Probabilistic Aspects of Fatigue

Variability

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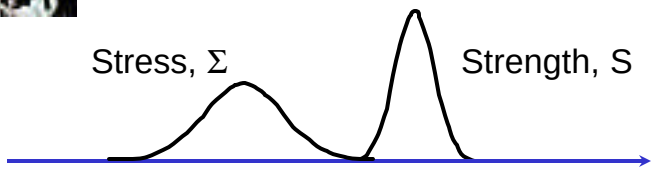
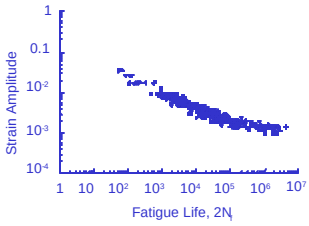
Probabilistic Aspects of Fatigue

- Introduction
- Basic Probability and Statistics
- Statistical Techniques
- Analysis Methods
- **Characterizing Variability**
- Case Studies
- FatigueCalculator.com
- GlyphWorks

Sources of Variability

customers ⟵ Stress ⟶ usage





materials ⟵ Strength ⟶ manufacturing

Variability and Uncertainty

Variability: Every apple on a tree has a different mass.

Uncertainty: The variety of the apple is unknown.

Variability: Fracture toughness of a material

Uncertainty: The correct stress intensity factor solution

Sources of Variability

- Stress Variables
  - Loading
  - Customer Usage
  - Environment
- Strength Variables
  - Material
  - Processing
  - Manufacturing Tolerance
  - Environment

Sources of Uncertainty

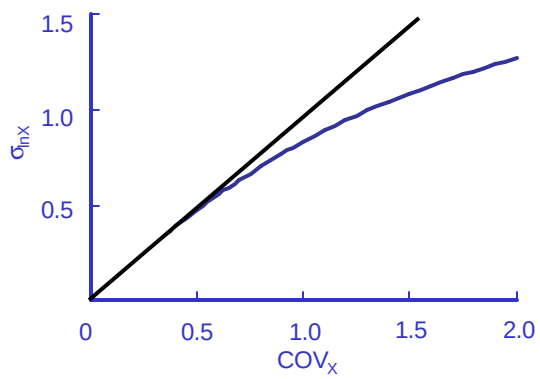
- Statistical Uncertainty
  - Incomplete data (small sample sizes)
- Modeling Error
  - Analysis assumptions
- Human Error
  - Calculation errors
  - Judgment errors

Modeling Variability

Central Limit Theorem:

Products: $Z = X_1 \bullet X_2 \bullet X_3 \bullet X_4 \bullet \ldots\ldots X_n$

$Z \rightarrow$ LogNormal as n increases



$\sigma_{lnX} \sim COV_X$

COV_X is a good measure of variability

COV and LogNormal Distributions

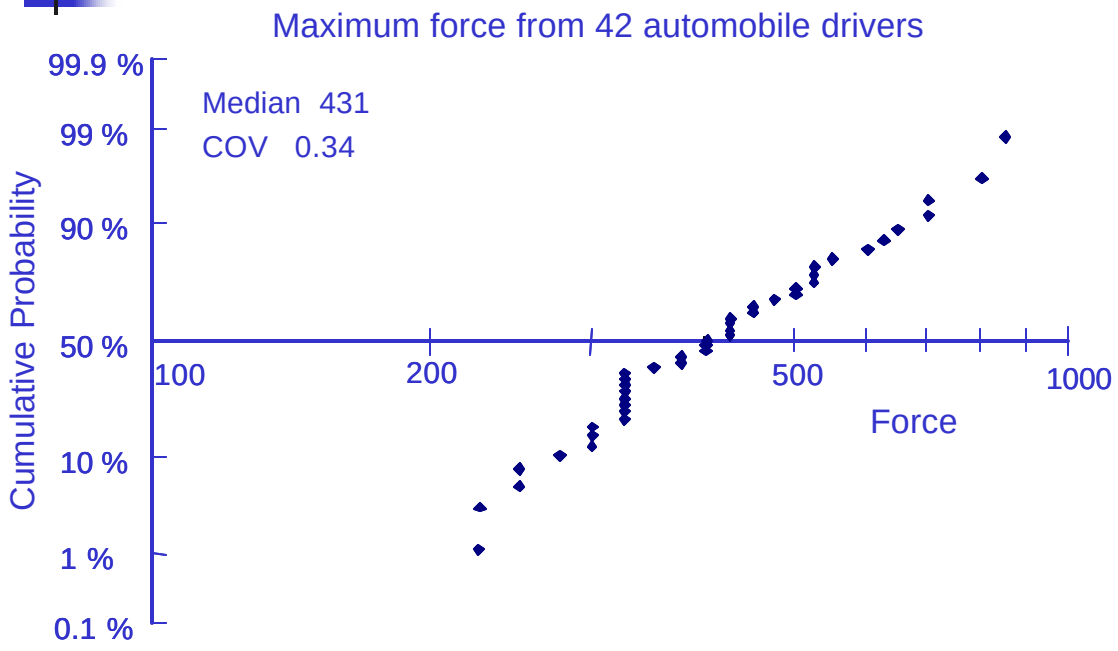
COV_X	Standard Deviation, lnx		
	1 68.3%	2 95.4%	3 99.7%
0.05	1.05	1.11	1.16
0.1	1.10	1.23	1.33
0.25	1.28	1.66	2.04
0.5	1.60	2.64	3.92
1	2.30	5.53	11.1

99.7% of the data is within a factor of $\pm$ 1.33 of the mean for a COV = 0.1

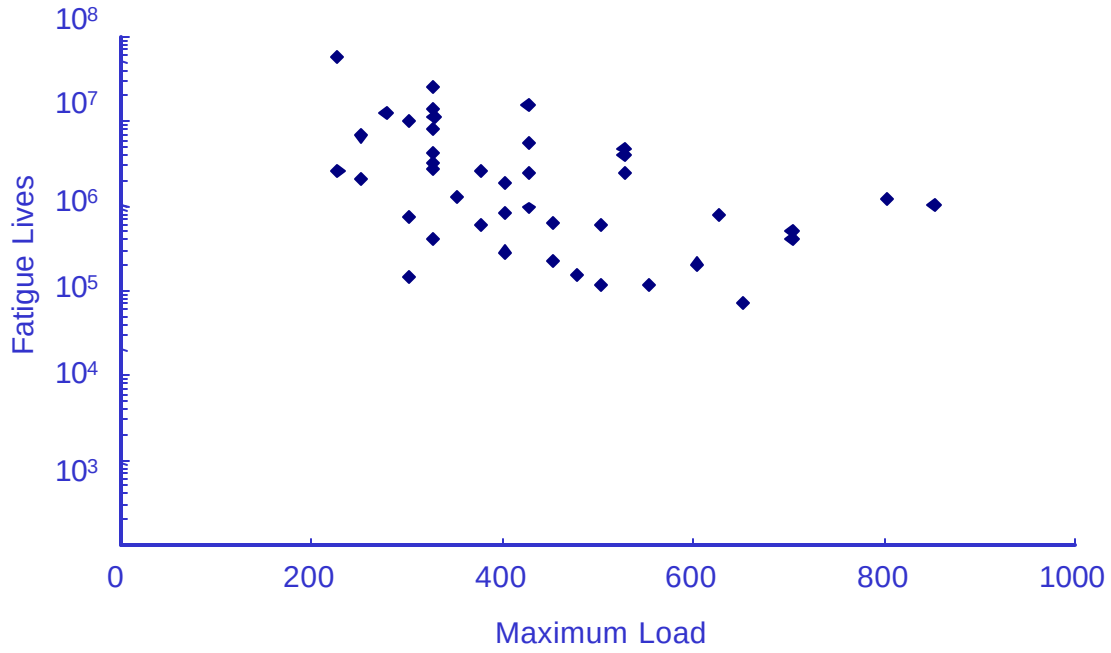
Variability in Service Loading

- Quantifying Loading Variability
  - Maximum Load
  - Load Range
  - Equivalent Stress

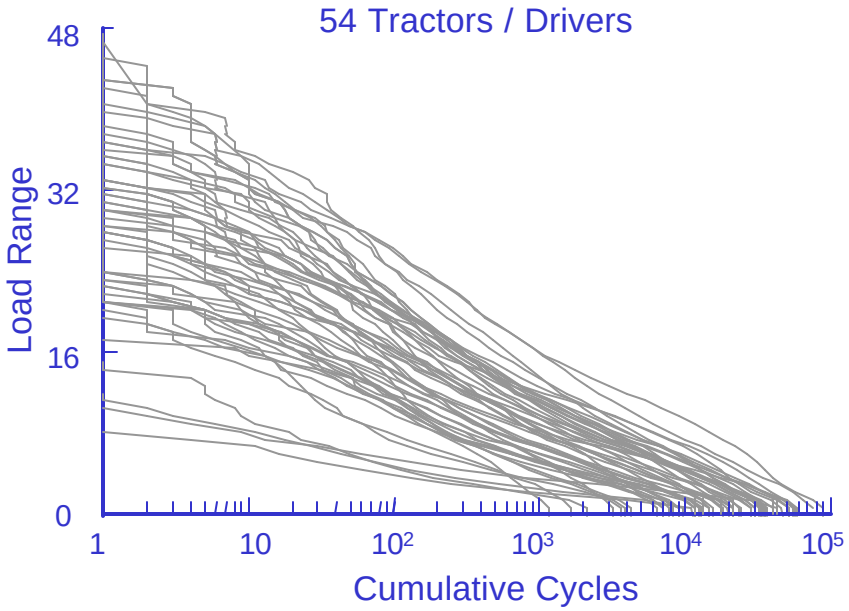
Maximum Force



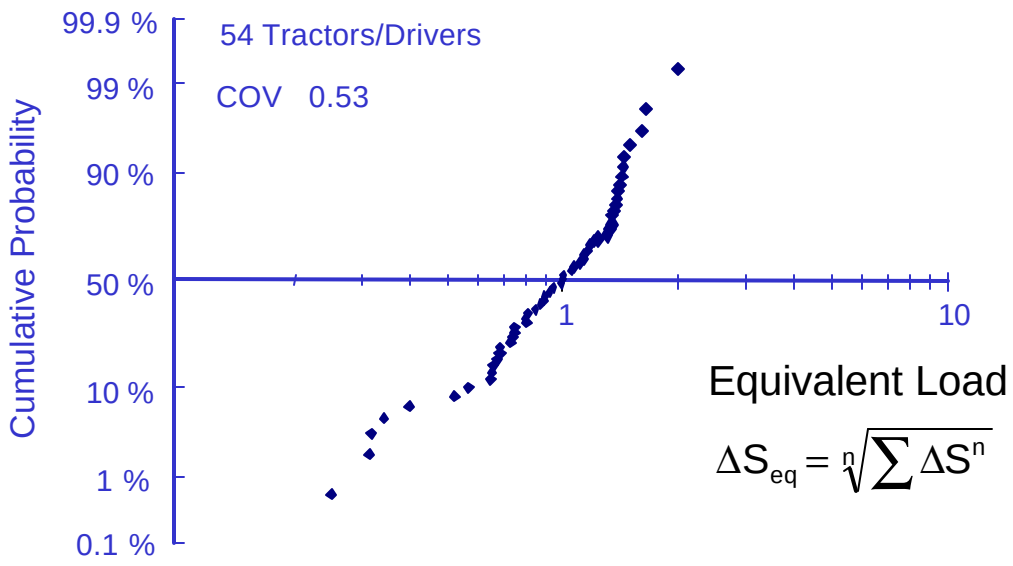
Maximum Load Correlation



Loading Variability

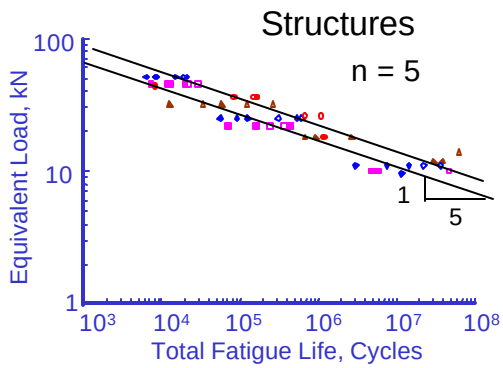


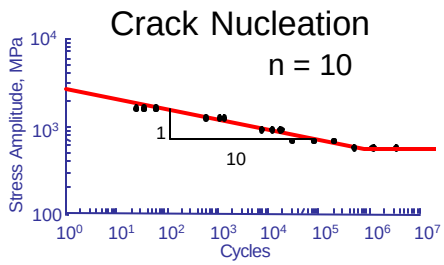
Variability in Loading

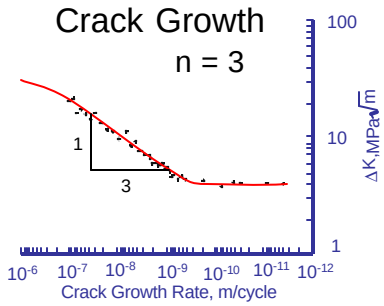


$$\Delta S_{eq} = \sqrt[n]{\sum \Delta S^n}$$

Mechanisms and Slopes

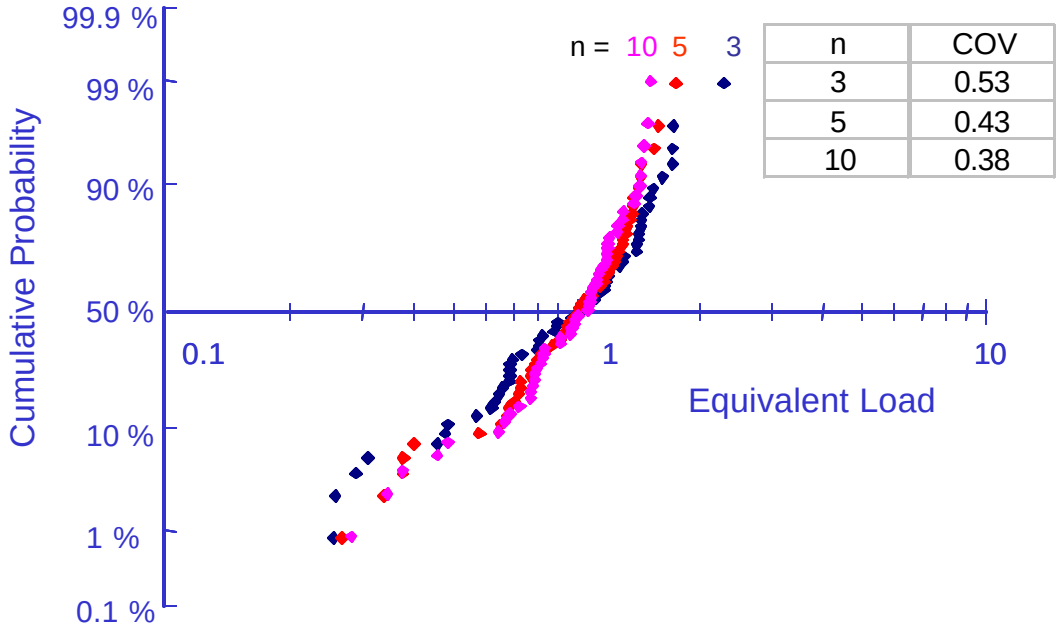






A combination of nucleation and growth

Effect of Slope on Variability

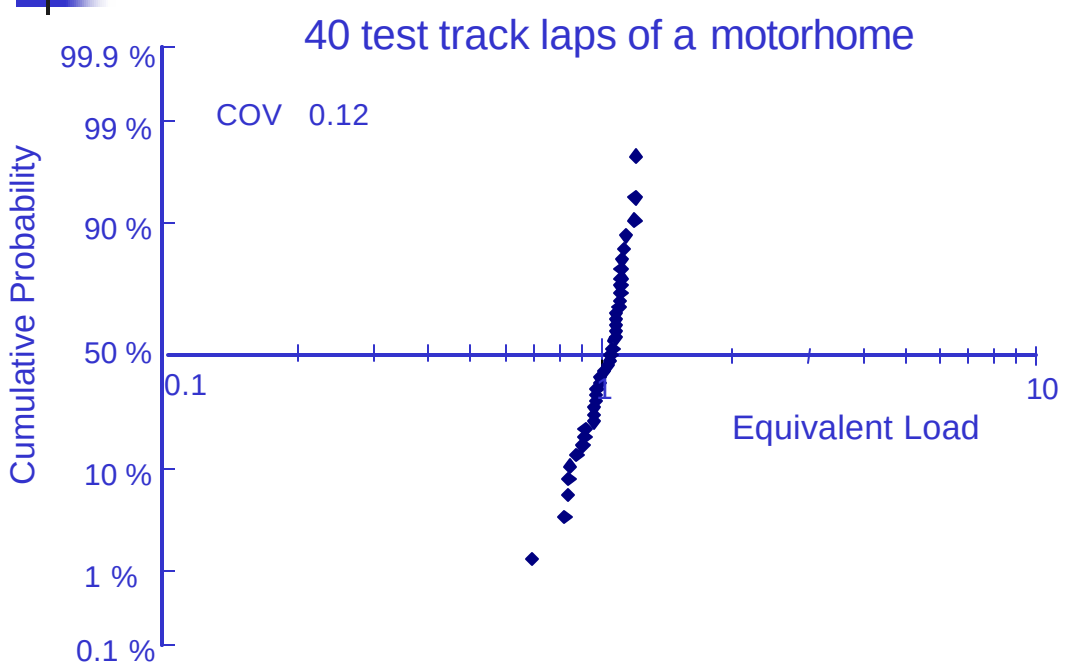


n	COV
3	0.53
5	0.43
10	0.38

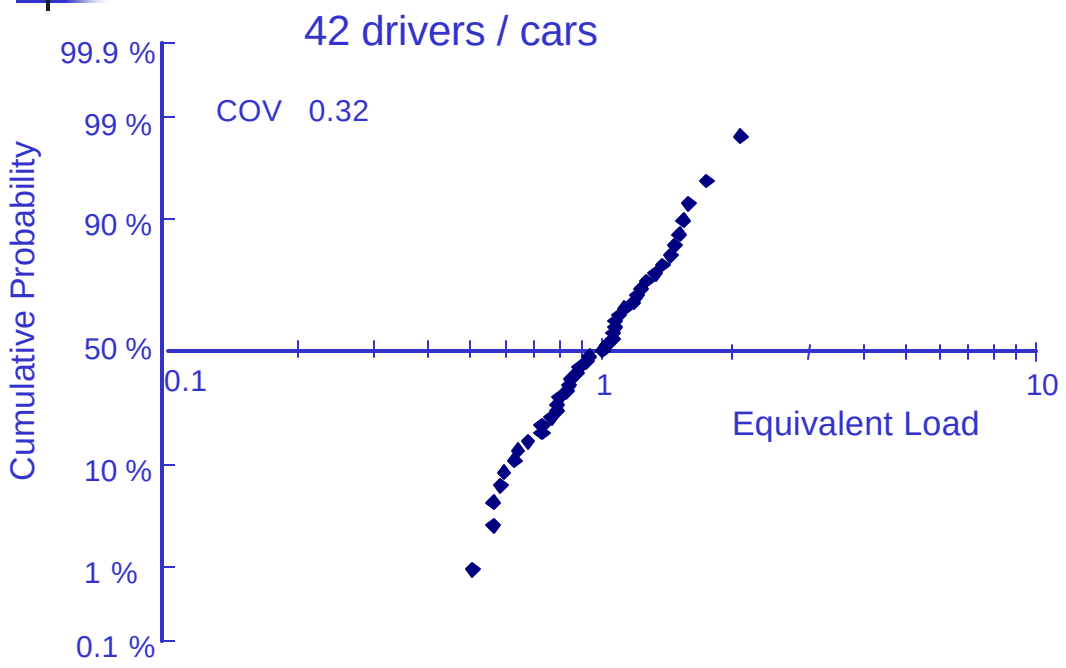
Loading History Variability

- Test Track
- Customer Service

Test Track Variability



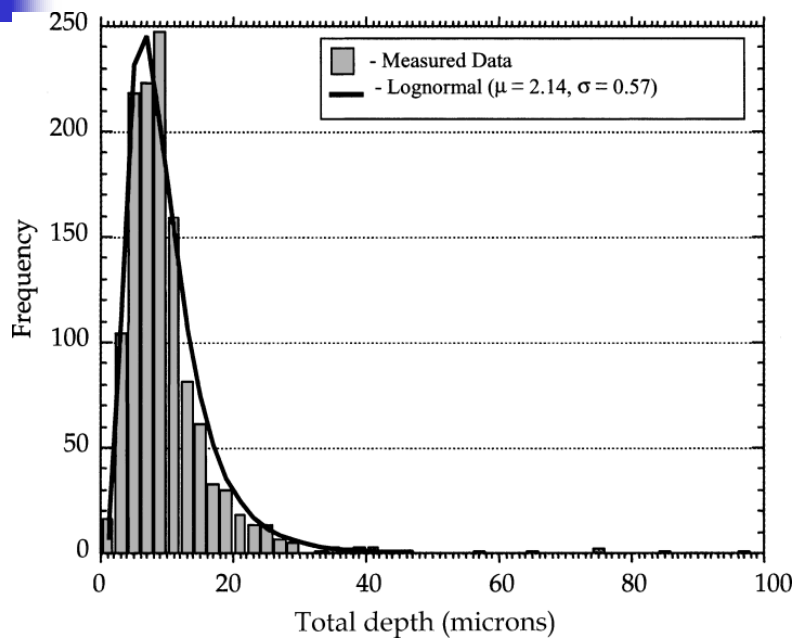
Customer Usage Variability



Variability in Environment

- Inclusions
- Pit depth

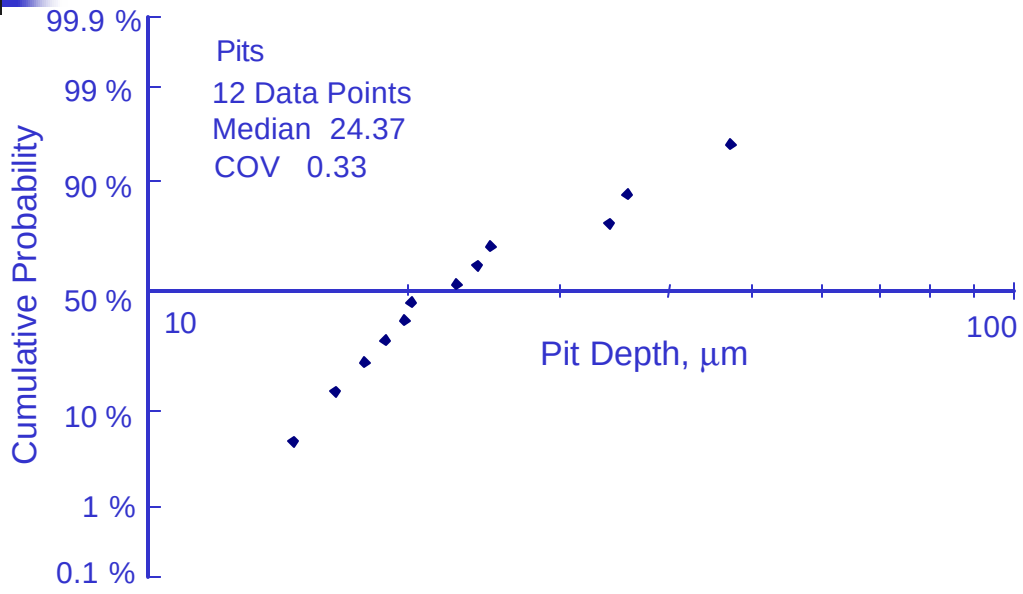
Inclusions That Initiated Cracks



COV = 0.27

Barter, S. A., Sharp, P. K., Holden, G. & Clark, G. "Initiation and early growth of fatigue cracks in an aerospace aluminium alloy", *Fatigue & Fracture of Engineering Materials & Structures* **25** (2), 111-125.

Pit Depth Variability

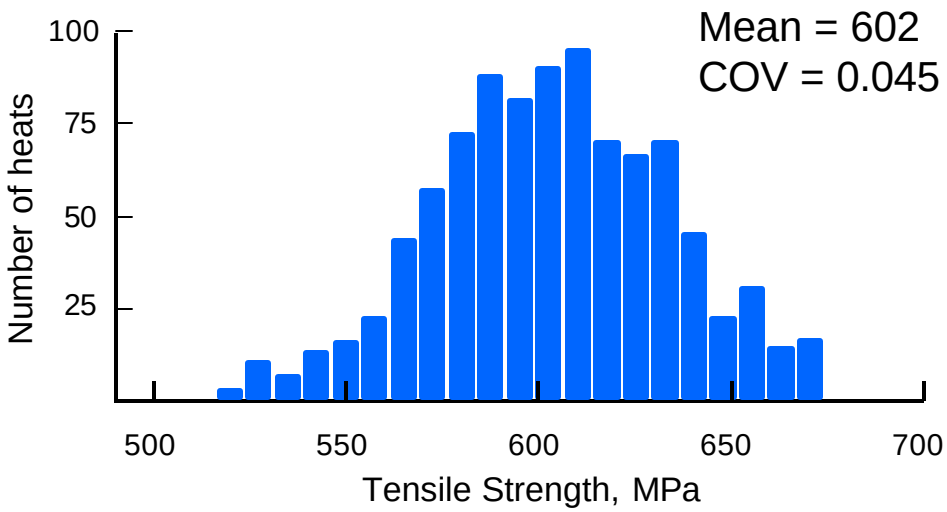


Dolly, Lee, Wei, "The Effect of Pitting Corrosion on Fatigue Life"
Fatigue and Fracture of Engineering Materials and Structures, Vol. 23, 2000, 555-560

Variability in Materials

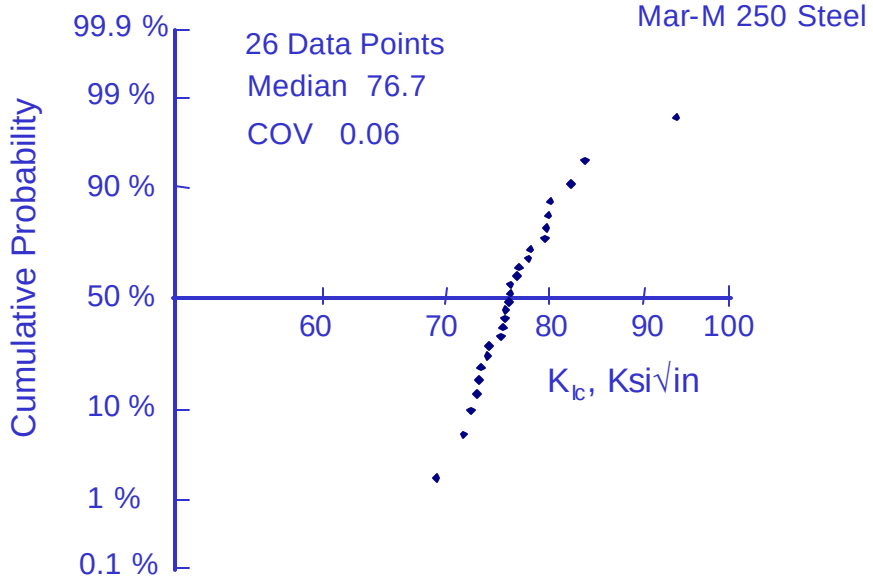
- Tensile Strength
- Fracture Toughness
- Fatigue
  - Fatigue Strength
  - Fatigue Life
- Strain-Life
- Crack Growth

Tensile Strength - 1035 Steel



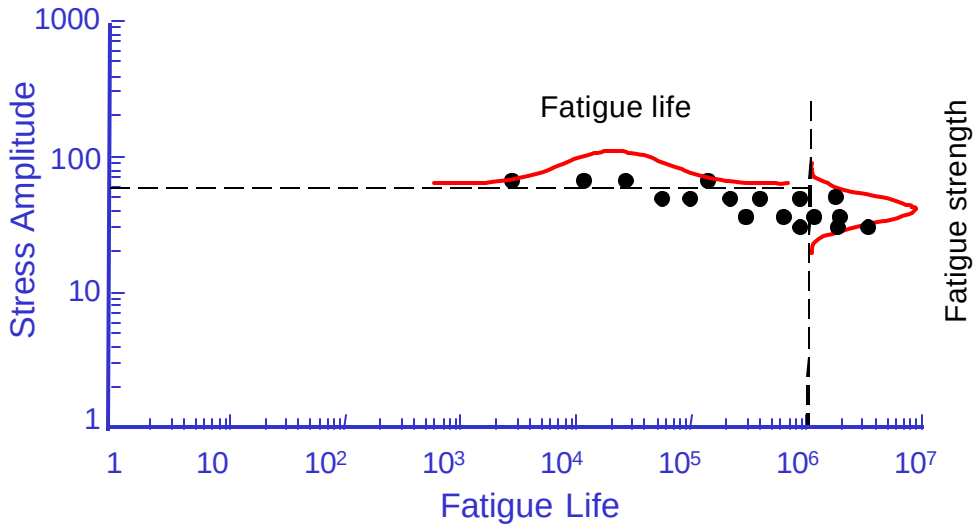
Metals Handbook, 8th Edition, Vol. 1, p64

Fracture Toughness

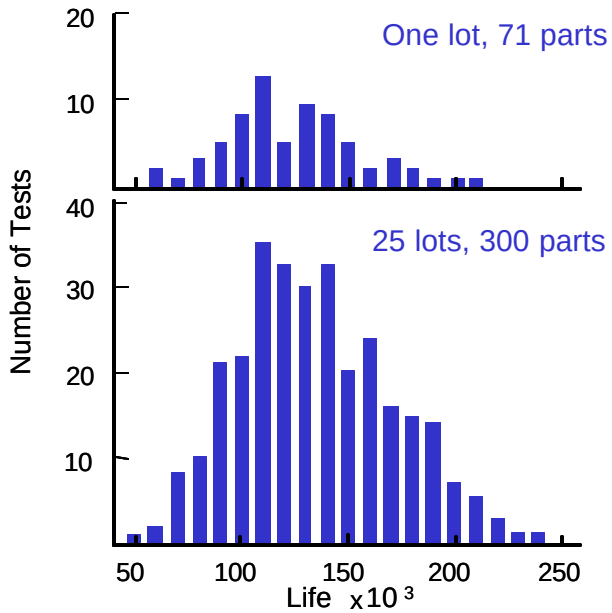


Kies, J.A., Smith, H.L., Romine, H.E. and Bernstein, H, "Fracture Testing of Weldments", ASTM STP 381, 1965, 328-356

Fatigue Variability



Fatigue Life Variability



Production torsion bars
5160H steel

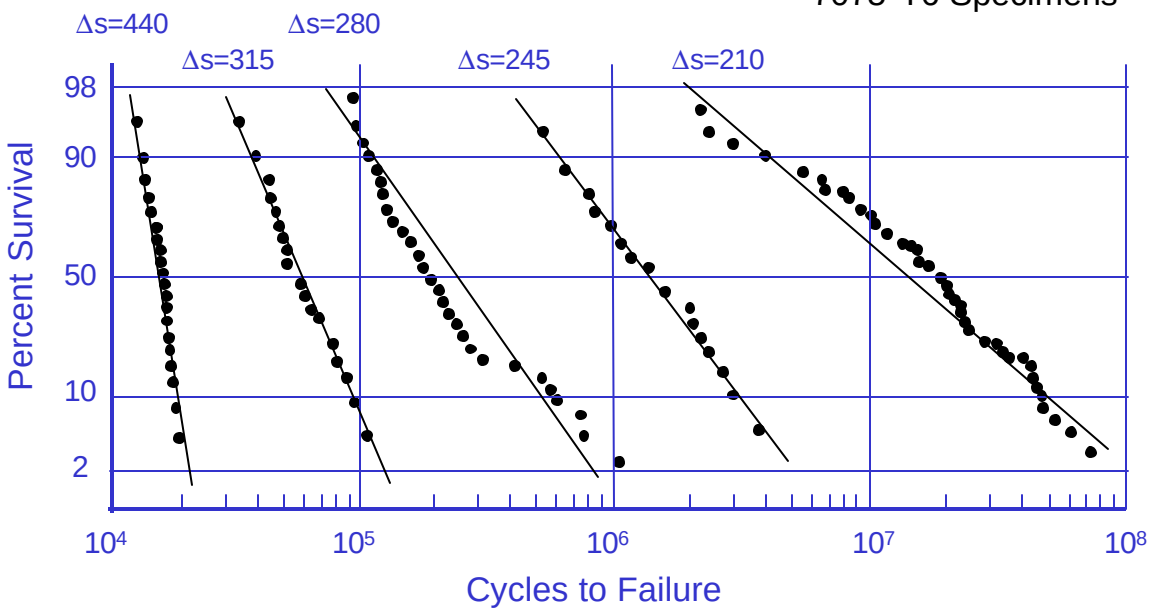
μ_x = 123,000 cycles
COV = 0.25

μ_x = 134,000 cycles
COV = 0.27

Metals Handbook, 8th Edition, Vol. 1, p219

Statistical Variability of Fatigue Life

7075-T6 Specimens



Sinclair and Dolan, "Effect of Stress Amplitude on the Variability in Fatigue Life of 7075-T6 Aluminum Alloy" Transactions ASME, 1953

COV vs Fatigue Life

ΔS	$\overline{X}$	COV
440	14,000	0.12
315	25,000	0.38
280	220,000	0.70
245	1,200,000	0.67
210	12,000,000	1.39

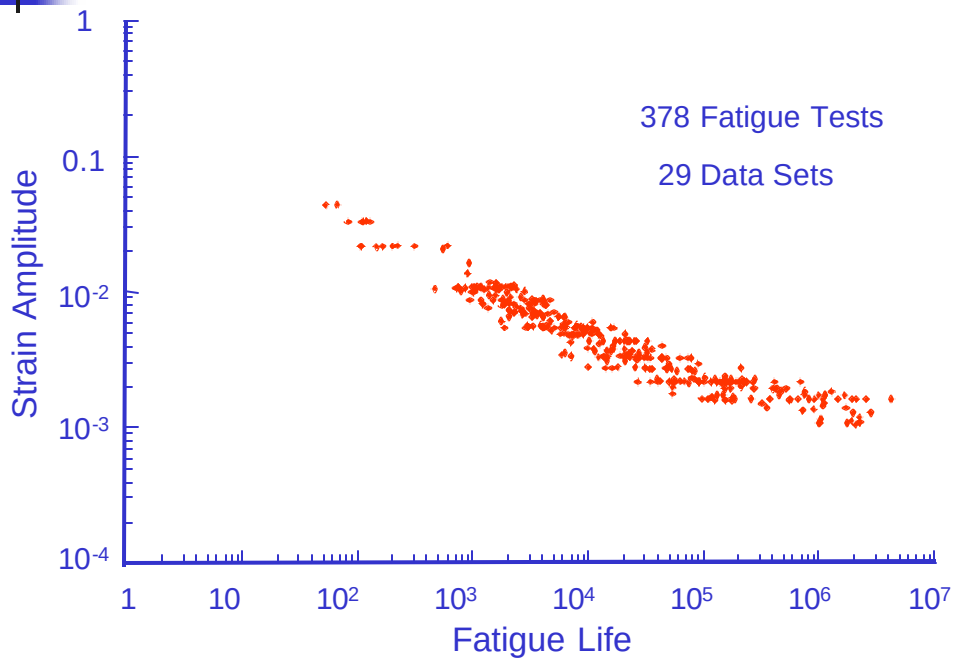
Variability in Fatigue Strength

$$\frac{\Delta S}{2} = S_f^{'}(N_f)^b \quad b \approx -0.085$$

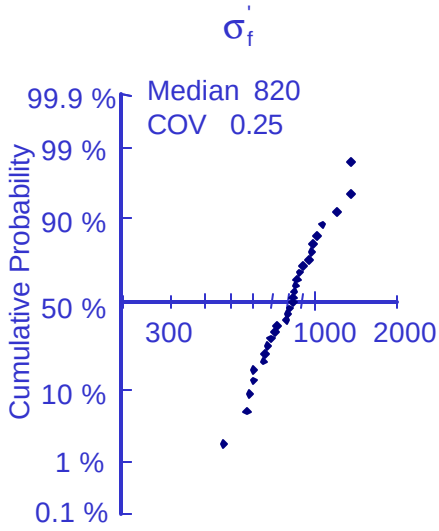
$$\text{COV} \quad C = \sqrt{\prod_{i=1}^{n}\left(1+C_{X_i}^{\ 2}\right)^{a_i^{\ 2}} - 1}$$

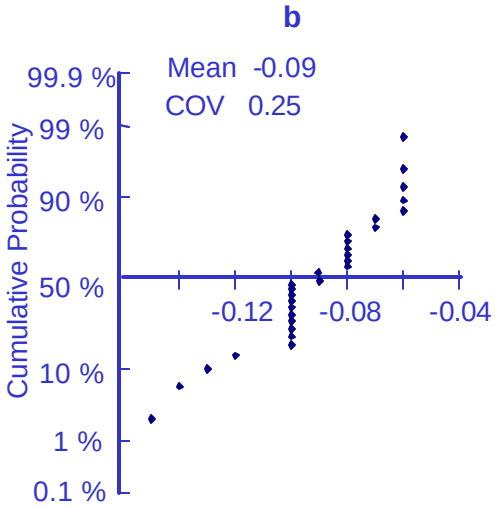
$$C_{S_f^{'}} = \sqrt{\left(1+1.39^2\right)^{(-.085)^2} - 1} = 0.088$$

Strain Life Data for 950X Steel

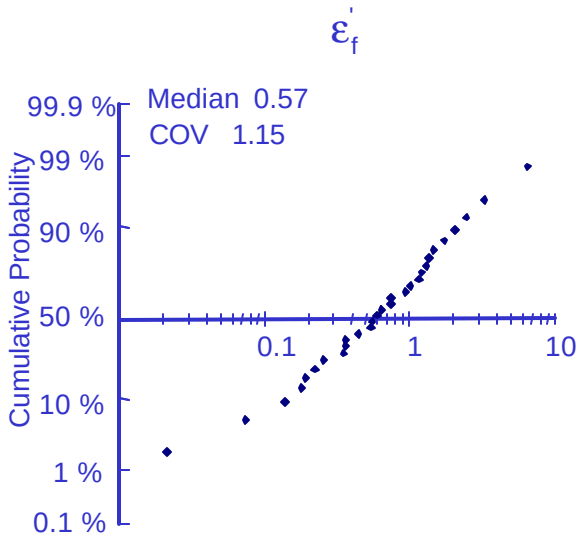


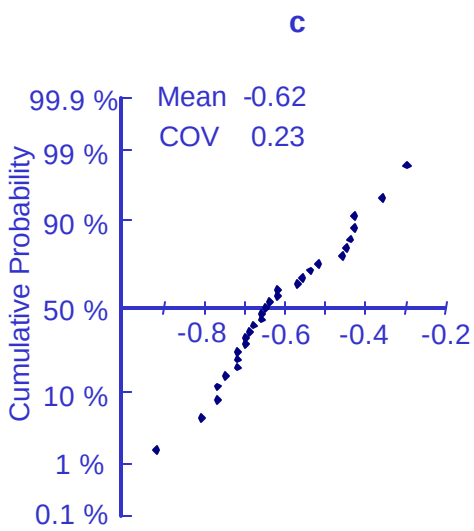
29 Individual Data Sets





29 Individual Data Sets (continued)

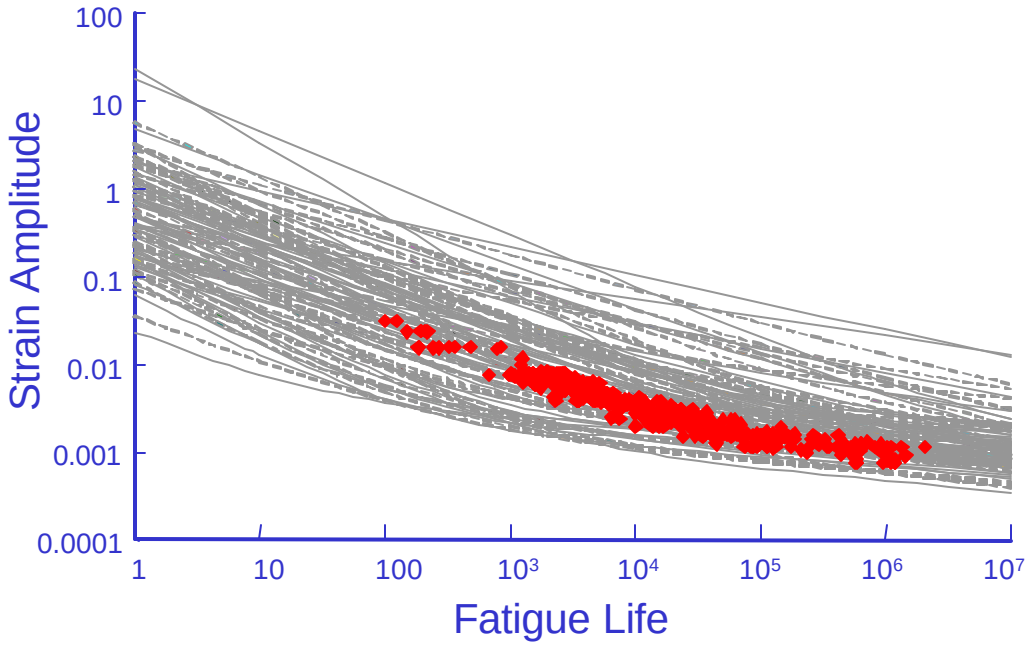




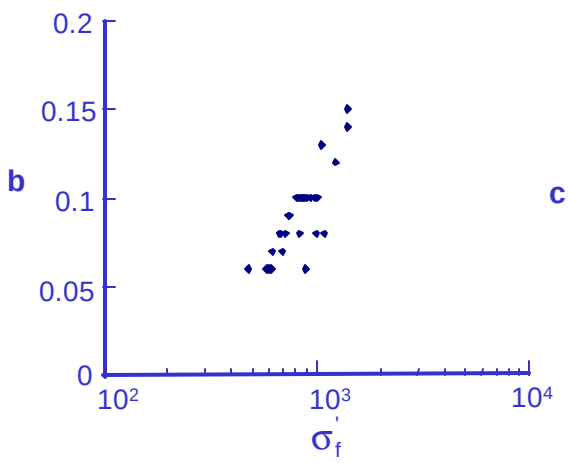
Input Data Simulation

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f^{'}\left(L, \mu_{\sigma_f}, \sigma_{\sigma_f}\right)}{E}\left(2N_f\right)^{b\left(N, \mu_b, \sigma_b\right)} + \varepsilon_f^{'}\left(L, \mu_{\varepsilon_f}, \sigma_{\varepsilon_f}\right)\left(2N_f\right)^{c\left(N, \mu_b, \sigma_b\right)}$$

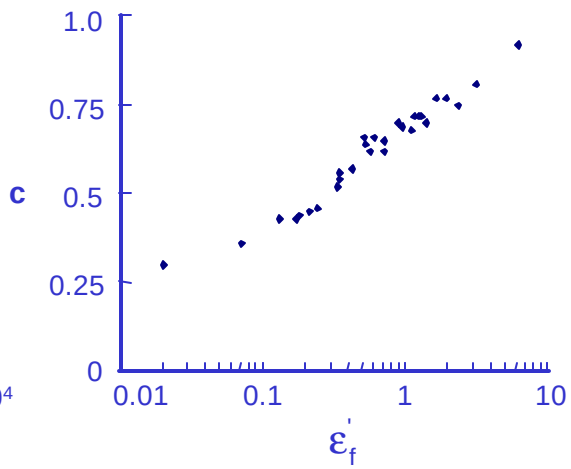
Simulation Results



Correlation



ρ = -0.828



ρ = -0.976

Generating Correlated Data

$$z_1 = \Phi(\text{ rand() }) \qquad z_1 = N(0,1)$$

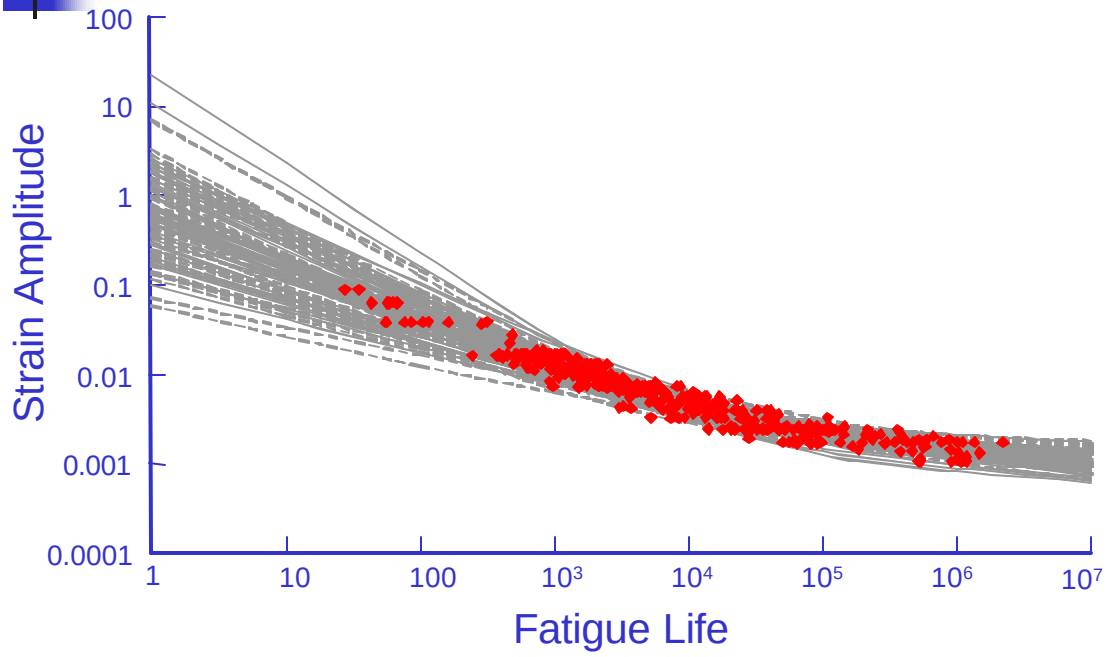
$$z_2 = \Phi(\text{ rand() })$$

$$z_3 = z_1 \rho + z_2 \sqrt{1-\rho^2}$$

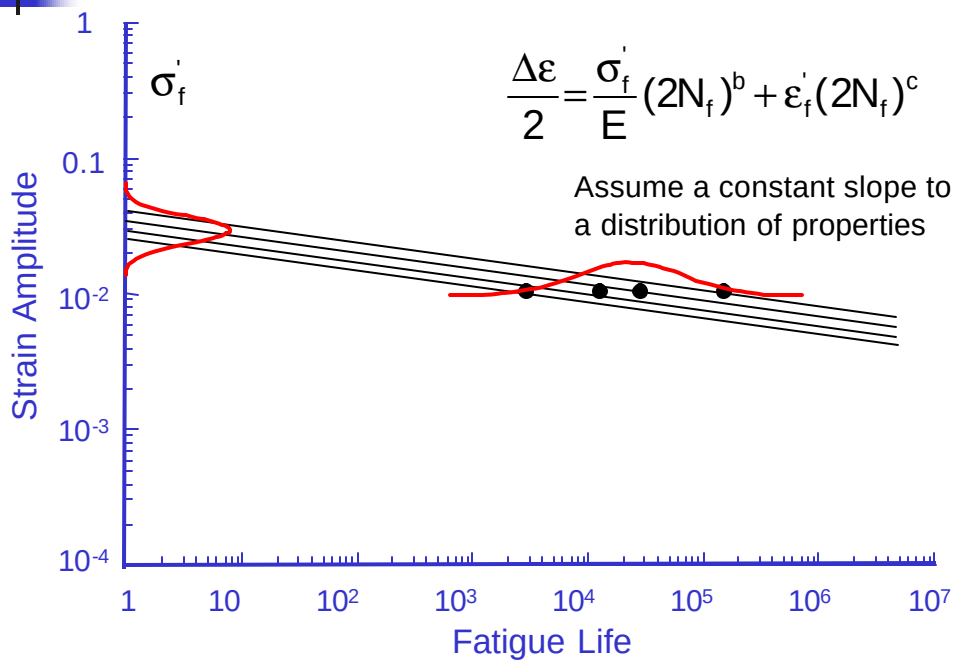
$$\sigma_f' = \exp(\mu_{\ln\sigma_f'} + \sigma_{\ln\sigma_f'} z_1)$$

$$b = \mu_b + \sigma_b z_3$$

Correlated Properties



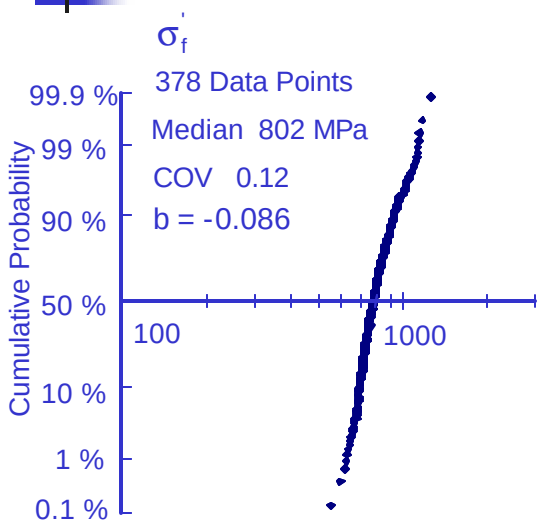
Curve Fitting

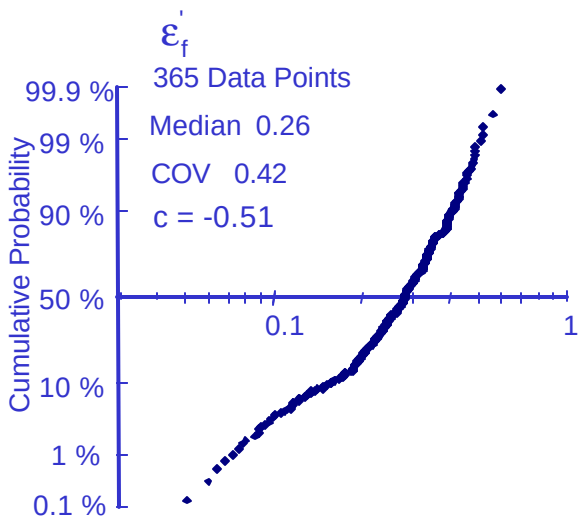


$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f'}{E}(2N_f)^b + \varepsilon_f'(2N_f)^c$$

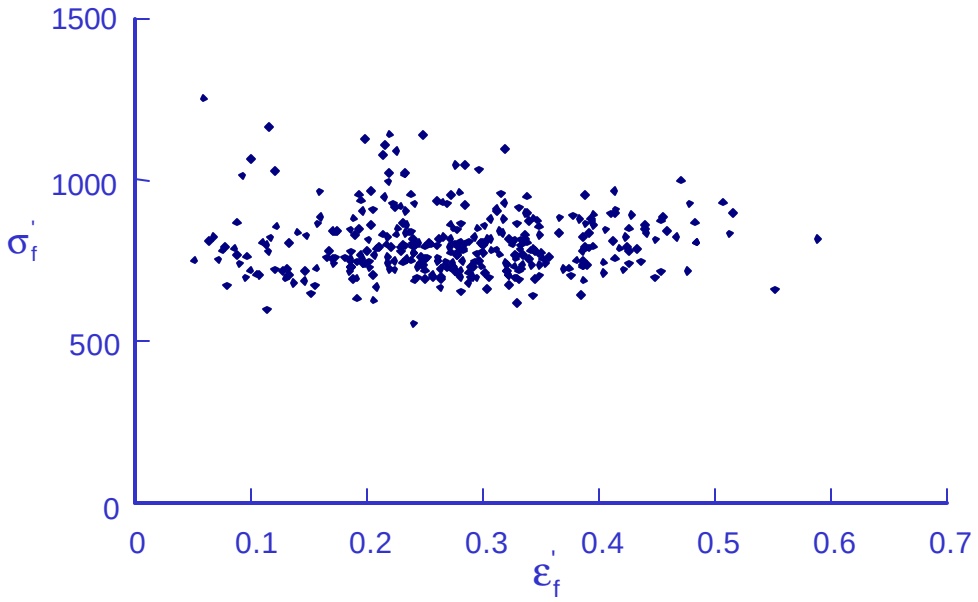
Assume a constant slope to get a distribution of properties

Property Distribution

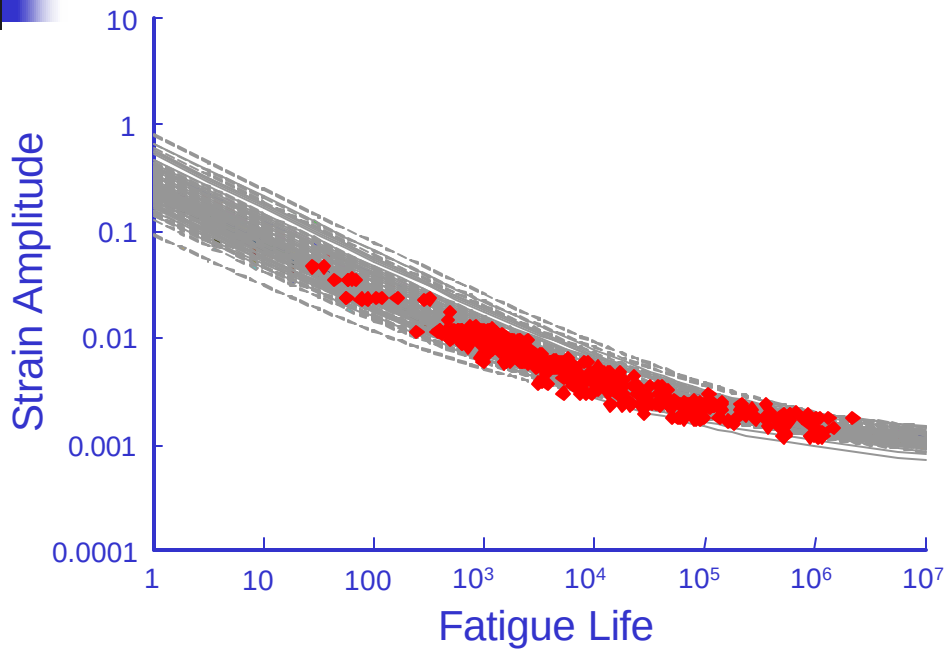




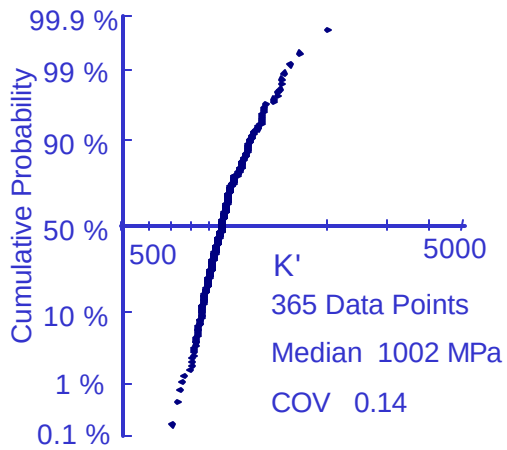
Correlation

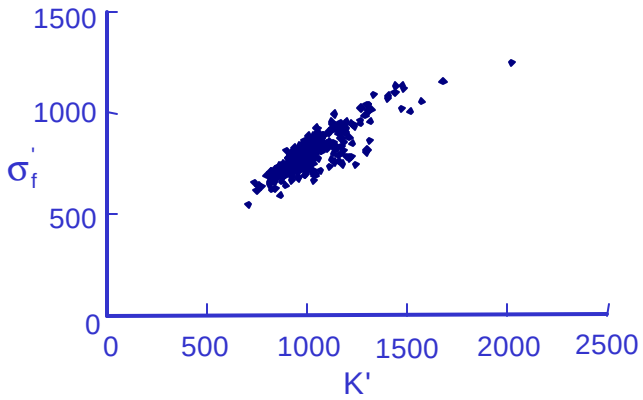


Simulation



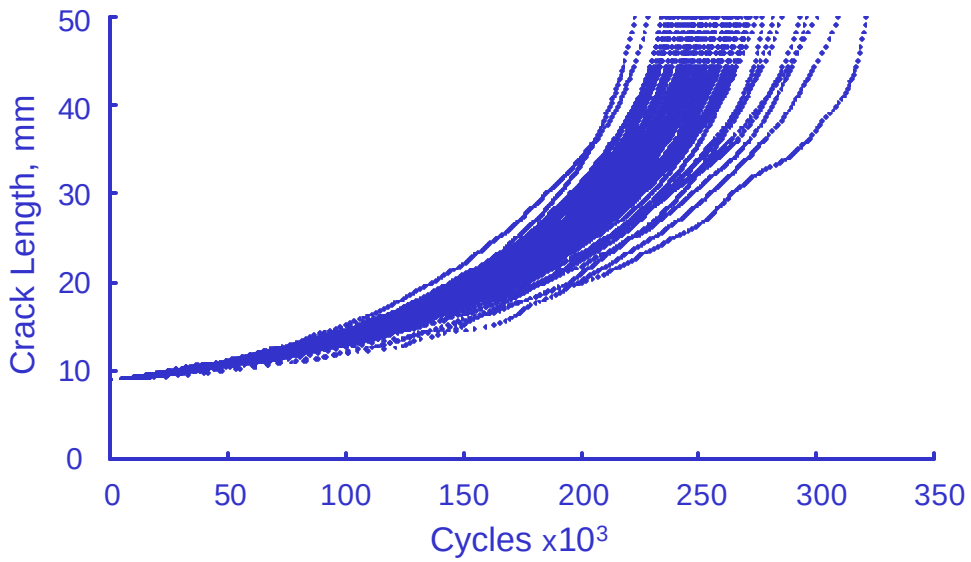
Strength Coefficient





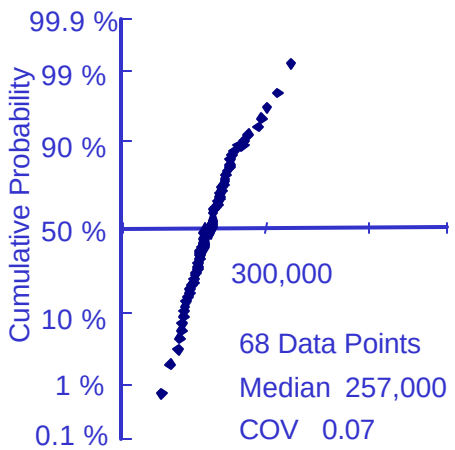
$\rho = 0.863$

Crack Growth Data

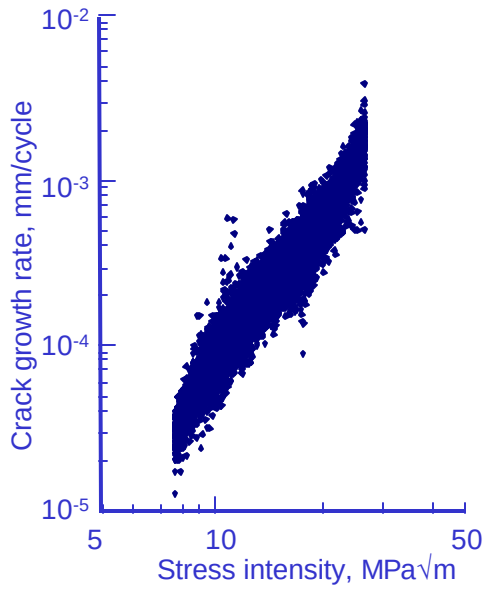


Virkler, Hillberry and Goel, "The Statistical Nature of Fatigue Crack Propagation", Journal of Engineering Materials and Technology, Vol. 101, 1979, 148-153

Crack Growth Rate Data



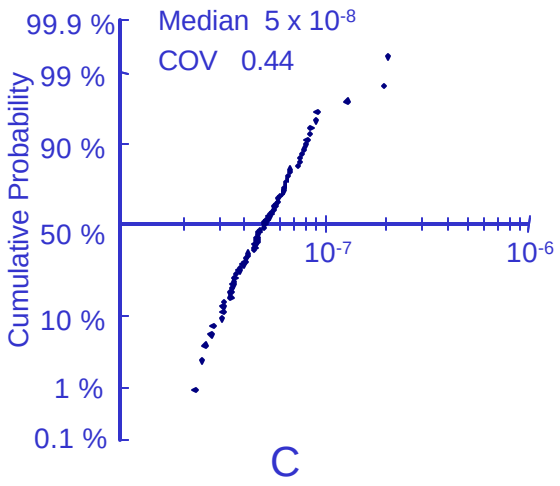
Fatigue Lives

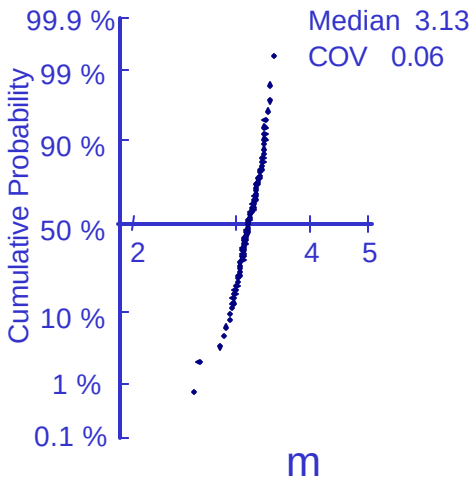


Crack Growth Rate

Crack Growth Properties

$$\frac{da}{dN} = C\,\Delta K^{m}$$





Beware of Correlated Variables

$$N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C\,\Delta S^m\,\pi^{\frac{m}{2}}\left(1-m/2\right)}$$

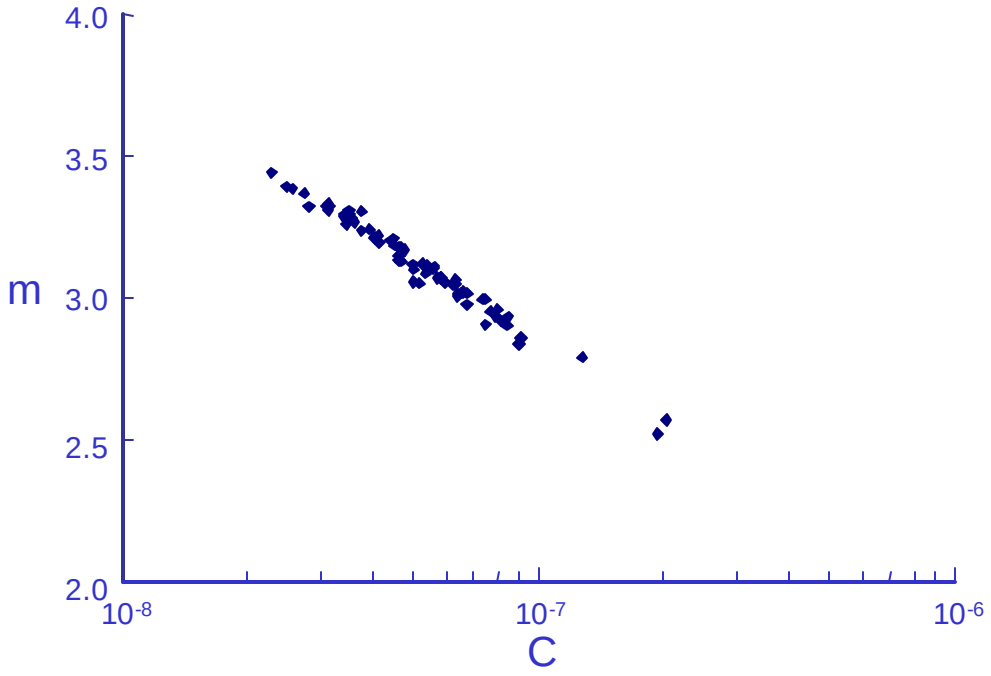
N_f and C are linearly related and should have the same variability, but

$$COV_{N_f} = 0.07$$

$$COV_C = 0.44$$

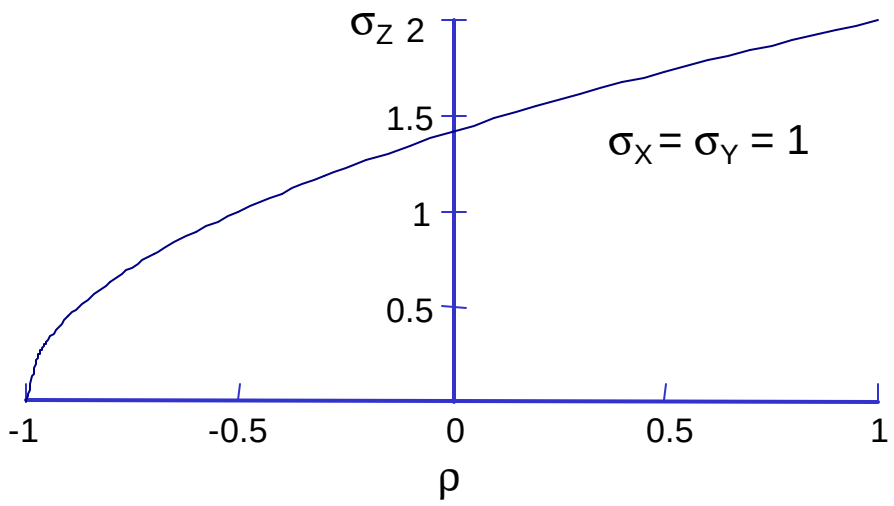
because C and m are correlated.

Correlation

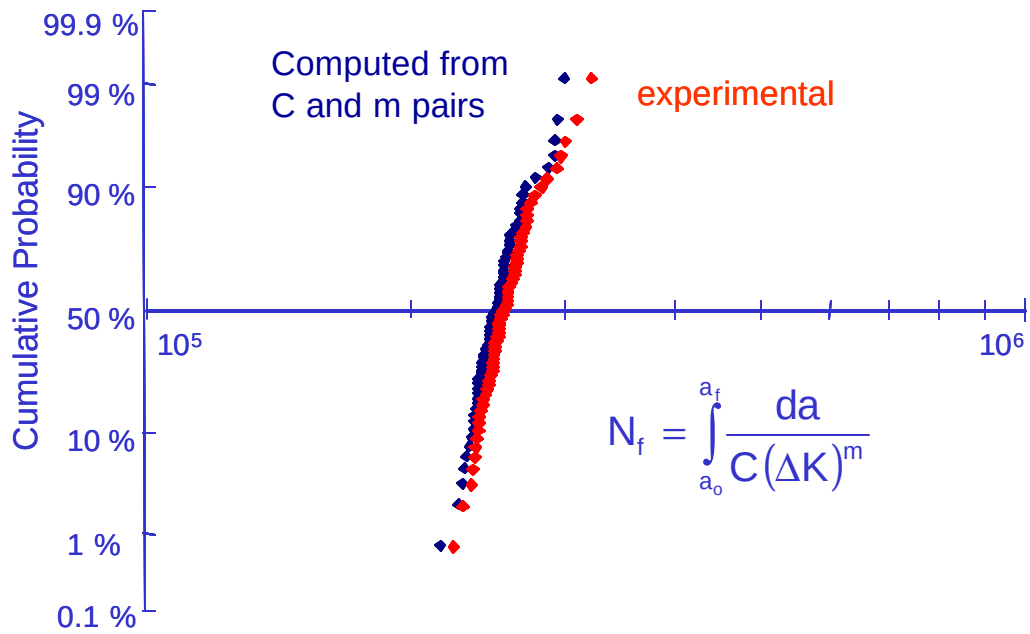


Correlated Variables

$$\sigma_Z = \sqrt{\sigma_X^{\ 2} + 2\rho\sigma_X\sigma_Y + \sigma_Y^{\ 2}}$$



Calculated Lives

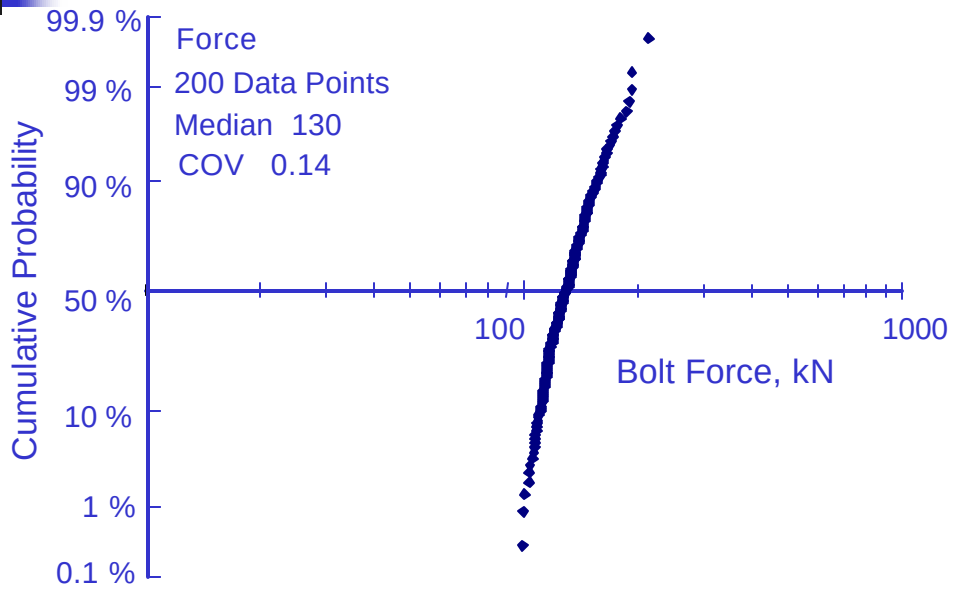


$$N_f = \int_{a_o}^{a_f} \frac{da}{C(\Delta K)^m}$$

Manufacturing/Processing Variability

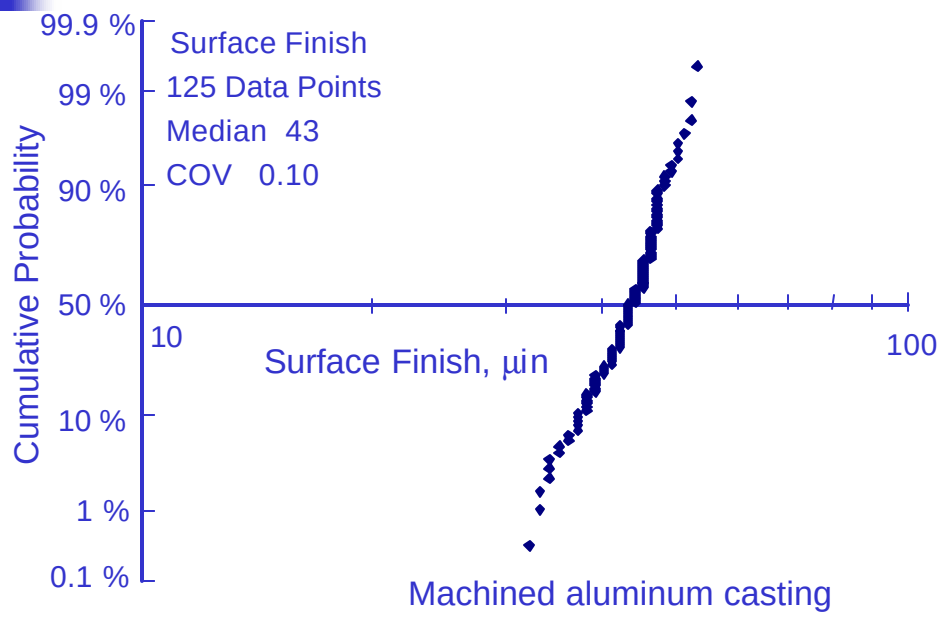
- Bolt Forces
- Surface Finish
- Drilled Holes

Variability in Bolt Force

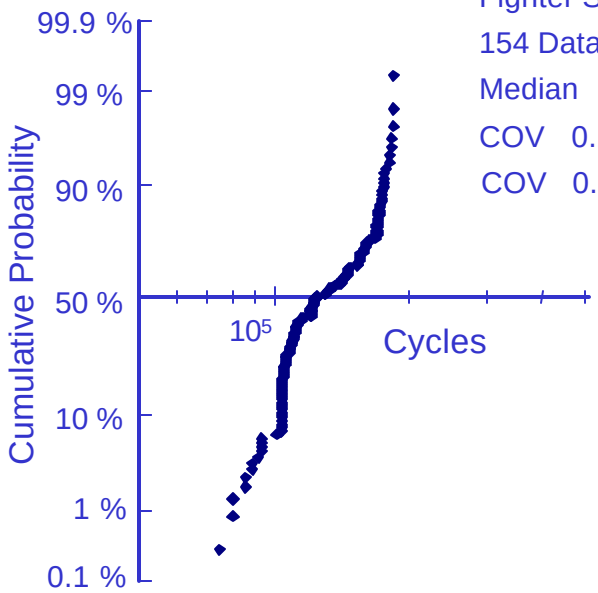


Preload force in bolts tightened to 350 Nm

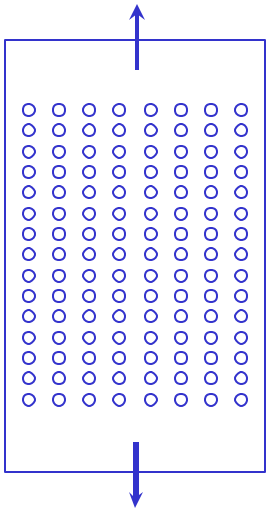
Surface Roughness Variability



Drilled Holes



Fighter Spectrum
154 Data Points
Median 126,750
COV 0.22 in life
COV 0.07 in strength



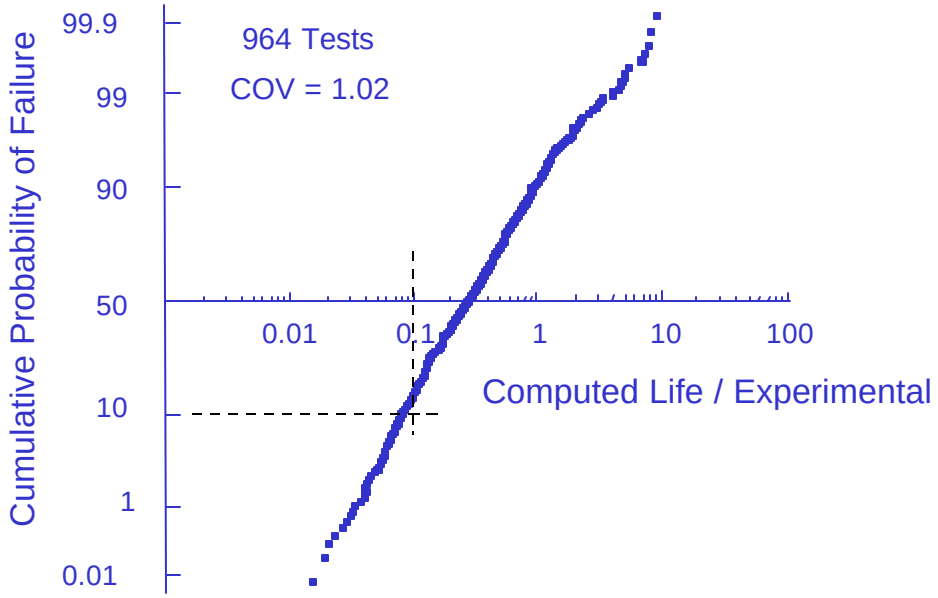
180 drilled holes in a single plate

From: J.P. Butler and D.A. Rees, "Development of Statistical Fatigue Failure Characteristics of 0.125-inch 2024-T3 Aluminum Under Simulated Flight-by-Flight Loading," ADA-002310 (NTIS no.), July 1974.

Analysis Uncertainty

- Miners Linear Damage rule
- Strain Life Analysis

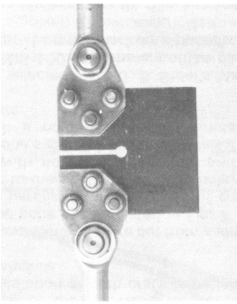
Miners Rule

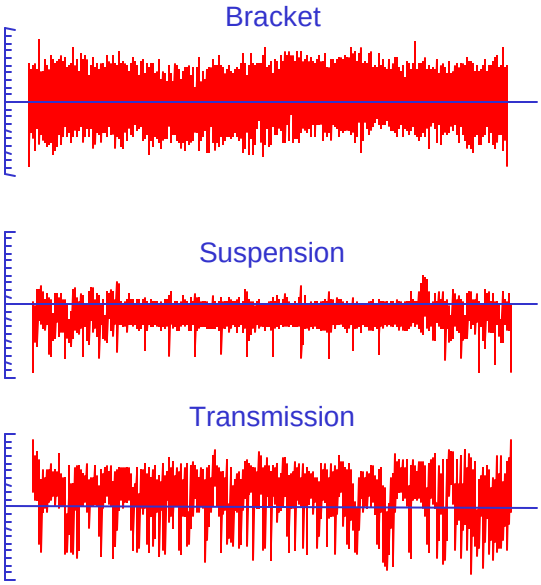


A safety factor of 10 in life would result in a 10% chance of failure

From Erwin Haibach "Betriebsfestigkeit", Springer-Verlag, 2002

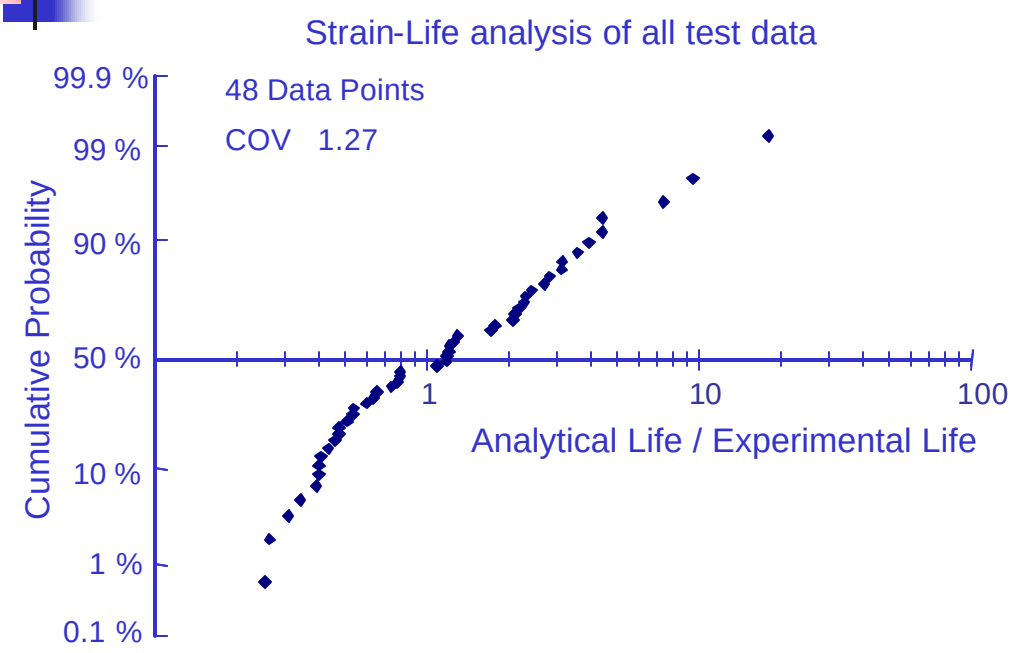
SAE Specimen



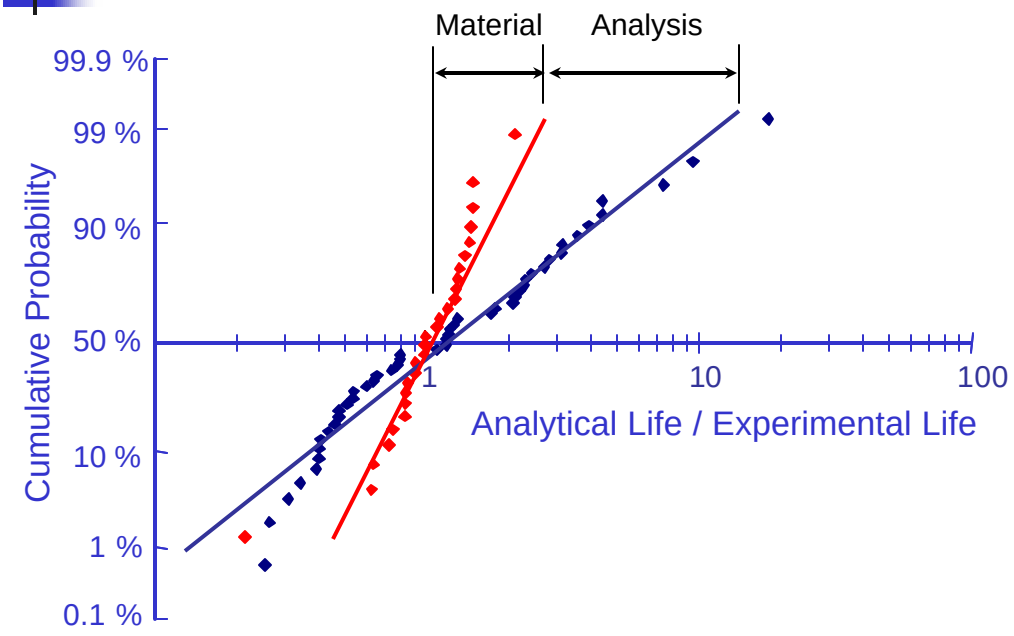


Fatigue Under Complex Loading: Analysis and Experiments, SAE AE6, 1977

Analysis Results



Material Variability



Strain-Life back calculation of specimen lives

Modeling Uncertainty

Analysis Uncertainty C_U = ?

The variability in reproducing the original strain life data from the material constants is $C_M \sim 0.44$

$$\text{COV} \quad C = \sqrt{\prod_{i=1}^{n}\left(1+C_{X_i}^{\ 2}\right)^{a_i^{\ 2}} - 1}$$

$$1+C_U^{\ 2} = \frac{1+C_{N_f}^{\ 2}}{1+C_M^{\ 2}}$$

$$C_U = 1.09$$

90% of the time the analysis is within a factor of 3 !
99% of the time the analysis is within a factor of 10 !

Variability from Multiple Sources

$$\text{COV} \quad C = \sqrt{\prod_{i=1}^{n}\left(1+C_{X_i}{}^{2}\right)^{a_i^{2}} - 1}$$

Suppose we have 4 variables each with a COV = 0.1

The combined variability is COV = 0.29

Suppose we reduce the variability of one of the variables to 0.05

The combined variability is now COV = 0.27

If all of the COV's are the same, it doesn't do any good to reduce only one of them, you must reduce all of them !

Variability from Multiple Sources

$$\text{COV} \quad C = \sqrt{\prod_{i=1}^{n}\left(1+C_{X_i}{}^{2}\right)^{a_i^{2}} - 1}$$

Suppose we have 3 variables each with a COV = 0.1 and one with COV = 0.4

The combined variability is COV = 0.65

Suppose we reduce the variability of these variables to 0.05

The combined variability is now COV = 0.60

If one of the COV's is large, it doesn't do any good to reduce the others, you must reduce the largest one !

Variability Summary

	Source	COV
Stress	Service Loading	0.5
	Environment	0.3
Strength	Materials	0.1
	Manufacturing	0.1
	Surface Finish	0.1
	Fatigue Lives	1.0
	Analysis Uncertainty	1.0

$$\text{Fatigue life} \propto \left(\frac{\text{Strength}}{\text{Stress}}\right)^5$$

Variability and Uncertainty

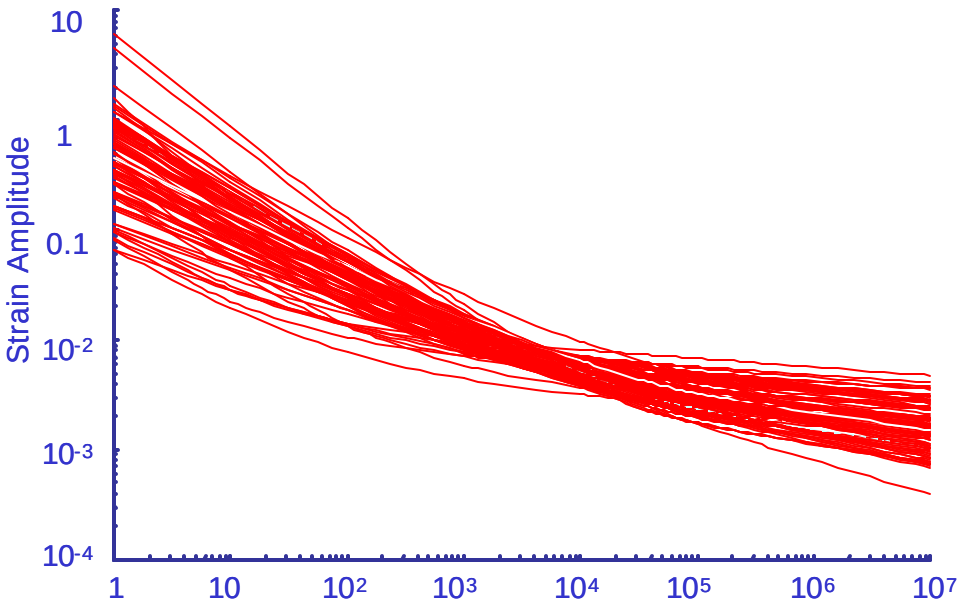
Variability: Every apple on a tree has a different mass.

Uncertainty: The variety of the apple is unknown.

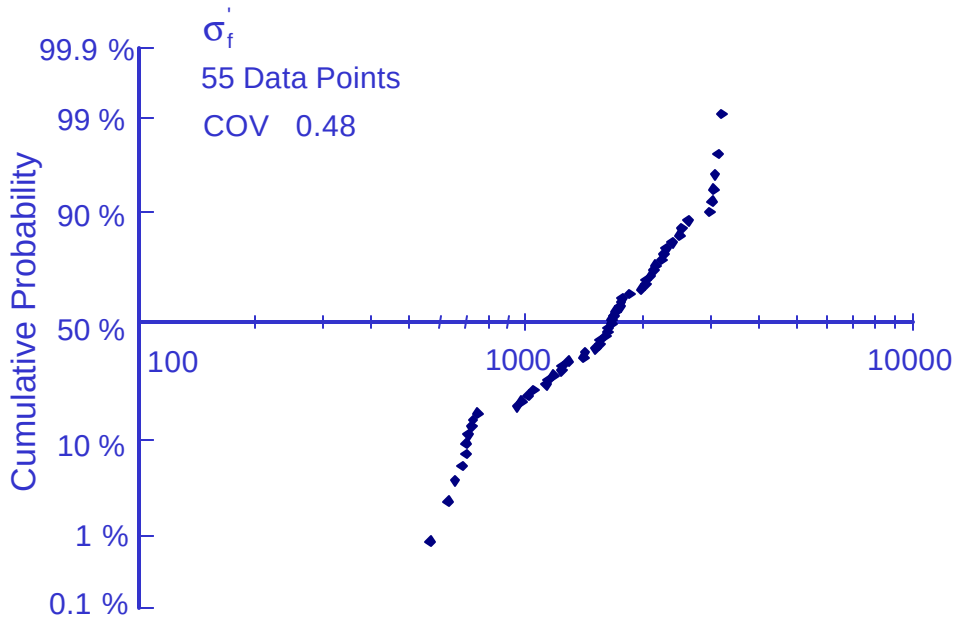
Variability: Multiple samples of the same material

Uncertainty: What is the material

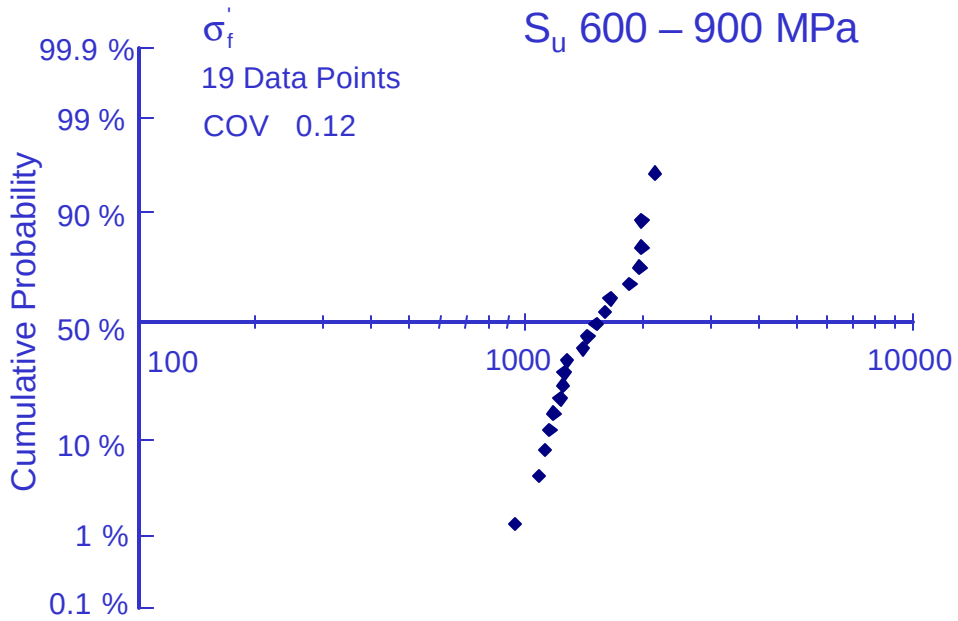
Strain Life Data for 93 Steels



Uncertainty for all Steels



Uncertainty for Structural Steels



Variability and Uncertainty

Fatigue Strength Coefficient

	Variability	Uncertainty	Combined
All Steels	0.12	0.48	0.75
Structural Steel	0.12	0.12	0.24

Quiz

At my last seminar everyone hit a golf ball and we recorded the maximum acceleration.

What is the expected variability ?

Results

	12.9
	7.7
	5.88
	11.1
	15.5
	10.3
	18.1
μ	11.64
σ	4.26
COV	0.37

Probabilistic Aspects of Fatigue