FCP Report No. 11

FATIGUE CRACK PROPAGATION LIFE PREDICTIONS FOR BUTT AND FILLET WELDS

by

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ABSTRACT

The philosophy, limitations and methods of fatigue crack propagation life calculations are discussed. Calculations are presented for the effects of bending stresses, undercut, weldment thickness, and internal defects upon the fatigue crack propagation life for butt welds. The methods have also been applied to a fillet weld geometry.

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LIST OF SYMBOLS USED

a	crack length for surface defect or half length for internal defect
$2a$	crack length for internal defect
a_o	initial crack length
a_i	instantaneous crack length
a_f	final crack length
$\frac{da}{dN}$	crack growth rate per cycle
A_o	nominal cross sectional area
A_r	cross sectional area through weld reinforcement
b	weld half thickness
$b_o,\ldots,b_4$	constants in stress polynomial
C,n	constants in crack growth power law
d	depth of undercut
h	height of weld reinforcement
ΔK	range in stress intensity
K_{1C} or K_C	critical value of stress intensity
ΔK_{th}	threshold value of ΔK for crack propagation
ℓ	length of fillet weld leg
N_T	total fatigue life
N_I	intiation portion of fatigue life
N_p	propagation portion of fatigue life
P	load
S	external stress applied to weld
t	plate thickness

w	width of weld reinforcement
x	distance inward from toe of weld
σ	stress at some internal point
θ,ϕ	angles defining weld geometry

1. INTRODUCTION

1.1 Background

The total fatigue life of a weldment can be divided into a fatigue crack initiation period and a fatigue crack propagation period. Future research may show that this separation is arbitrary and that one theory can be used to describe both phenomena. At present, however, the developed theories which treat crack initiation and propagation are dissimilar, and consequently predicting the total fatigue life requires a separate analysis for the initiation and the propagation portions.

Recent work by the writer has been directed toward predicting the fatigue crack propagation life of weldments [1,2]. Either of two rationale for this work can be used: The results of such calculations can be added to estimates of the initiation period when available or, particularly in weldments with preexisting discontinuities, crack propagation may be considered to comprise the major portion of the fatigue life.

In this report, the methods developed for calculating the fatigue crack propagation life in Fracture Control Program Report No. 6 [1,2] have been extended to treat additional aspects of butt and fillet welds. As in the former work, it is necessary to assume some initial size of the propagating fatigue crack. An initial crack size (a) of 0.01 inches has been the standard assumption. There are several justifications for this choice: smaller initial crack sizes may not obey the power law idealization of fatigue crack propagation behavior. Cracks 0.01 inches in half width (a) or larger are probably just barely detectable visually or with the use of standard NDT techniques such as ultrasonics and radiography.

1.2 Fatigue Crack Propagation Calculations

Predictions of the fatigue crack propagation life are based upon empirical relationships between the range in stress intensity (ΔK) and the resulting steady state fatigue crack growth rate (da/dN). It is important to realize that the predictions are no better than the data on which they are based. For instance, if one uses fatigue crack growth data for a particular heat of steel, tested at 20 Hz in dry air, error may result if the weld is subjected to a corrosive environment and/or different loading conditions. At present, most collected crack propagation data are for laboratory conditions.

The basis of all fatigue crack propagation life predictions is an empirically established relationship between ΔK and da/dN.

$$\frac{da}{dN} = f(\Delta K) \quad (1)$$

Estimates of the fatigue crack propagation life N_p can be obtained by integrating Eq. 1.

$$N_p = \int_{a_o}^{a_f} f(\Delta K)\, da \quad (2)$$

A good approximation of $f(\Delta K)$ is the so-called power law [3]; see Fig. 1.

$$\frac{da}{dN} = f(\Delta K) = C(\Delta K)^n \quad (3)$$

From Fig. 1 it is apparent that Eq. 3 fits the observed da/dN data over a wide range of ΔK values. Deviations from the power law are experienced at large ΔK values approaching K_{IC} on K_C and at very low values approaching ΔK_{th}. The deviations at high ΔK are of little consequence for present purposes since at high ΔK values the crack is propagating so rapidly that the error in the use of the power law approximation is small, i.e., only a small fraction of life is spent in the high ΔK regime. The deviations from power law behavior at low ΔK are potentially very serious since the majority of the fatigue crack propagation life occurs at low values of ΔK. In recognition of this difficulty all calculations of N_p have been performed using a_o of 0.01 inches or larger, i.e., over ranges of ΔK for which the power law should be a valid approximation. Future work using approximations which better model the low ΔK regime may allow more accurate N_p predictions and may permit an even greater fraction of the total fatigue life N_T to be considered as crack propagation. At present using a_o of 0.01 inches and the power law (Eq. 3) leads to N_p predictions between one-third to two-thirds of N_T [1]. This point will be discussed in detail in a sequel Fracture Control Report.

1.3 The Superposition Method of Determining ΔK

To evaluate Eq. 2 and thereby obtain N_p for a weld, ΔK must be known:

$$\Delta K = \sigma\sqrt{\pi a}\, f(a) \tag{4}$$

The exact form of Eq. 4 will vary with geometry and loading condition. Closed form solutions are available for a number of simple cases but solutions for more complex situations are generally unavailable. A powerful method for obtaining ΔK for arbitrary geometries and loadings was developed in a previous report [1,2] and will be outlined below.

One wishes to find the stress intensity factor for a crack in a loaded body. If a closed form solution is unavailable, the stress intensity factor may be found indirectly using the principle of superposition, see Fig. 2.

The stress field in a loaded body (1) can be considered to be the sum of the stress field in a similar body (2) with the crack held closed (really, an uncracked body) by a system of forces (which must be found) and the stress field in a similar but unloaded body (3) with the interior surface of the crack held open by the negative of the system of forces which were determined for body 2. The crucial point is that ΔK associated with a loaded crack in Body (3) is identical with the ΔK sought in body (1).

Operationally, one proceeds as follows: (a) Idealize the weld geometry as in Fig. 3 and (b) find the system of stresses necessary to hold the crack closed using finite element methods (see Fig. 4), and (c) fit the stresses along the crack with a polynomial,

$$\frac{\sigma}{S} = b_0 + b_1\left(\frac{x}{t}\right) + b_2\left(\frac{x}{t}\right)^2 + b_3\left(\frac{x}{t}\right)^3 + b_4\left(\frac{x}{t}\right)^4 \qquad (5)$$

where

x = coordinate along crack surface

so that one has an analytical function for stress along the crack interface. (The weld is at this point considered to be body 2). With the system of forces necessary to hold the crack closed (Eq. 5) now determined one can (d) place these stresses along the internal surface of an unloaded body and determine the stress intensity factor from the relationship given by Emery [4,5]; see Fig. 5.

$$K = \Delta K = \sqrt{\pi a}\,\{1.1\,\sigma_a - \int_0^a f(\tfrac{x}{a})\,\frac{d\sigma}{dx}\,dx\} \qquad (6)$$

where

$$f(\tfrac{x}{a}) = 0.8(\tfrac{x}{a}) + 0.04(\tfrac{x}{a})^2 + 0.362 \times 10^{-5} \exp 11.18(\tfrac{x}{a})$$

σ_a = stress at crack tip

a = crack length

x = coordinate along crack surface

$\frac{d\sigma}{dx}$ = determined from Eq. 5

Since K (and therefore ΔK for zero to tension loading conditions) can be found, N_p can be evaluated by substituting the expression for ΔK (Eq. 6) into the power law (Eq. 3) and solving Eq. 2.

$$N_p = \int_{a_o}^{a_f} \frac{da}{C[\sqrt{\pi a}\,\{1.1\sigma_a - \int_0^{a_i} f(\tfrac{x}{a_i})\,\frac{d\sigma}{dx}\,dx\}]^n} \qquad (7)$$

The final expression is difficult to evaluate directly and requires the use of numerical integration techniques.

2. RESULTS

2.1 Scope

The methods outlined above have been applied to several common problems encountered in the fatigue of welds. For butt welds the effect on N_p of bending stresses, undercut, weldment thickness and internal defects will be discussed. Finally N_p predictions for a second weld geometry, a double fillet weld will be presented. Throughout this report, calculations will be based upon the crack propagation constants (C,n) reported by Barson [6,7] as average for ferritic pearlitic steels ($C = 0.36 \times 10^{-9}$, $n = 3.0$). Unless otherwise noted the initial crack size is assumed to be 0.01 inches ($a_o = 0.01$ inches). Also, a typical weld profile for a butt weld is taken to be $\theta = 20°$ and $\phi = 60°$.

2.2 Effect of Bending on N_p

In FCR #6 [1] the N_p for butt welds subjected to 0 to tension direct stress were determined for various geometries (θ and ϕ), varying assumptions of initial flaw size (a_o) and material properties. The N_p calculated for a butt weld subjected to pure bending is given in the table below and is plotted in the S-N diagram of Fig. 6. A comparison is made between direct stress (S) and pure bending which causes an extreme fiber stress of S. The conditions for this set of calculations are a typical butt weld geometry ($\theta = 20°$, $\phi = 60°$, $t = 1.0$ inch) a ferritic pearlitic steel ($C = 0.36 \times 10^{-9}$, $n = 3.0$) and an initial flaw size (a_o) of 0.01 in.

For a given value of stress (S) and for the above stated conditions, N_p for pure bending is approximately four times the N_p for direct stress. This result is of considerable value and permits the general case of combined direct stress and bending to be treated by superposition of stresses.

Table 1

N_p Values for Bending and Direct Stresses
(Double-V butt welds)

$\phi = 60°$, $\theta = 20°$, t = 1:, $a_o = 0.01"$

Stress (S) ksi	N_p (thousands of cycles)	
	Direct Stress	Bending Stress
20	376.5	724.6
32	91.9	176.9

2.3 Effect of Plate Thickness on N_p

The weld reinforcement geometry produces a stress concentration which most greatly influences the rate of fatigue crack growth during the time that the (toe) fatigue crack is very small. Since the major part of the fatigue crack propagation life N_p is accumulated at small crack sizes, it follows that the nature of the weld reinforcement can exert a large influence on N_p. Is this influence the same in large and small thickness weldments of identical geometry?

The stress field associated with the toe of the weld is geometrically similar in geometrically similar welds (of various thicknesses). The rate of fatigue crack growth depends only upon local values of stress.

Consently, an initial 0.01 inch crack will find itself in a higher stress field which does not decay as rapidly with distance inward from the toe of the weld in a large weldment. Assuming the existence of 0.01 in. initial cracks in large (4 in. thick) welds leads to shorter calculated N_p values than the same 0.01 in. initial crack in smaller thickness welds (3/8 in.) even though the thickness of material available for propagation is larger. This trend is shown in the table below which has been calculated assuming various plate thicknesses (t) (3/8 in. to 4 in.) and two reinforcement flank angles ($\theta = 20°$ and $\theta = 30°$). N_p is defined as the number of cycles required for propagation from 0.01 in. (a_o) to 0.2t (a_f). The results are also plotted on an S-N diagram shown in Fig. 7.

Table 2

N_p Value for Various Thickness Butt Welds
$\phi = 60°$, $a_o = 0.01"$

Thickness t (inches)	N_p (thousands of cycles)			
	$\theta = 20°$		$\theta = 30°$	
	S = 20 ksi	32 ksi	S = 20 ksi	32 ksi
3/8	380.5	92.9	346.4	84.6
5/8	386.4	94.3	343.9	84.0
1	376.5	91.9	327.0	79.8
2	375.7	91.7	320.6	78.3
4	368.5	89.9	309.4	75.6

The differences between N_p calculated for 4 in. and 3/8 in. weldments are not large but the 10 percent difference points out the following important point: The fatigue life of a weld (in which propagation plays a major role) is not necessarily increased by increasing its thickness particularly if the geometry of the weld reinforcement is scaled up proportionately. In the latter case even a reduction in N_p is possible.

2.4 Effect of Undercut on N_p

A common problem associated with V-groove butt welds and also with fillet welds is undercutting and/or incomplete filling of the V-groove. This problem can be easily approximated: see Fig. 8. Undercutting can be modeled by letting the weld reinforcement start at a point below the top surface of the plate forming a pseudo V-notch defined by the angle of edge preparation (ϕ) and the flank angle (θ). The approximate angle of the notch associated with the undercut is $90° - \theta + \phi/2$ or in the present instance 100°. The condition of a weld without any undercut is d equal to 0. A 0.1 in. undercut

Table 3

Effect of Undercut Depth, d, on Fatigue Life, N_p
$\phi = 60°$, $\theta = 20°$, $t = 1"$, $a_o = 0.01"$

Depth of Undercut (d)	N_p (thousands of cycles)	
	S = 20 ksi	32 ksi
0.0"	442.2	108.5
0.02"	324.3	79.2
0.05"	266.4	65.1
0.10"	171.6	41.9

results in a large decrease in N_p (from 442,000 to 171,600 at 20 ksi). If one compares the 0.1 in. undercut (plus a 0.01 in. (a_0) crack) with an initial crack size (a_0) of 0.1 in., one finds that the undercut of length (d) is nearly as serious a defect as an initial crack of that length. (N_p for a_0 = 0.1 in. is ~ 80,000 cycles). Undercuts (d) small relative to the initial crack length (a) should be of little consequence.

2.5 Internal Defects

The problem of internal weld defects is common and troublesome in practice. Welds containing an internal lack of fusion or lack of penetration may be considered as having two competing sites for fatigue crack initiation and propagation: the toe of the weld and the internal defect. One is really concerned only with the site which first causes failure. Consequently, internal defects below a certain, small dimension are of little consequence since failure will be caused by initiation and propagation from the toe of the weld. Identification of this internal flaw size would be of considerable practical interest, and we have made a first attempt at applying the fatigue crack propagation analysis to the problem.

The conditions assumed in the calculations performed are sketched in Fig. 9. The N_p for propagation from the toe of the weld has been calculated for various assumptions of internal flaw size: see also Table 4 below and Fig. 10. It can be seen that the presence of the internal flaw lowers the N_p for propagation from the toe of the weld due to the fact that the internal flaw further concentrates stress near the toe of the weld. For example a 0.12 in. (2a) lack of penetration defect in a butt weld of

Table 4

N_p Values for Various Sized Internal Flaws

S (ksi) \ 2a (in.)	N_p (thousands of cycles) 0.044"	0.07"	0.10"	0.12"	0.15"	0.20"
	1. Origin of crack at toe of weld					
24	374.2	354.6	326.9	307.6	278.8	227.5
16	1426.3	1351.5	1245.9	1172.5	1062.6	867.1
	2. Origin of crack at internal flaw					
	a) $\Delta K = \sigma\sqrt{\pi a}\,(\sec \frac{\pi a}{2b})^{1/2}$; $\sigma = P/A_r$					
24	1304	676.0	402.7	277.4	257.0	53.5
16	4967	2578.0	1533.4	1056.0	598.0	203.8
	b) $\Delta K = \sigma\sqrt{\pi a}\,(\sec \frac{\pi a}{2b})^{1/2}$; $\sigma = P/A_o$					
24	429.0	237.8	132.6	91.3	51.7	17.6
16	1636.0	906.5	505.4	348.1	197.1	67.2
	c) Superposition principle					
24	1001.0	655.6	448.3	357.3	256.3	137.1
16	3814.0	2498.8	1708.7	1361.7	976.8	522.7
	d) Superposition principle with secant term					
24	769.0	440.4	256.6	182.7	109.4	41.5
16	2931.0	1678.7	978.0	696.4	416.8	158.0

1/4 in. plate reduces the calculated N_p from approximately 400,000 cycles (zero sized internal defect) to about 300,000 cycles.

A variety of conditions were assumed for the calculation of N_p from the internal defect. One reasonable model is to assume that:

$$\Delta K = \Delta\sigma \sqrt{\pi a} \left(\sec \frac{\pi a}{2b}\right)^{1/2} \qquad (8)$$

where

$$\Delta\sigma = \frac{P}{A_r}$$

A_r is the cross sectional area of the weld in the plane containing the defect and the weld reinforcement: see Fig. 9.

A second and perhaps less reasonable assumption is to assume that the stress (σ) in Eq. 8 is given by $\sigma = \frac{P}{A_o}$ where A_o is the cross sectional area of the plates.

Both of the above expressions may suffer from inaccuracy resulting from the complex distribution of stress in the vicinity of the defect due to the discontinuities associated with the weld reinforcement. To test this idea, N_p was calculated in a manner similar to that used to calculate N_p for the external flaws (Eq. 7 less the factor 1.1 which is the free surface correction). This assumption neglects the fact that the crack is propagating in a finite body, so a final calculation was made assuming ΔK to be given by Eq. 6 (as above) but multiplied by a finite body correction factor as in Eq. 8 which is given below:

$$\left(\sec \frac{\pi a}{2b}\right)^{1/2} \qquad (9)$$

This last assumption is believed to most accurately represent the physical situation. A listing of the results is given below and in Fig. 10.

If one assumes that the superposition principle with the secant term (curve 2d in Fig. 10) is the most accurate estimate of N_p for an internal defect, then the condition that N_p for propagation from the toe of the weld equals N_p for the internal defect (i.e., the point at which the internal flaw begins to dominate) occurs for internal defects of 0.08 inches (2a) or larger.

These results are preliminary and the problem requires further investigation. The triangles in Fig. 10 represent the data available at this writing. The total fatigue lives represented by these data are less than any of the predictions (except curve 2b which is clearly based on a crude assumption). Since the fatigue crack propagation is occurring through weld metal in this instance, it is likely that the C and n values for base metal which were used in all calculations are inappropriate. Research is continuing on this point and on the effects of residual stresses on N_p from internal defects.

2.6 Fillet Welds

The methods developed for calculating the fatigue crack propagation life N_p from the toe of the reinforcement in butt welds can be applied to the fillet weld problem shown in Fig. 11. The crusiform joint is considered to be welded with four, full-penetration welds. It was assumed that the plates were all of 1 in. thickness and that the joint was loaded in tension along one axis (x-axis) only. The size of the fillet (ℓ) and the weld profile (as determined by the angle (θ)) were allowed to vary. The weld profile was assumed to be triangular in cross section when θ = 45° and a segment of a circle for θ larger or smaller than 45°. θ = 15° defines a concave profile θ = 75° defines a convex profile, for example.

Table 5

N_p Values for Various Fillet Weld Geometries

a_o = 0.01", t = 1"

Length of Weld leg ℓ	Stress S (ksi)	N_p (thousands of cycles) θ 15°	30°	45°	60°	75°
1/4"	20	215.40	182.60	177.80	187.60	193.50
	32	52.58	44.58	43.41	45.80	47.20
1/2"	20	211.70	183.30	167.60	176.00	179.50
	32	51.68	44.75	40.91	42.96	43.82
5/8"	20	219.70	182.90	167.30	179.90	172.90
	32	53.64	44.67	40.85	43.91	52.22

The results of the above calculations indicate that N_p for the fillet weld is sensitive to changes in θ (a measure of the weld contour) and for θ less than 45° (concave weld profile) N_p becomes larger. For example for ℓ = 1/4" S = 20 ksi, N_p = 177,800 cycles when θ = 45° and N_p = 215,400 cycles when θ = 15°. The length of the weld leg (ℓ) was found to have only a very small effect on N_p.

The influence of θ and ℓ upon N_p was much smaller than anticipated. Furthermore it was not anticipated that N_p would rise for θ larger than 45°

(convex weld profile). This rise may be spurious and a result of inaccuracies in the finite element program for θ larger than 45°. It is also possible that the result is due to increases in the gradient of stress inward from the toe. To resolve this problem, a finer finite element mesh in the vicinity of the toe of the weld would be required. Unfortunately this would require a larger computer capacity than is currently available on a routine basis.

The small influence of θ and ℓ upon N_p leads one to speculate that N_p may be more influenced by the thickness of the attached plates or whether or not the welds are full penetration or only partial penetration. These variables will be considered at a later time.

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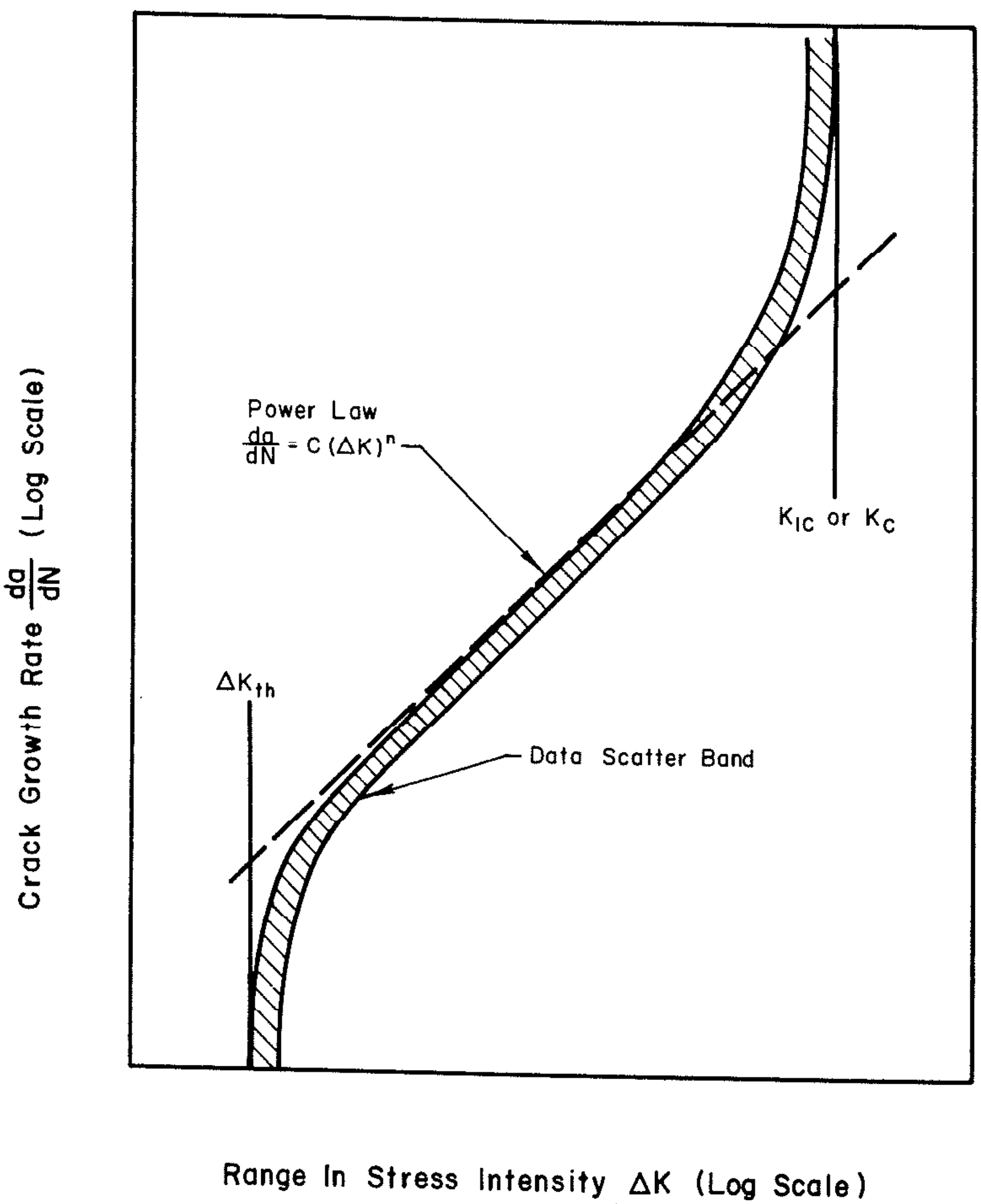


Fig. 1 Crack Growth Rate as a Function of Range in Stress Intensity

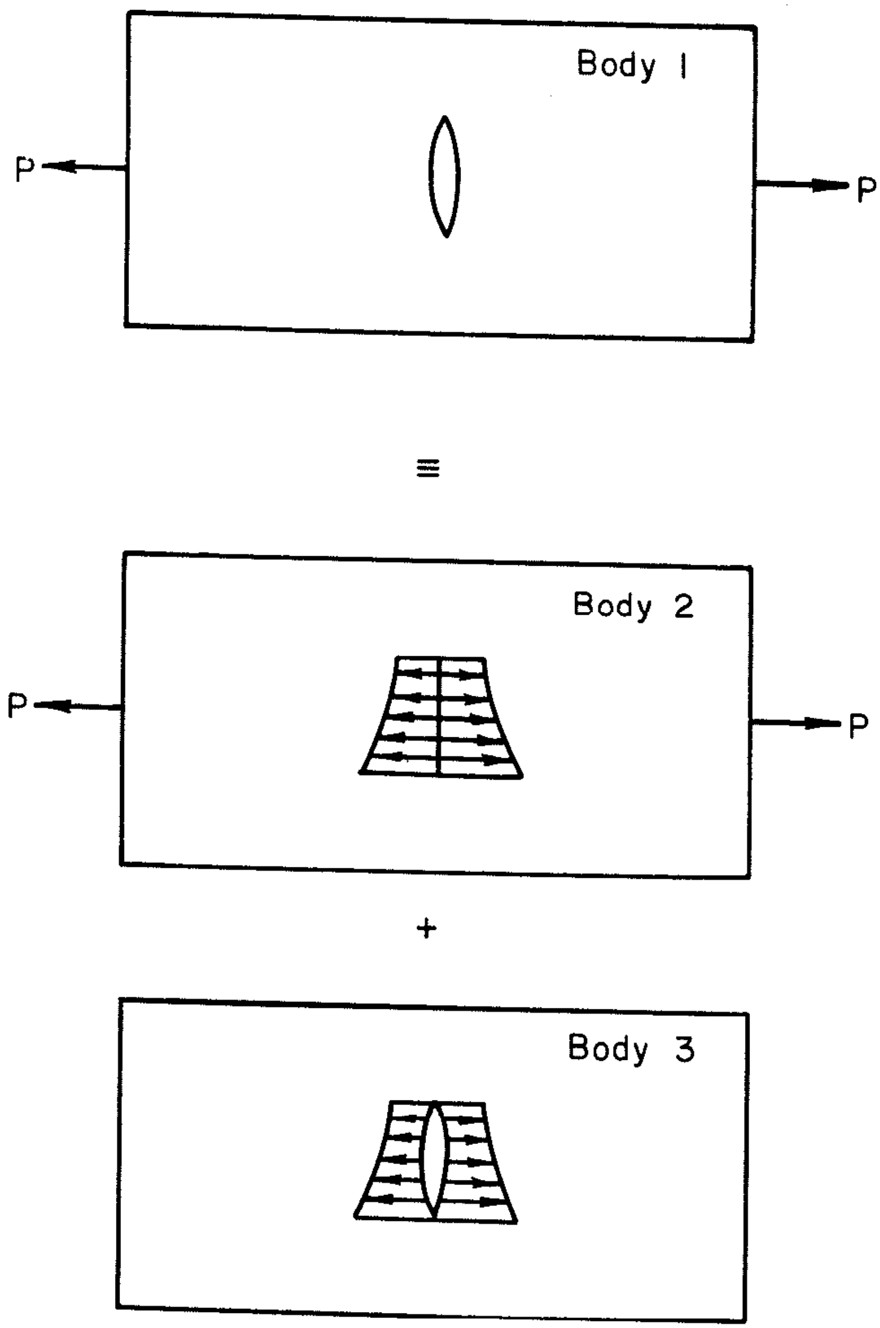
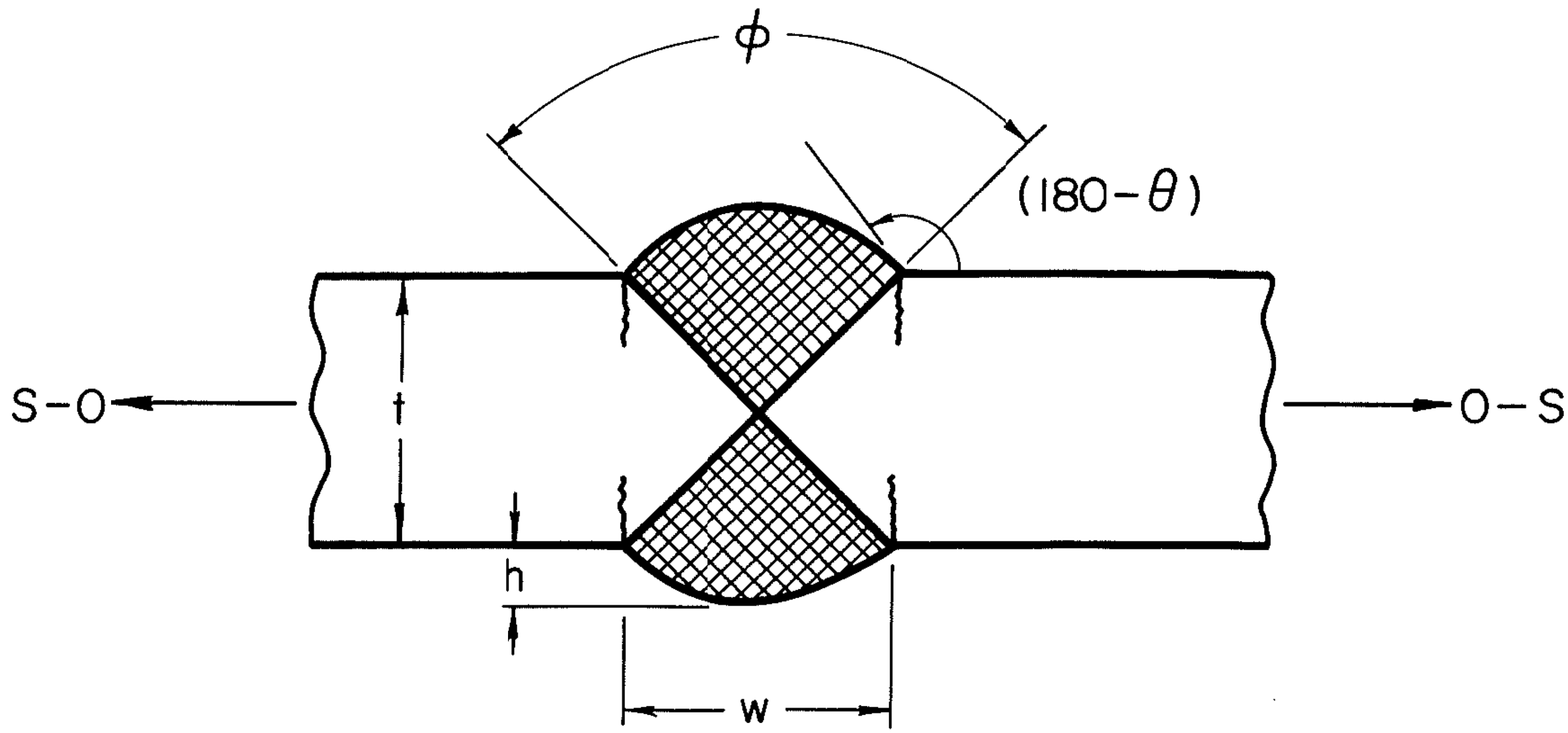


Fig. 2 Superposition of Stresses in a Flawed Body



$$h/w = 1/2 \tan \theta/2$$

$$w/t = \tan \phi/2$$

Fig. 3 Geometric Variables for a Double-V Butt Weld

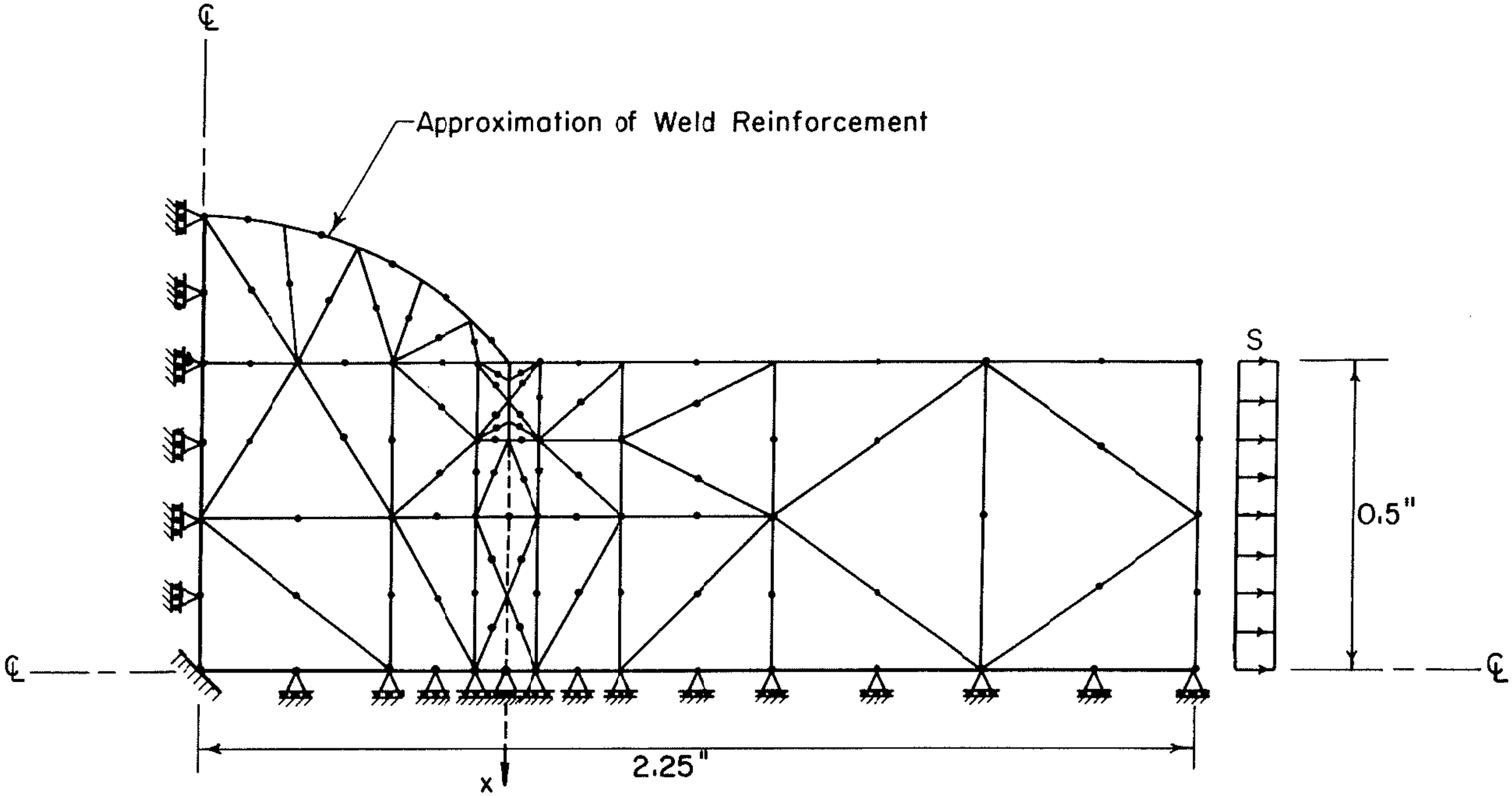
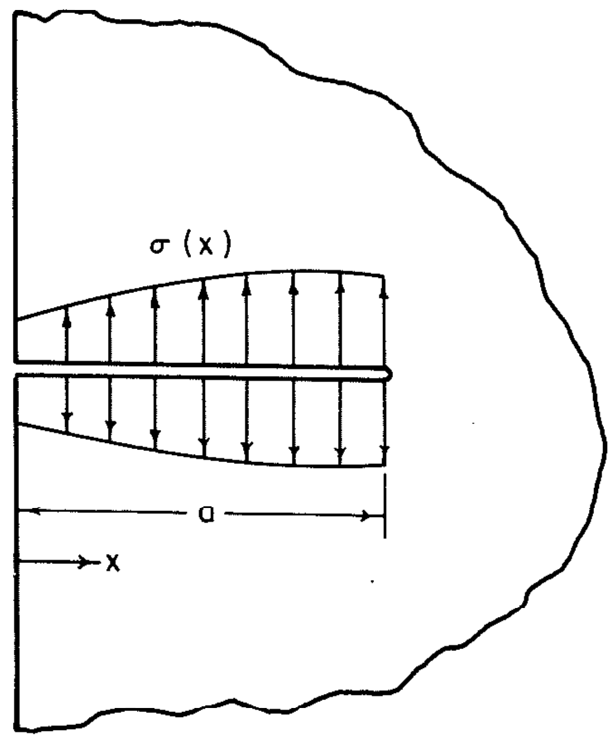


Fig. 4 Finite Element Network for Determining Stress Distribution Near the Toe of a Butt Weld



$$K = \sqrt{\pi a}\left\{1.1\,\sigma - \int f\left(\frac{x}{a}\right)\cdot\frac{d\sigma}{dx}\,dx\right\}$$

$$f\left(\frac{x}{a}\right) = 0.8\left(\frac{x}{a}\right) + 0.04\left(\frac{x}{a}\right)^2 + 3.62 \times 10^{-6} \times e^{11.18\left(\frac{x}{a}\right)}$$

Fig. 5 Stress Intensity Factor for an Edge Crack Loaded with an Arbitrary System of Internal Stresses

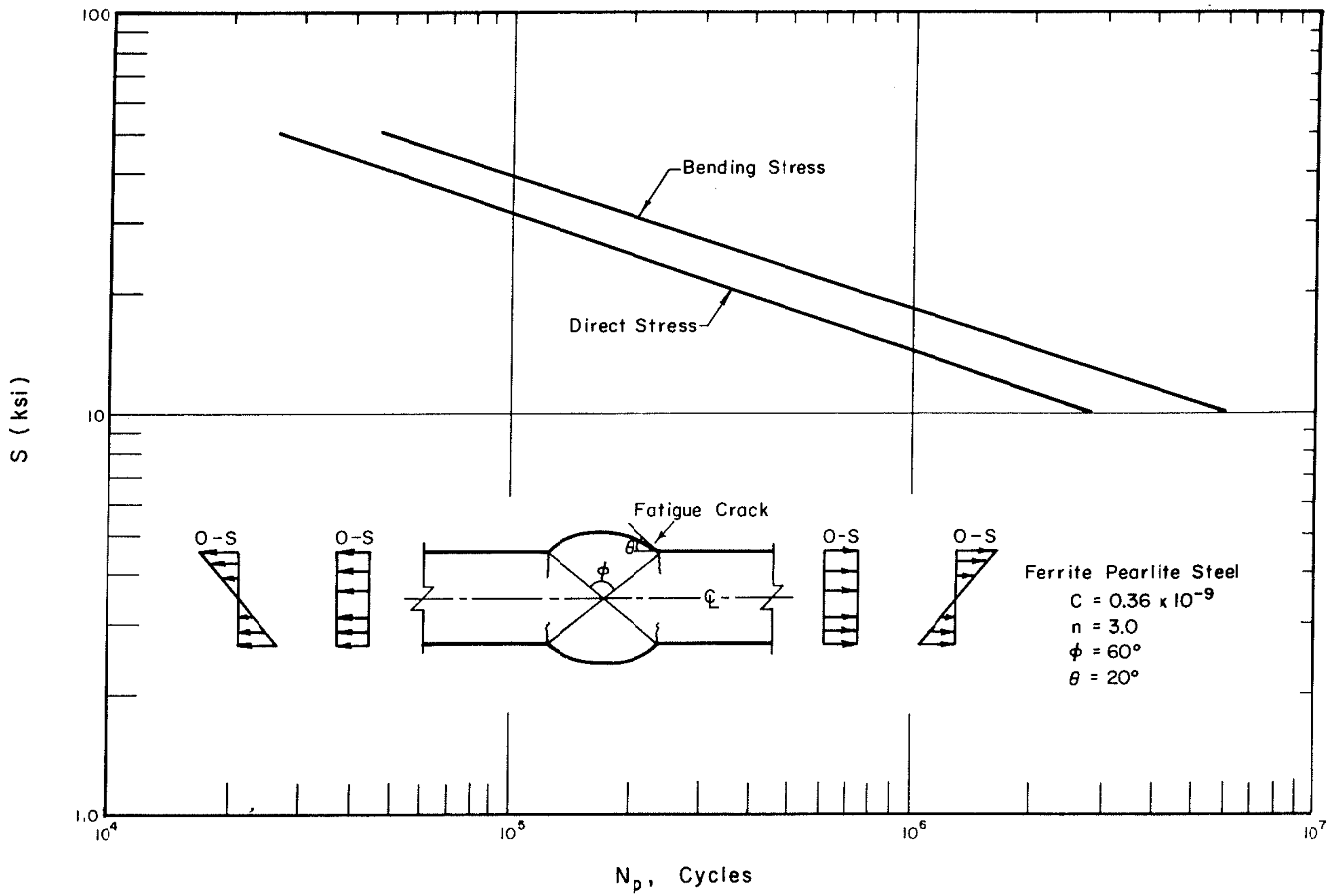


Fig. 6 Effect of Bending Stresses on N_p for a Butt Weld

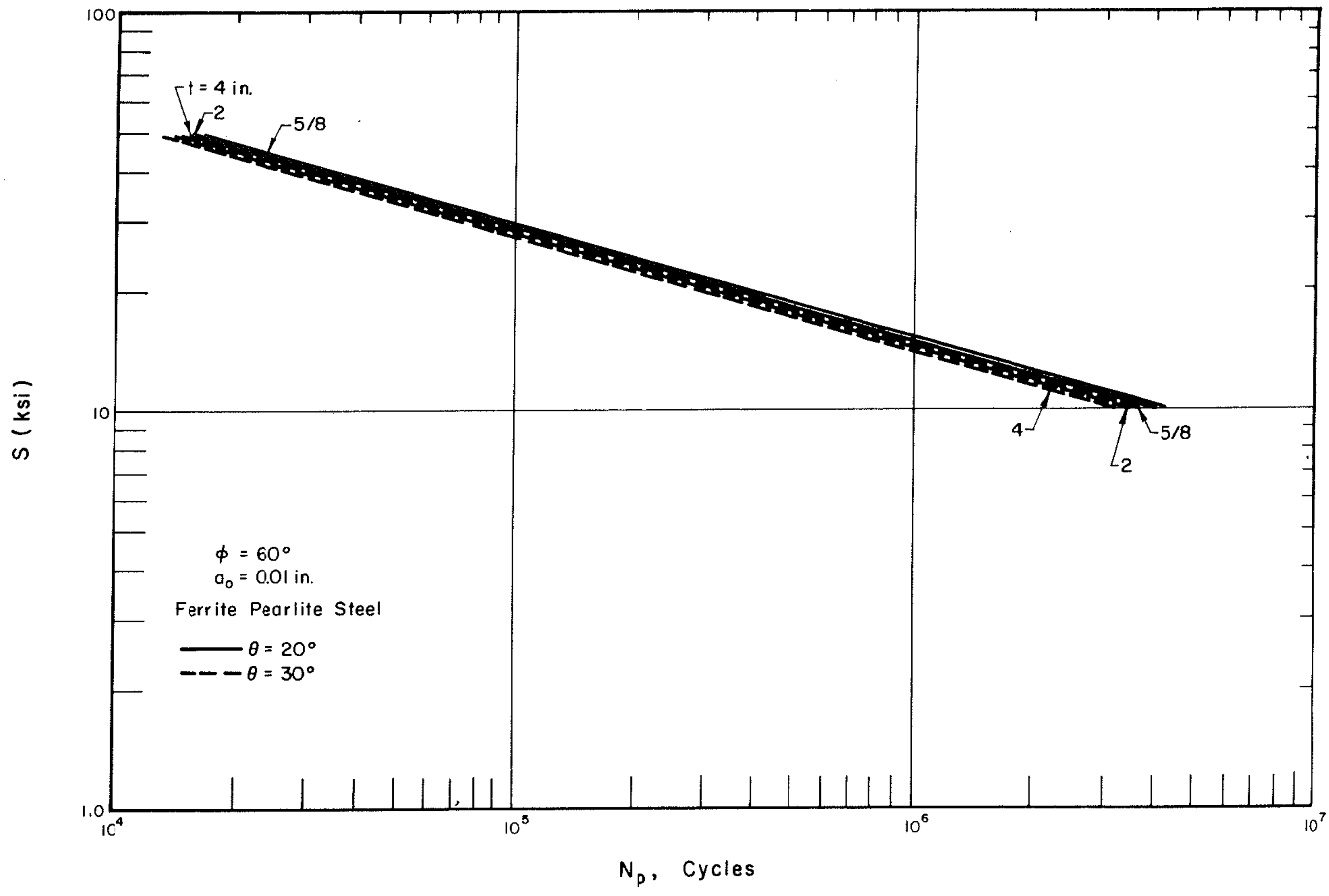


Fig. 7 Effect of Thickness on N_p for a Butt Weld

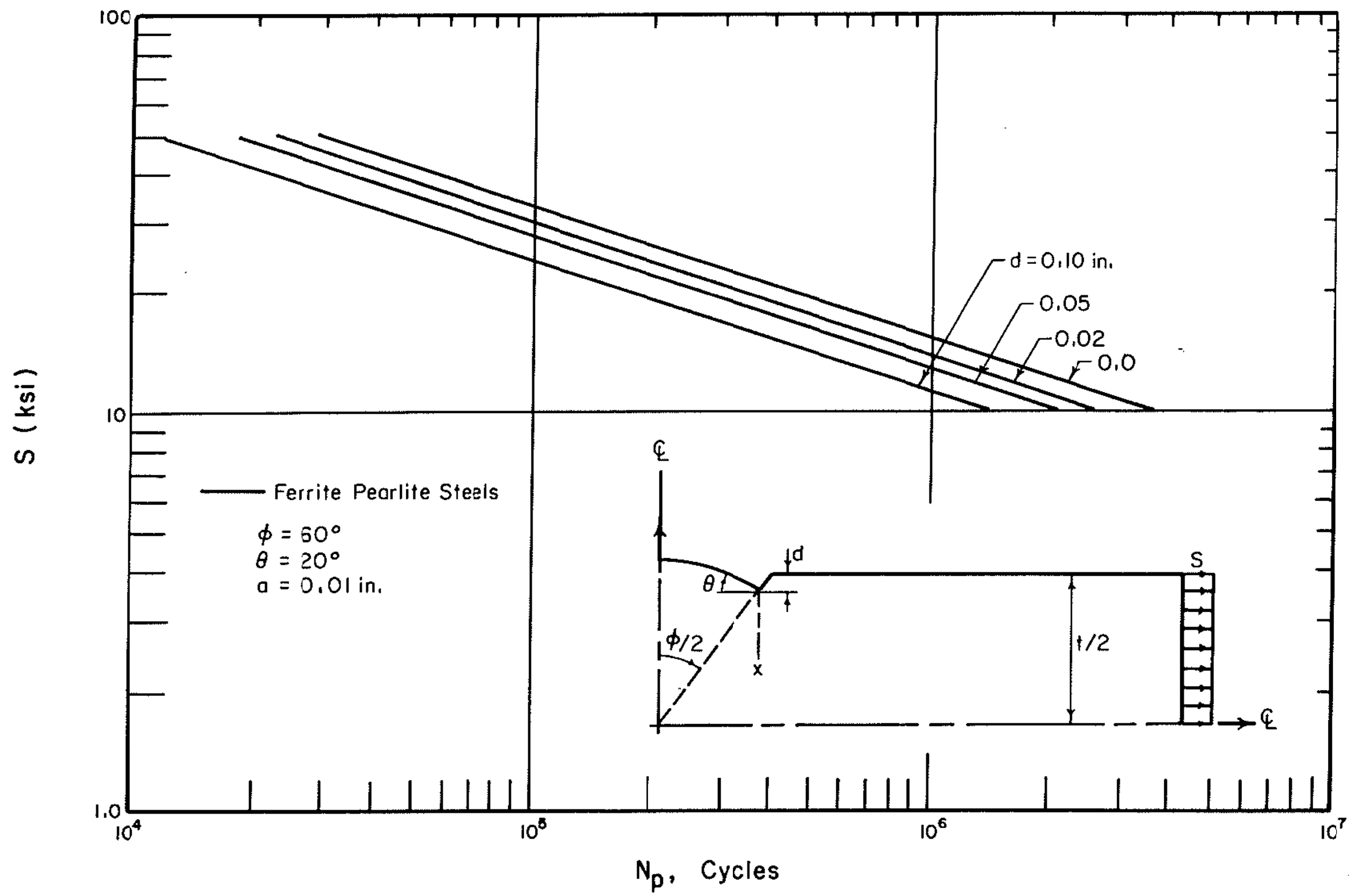


Fig. 8 Effect of Undercut on N_p for a Butt Weld

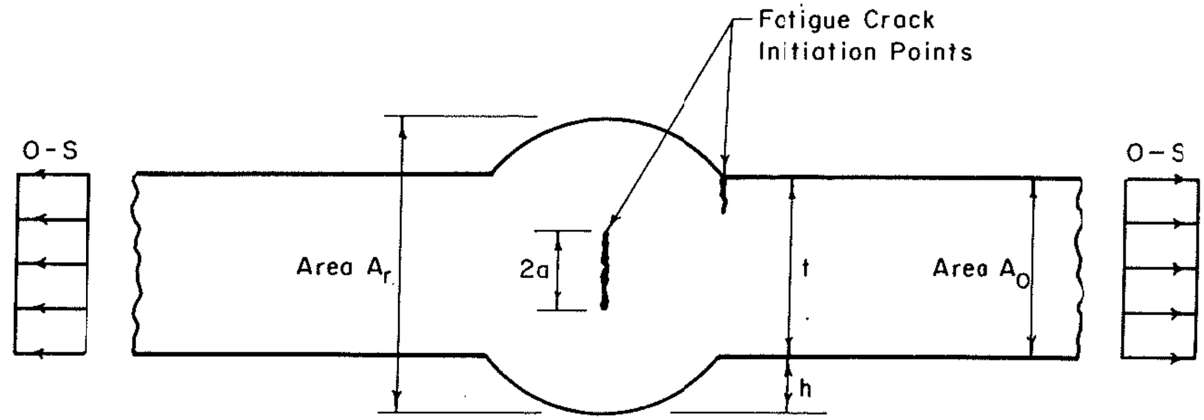


Fig. 9 Geometry Considered in the Investigation of N_p from Internal Weld Defects

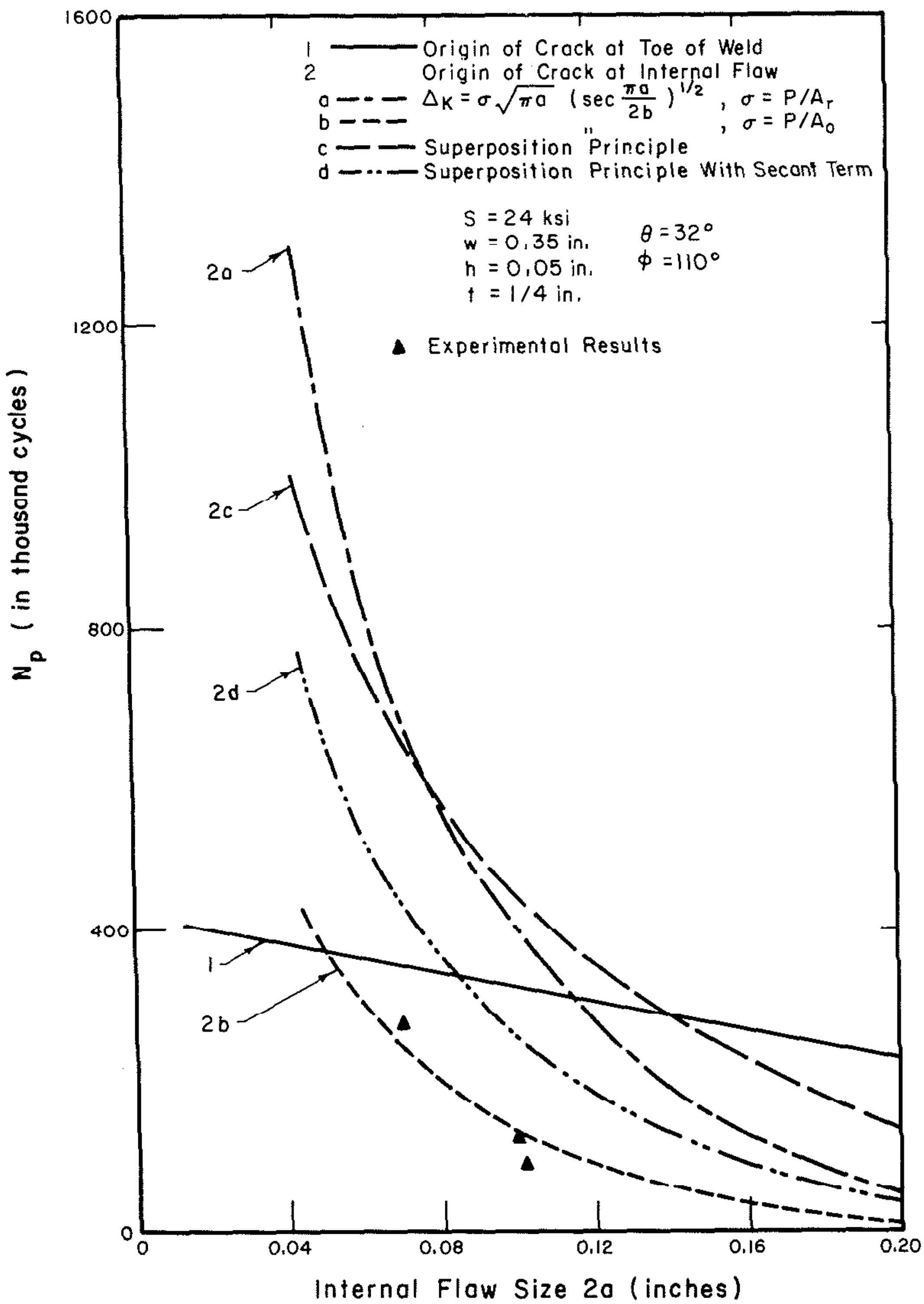


Fig. 10 Calculated Fatigue Crack Propagation Life versus Internal Defect Size

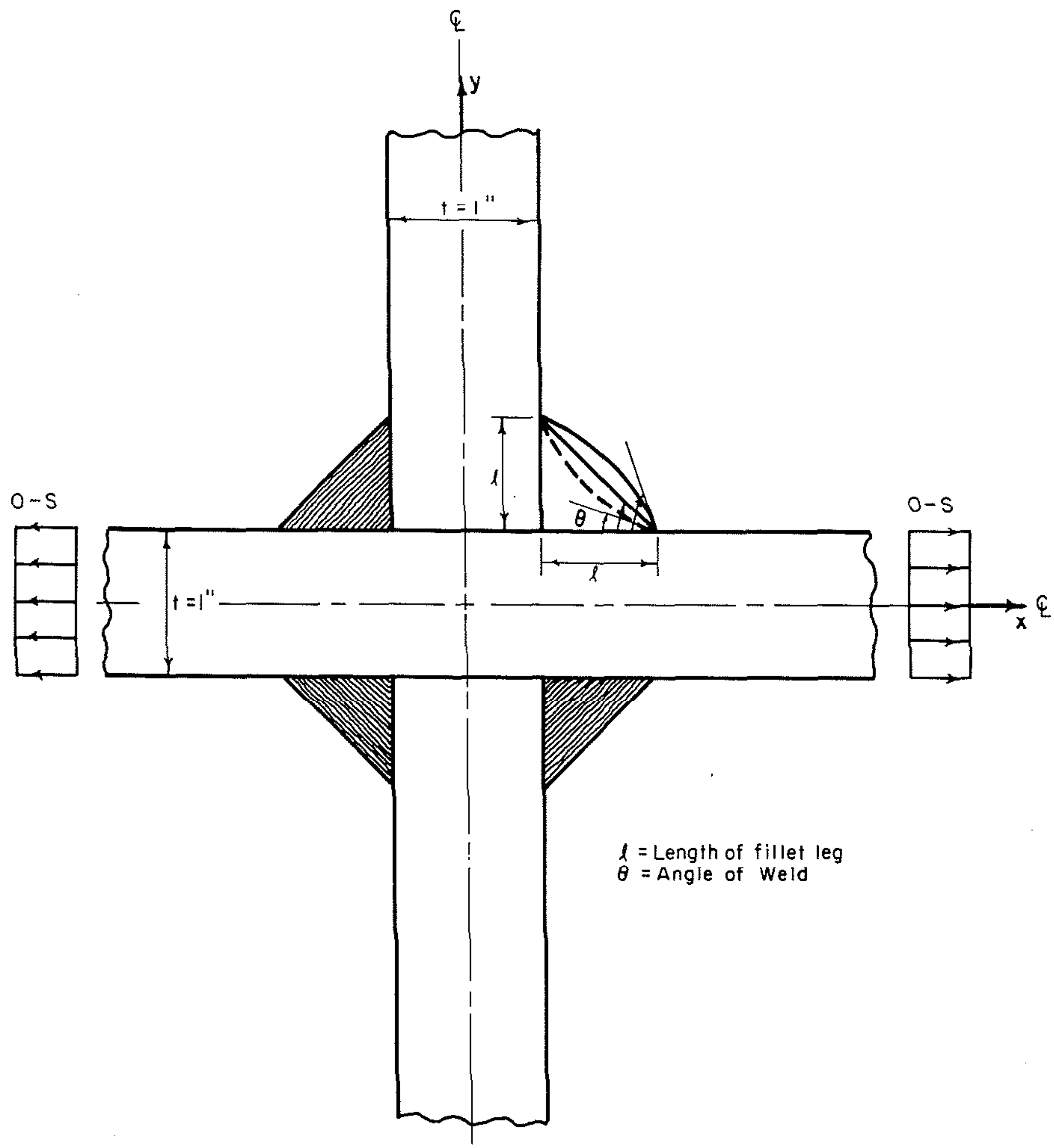


Fig. 11 Fillet Weld Geometry

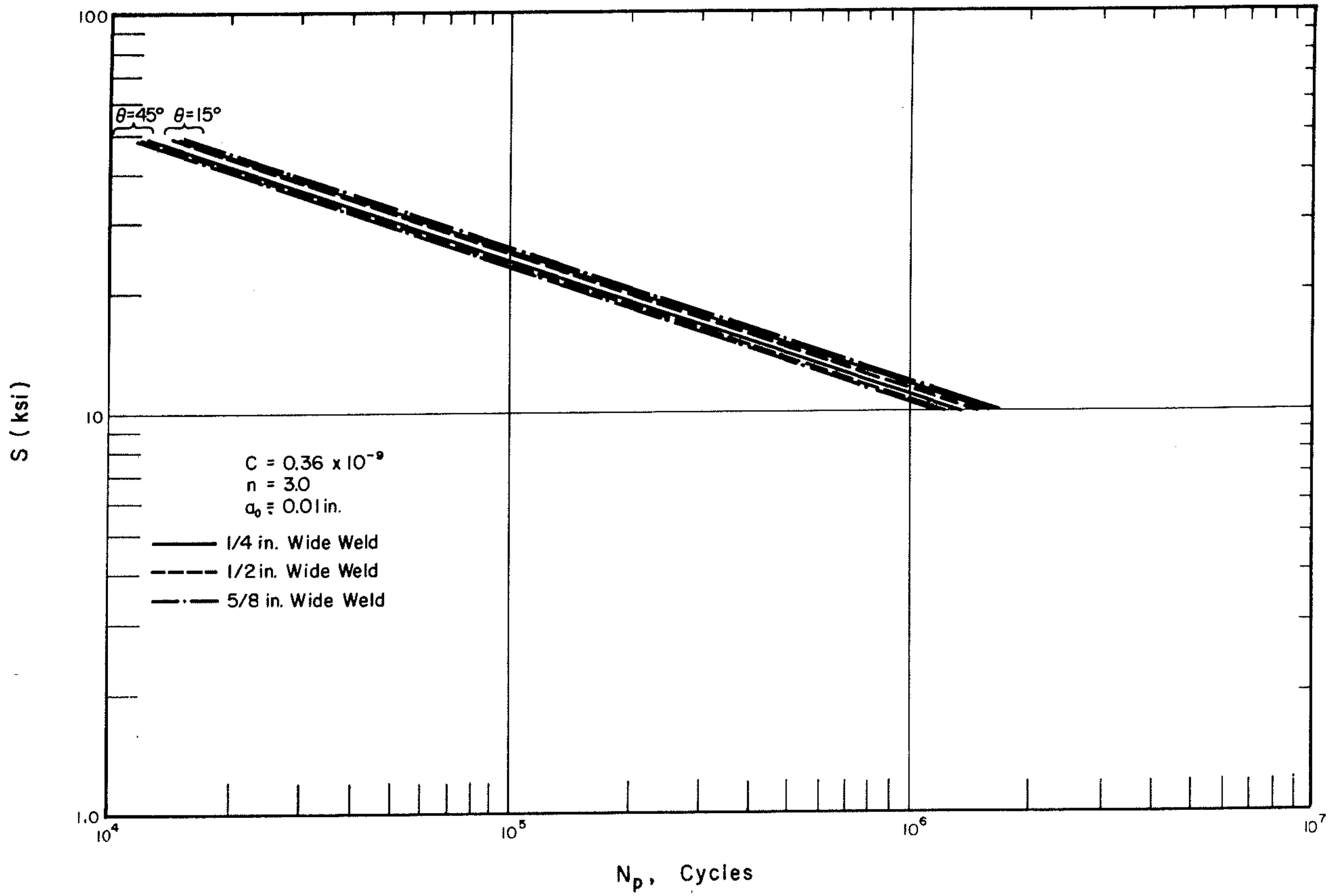


Fig. 12 Effect of Fillet Weld Leg Length and Profile on N_p