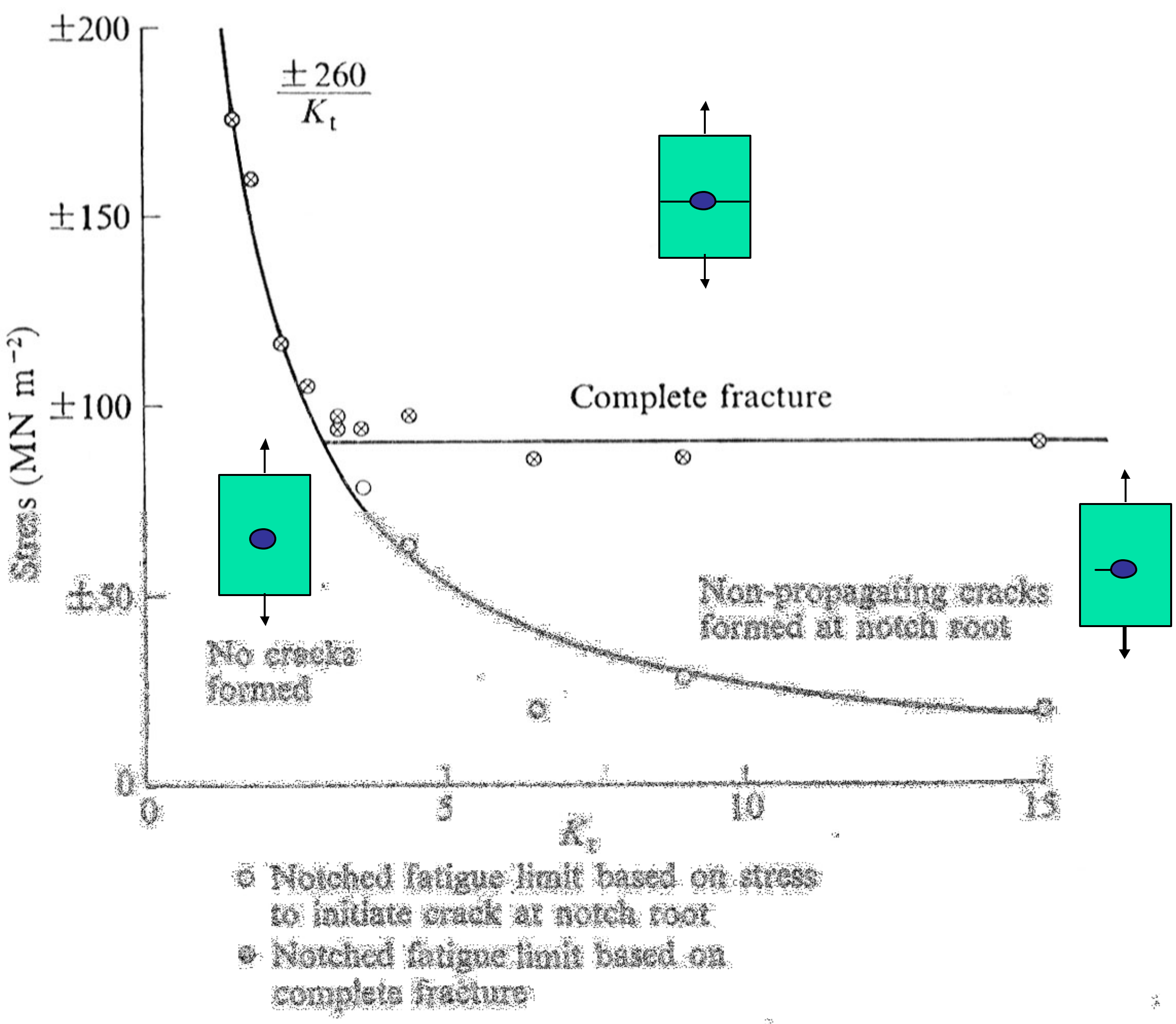
- Sources of knowledge
- Modelling
  - Crack nucleation
  - Non propagating cracks
  - Crack growth

# The three BIG fatigue questions



1. Will a crack nucleate?
2. **Will it grow?**
3. How fast will it grow?

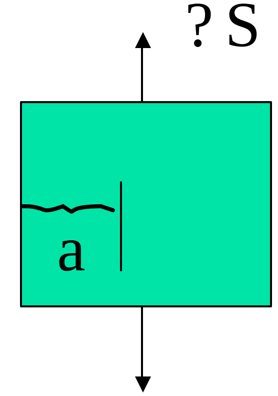
# Non-propagating cracks



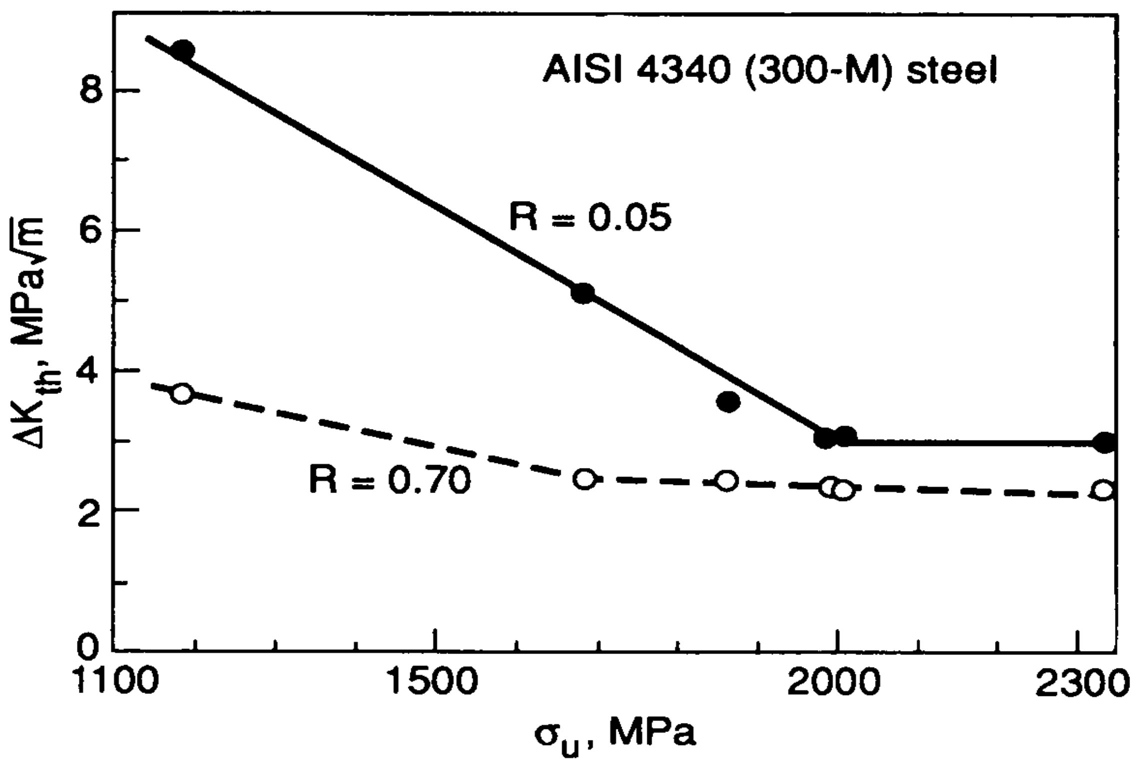

Sharp notches may nucleate cracks but the remote stress may not be large enough to allow the crack to leave the notch stress field.

# Threshold Stress Intensity

$$\Delta K = Y\ \Delta S\sqrt{\pi a}$$

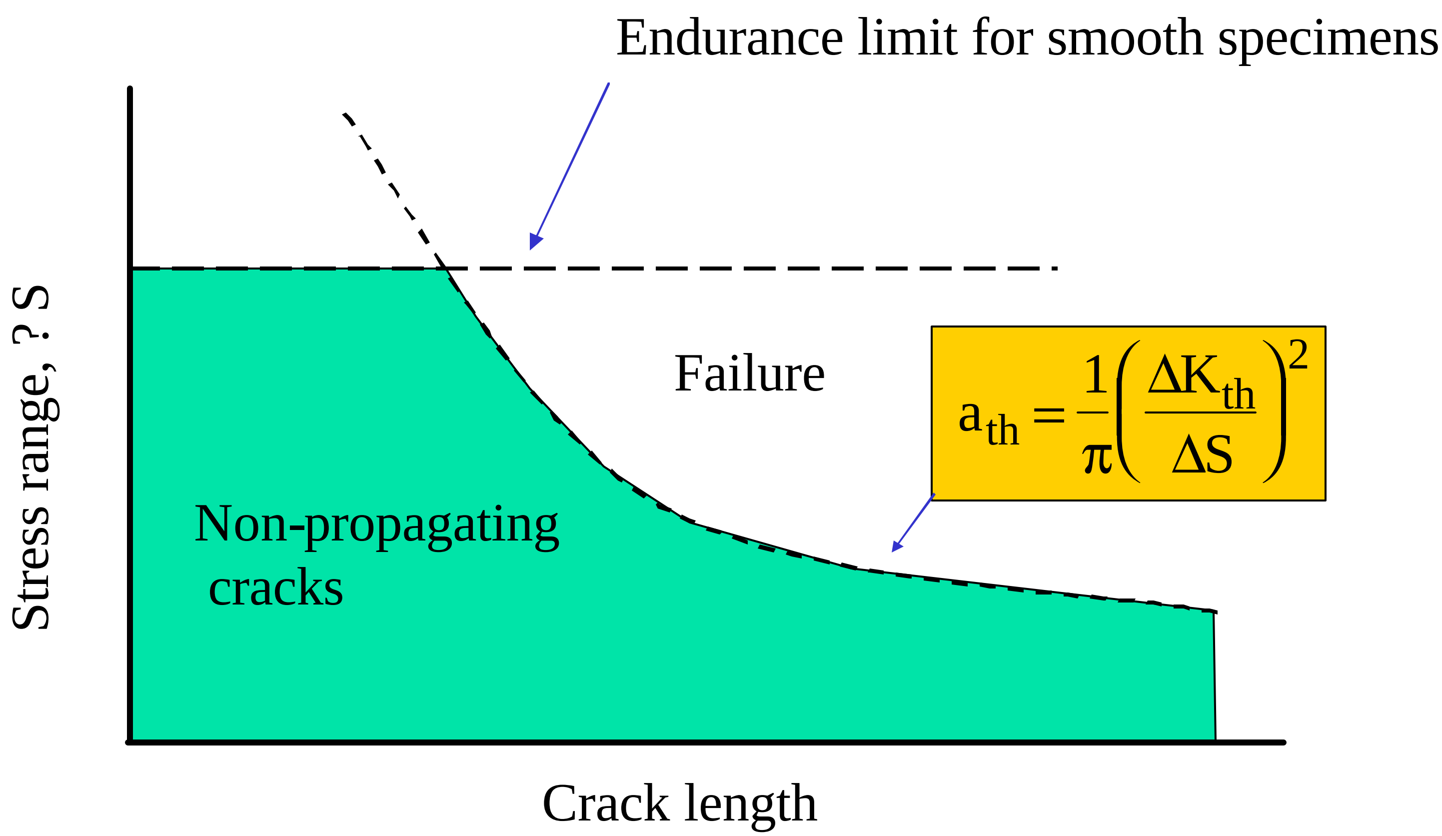



The forces driving a crack forward are related to the stress intensity factor



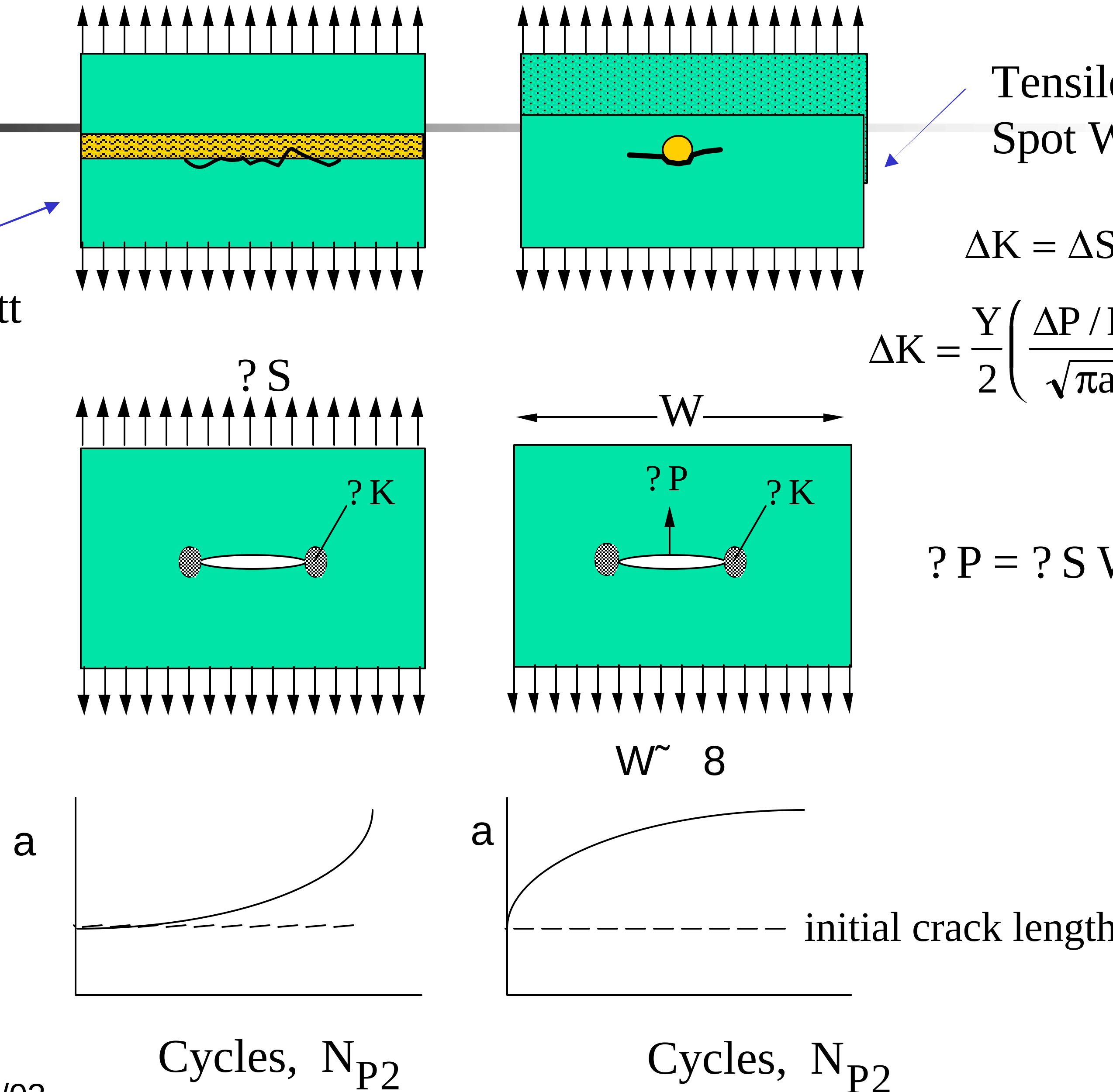

At or below the threshold value of ? K, the crack doesn't grow.

# MODEL: Non-propagating cracks



Groove Welded Butt Joint

Tensile-Shear Spot Weld

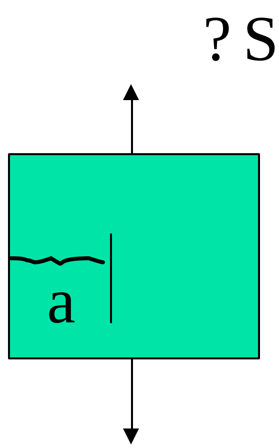



$$\Delta K = \Delta S\sqrt{\pi a}$$

$$\Delta K = \frac{Y}{2}\left(\frac{\Delta P / B}{\sqrt{\pi a}} + \Delta S\sqrt{\pi a}\right)$$

? P = ? S W

# MODEL USES



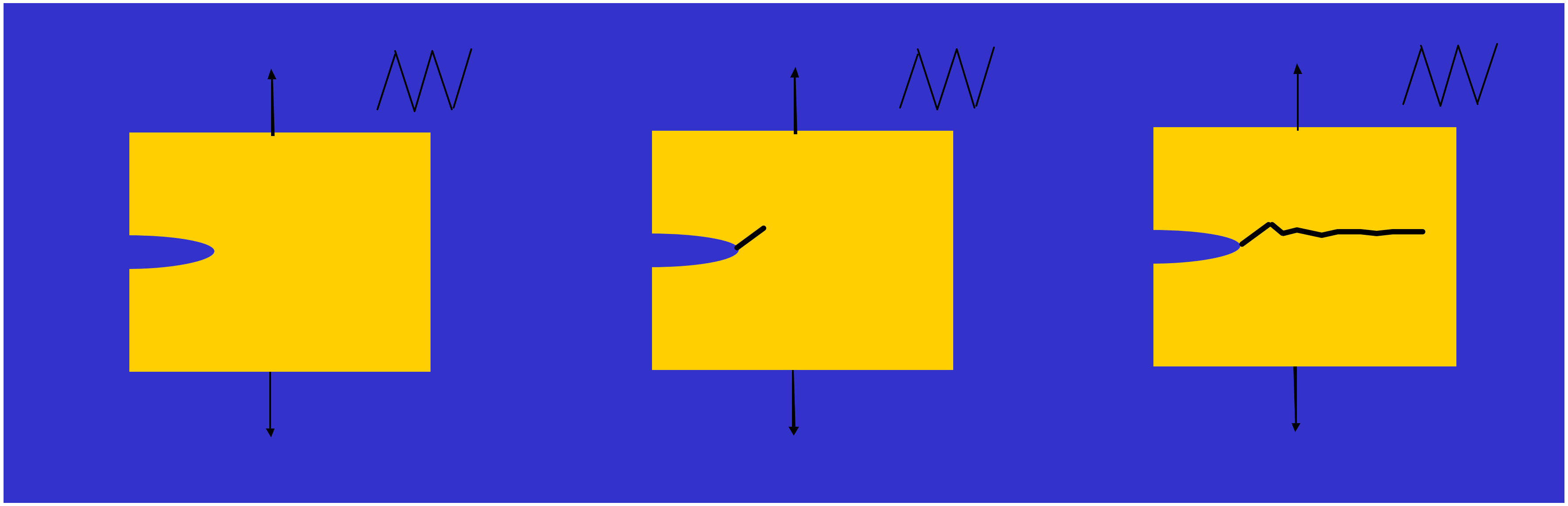

$$a_{th} = \frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta S}\right)^2$$

- Will the defect of length a grow under the applied stress range ?S?
- How sensitive should our NDT techniques be to ensure safety?

# Outline

- Fatigue mechanisms
- Sources of knowledge
- Modelling
  - Crack nucleation
  - Non propagating cracks
  - Crack growth

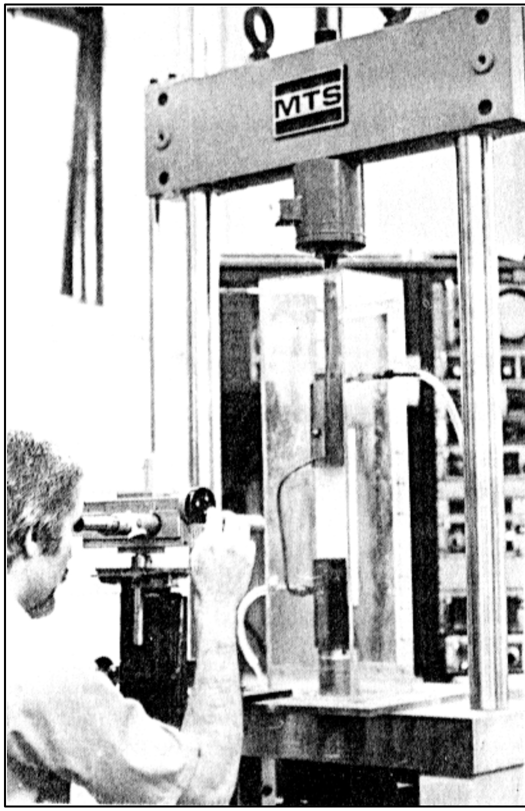
# The three BIG fatigue questions

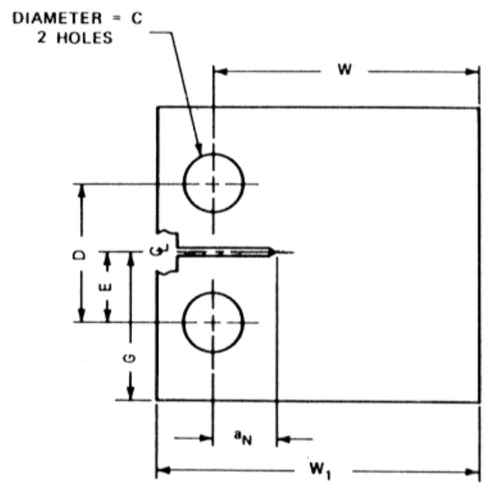


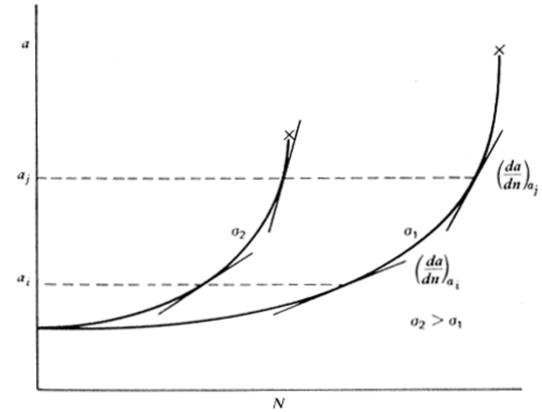
1. Will a crack nucleate? 2. Will it grow? **3. How fast will it grow?**

# How fast will it grow???

Measuring Crack Growth - Everything we know about crack growth begins here!

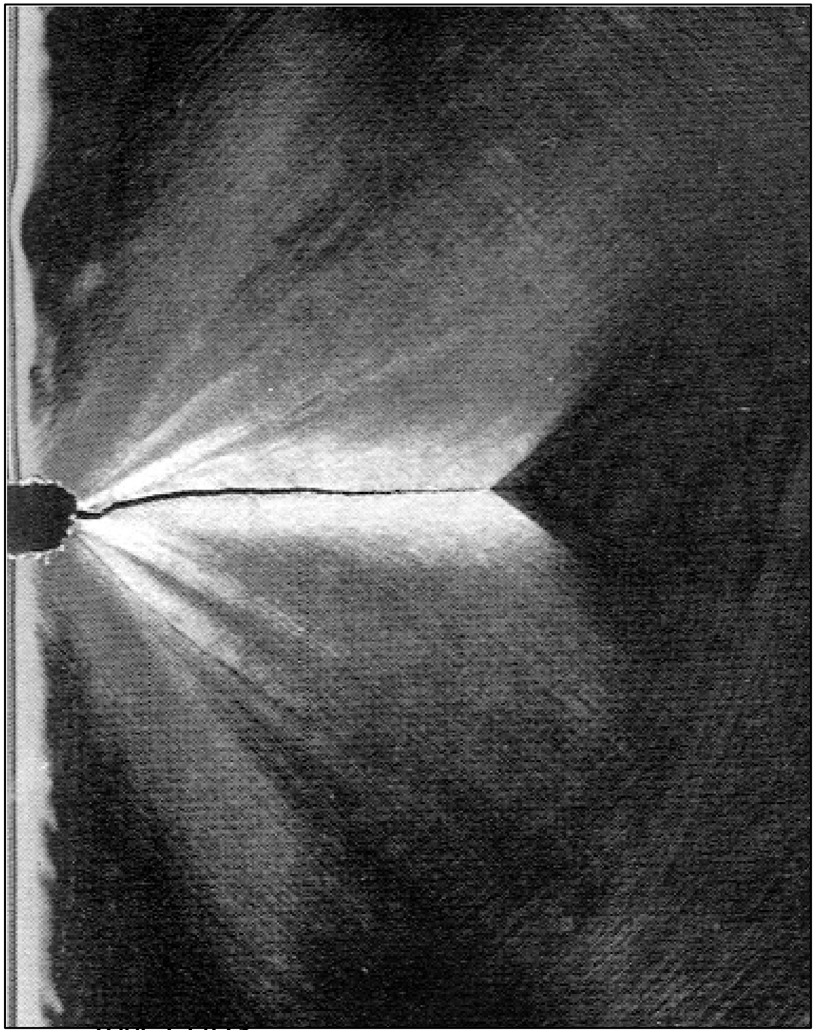



Cracks grow faster as their length increases.

# MODEL: Paris Power Law



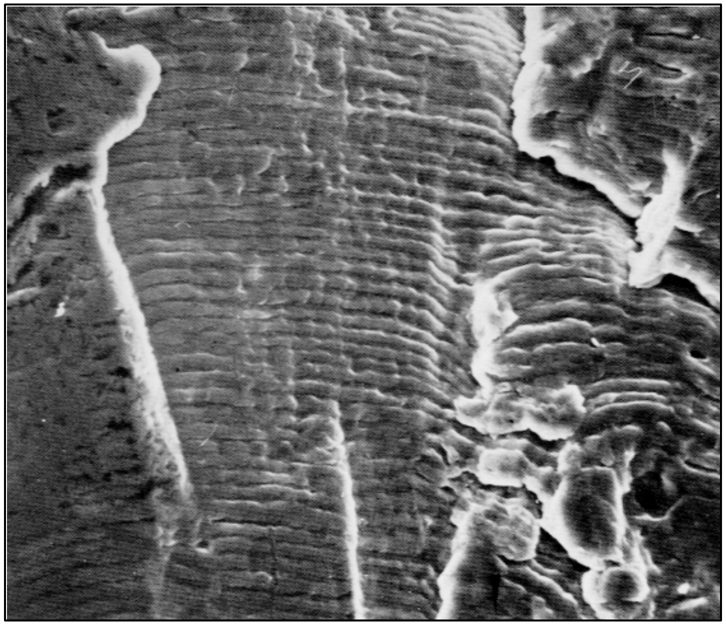
Growth of fatigue cracks is <u>correlated</u> with the range in stress intensity factor: ? K.

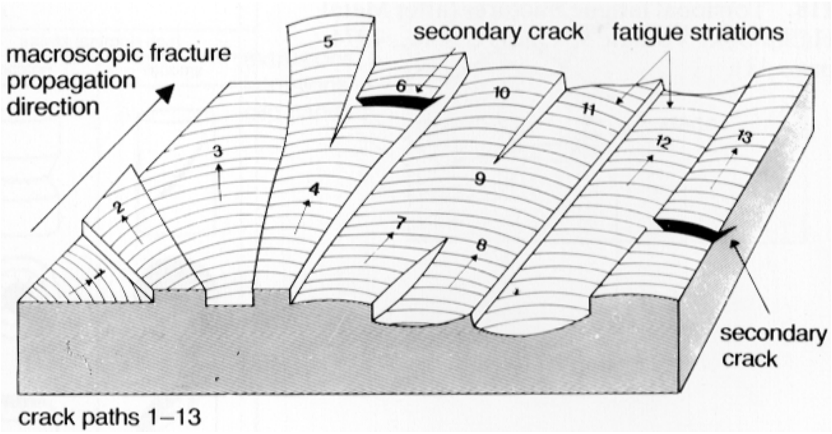
$$\Delta K = Y\ \Delta S \sqrt{pa}$$

$$\frac{da}{dN} = C(\Delta K)^n$$

# Fatigue fracture surface

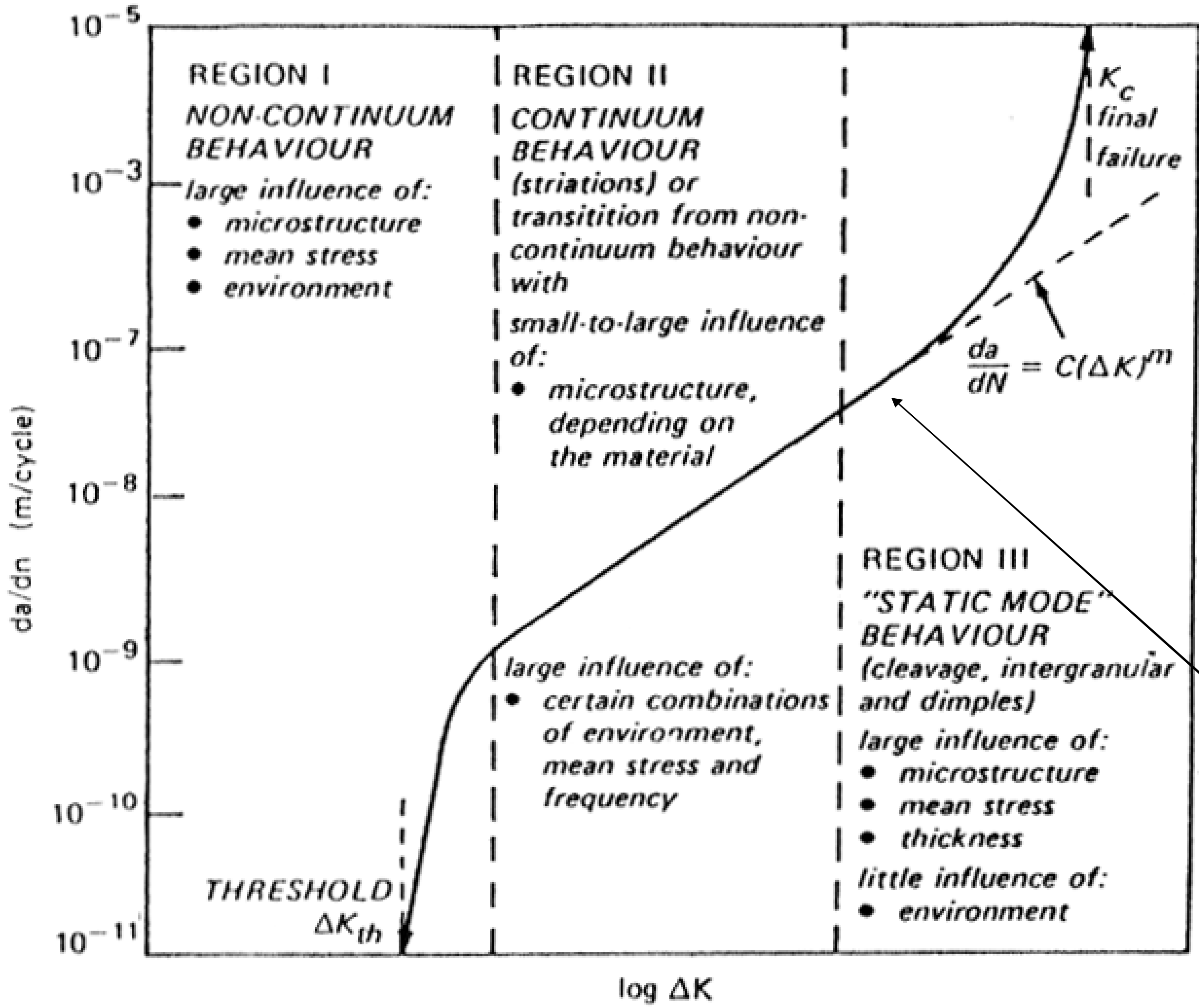
*Scanning electron microscope image - striations clearly visible*






*Schematic drawing of a fatigue fracture surface*

# MODEL: Crack growth rate versus ?K

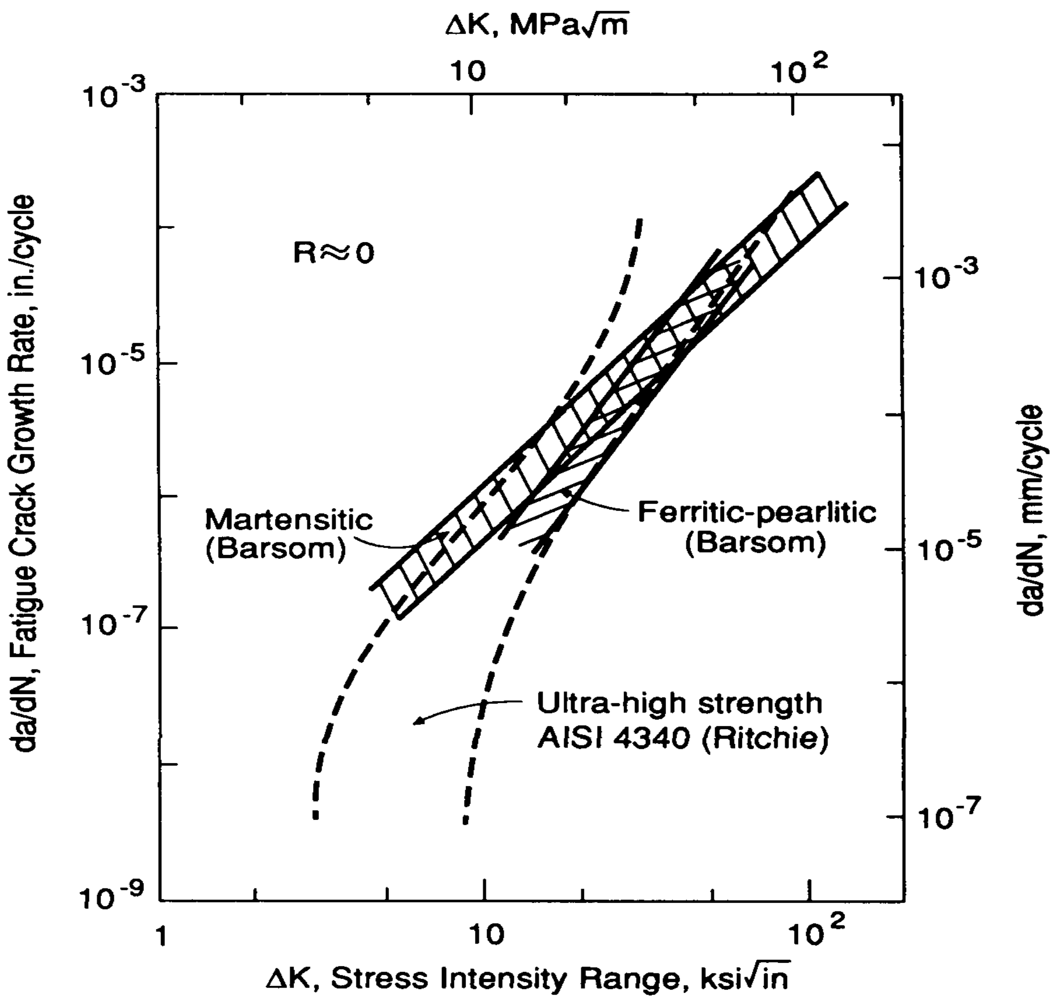



Crack growth rate (da/dN) is related to the crack tip stress field and is thus strongly correlated with the range of stress intensity factor:
(? K=Y? Svpa).

$$\frac{da}{dN} = C(\Delta K)^n$$

Paris power law

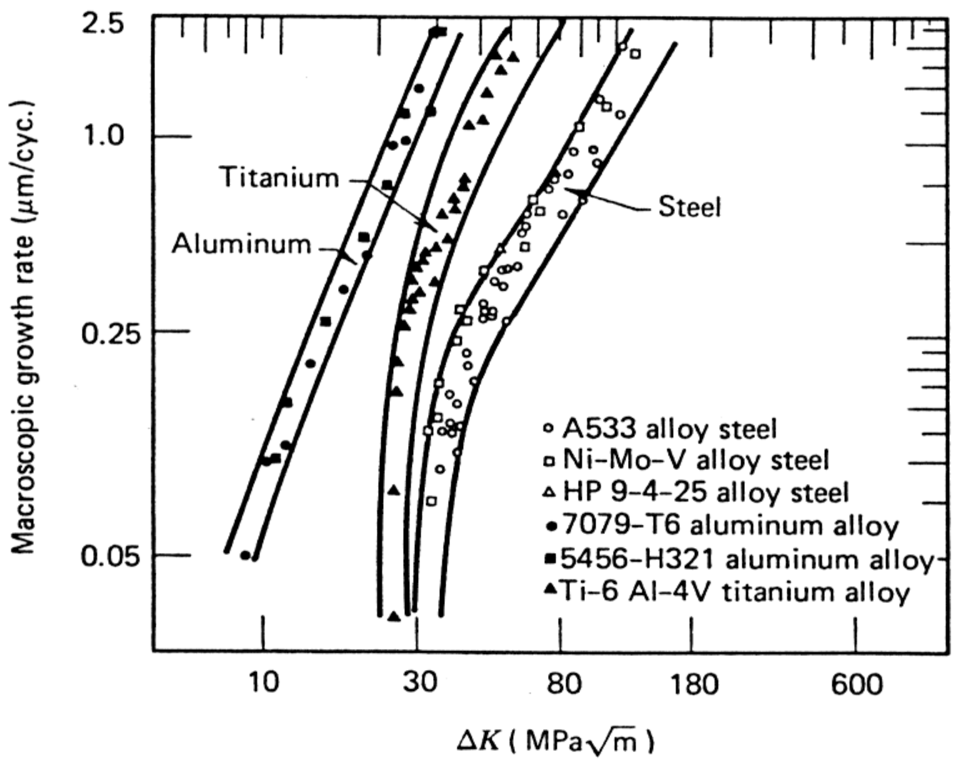
# Crack Growth Data

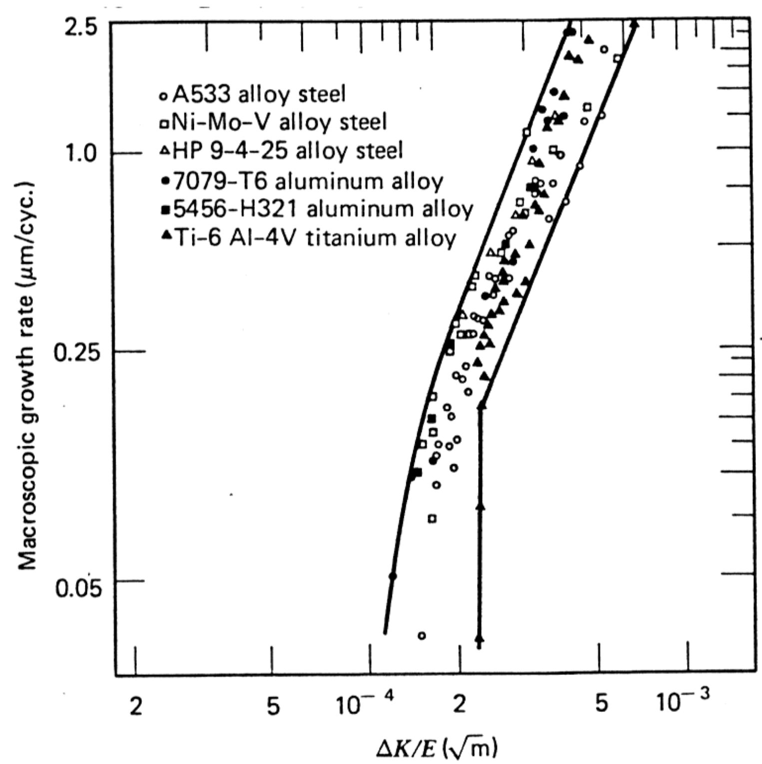



All ferritic-pearlitic, that is, plain carbon steels behave about the same irrespective of strength or grain size!

# Crack growth rates of metals

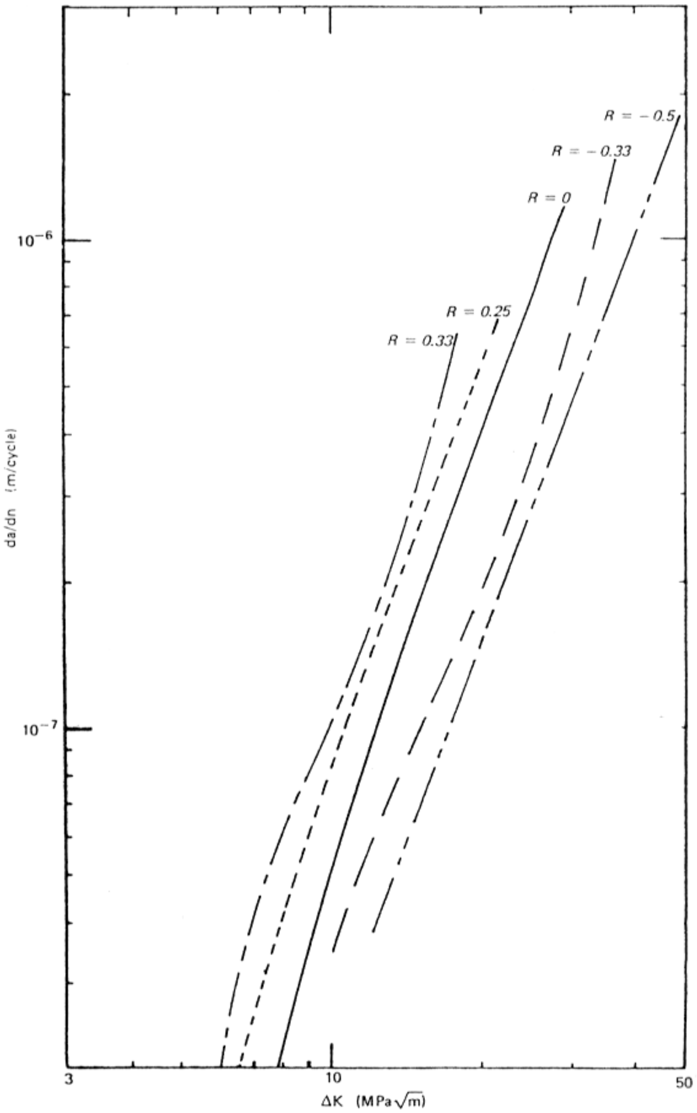
$$\frac{\Delta K}{E}$$




The fatigue crack growth rates for Al and Ti are much more rapid than steel for a given ? K. However, when normalized by Young's Modulus all metals exhibit about the same behavior.

# MODEL:  Mean stresses (R ratio)




$$\frac{da}{dN} = \frac{C\,\Delta K^{m}}{(1-R)\,K_c - \Delta K}$$

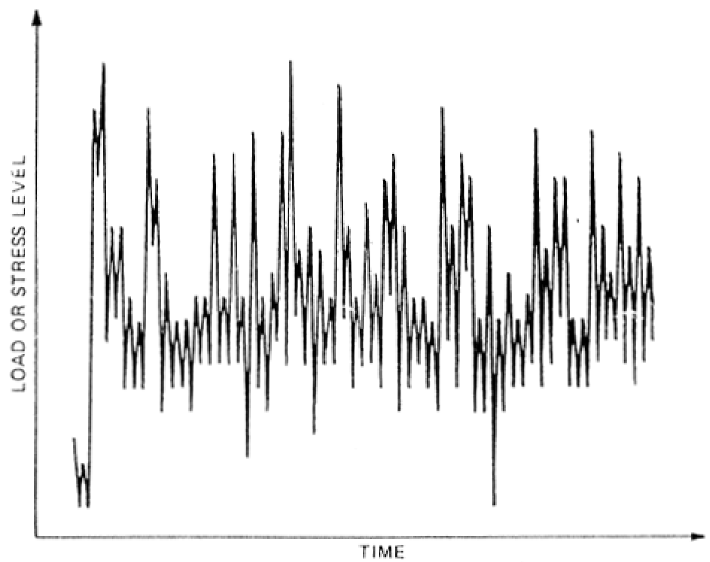
Forman's equation which includes the effects of mean stress.

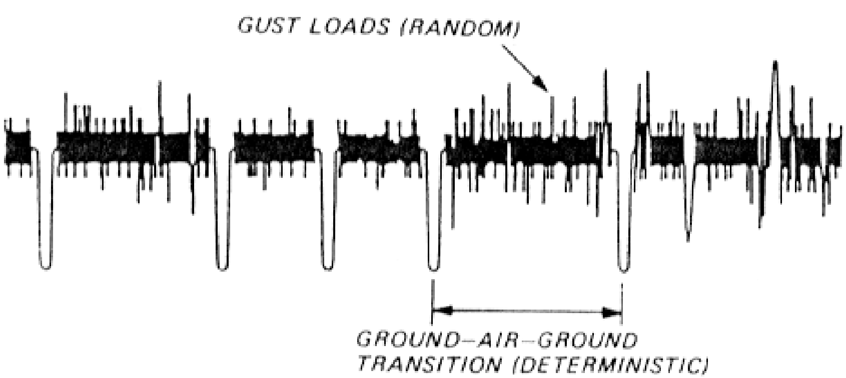
$$\frac{da}{dN} = \frac{C\,\Delta K^{m}}{(1-R)^{g}}$$

Walker's equation which includes the effect of mean stress.

# PROBLEM: Variable load histories?

While some applications are actually constant amplitude, many or most applications involve variable amplitude loads (VAL) or variable load histories (VLH).

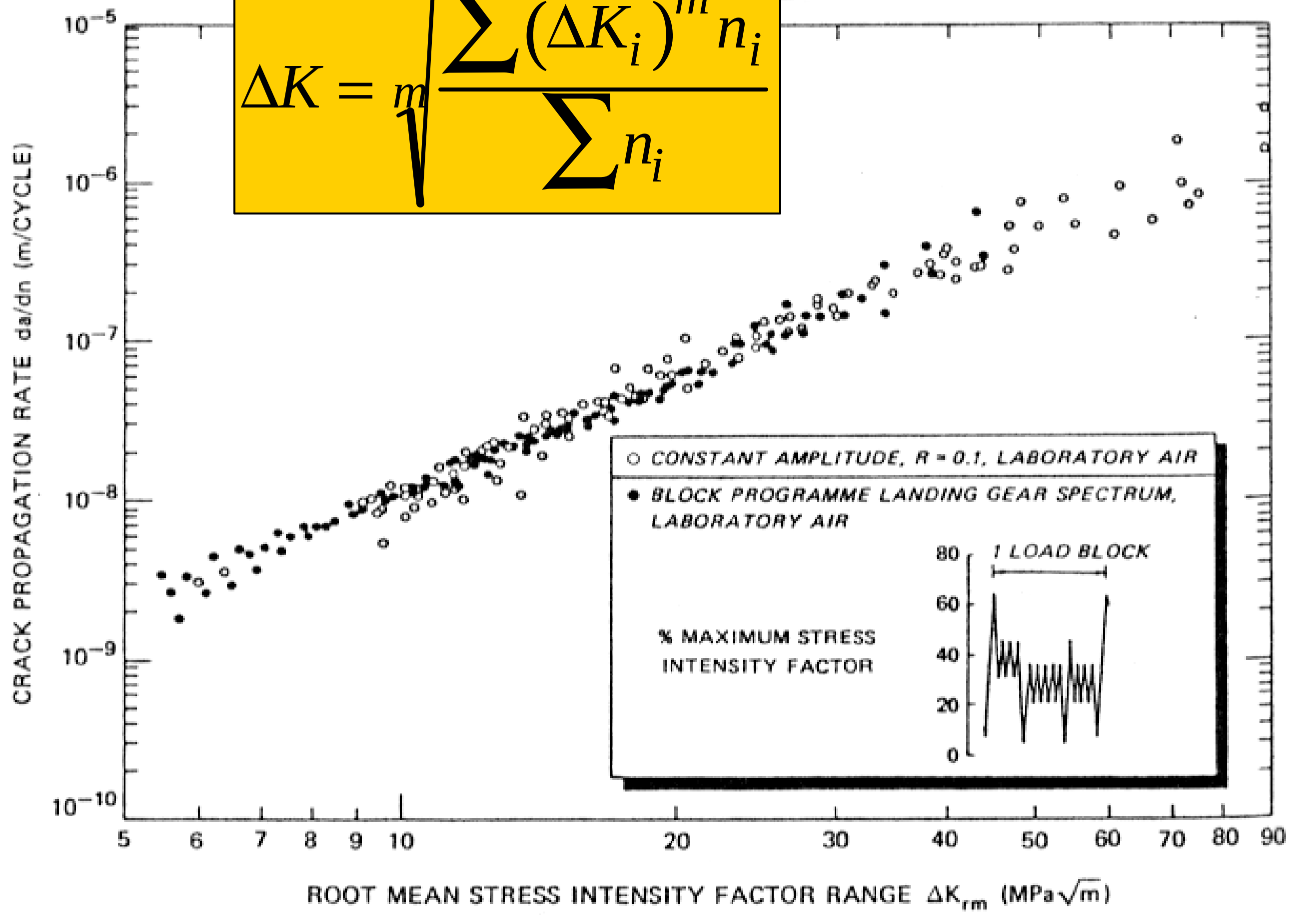


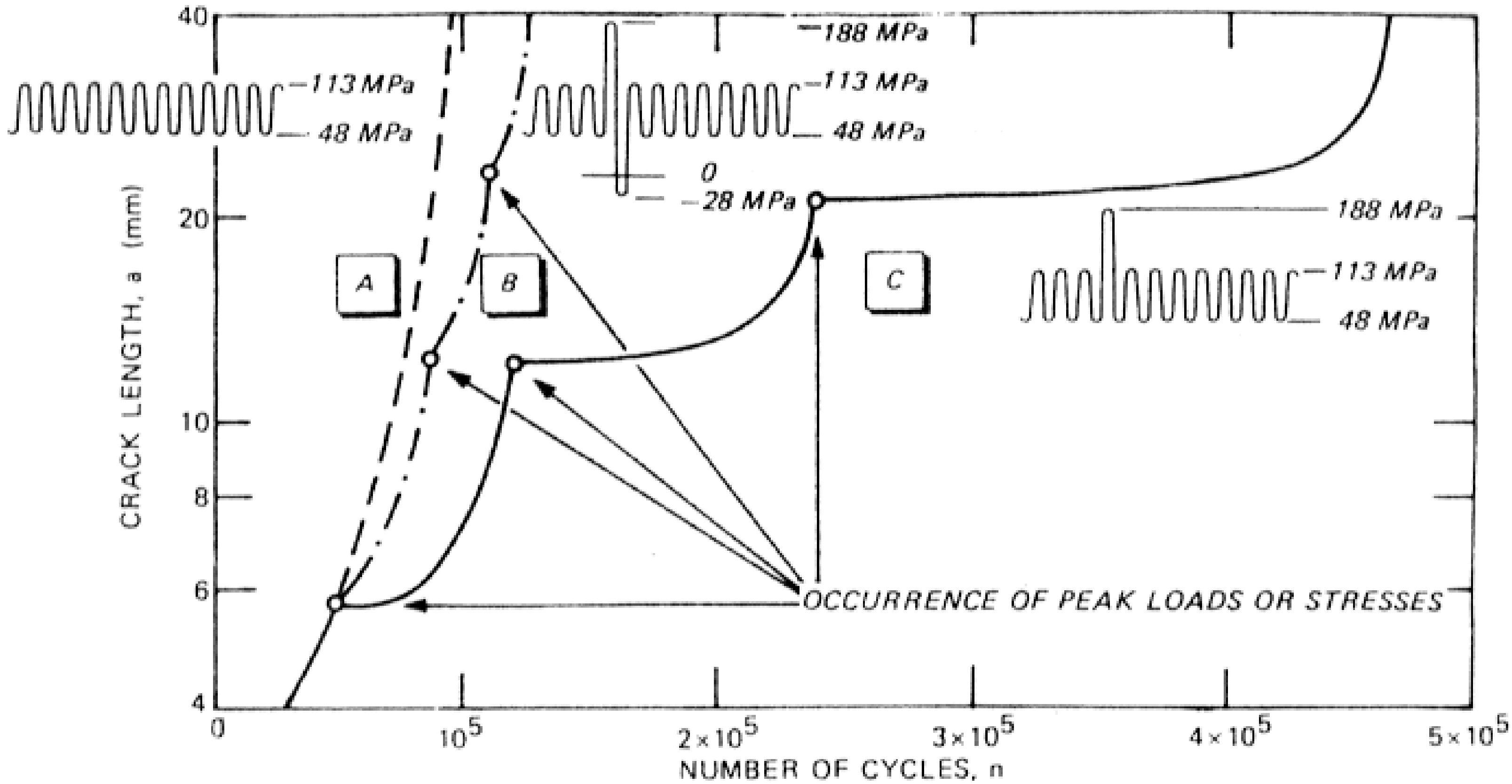

Aircraft load histories

# FIX: Root mean cube method

$$\Delta K = \sqrt[m]{\frac{\sum (\Delta K_i)^m n_i}{\sum n_i}}$$

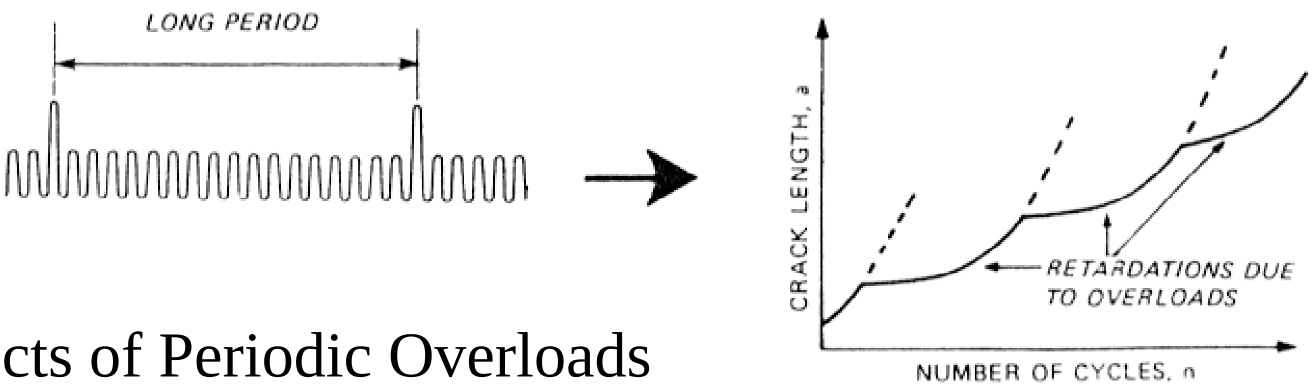


# PROBLEM: Overloads, under-loads?

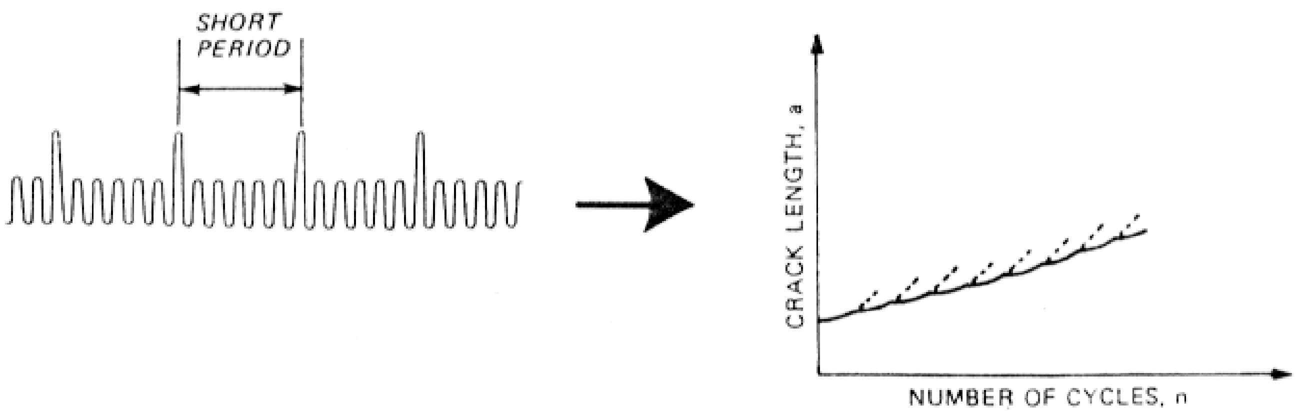



Overloads retard crack growth, under-loads accelerate crack growth.

# PROBLEM: Periodic overloads?



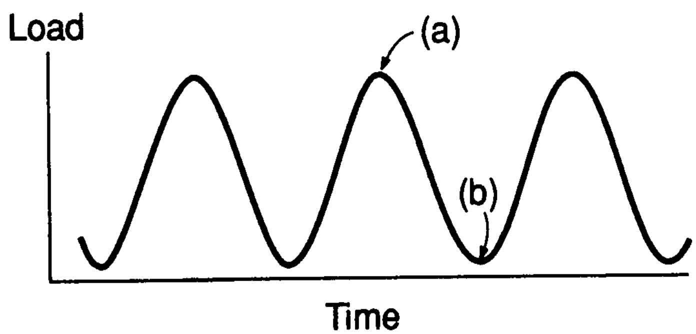

Effects of Periodic Overloads

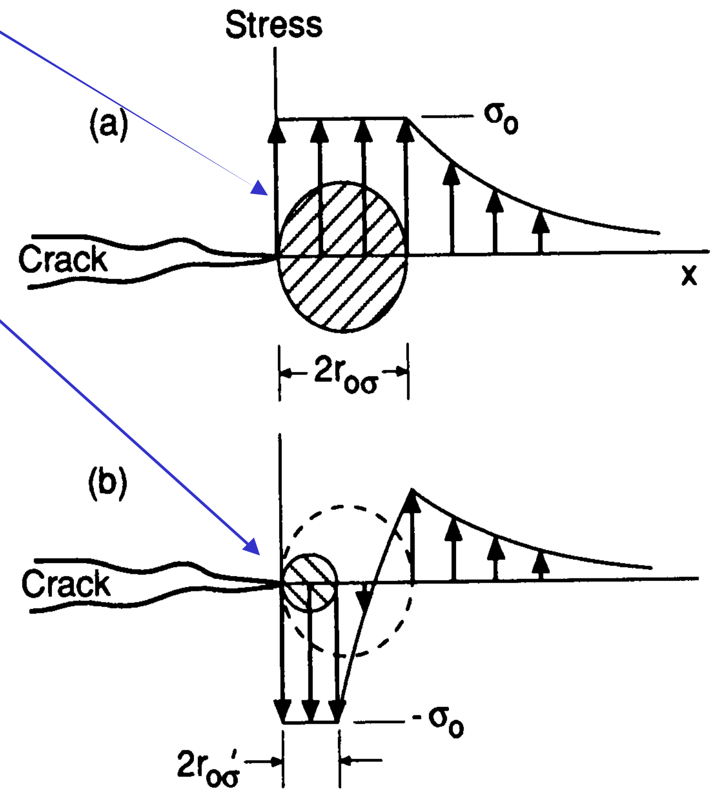



Infrequent overloads help, but more frequent may be even better. Too frequent overloads very damaging.

# FIX: MODEL for crack tip stress fields

Monotonic plastic zone size
Cyclic plastic zone size

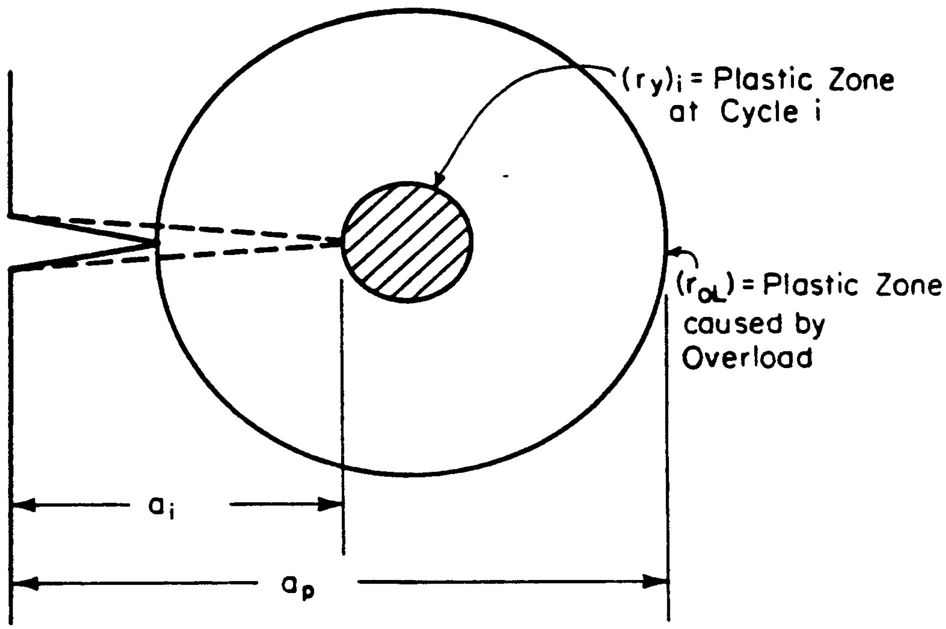



$$r_y = \frac{1}{bp}\left(\frac{K}{s_{\ y}}\right)$$

ß is 2 for (a) plane stress and 6 for (b)

# MODEL: Overload plastic zone size



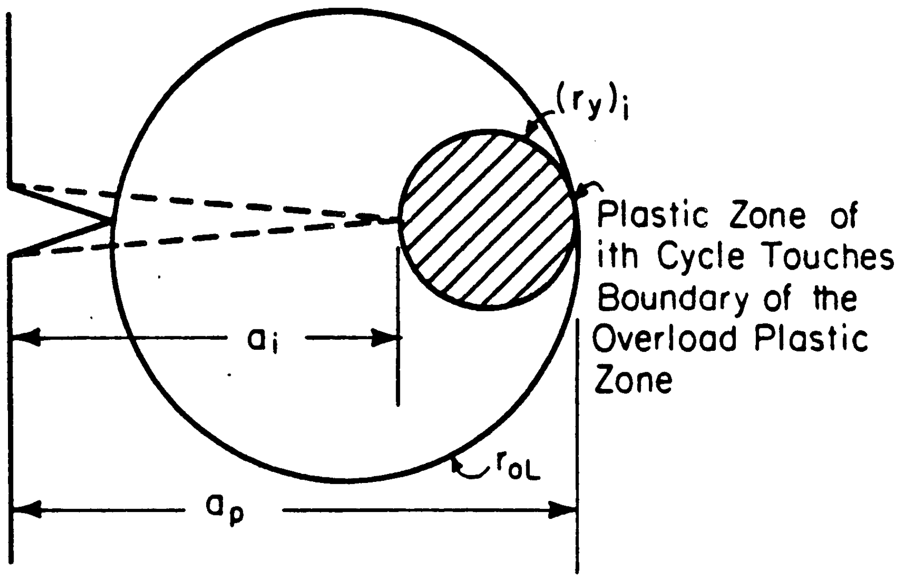

$$r_y = \frac{1}{bp}\left(\frac{K}{s_y}\right)$$

Crack growth retardation ceases when the plastic zone of ith cycle reaches the boundary of the prior overload plastic zone.

ß is 2 for plane stress and 6 for plain strain.

# MODEL: Retarded crack growth

Wheeler model

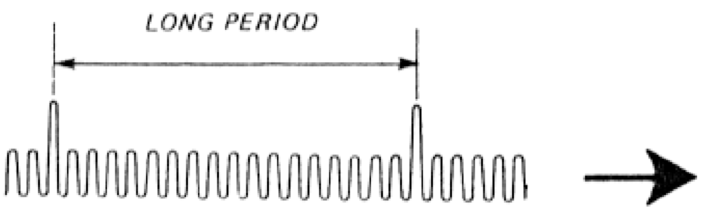


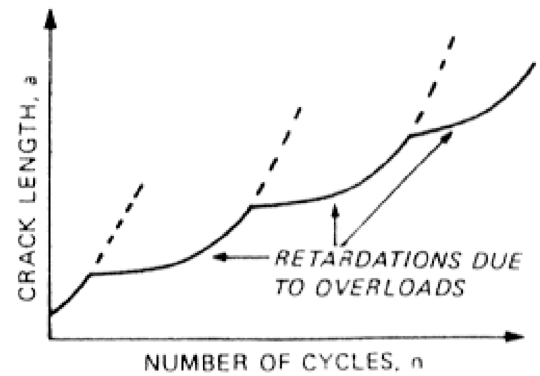

$$\left(\frac{da}{dN}\right)_{VA} = b\left(\frac{da}{dN}\right)_{CA}$$

$$b = \left(\frac{2\,r_y}{a_p - a_i}\right)^k$$

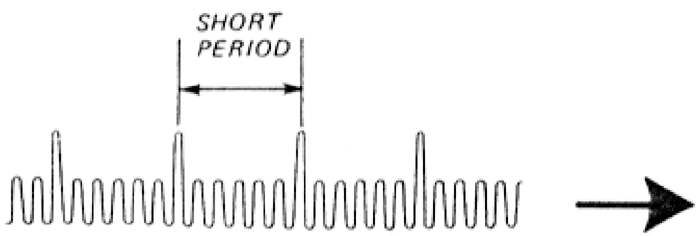
$$r_y = \frac{1}{2p}\left(\frac{K}{s_{ys}}\right)^2$$

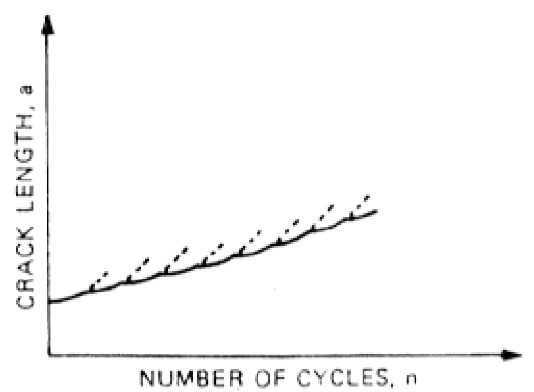
# Retarded crack growth



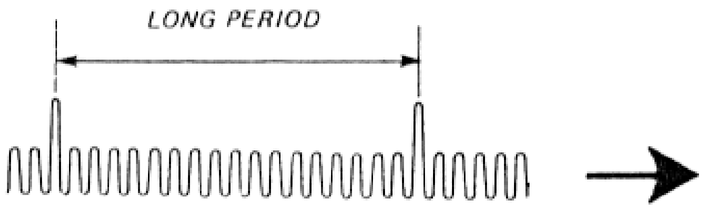

Effects of Periodic Overloads

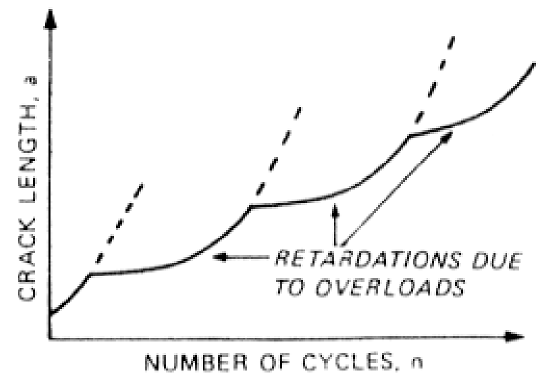


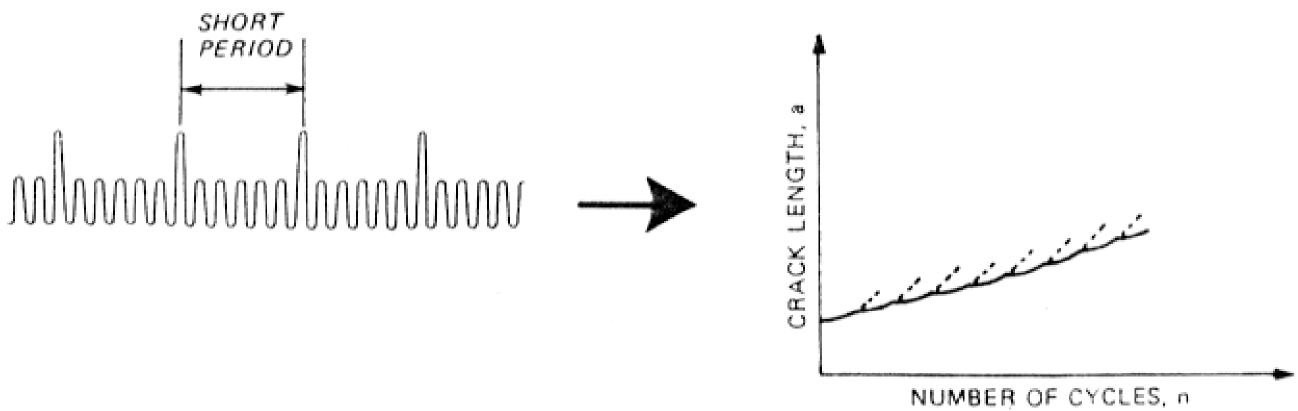

Infrequent overloads help, but more frequent may be even better.  Too frequent overloads very damaging.

# PROBLEM: Periodic overloads?



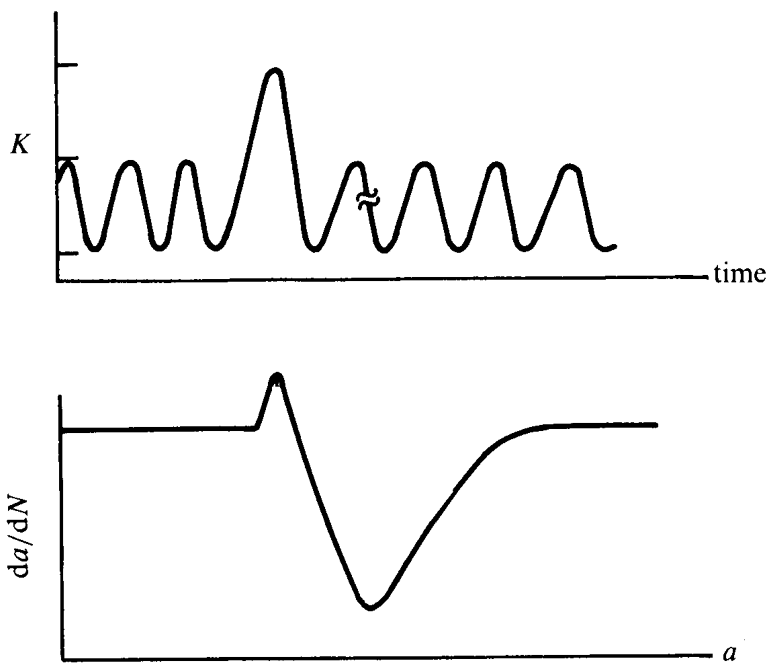




Effects of Periodic Overloads




Infrequent overloads help, but more frequent may be even better. Too frequent overloads very damaging.

# Sequence Effects



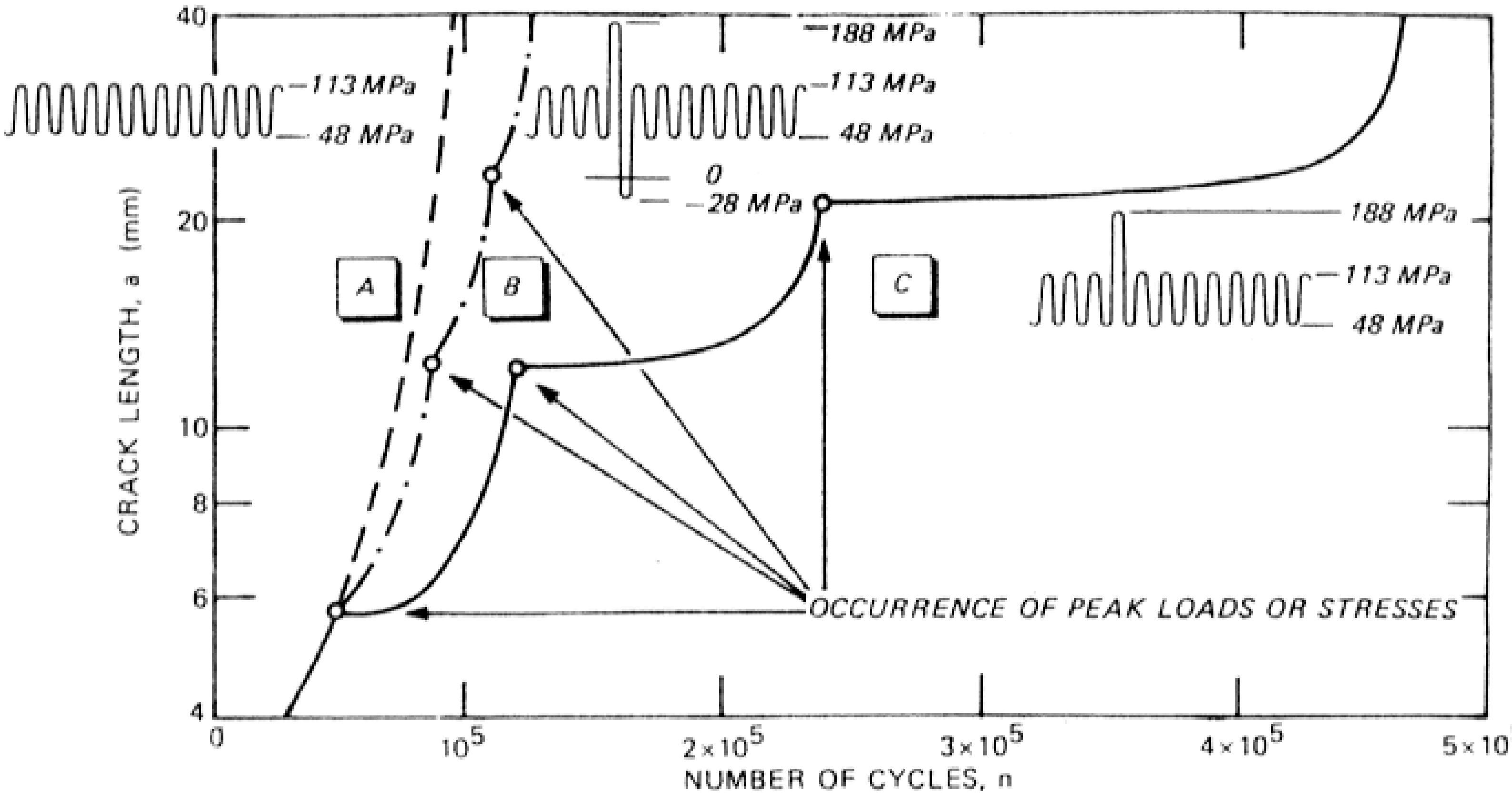

Last one controls!

Compressive overloads accelerate cdrack growth by reducing roughness of fracture surfaces.

Compressive followed by tensile overloads…retardation!

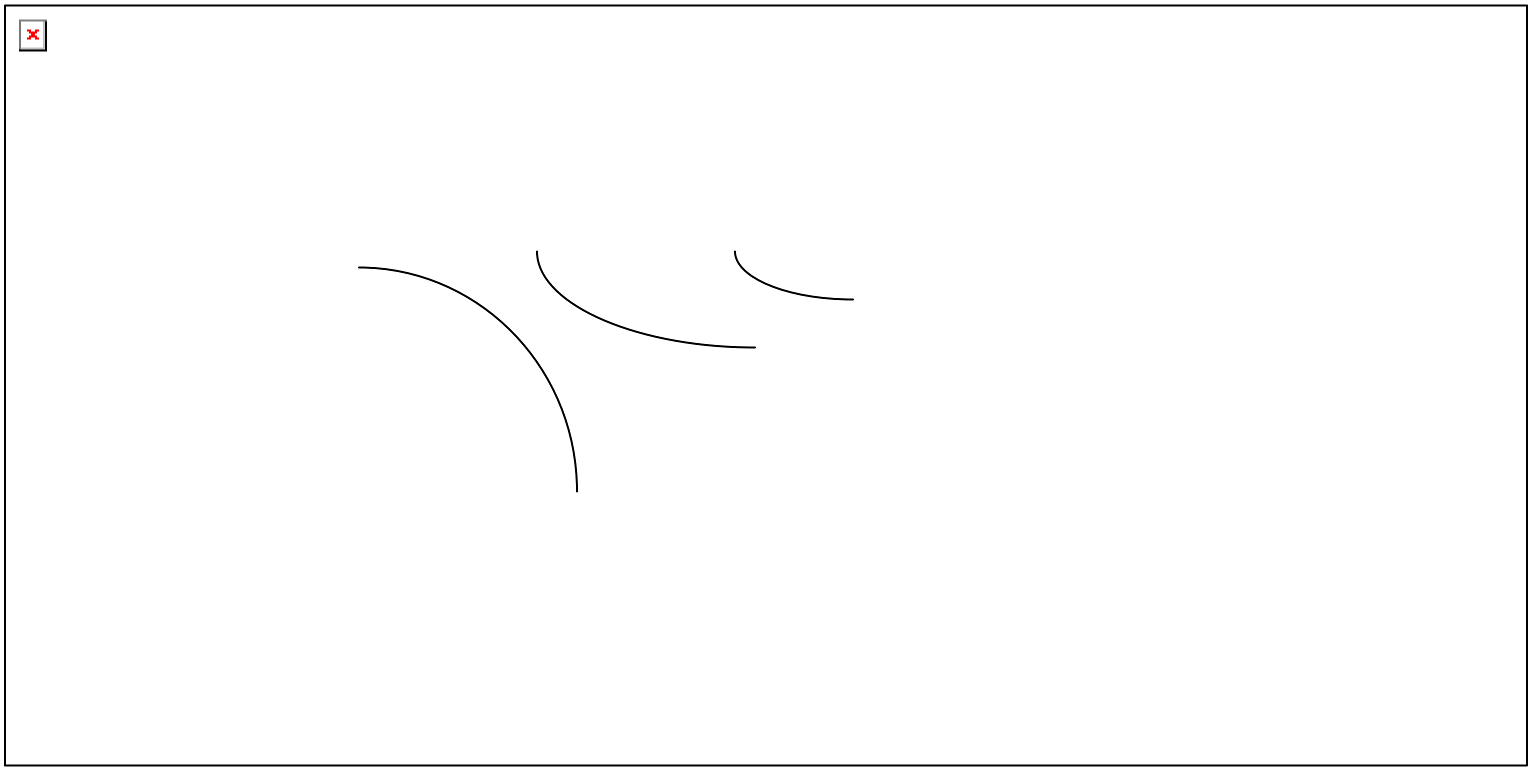
Tensile followed by compressive, little retardation

# Sequence effects



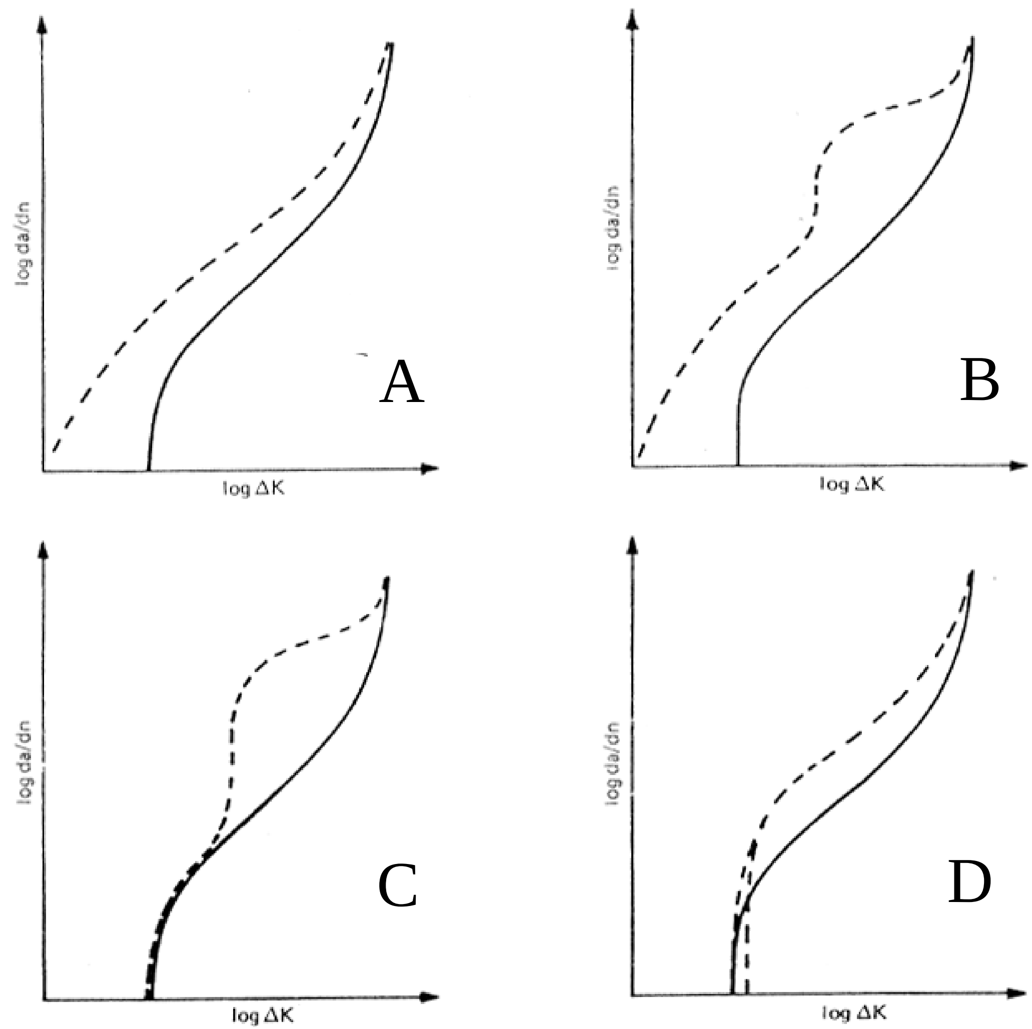

Overloads retard crack growth, under-loads accelerate crack growth.

# PROBLEMS: Small Crack Growth?



Small cracks don't behave like long cracks!  Why?

# PROBLEMS: Environment?



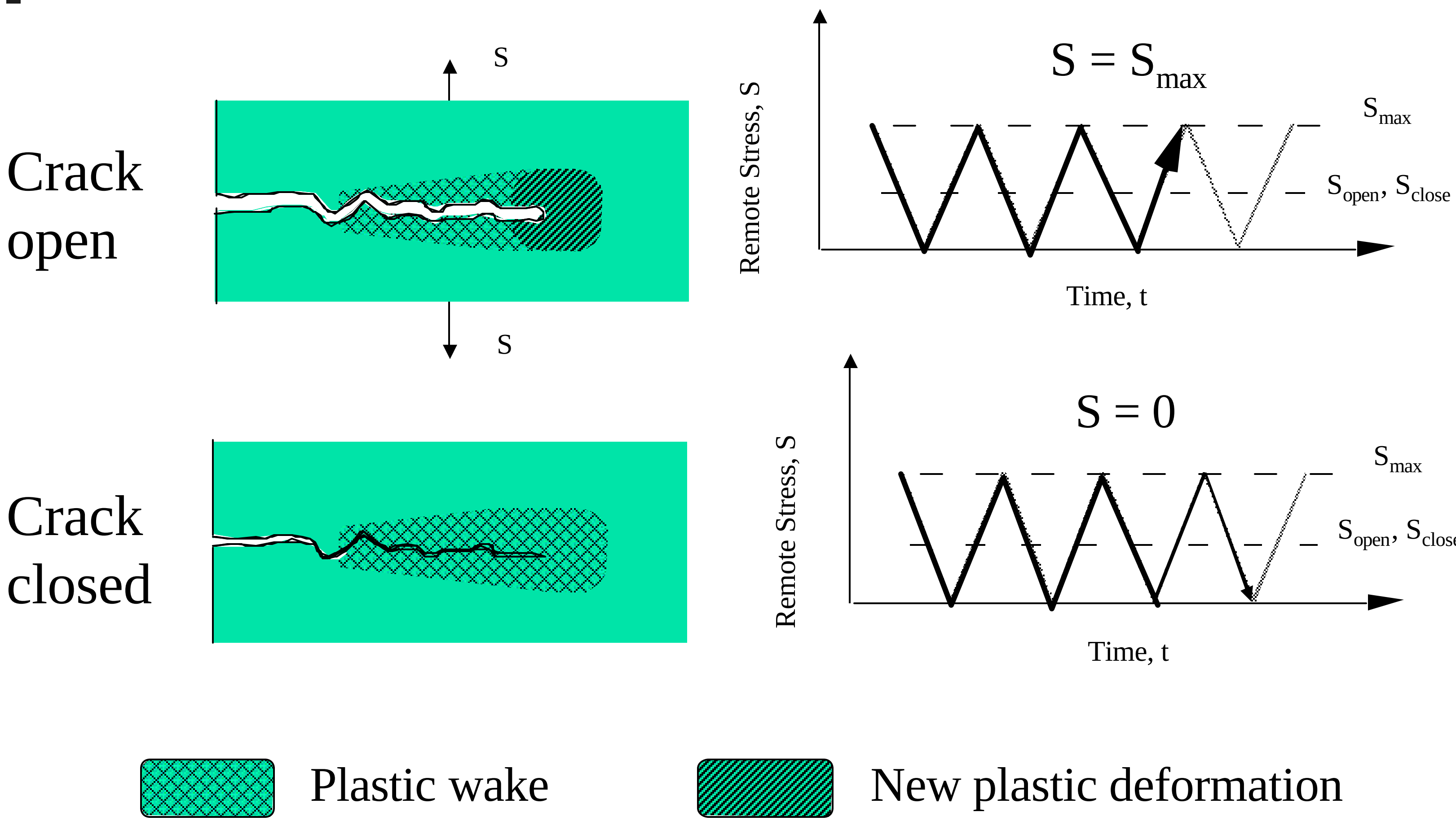

A. Dissolution of crack tip.

B. Dissolution plus H+ acceleration.

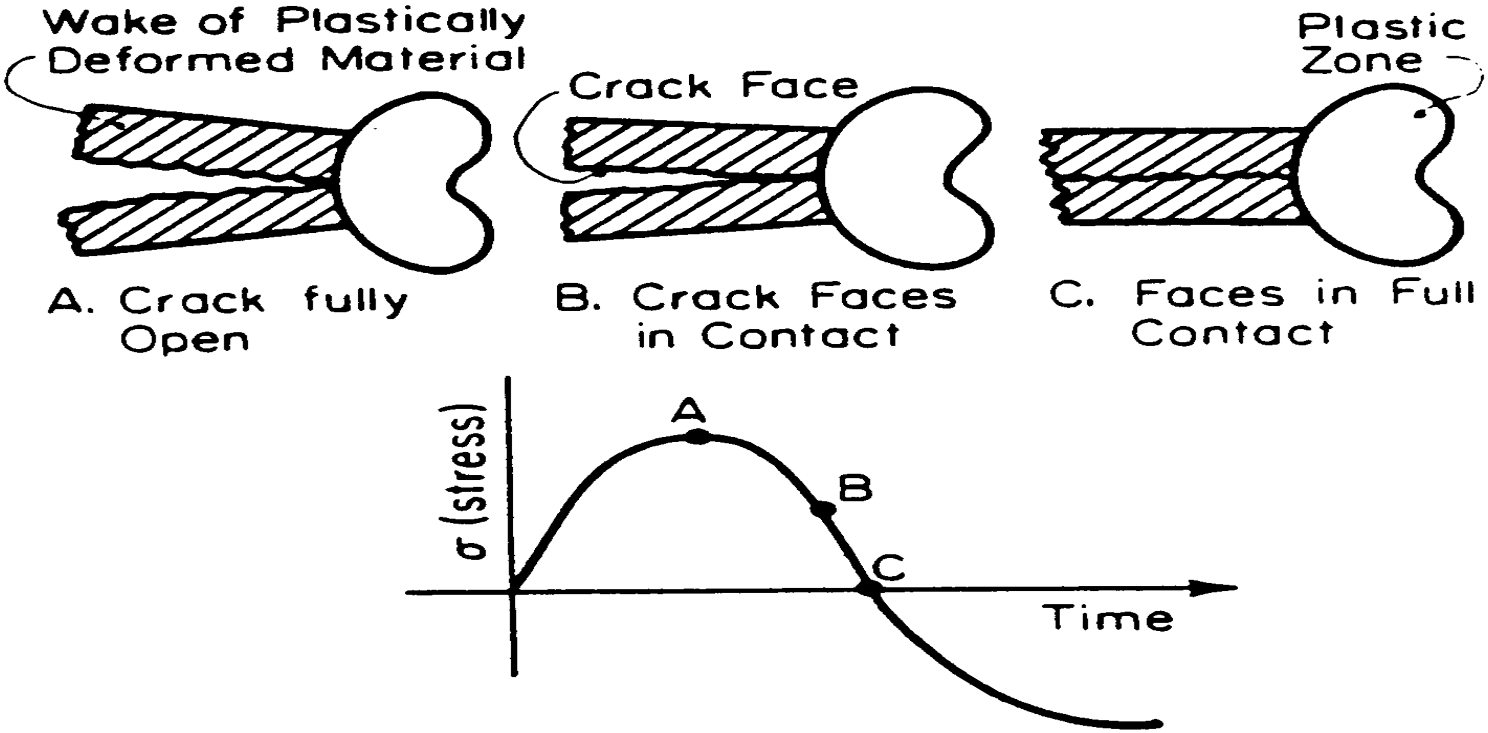
C. H+ acceleration

D. Corrosion products may retard crack growth at low ? K.

# FIX: Crack closure

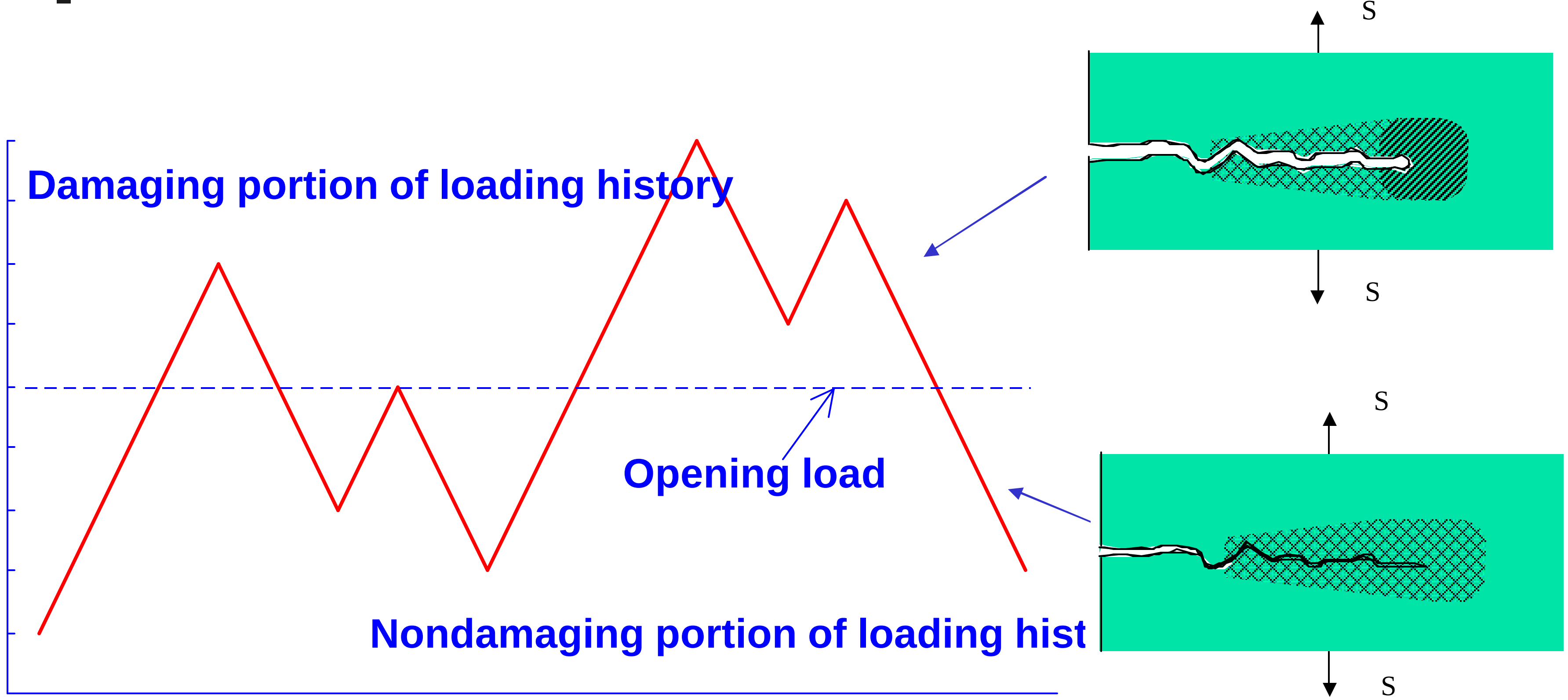


# CAUSE: Plastic Wake

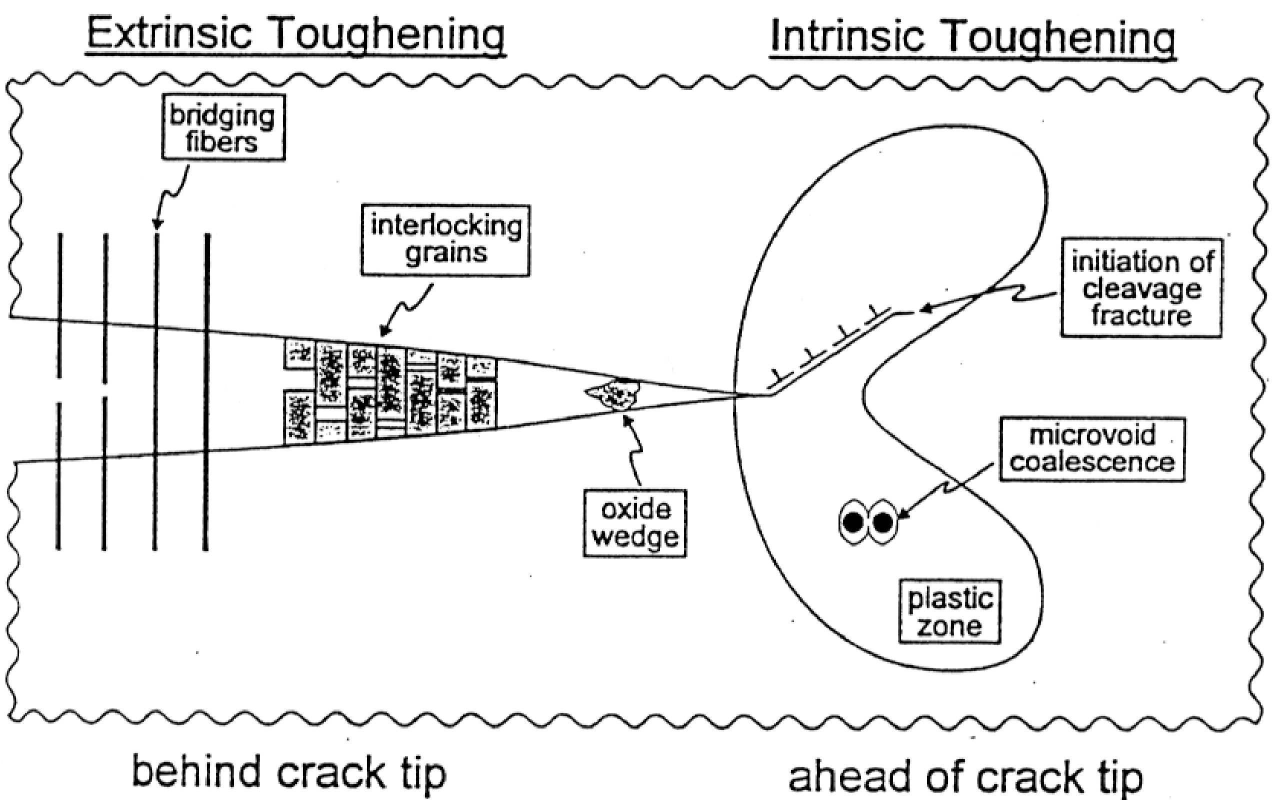



Short cracks can’t do this so they behave differently!

# “Effective” part of load history



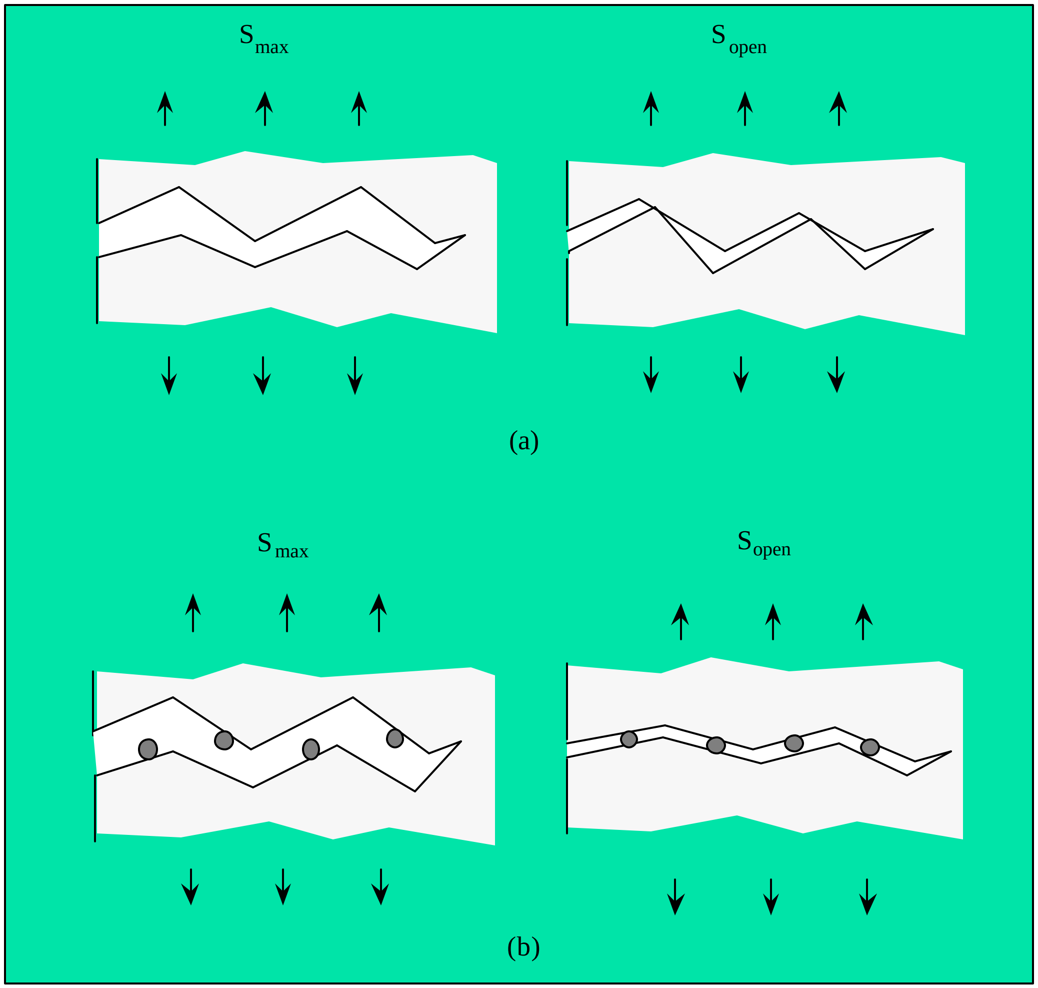
# MODEL: Crack closure




$$\frac{da}{dn} = C\ (\Delta K)^m (K_{max})^p$$

Extrinsic
Intrinsic

# Other sources of crack closure




Roughness induced crack closure (RICC)

Oxide induced crack closure (OICC)

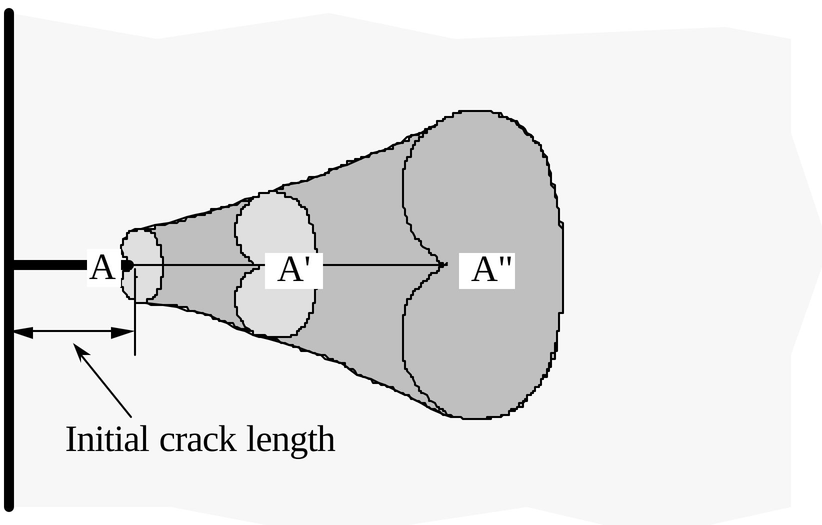
# MODEL: Crack closure

$? K_{eff} = U \, ? K$

$$U = \frac{\Delta K_{eff}}{\Delta K} = \frac{S_{max} - S_{open}}{S_{max} - S_{min}} = \frac{1}{1-R}\left(1 - \frac{S_{open}}{S_{max}}\right)$$

$U \approx 0.5 + 0.4R$ (sometimes)

Plasticity induced crack closure (PICC)



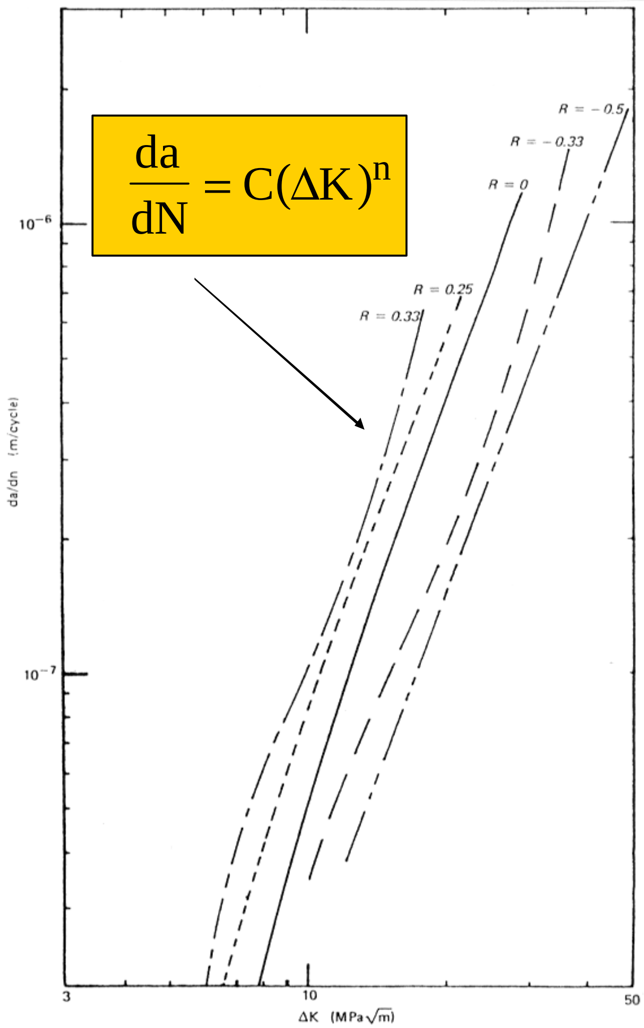

A, A', A" Crack tip positions

Plastic zones for crack positions A...A”

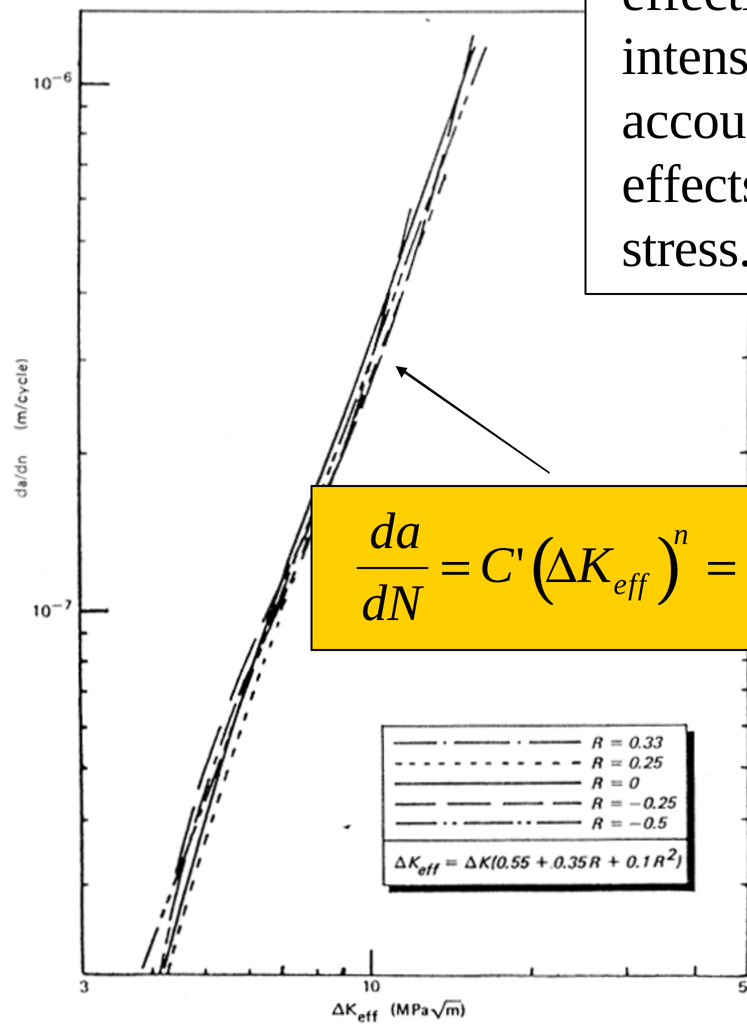
Plastic wake

# MODEL USE: ?K$_{effective}$

The use of the effective stress intensity factor accounts for the effects of mean stress.




$$\frac{da}{dN} = C(\Delta K)^n$$




$$\frac{da}{dN} = C'\left(\Delta K_{eff}\right)^n = C'(U\Delta K)^n$$