FCP Report No. 5

DETERMINATION OF THE FATIGUE NOTCH FACTOR IN THIN PLATES

by

D. T. Raske
Department of Theoretical and Applied Mechanics

ABSTRACT

A method is presented to determine the fatigue strength reduction factor, K_f, for uniaxially loaded notched plates. This includes an equation for K_f which is derived by correcting a critical zone of metal at the notch root for the effects of the stress gradient and the volume of this zone. The effects of local plastic action are included to extend the usefulness of this equation to all fatigue lives. Sample calculations to illustrate the use of this equation are also given.

A Report of the
FRACTURE CONTROL PROGRAM

College of Engineering, University of Illinois
Urbana, Illinois
October, 1972

INTRODUCTION

It has been well established that small geometrical stress concentrations such as notches, etc., do not reduce fatigue strength of metals as much as predicted by the theoretical elastic stress concentration factor, K_t. To take advantage of this difference in terms of higher loads or longer fatigue lives, designers must either estimate the actual fatigue notch factor, K_f, analytically or determine its value by experiment. Even in those cases where K_f is determined solely by experiment, an initial estimate is often required for "sizing" the part of interest.

This report, therefore, presents a systematic means by which K_f can be accurately assessed for uniaxially loaded plates. In order to apply this technique, the designer must have access to the unnotched fatigue performance of the metal, and an elastic-plastic finite element computer program.

DETERMINATION OF THE FATIGUE NOTCH FACTOR

To derive an equation to predict K_f, the following premises were assumed valid (1)*:

(1) The stress distribution over a distance δ from the notch root (rather than the peak stress) governs the fatigue initiation process;

(2) The significant stress over this zone is the mean of the peak stress and the stress at the depth δ;

(3) Statistical effects such as inherent flaws, etc., are of a volumetric nature;

(4) The volume effect of the material defined by the depth δ must be corrected by comparison with the volume of an unnotched specimen.

Determination of the distance δ is divided into two cases: (a) when the local stress is elastic, and (b) when local plastic action is present. In the elastic case, the means for selecting δ is shown in Fig. 1. Here the depth $\delta/2$ is defined as the point of intersection of the local stress distribution curve, and the curve determined from a relation between fatigue strength, and volume derived by Kuguel (2). This relation is given as

$$\sigma_i = \sigma_u \, (V_i/V_u)^k \tag{1}$$

where σ_u is the fatigue strength, and V_u the volume, of an unnotched specimen. The term V_i refers to the volume of a zone which extends a distance x_i below the notch root. The effective stress σ_i for fatigue strength of the notched specimen, is computed from Eq. 1 (which gives the correction for critical volume as compared with an unnotched specimen). The value of the exponent, k, depends on

*Numbers in parentheses refer to the list of references.

the metal used and the fatigue life. By successively increasing x_i, the curve shown in Fig. 1 is generated. The variable x_i is divided by 2 when plotting σ_i, because the zone defined by x_i is volume corrected to the mean stress over that zone. Once the depth δ is established for a given metal, it is assumed constant for all notch radii and fatigue lives as long as elastic conditions prevail.

When local plastic action is present, the distance δ is taken to be the extent of the plastic zone, δ_p. If, however, the plastic zone is smaller than the δ calculated for elastic conditions, the latter is used instead.

Now by applying corrections for the effects of the stress gradient and the critical volume in this zone, the following equation for K_f results.*

$$K_f = K_t \left(1 - \frac{c\delta}{2r}\right) (V_u/V_\delta)^k \qquad (2)$$

Here c is a dimensionless gradient term, r the radius of the notch, V_u the volume of an unnotched specimen, V_δ the critical volume at the notch root, and again, k is Kuguel's exponent. The quantity $(1 - \frac{c\delta}{2r})$ is the correction for the stress gradient and $(V_u/V_\delta)^k$ is the volume correction.

In order to determine δ and the value of c, the local stress distribution for the given notch is calculated by an elastic-plastic finite element computer program. Then, using the technique previously discussed for elastic conditions, δ can be determined. When the results of the computer program indicate that local plastic action is present, δ_p is determined as the depth where the octahedral shear stress at first yielding is exceeded. From the local stress distribution, the value of c is given by the relation

$$c = \frac{r}{\delta} \left(1 - \frac{\sigma_\delta}{\sigma_p}\right) \qquad (3)$$

*The derivation of this equation is given in Ref. 1.

where σ_δ is the stress at the depth δ and σ_p is the peak stress at the notch root.

To calculate the volume of the critical zone defined by δ (or δ_p), the value of the octahedral shear stress, τ_o, at that depth is used to determine the shape of the zone. This is done by plotting the constant values of this stress around the notch root. Then, V_δ is determined by the product of the area enclosed by this curve and the plate thickness. The volume of the axially loaded unnotched specimen, V_u, is calculated using one of the following equations. For specimens made from plates, V_u in terms of the thickness t, the minimum width b, and the profile radius R is given as (2)

$$V_u \approx 0.46\, t b \sqrt{R b}\ \text{in.}^3 \tag{4}$$

For longitudinal specimens made from round bars, V_u is given as

$$V_u \approx \frac{\pi}{4}\, d^2 \left(L + 0.322\sqrt{d}\right)\ \text{in.}^3 \tag{5}$$

where d is the diameter and L the gage length. When the round specimens have an hour-glass shape

$$V_u \approx 0.253\sqrt{R\, d^5}\ \text{in.}^3 \tag{6}$$

Here again R is the profile radius, and d the minimum diameter. In all cases, these equations were determined by the volume of material subjected to 95 percent or more of the peak stress.

Values of Kuguel's exponent, k, must be determined from the results of fatigue tests on unnotched specimens of several different sizes. However, if such data are unavailable, values of k in terms of stress for aluminum alloys and steels can be approximated from the results shown in Fig. 2. The curves in this figure were determined for an aluminum alloy, 7075-T6, and SAE 4340 steel with an ultimate tensile strength of 170 ksi. When the stresses in Eq. 1 are replaced by corresponding strains, the value of k was found to be approximately constant with fatigue life and equal for both metals (1); thus, the average value of k was found to be about -0.024.

When the nominal stress is high enough to produce local yielding, K_t in Eq. 2 is replaced by K_σ which is the peak stress divided by the nominal stress. When K_f is predicted in terms of strain, K_t is replaced by K_ϵ which is similarly the peak strain divided by the nominal strain.

ESTIMATING K_f -- AN EXAMPLE

The following example will demonstrate the step by step procedure to estimate K_f for notched SAE 4340 steel specimens (1). The baseline data for the unnotched specimens were generated for two specimen types as shown in Fig. 3. The resulting stress-life curves are shown in Fig. 4. The upper two curves are the results for the unnotched specimens, and the lower curve are the results for the notched specimens shown in Fig. 1c. For the unnotched specimens, each data point represents the mean value from a minimum of six specimens. Each data point for the notched plates represents the mean value of at least four specimens. In terms of strain, these data are shown in Fig. 5. Equation 2 was used to estimate values of K_f for each of the four load levels shown in these figures. The loadings were fully reversed stress controlled tests at amplitudes of 60, 50, 40, and 33 ksi.

The local stress and strain distributions were generated by an elastic-plastic plane stress finite element computer program. The program employed uniform strain triangular elements which are loaded incrementally. Linear strain hardening is used in the plastic range. The grid pattern employed for a plate with a central hole is shown in Fig. 6.

Input parameters included a plastic modulus and Poisson's ratio. The plastic moduli were determined from a straight line passed thru that portion of the cyclic stress-strain curve of interest. As shown in Fig. 7, two different plastic moduli were used, one for small and one for large plastic deformations. For all calculations, the plastic Poisson's ratios were assumed to be equal to the elastic values.

The output of this program consists of stresses and displacements at the centroid of each element. The stresses were then averaged between adjacent elements to obtain a more accurate stress distribution curve. Local stress and strain distributions are shown in Fig. 8. These were then used to determine δ, c, K_σ, and K_c for use in Eq. 2.

The distance δ for this metal was found to be approximately 0.002 in. which, in all cases, was less than the extent of the plastic zone δ_p. As discussed previously, this depth was determined by the point on the local octahedral shear stress distribution curve where the initial yield strength was exceeded. These curves are shown in Fig. 9a. The curves shown in Fig. 9b were used to determine the extent of the critical zone in the direction of loading. The shape of the zone was determined with the theoretical equations for an infinite plate with a hole (5). Using these equations, stresses were calculated along radial lines at successive angular positions around the hole. The extent of the zone along any line was then determined when the critical value of the octahedral shear stress was obtained. Zone shapes determined in this manner and a comparison with the elements that have yielded are shown in Fig. 10.

The results of these calculations to estimate K_f are summarized in Table 1. For these data with four test levels, a total of sixteen K_f predictions were made; that is, four values were calculated in terms of stress, and four in terms of strain for each of the two unnotched specimen sizes. The average errors (without regard to sign) were 5 percent for the predictions in terms of both stress and strain.

Figures 11 and 12 summarize the results of using Eq. 2 to predict K_f in terms of stress and strain for the data presented in this report and for data on several other metals. For each set of data indicated in these figures, enough fatigue tests were conducted to provide a reasonable degree of statistical accuracy. Thus, for fatigue lives from 10^2 to 10^6 cycles, the average errors in predicting K_f for the seven sets of data were 3 percent in terms of stress and 8 percent in terms of strain.

Table 1 Comparison of Experimentally Observed and Predicted Fatigue Notch Factors--SAE 4340 Steel Plates with $K_t = 2.37$

Nominal Stress S, ksi	Cycles to Failure, N_f	K_σ, Stress Cons. σ_p/s	K_ε, Strain Cons. ε_p/e	Plastic Zone Size δ_p, in.	δ, in.	Kuguel's Exponent for Stress, k	Local Volume[a] V_δ, in.^{3}	Gradient Factors c, Stress	Gradient Factors c, Strain	Volume Corrections, Stress	Volume Corrections, Strain	Unnotched Spec. Dia., in.	Observed & Predicted K_f, Stress Obs.	Stress Pred.	Stress Error, %	Strain Obs.	Strain Pred.	Strain Error, %
60	15000	1.72	2.94	0.060	--	-0.010	1.44×10^{-3}	0.43	0.94	1.000	1.014	0.100	1.63	1.51	-7	2.22	2.16	-2
										0.968	0.923	0.250	1.56	1.45	-6	1.99	1.96	-1
50	35000	1.93	2.70	0.037	--	-0.012	6.88×10^{-4}	0.63	1.28	0.998	0.960	0.100	1.81	1.70	-5	2.20	2.07	-6
										0.954	0.907	0.250	1.72	1.63	-5	1.96	1.88	-3
40	92000	2.15	2.58	0.018	--	-0.015	2.30×10^{-4}	0.90	1.82	0.981	0.970	0.100	2.06	1.94	-5	2.35	2.10	-10
										0.927	0.883	0.250	1.94	1.83	-5	2.04	1.91	-6
33	250000	2.39	2.61	0.007	--	-0.016	5.00×10^{-5}	1.67	2.31	0.956	0.935	0.100	2.24	2.15	-3	2.38	2.25	-5
										0.899	0.852	0.250	2.09	2.02	-3	2.17	2.05	-5

[a] The unnotched specimen volumes were 8.1×10^{-4} in.^{3} and 4×10^{-2} in.^{3}

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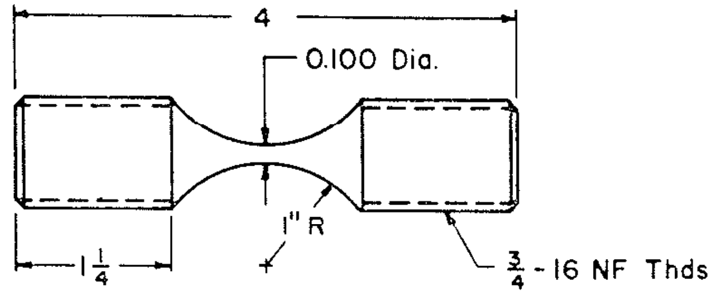
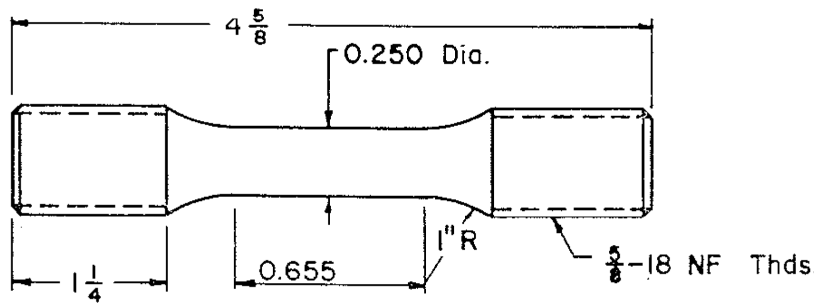
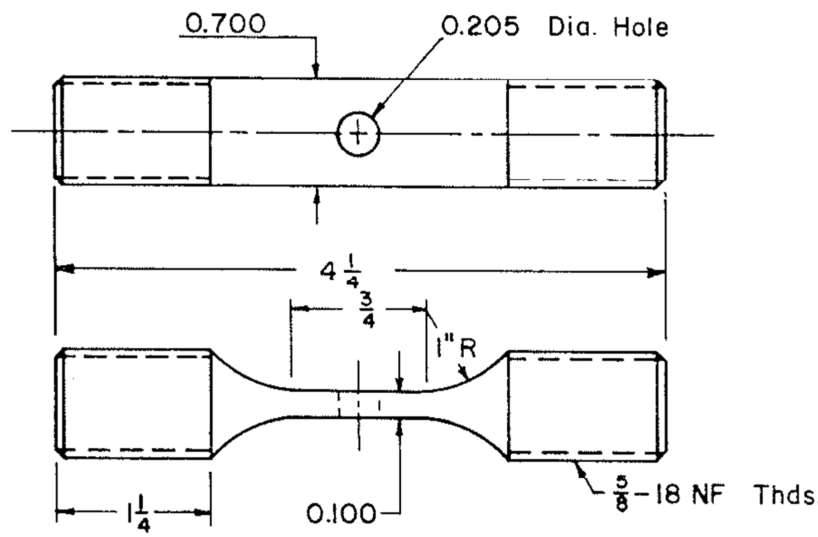


(a) Hourglass Specimen



(b) Longitudinal Specimen



(c) Notched Specimen, $K_t = 2.37$

Fig. 3 Test Specimens

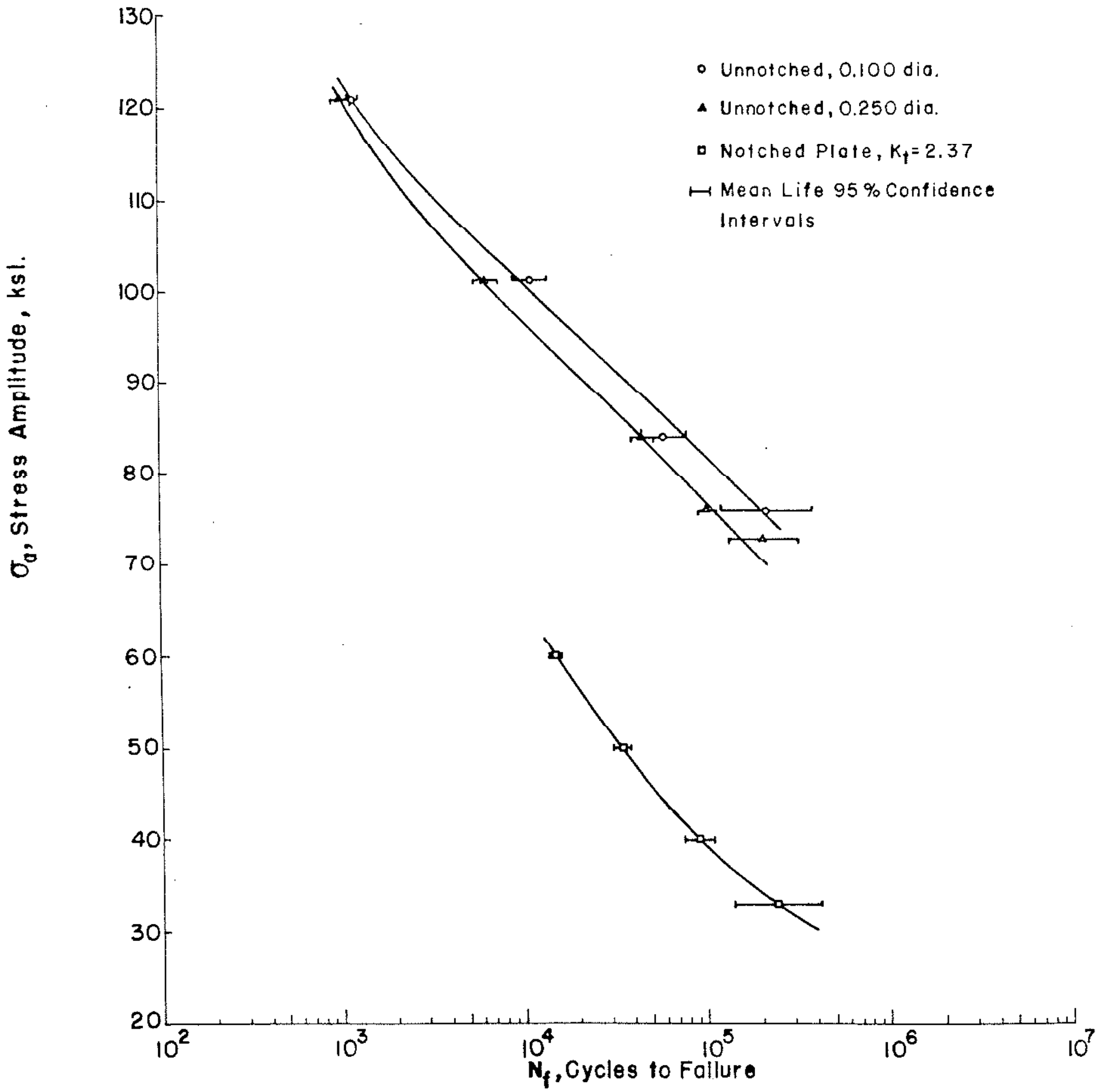


Fig. 4 Stress-Life Curves, SAE 4340 Steel

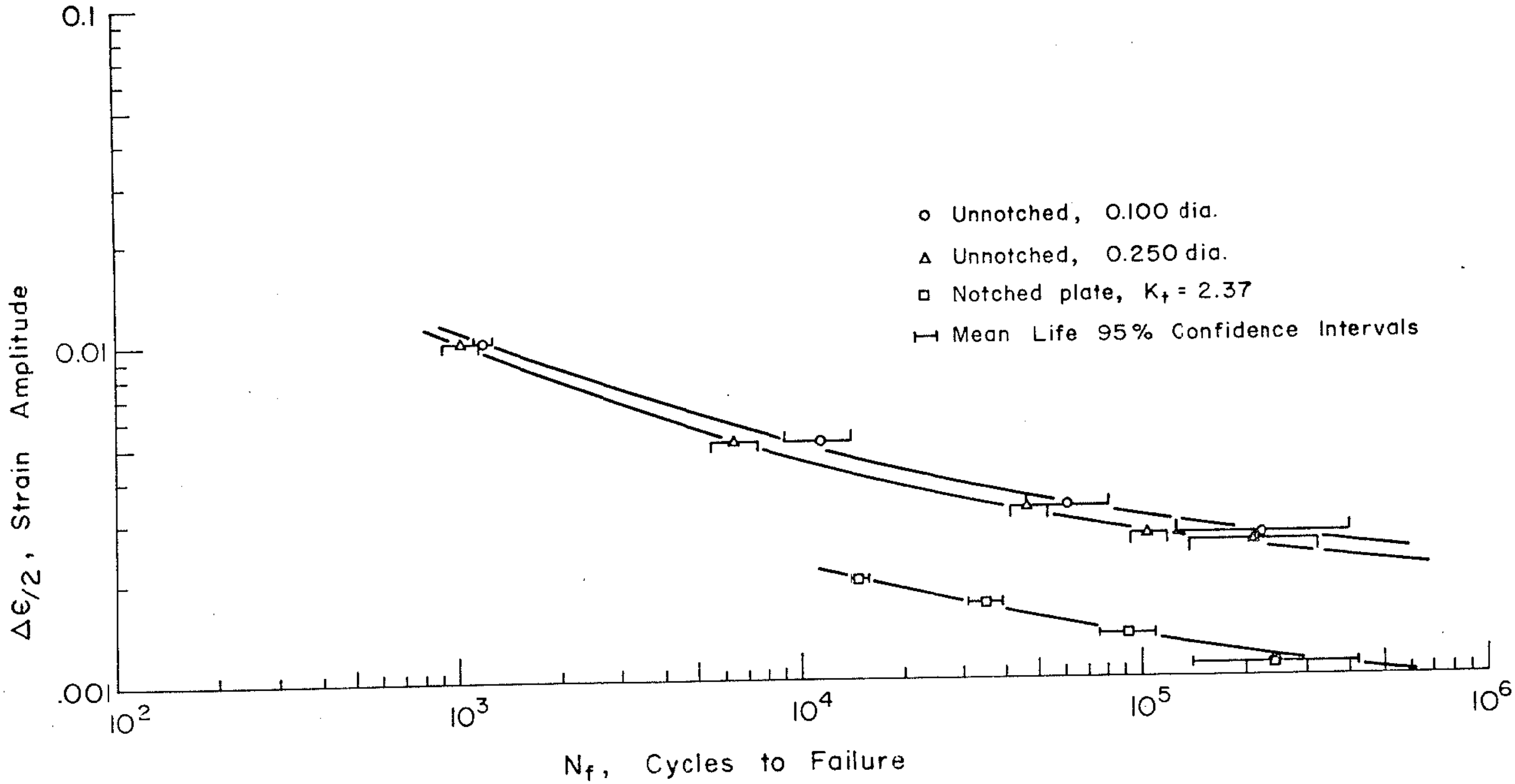


Fig. 5 Strain-Life Curves, SAE 4340 Steel

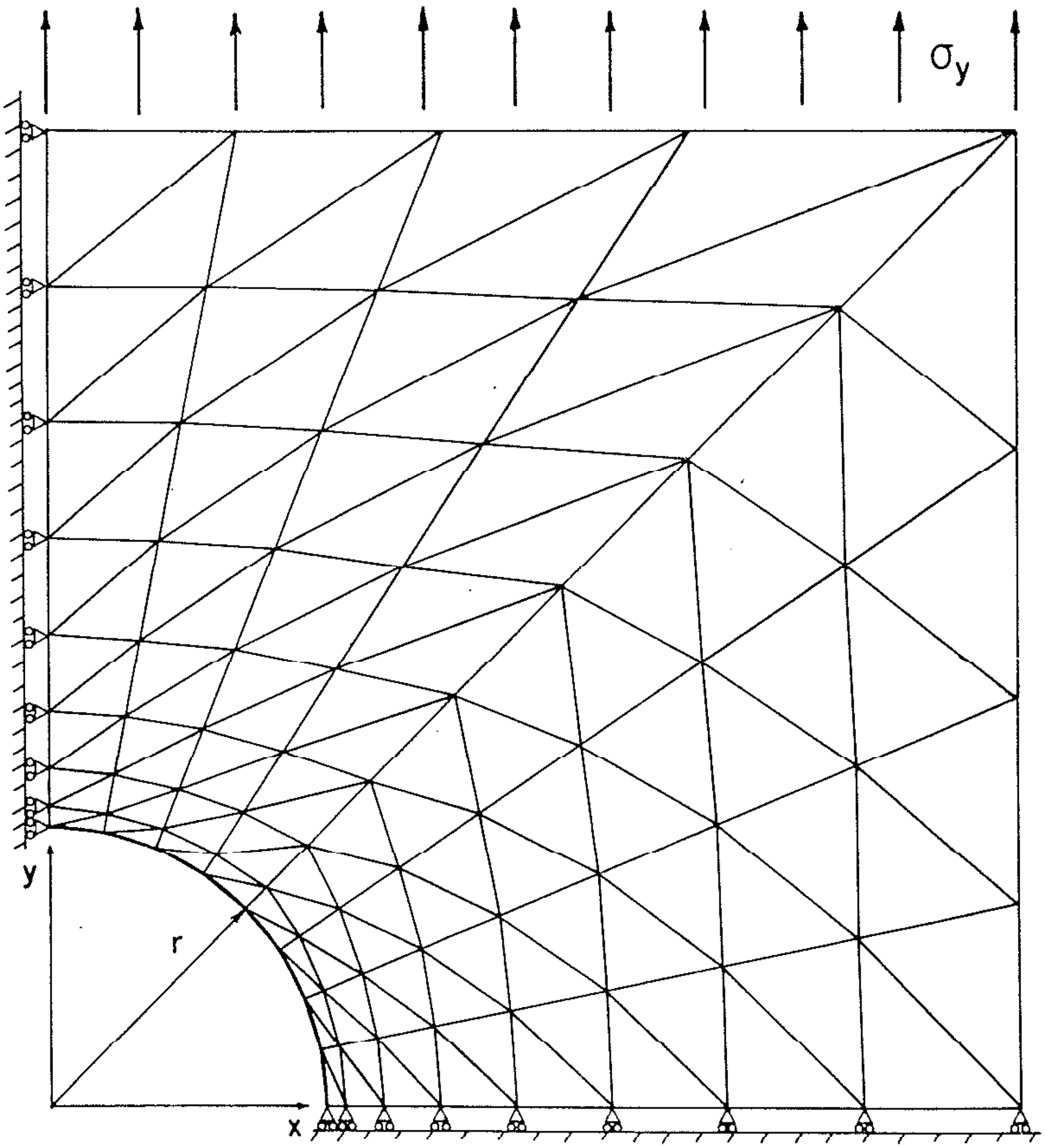


Fig. 6 Finite Element Grid Pattern for a Plate with a Central Hole

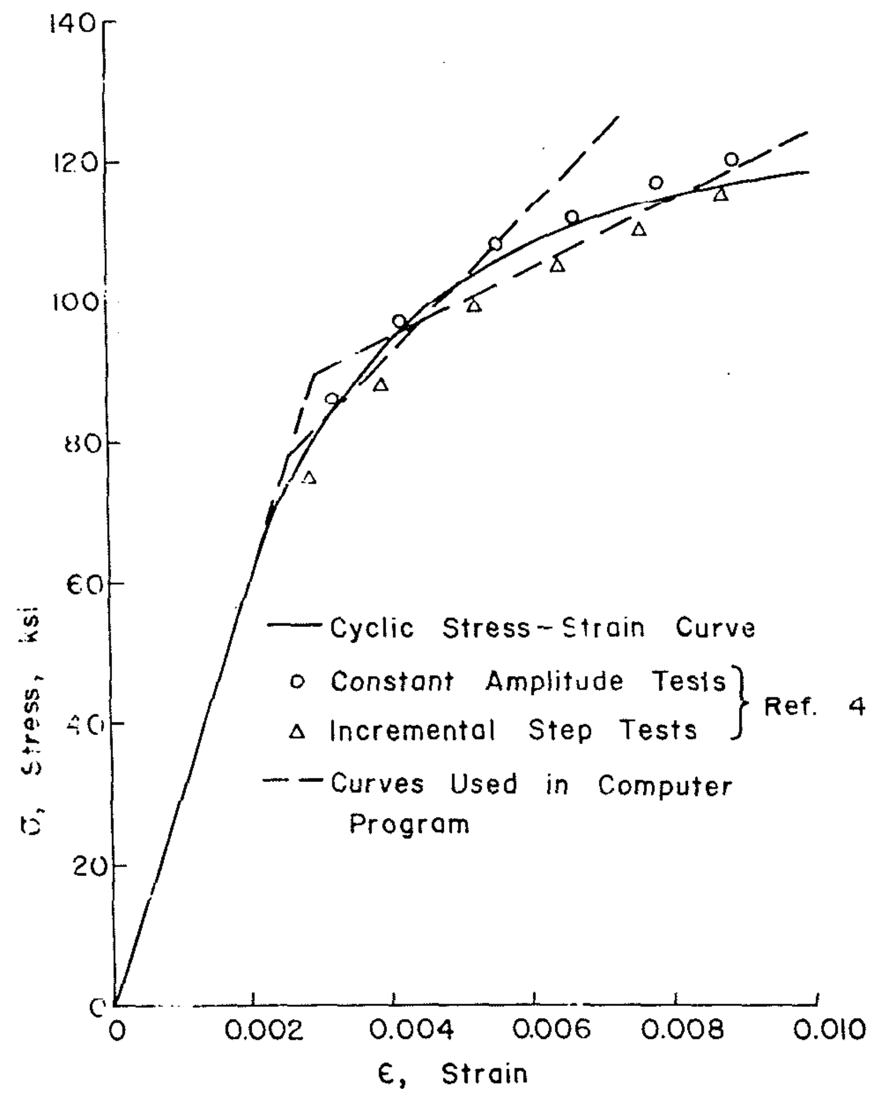
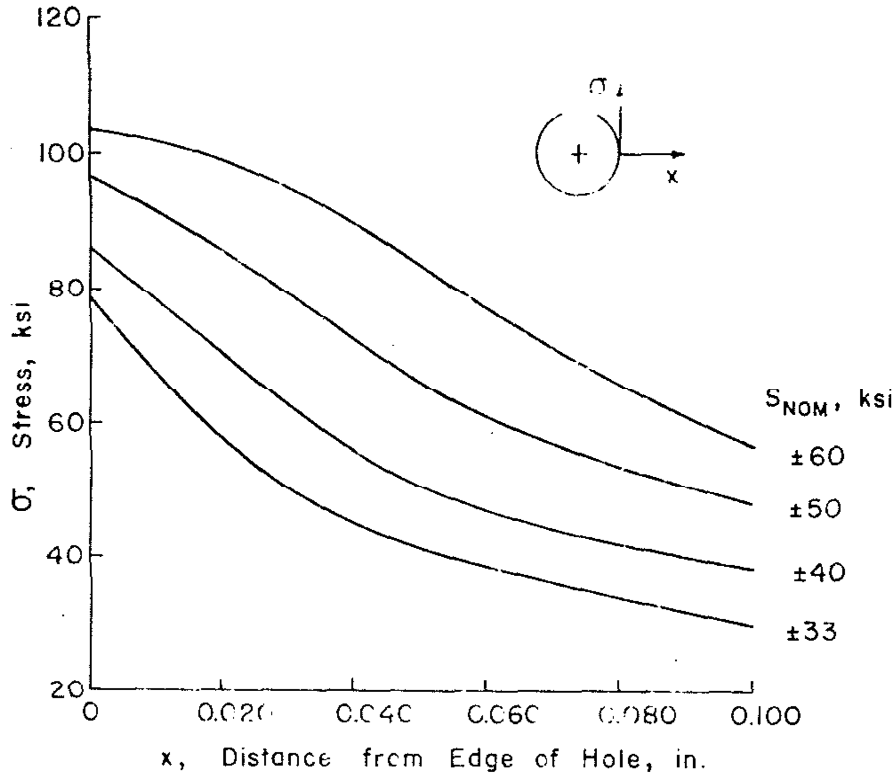
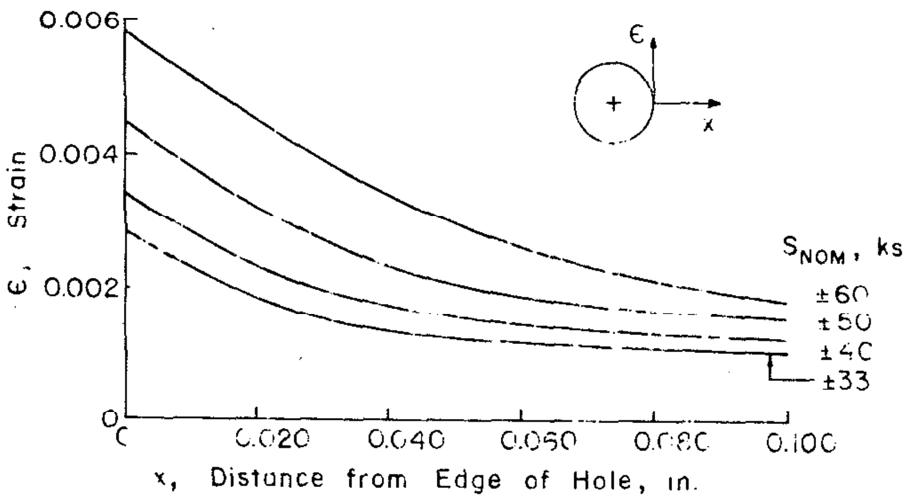


Fig. 7 Cyclic Stress-Strain Curve, SAE 4340 Steel



(a) Local Stress Distribution



(b) Local Strain Distribution

Fig. 8 Local Stress and Strain Distributions—
SAE 4340 Steel Plates with a Hole

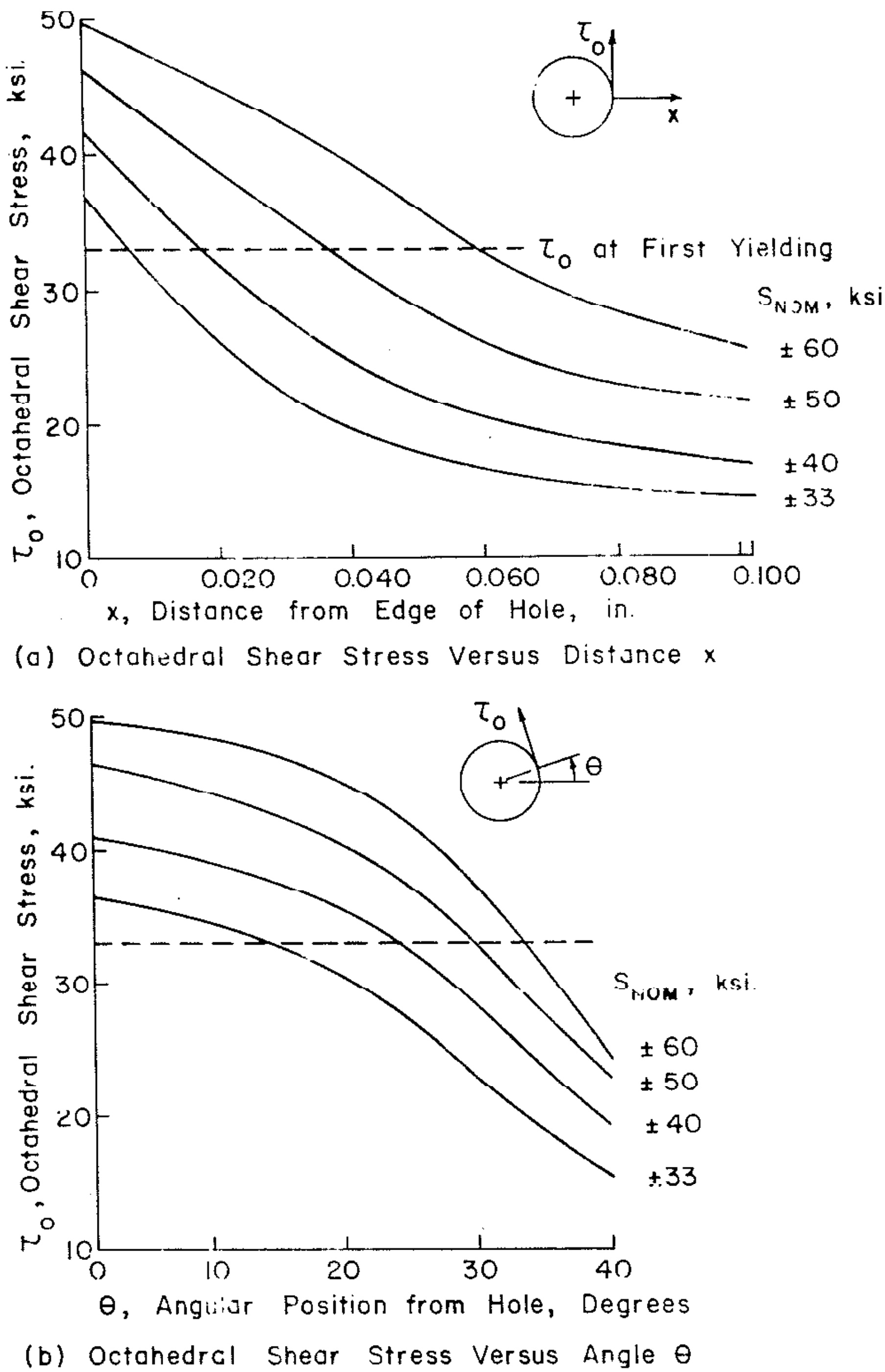
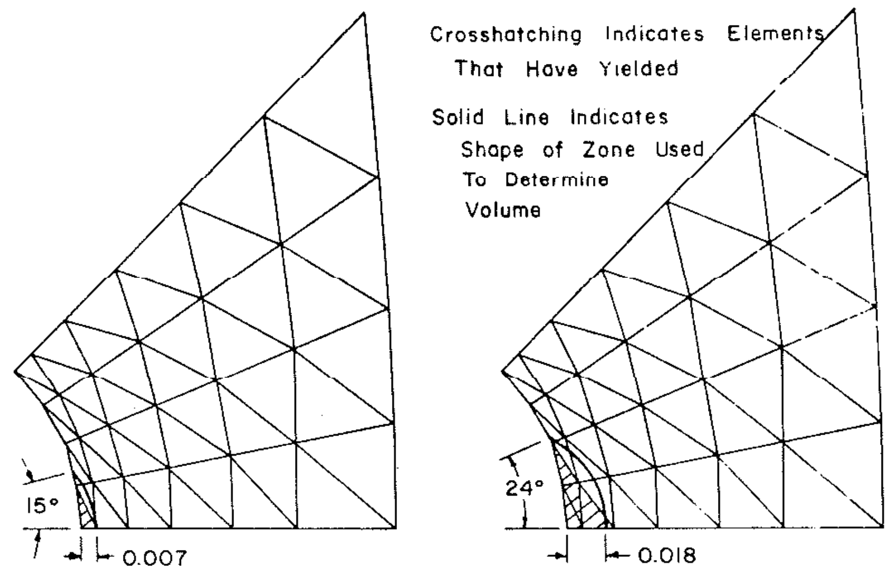
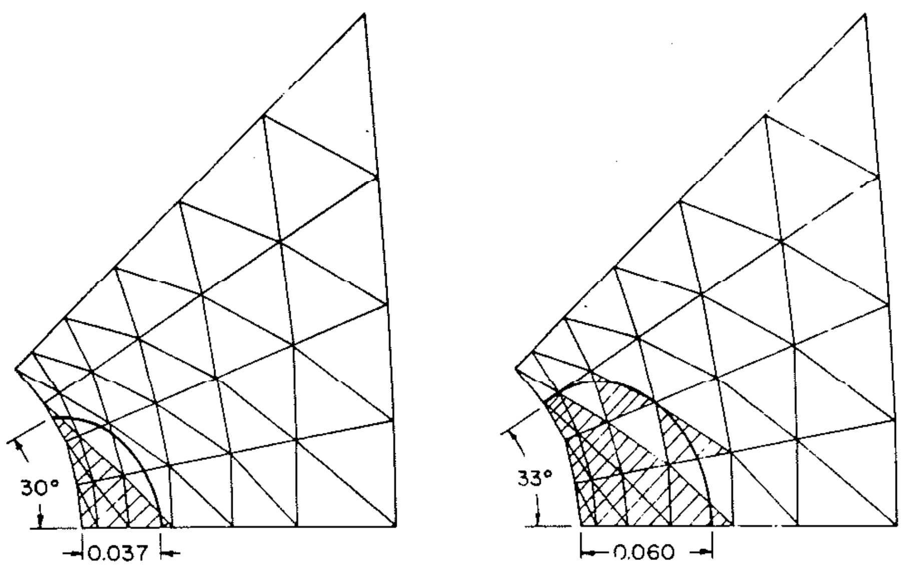


Fig. 9 Local Octahedral Shear Stress Distribution— SAE 4340 Steel Plates with a Hole



(a) Nominal Stress = 33 ksi. (b) Nominal Stress = 40 ksi.



(c) Nominal Stress = 50 ksi. (d) Nominal Stress = 60 ksi.

Fig. 10 Portion of Finite Element Grid Showing Plastic Zones, SAE 4340 Steel

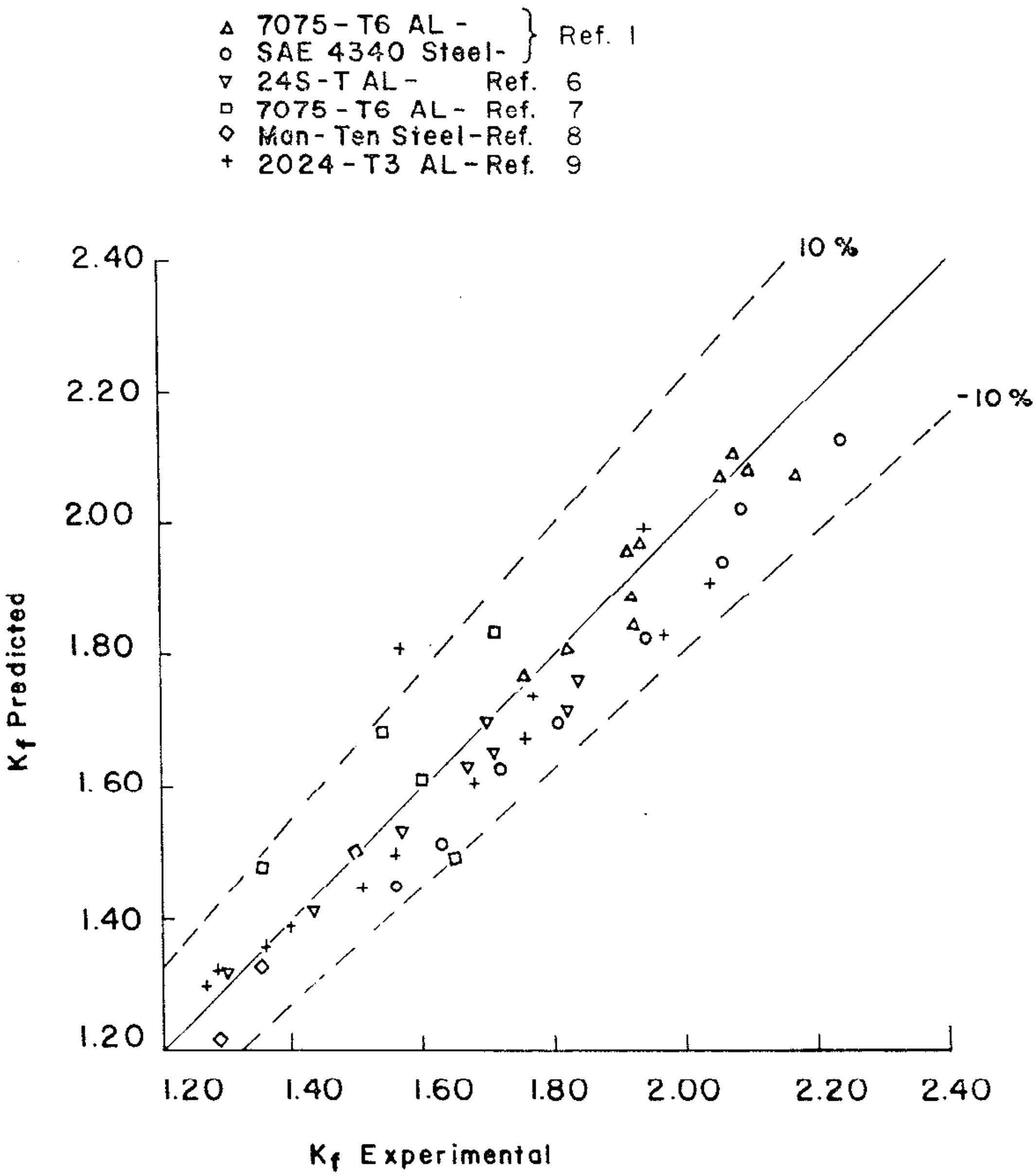


Fig. 11 Experimental Versus Predicted Values of K_f in Terms of Stress

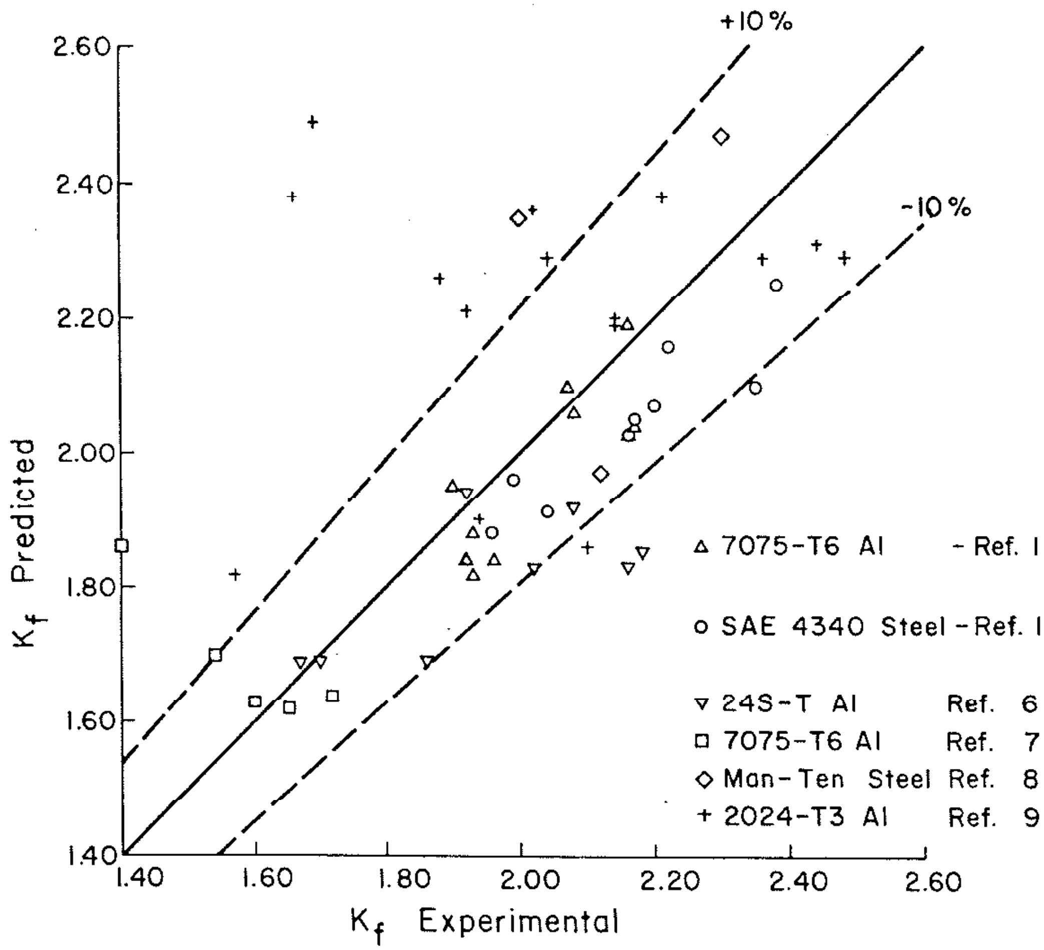


Fig. 12 Experimental Versus Predicted Values of K_f in Terms of Strain