Fatigue and Fracture
(Basic Course)

Factors Influencing Fatigue

Mean Stress

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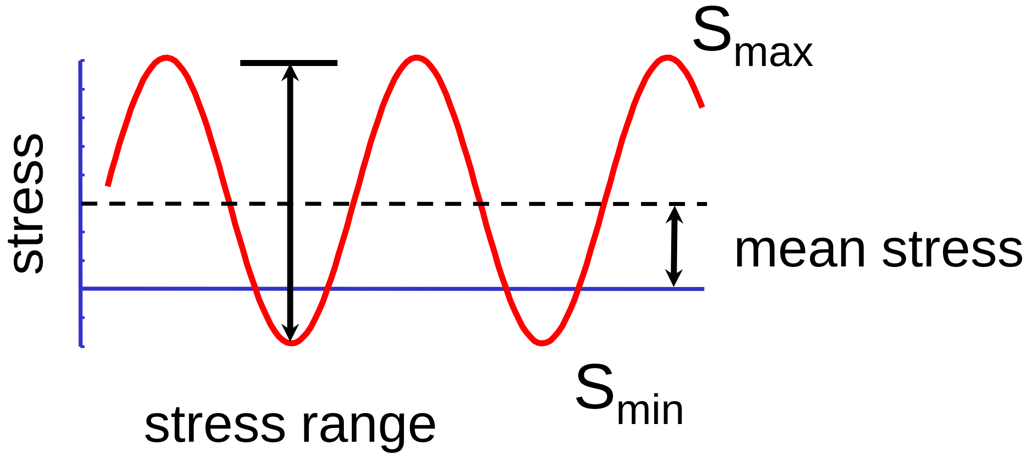
Factors Influencing Fatigue

- **Mean Stress**
- Variable Amplitude
- Stress Concentrations
- Surface Finish

Mean Stresses



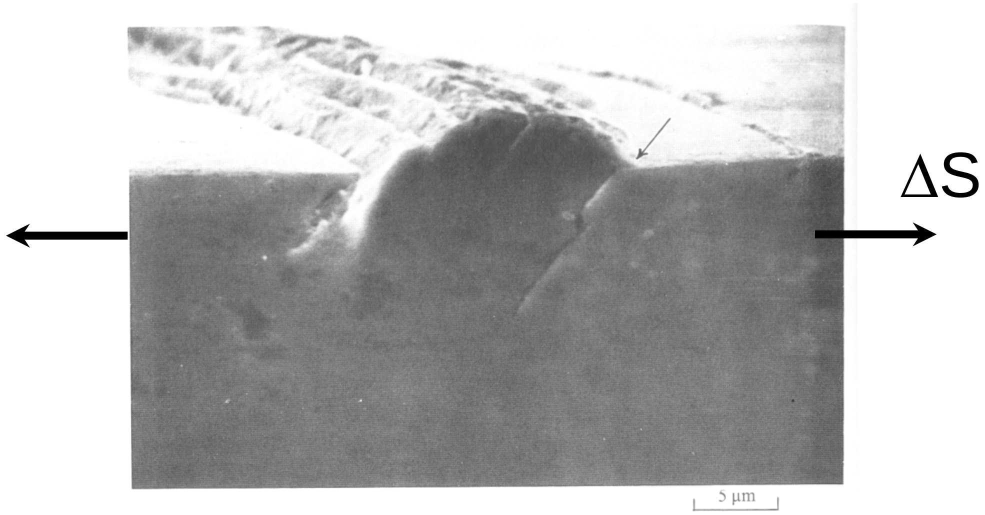
$$S_{mean} = \frac{S_{max} + S_{min}}{2}$$

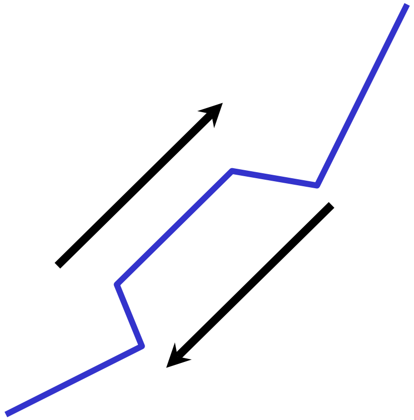
$$R = \frac{S_{min}}{S_{max}}$$

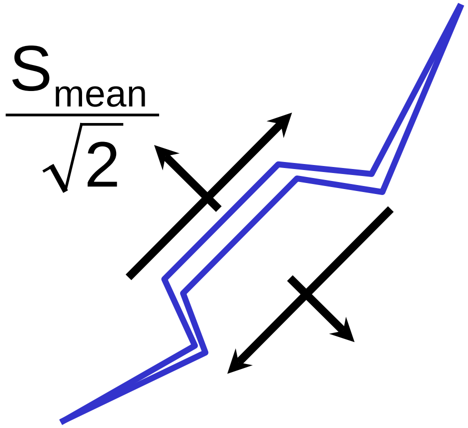
General Observations

- Tensile mean stresses reduce the fatigue life or decrease the allowable stress range
- Compressive mean stresses increase the fatigue life or increase the allowable stress range

Mechanism







Fatigue damage is a shear process

Tensile mean stresses open microcracks and make sliding easier

Goodman 1890

Mechanics Applied to Engineering
John Goodman, 1890

“.. whether the assumptions of the theory are justifiable or not …. We adopt it simply because it is the easiest to use, and for all practical purposes, represents Wöhlers data.

$$S_{ultimate} = S_{min} + 2\,\Delta S$$

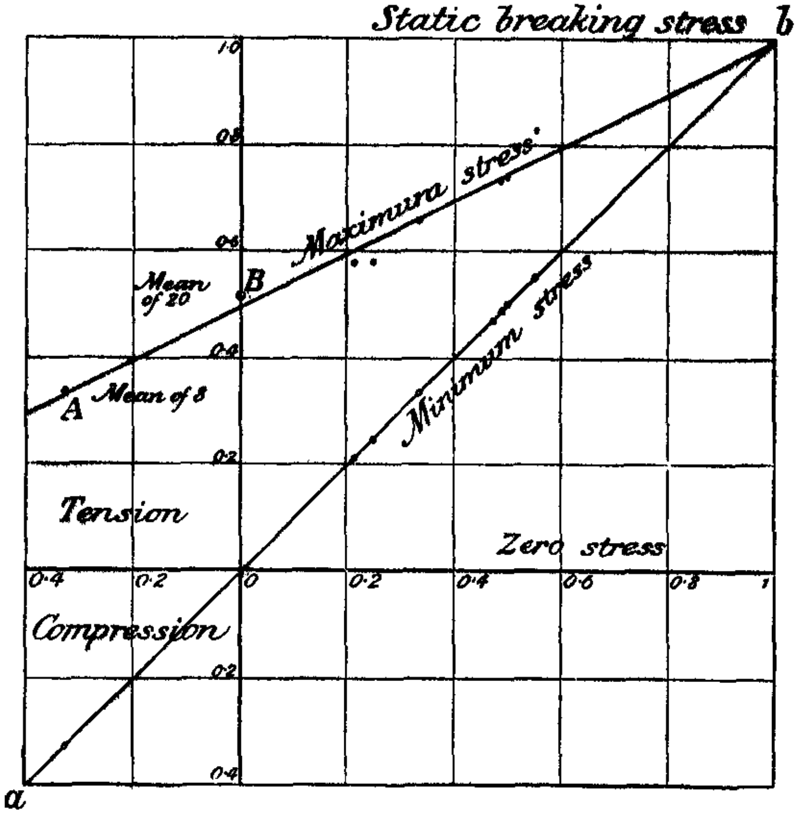
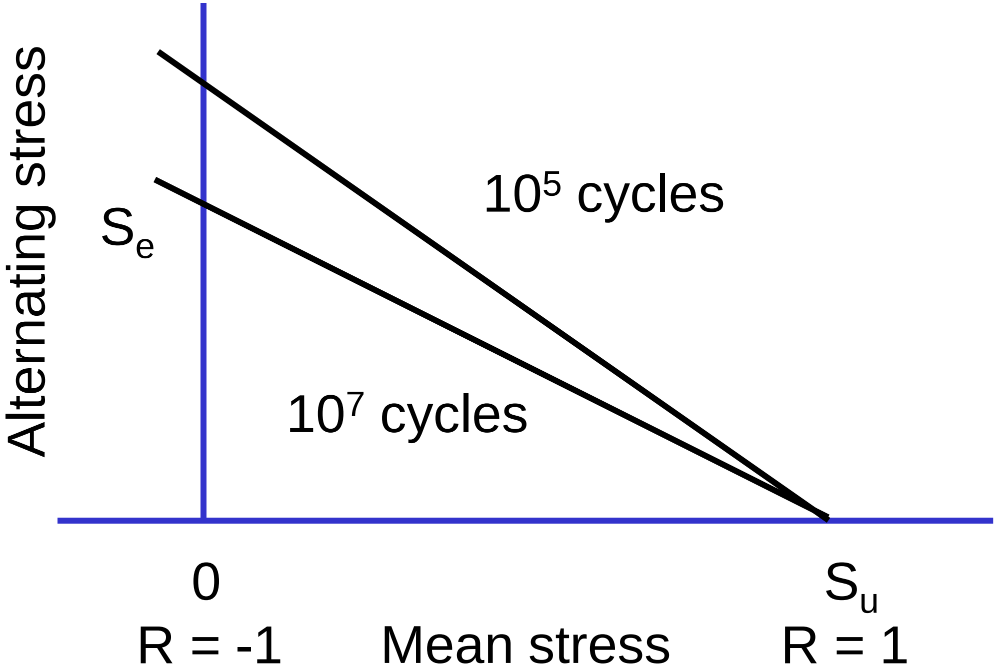


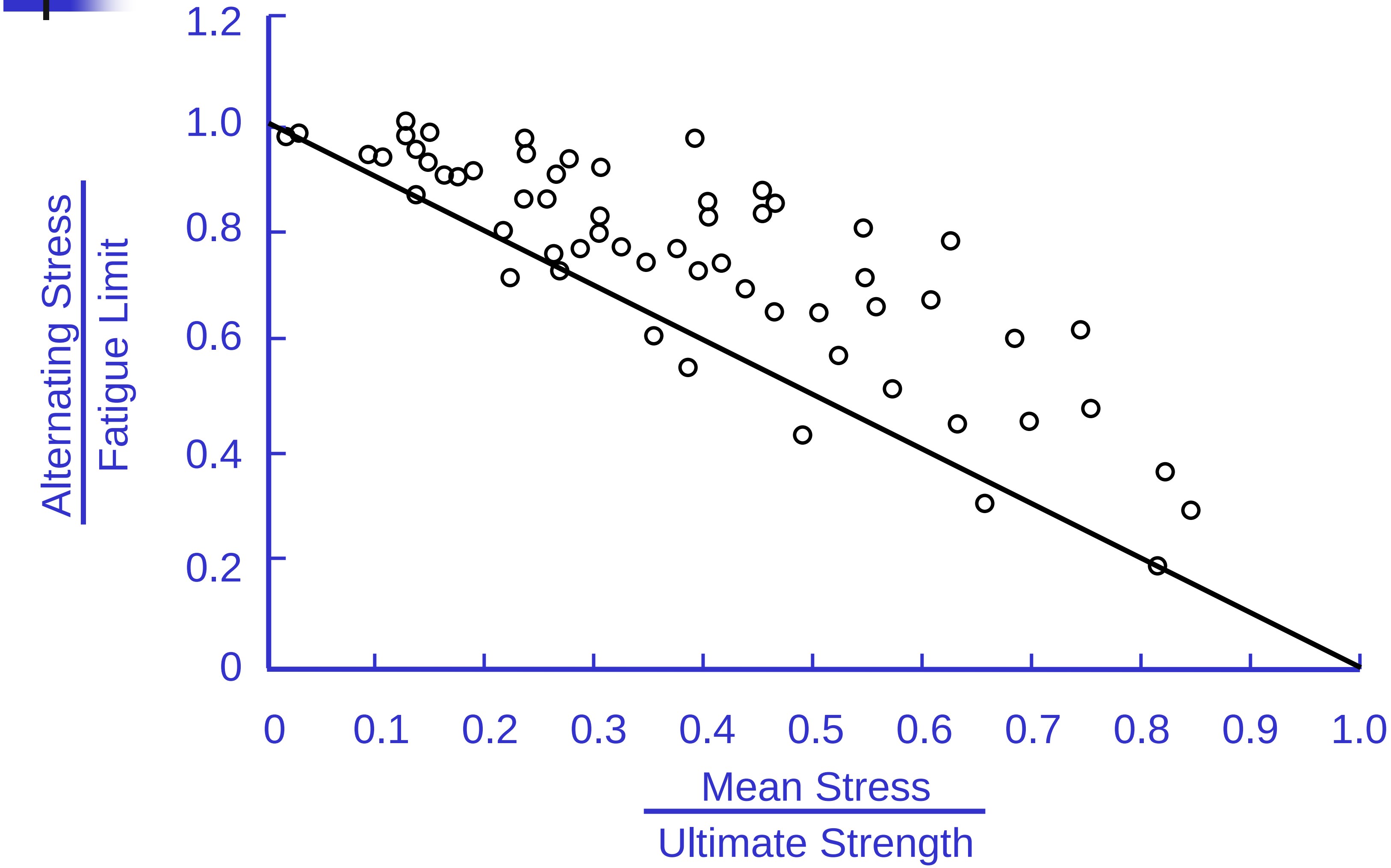
FIG. 517.

Goodman Diagram



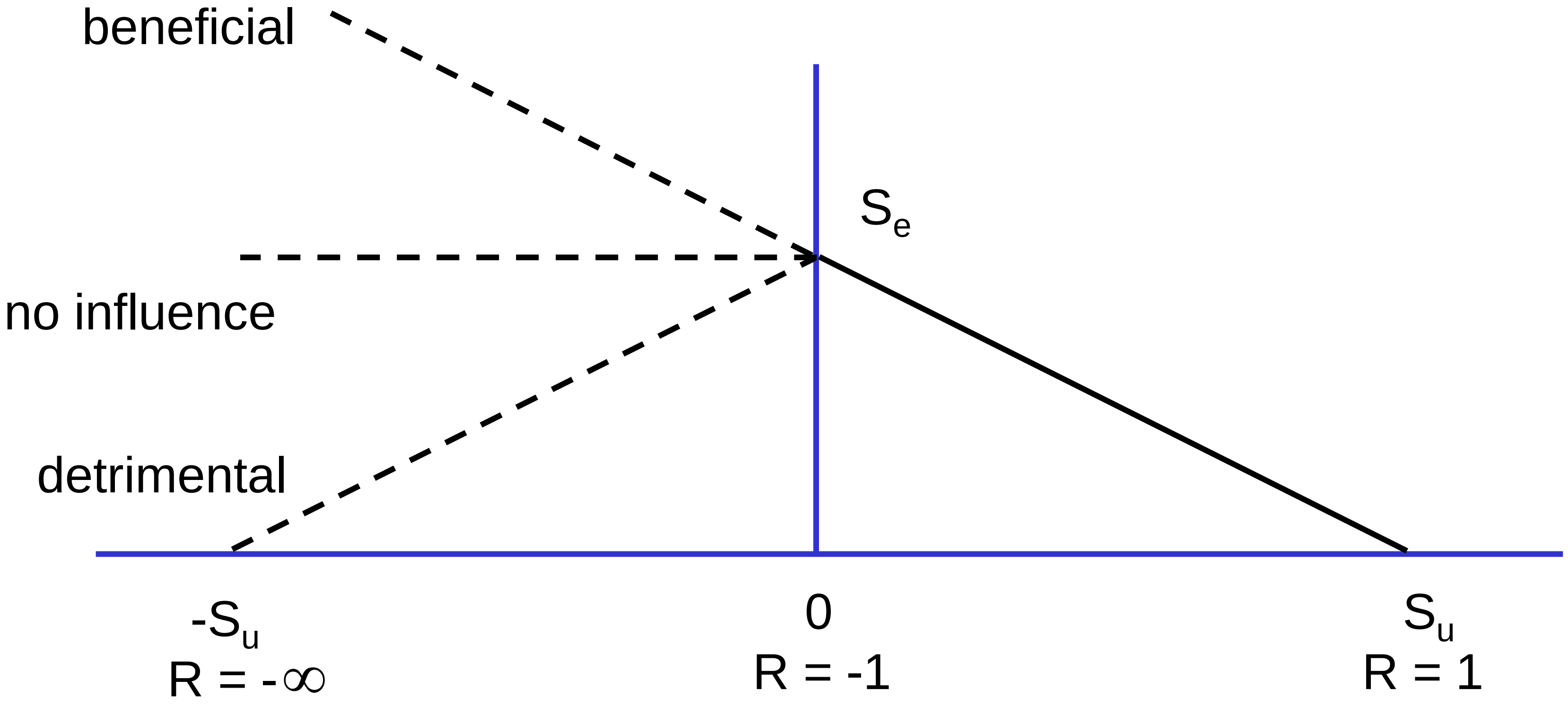
$$\frac{\Delta S}{2} = \left(\frac{\Delta S}{2}\right)_{R=-1}\left(1 - \frac{S_{mean}}{S_{ultimate}}\right)$$

Test Data (1941)

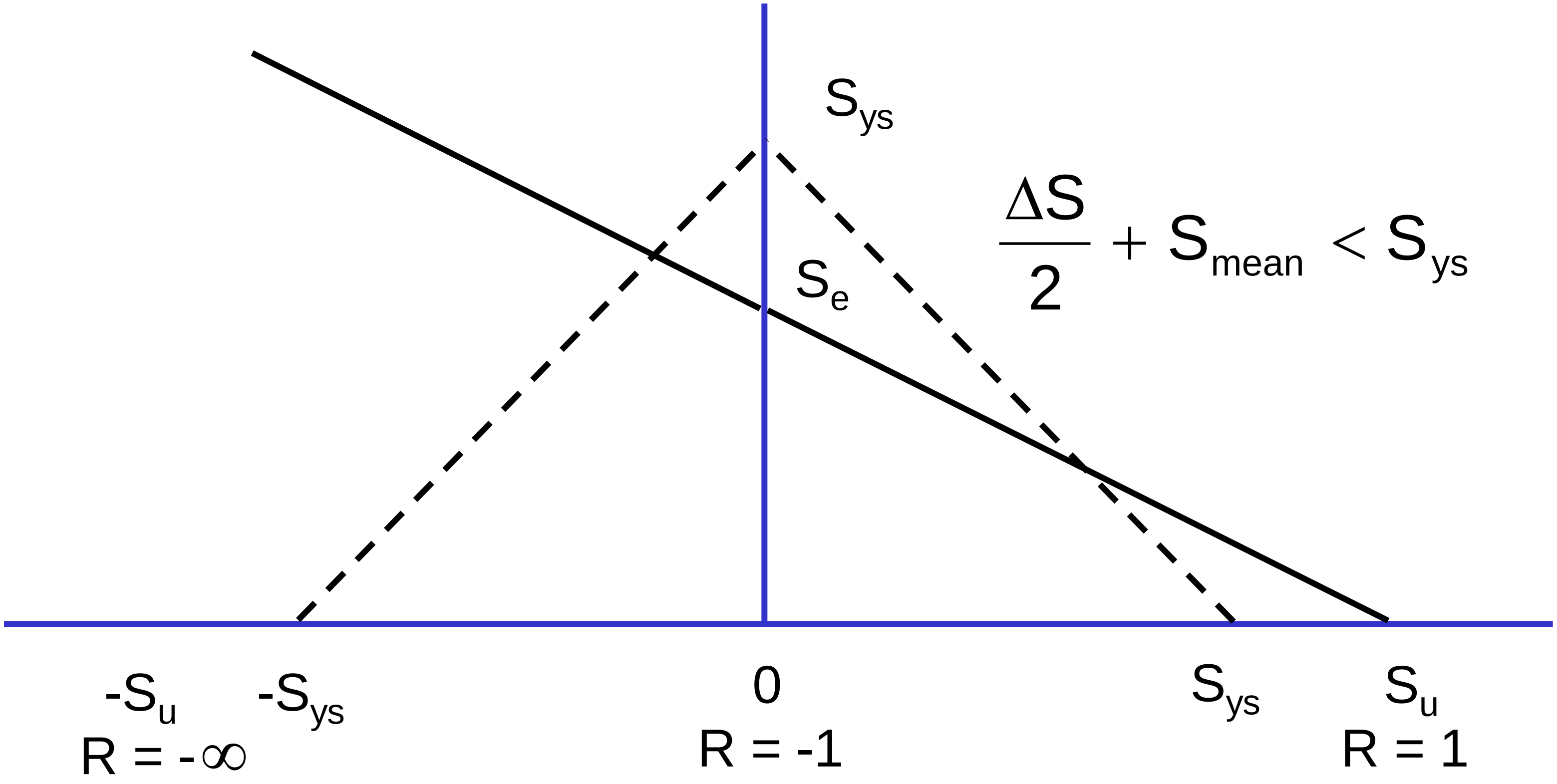


J.O. Smith, The Effect of Range of Stress on the Fatigue Strength of Metals, Engineering Experiment Station Bulletin 334, University of Illinois, 1941

Compression

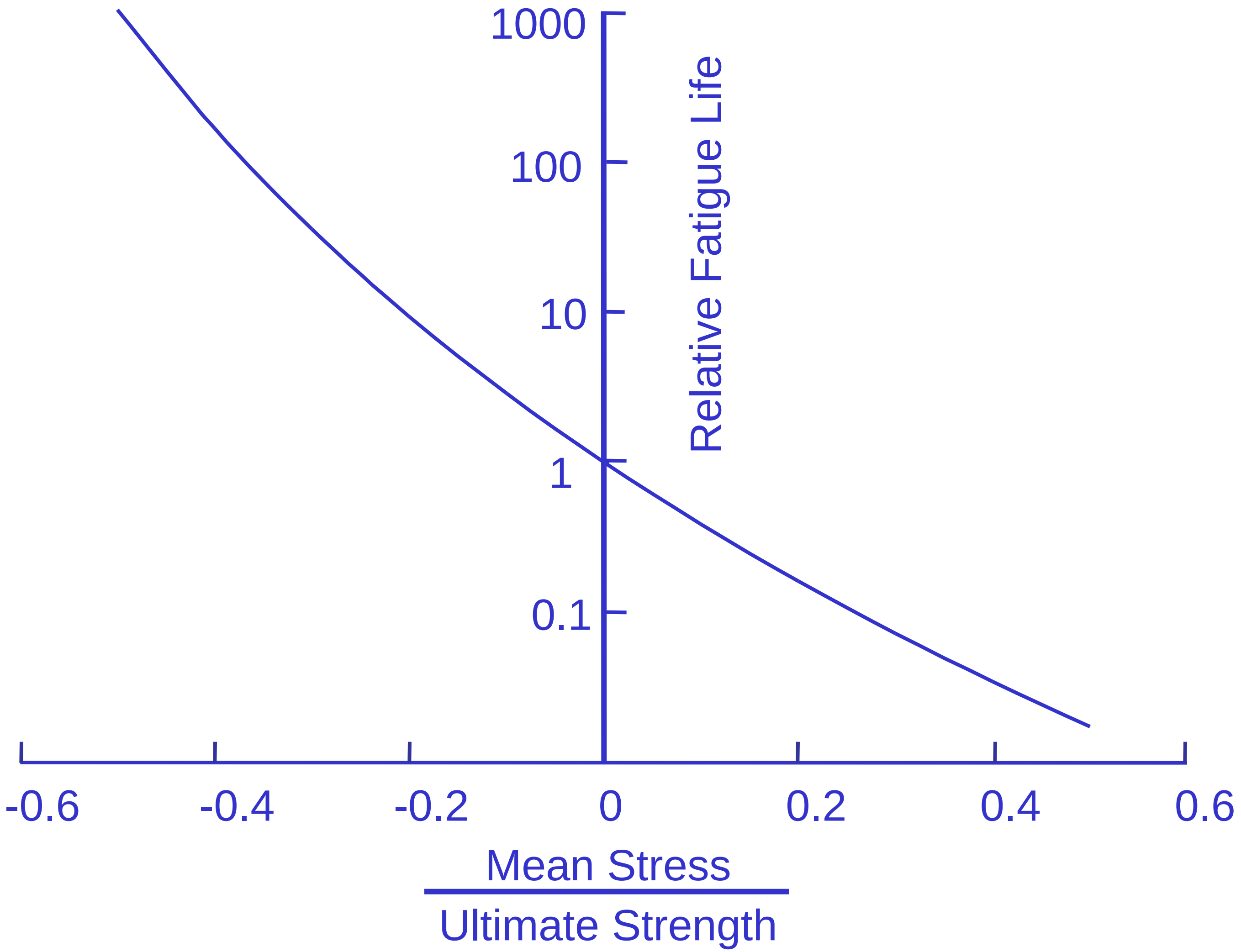


Modified Goodman (no yielding)

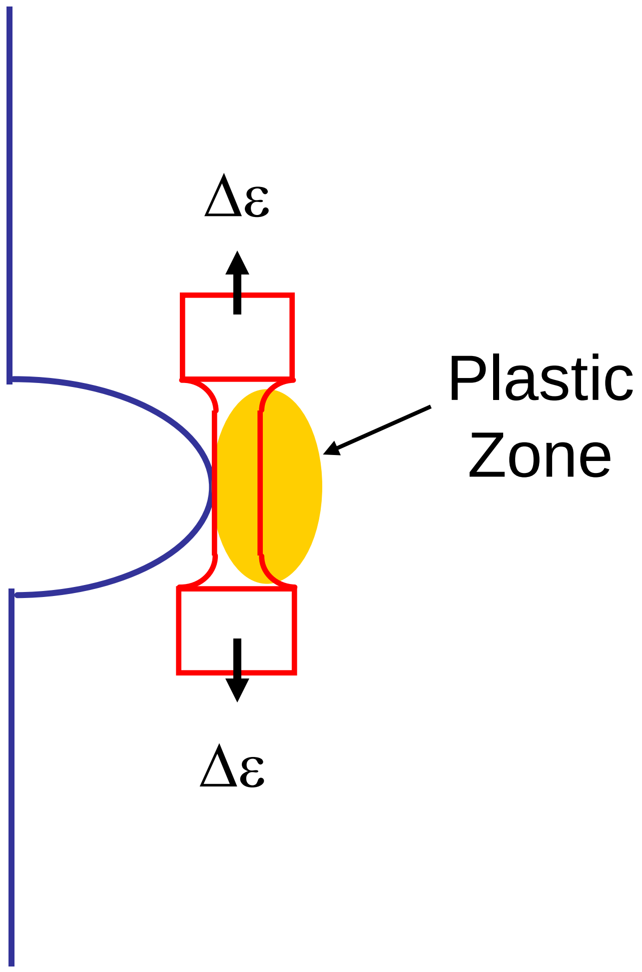


$$\frac{\Delta S}{2} + S_{mean} < S_{ys}$$

Mean Stress Influence on Life



Stress Concentrations

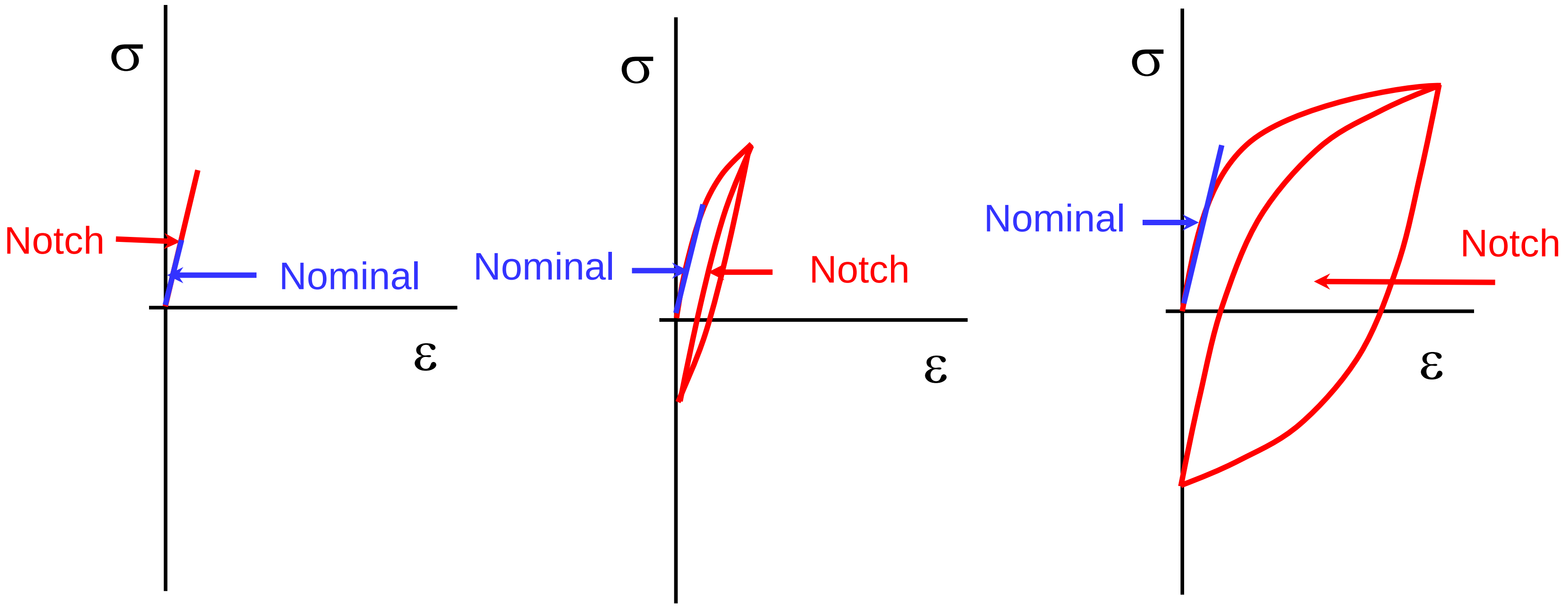


The elastic material surrounding the plastic zone around a stress concentration forces the material to deform in strain control

Mean Stresses at Notches

elastic

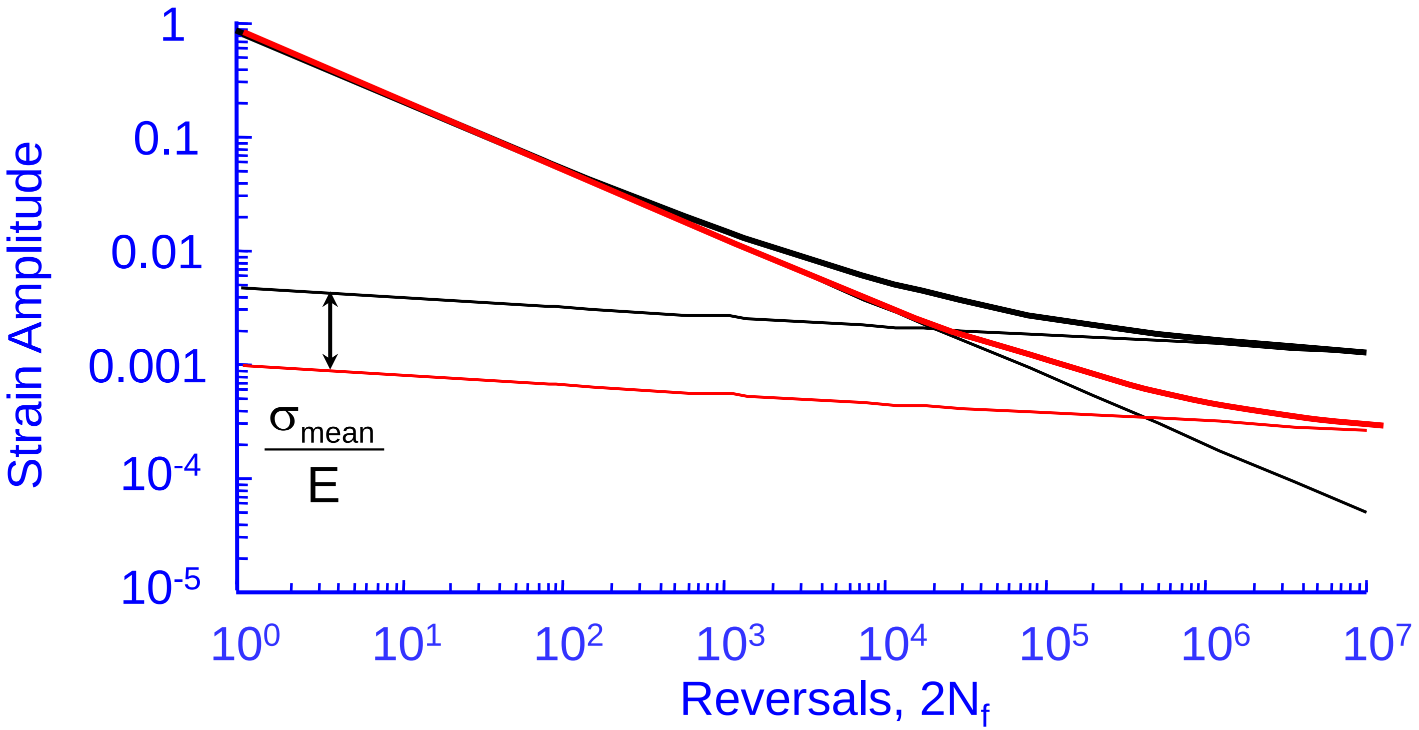
plastic



Nominal mean stress is less than notch mean stress

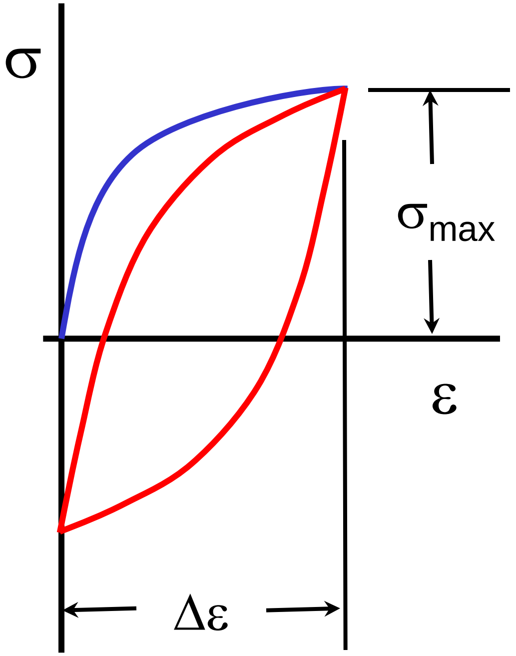
Nominal mean stress is greater than notch mean stress

Morrow Mean Stress Correction



$$\frac{\Delta\varepsilon}{2} = \frac{\sigma_f^{'} - \sigma_{mean}}{E}(2N_f)^b + \varepsilon_f^{'}(2N_f)^c$$

Smith Watson Topper



$$\sigma_{max}\frac{\Delta\varepsilon}{2}=\frac{{\sigma'_f}^2}{E}(2N_f)^{2b}+\sigma'_f\varepsilon'_f(2N_f)^{b+c}$$

Mean Stress Relaxation

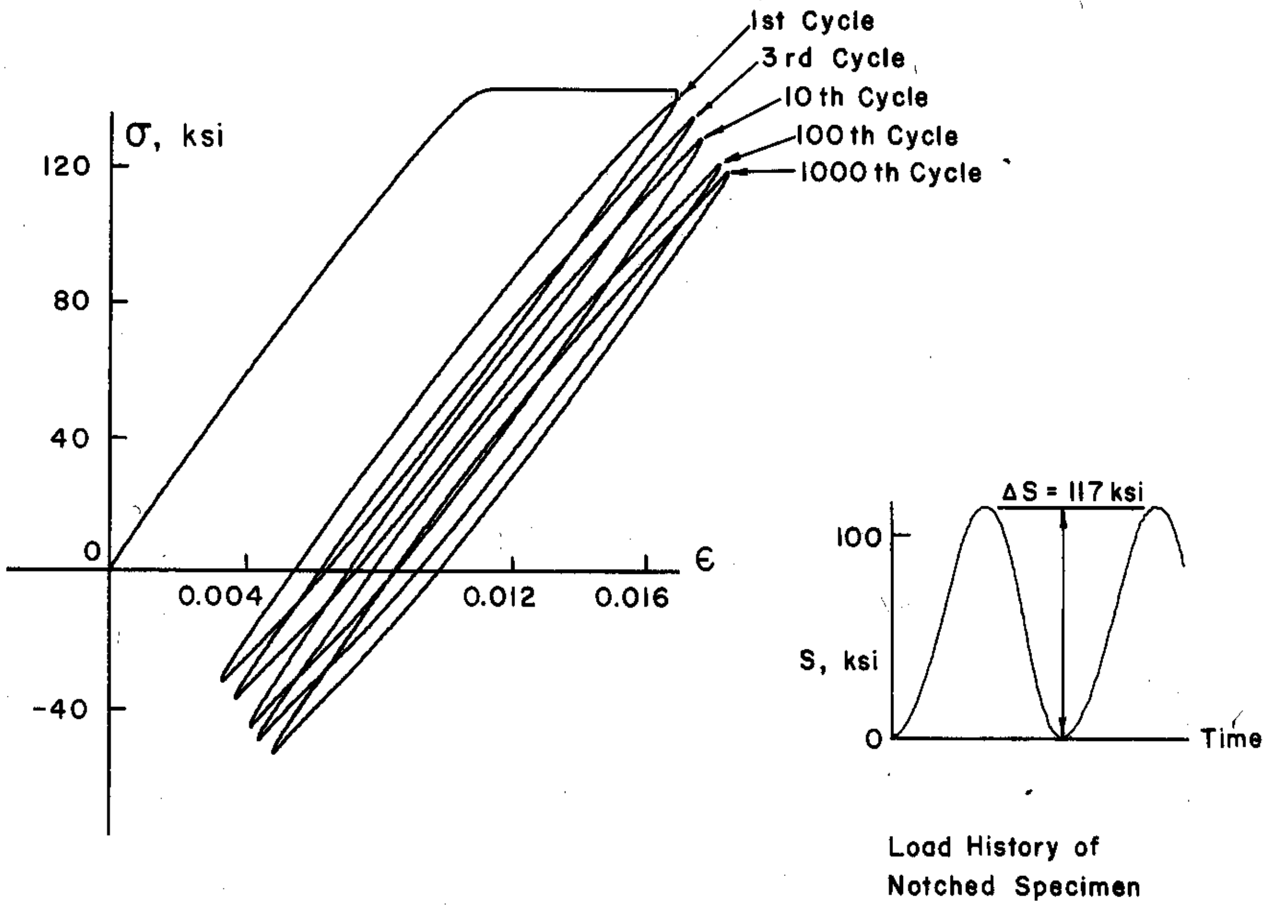
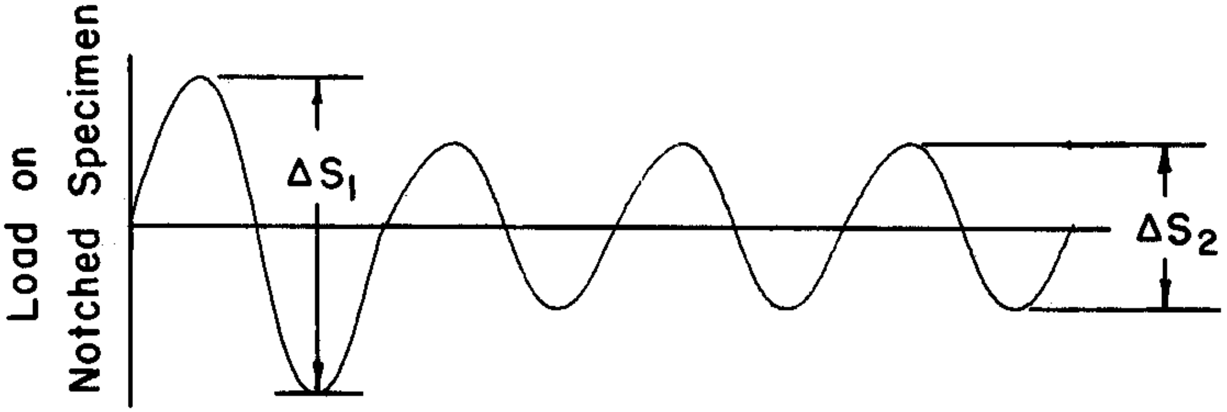


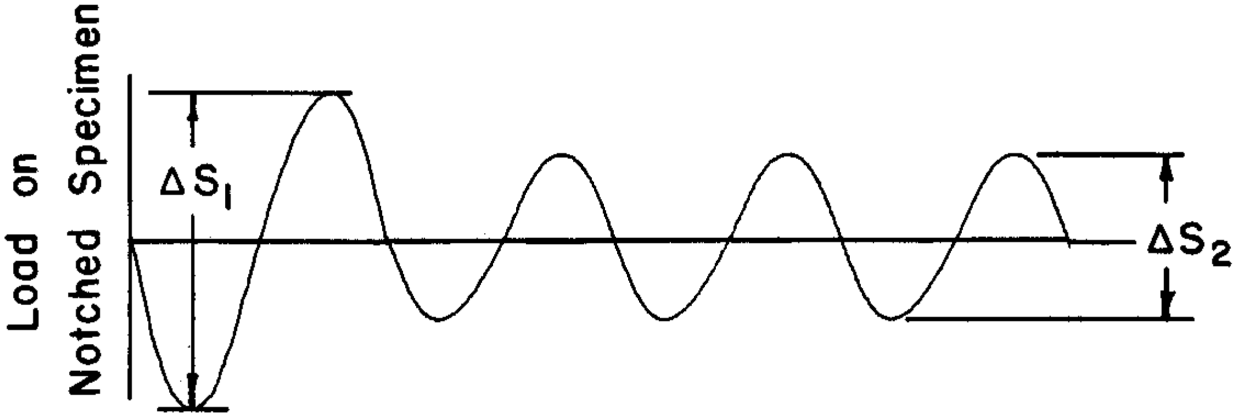
FIG. 7—*Cyclic softening and relaxation of mean stress under Neuber control (Ti-8Al-1Mo-1V, $K_t = 1.75$).*

Stadnick and Morrow, “Techniques for Smooth Specimen Simulation of Fatigue Behavior of Notched Members” ASTM STP 515, 1972, 229-252

Loading Histories

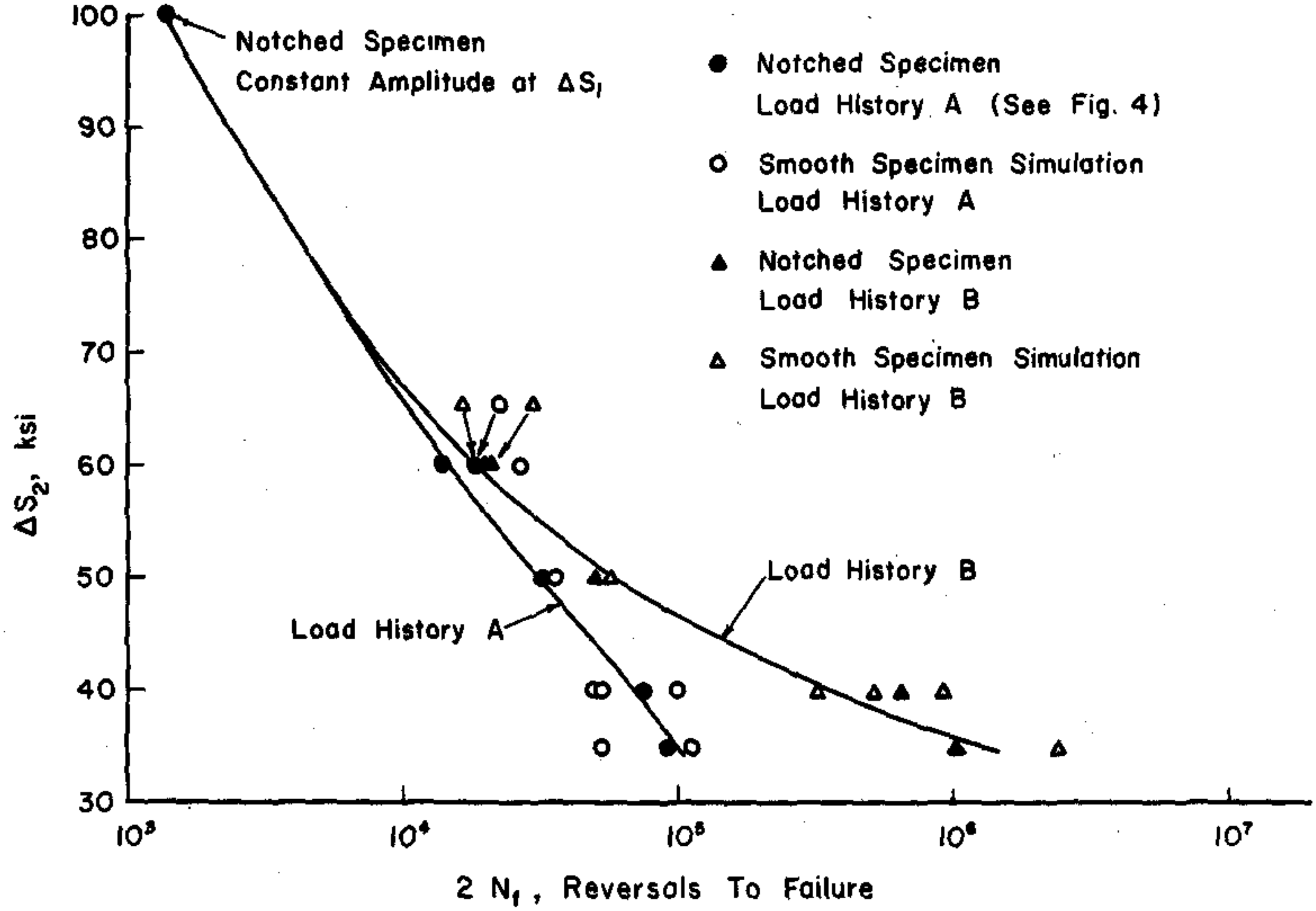


Load History A

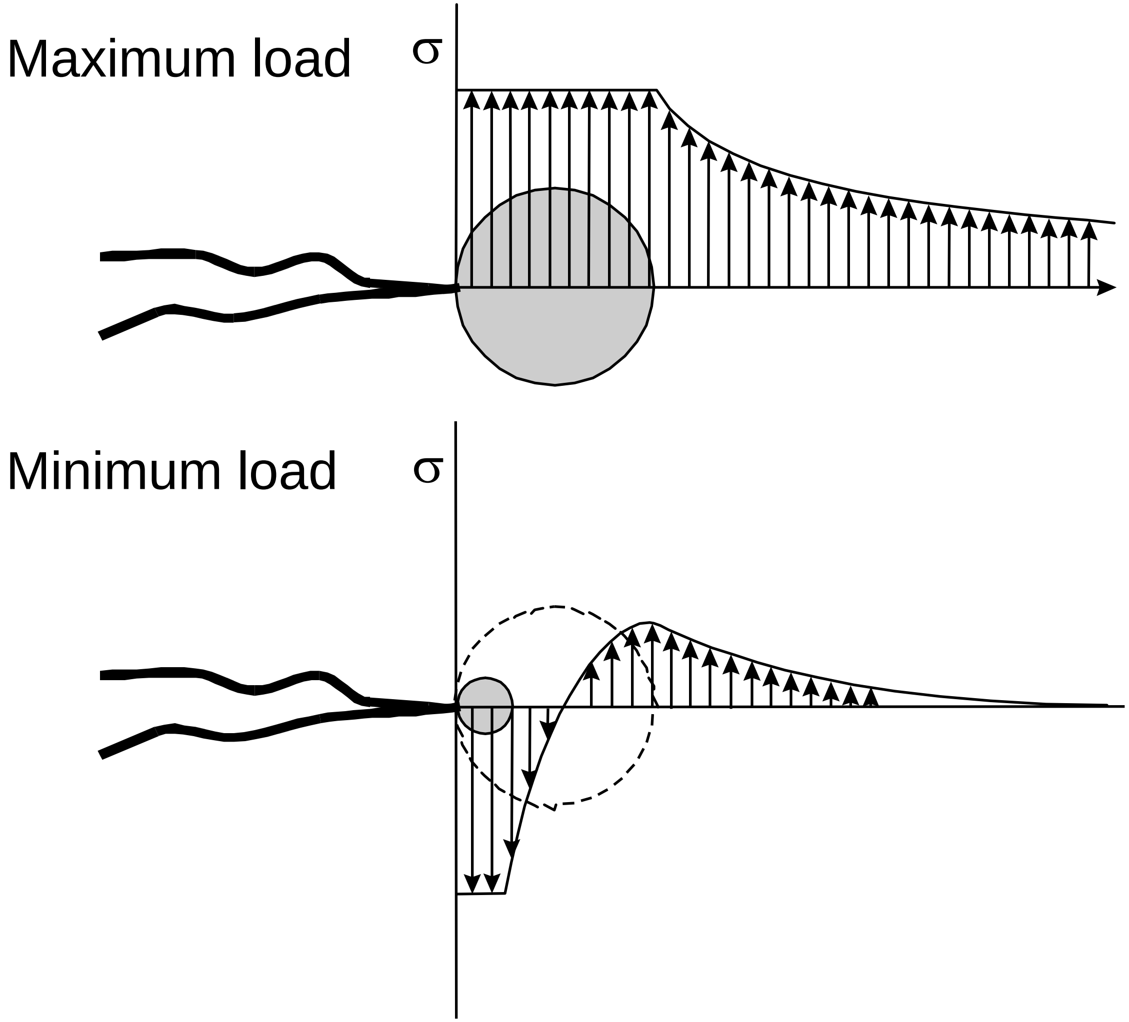


Load History B

Test Results

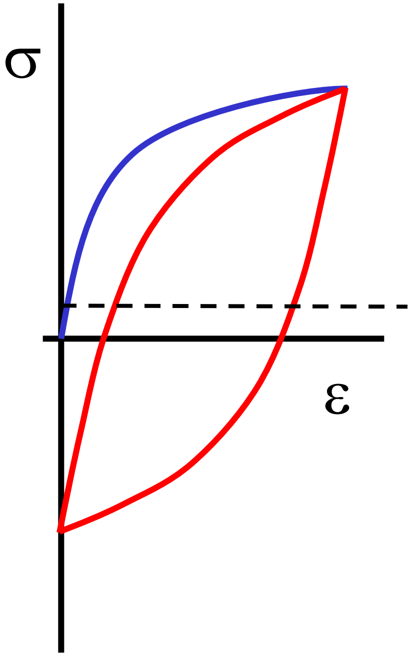


Crack Growth Physics



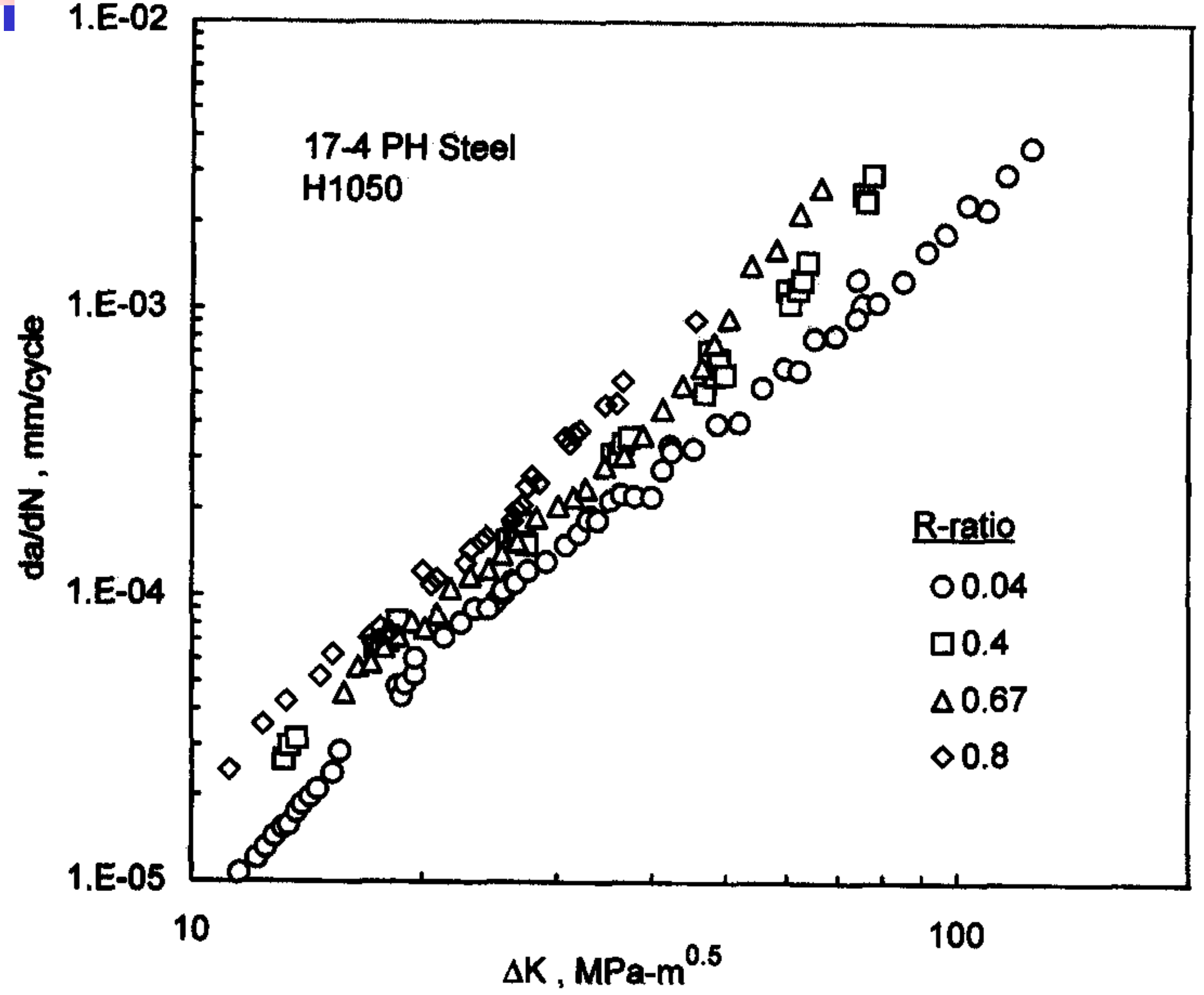
Maximum load

Minimum load



Mean stresses in plastic zone are small

Mean Stress Effects

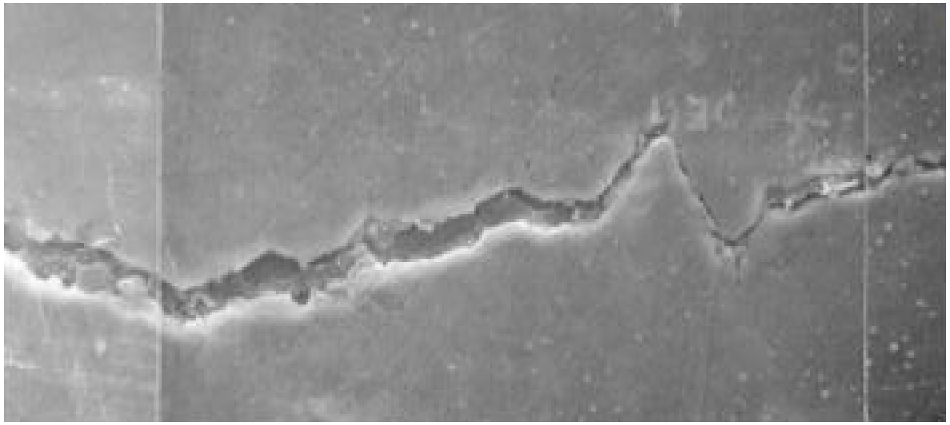


$$\frac{da}{dN} = \frac{C\,\Delta K^{m}}{\left(1-R\right)^{\gamma}}$$

$$0 < \gamma < 0.5$$

From: Dowling and Thangjitham, An Overview and Discussion of Basic Methodology for Fatigue, ASTM STP 1389,2000, 3-38

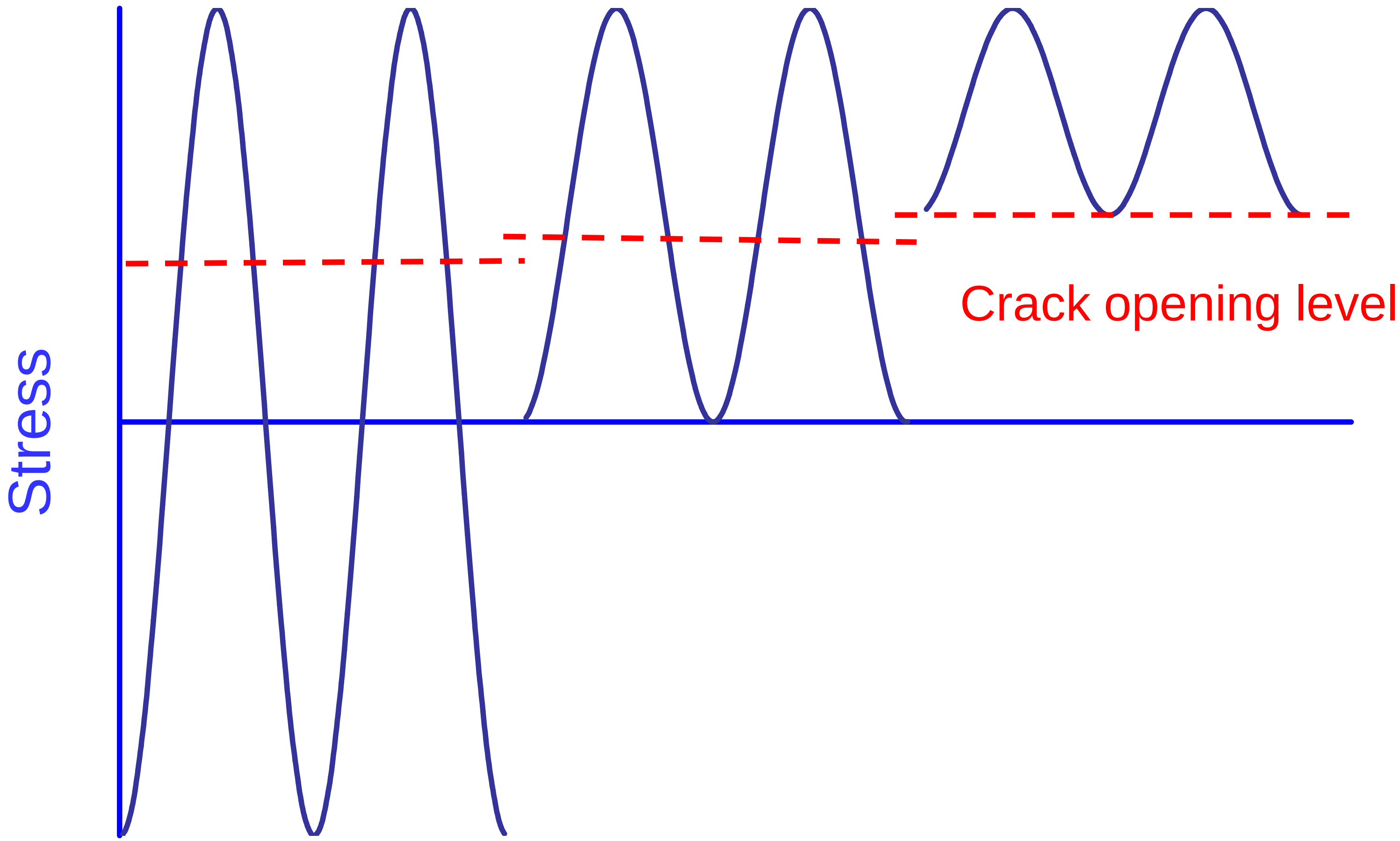
Compression



Crack open

Crack closed

Compressive Stresses



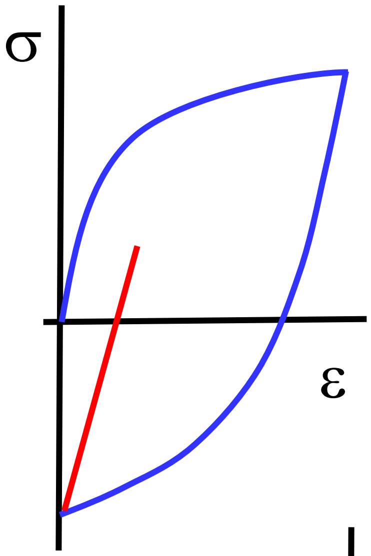
Compressive stresses are not very damaging in crack growth

Sources of Mean/Residual Stress

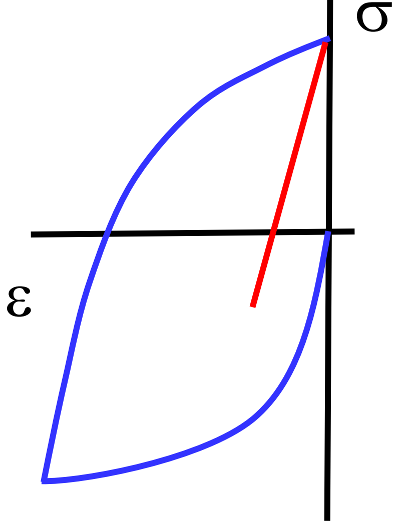
- Loading History
- Fabrication
- Shot Peening
- Heat Treating

Loading History

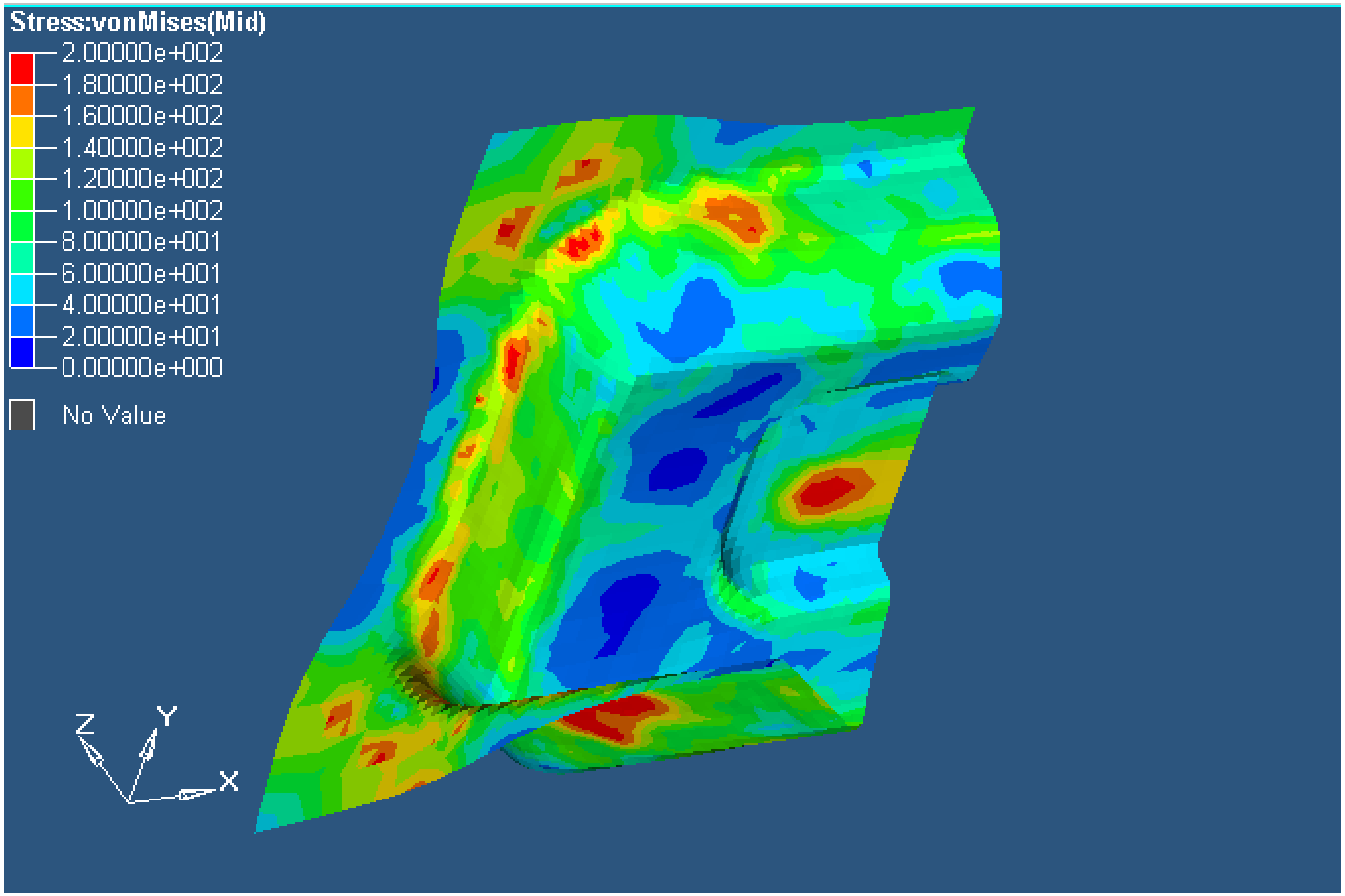
Tension overloads produce favorable compressive residual stress



Compressive overloads produce unfavorable tensile residual stress

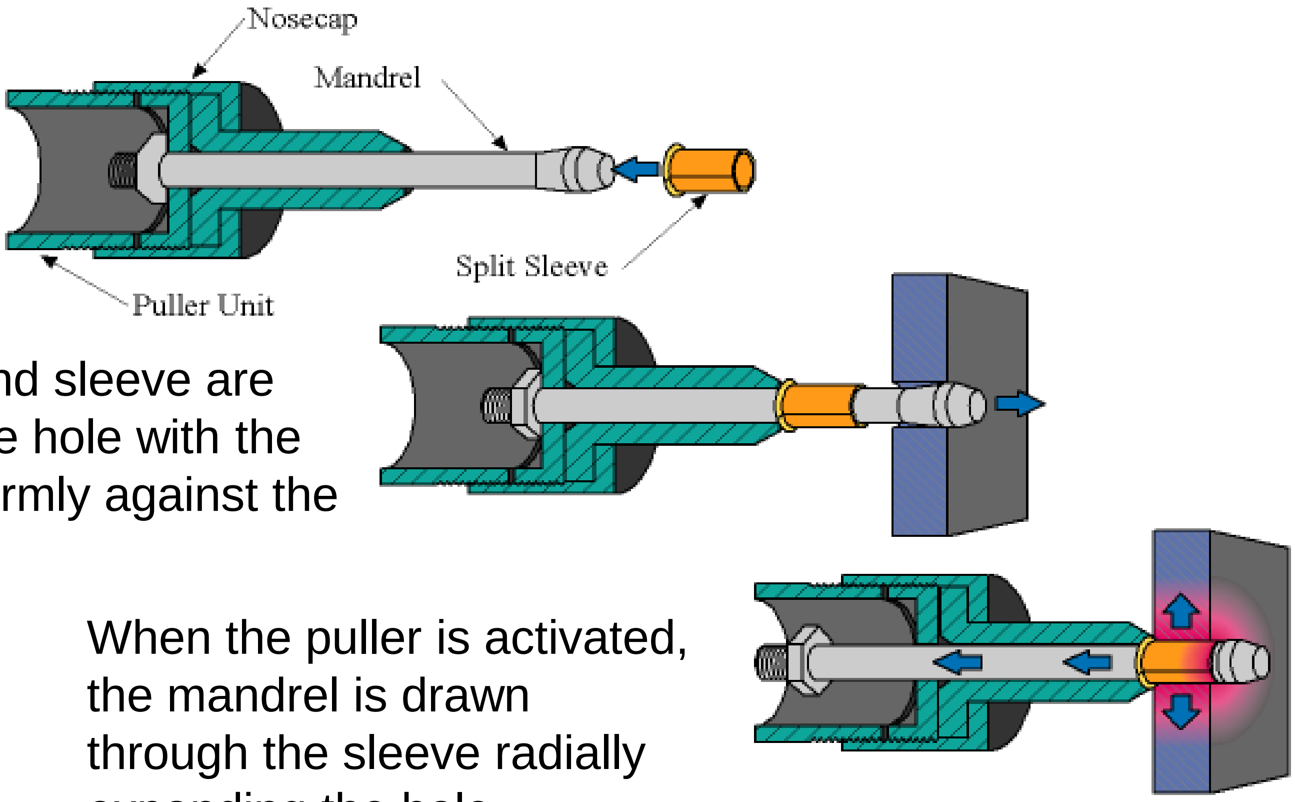


Fabrication



Cold Expansion

1965 Basic Cx process conceptualized (Boeing)



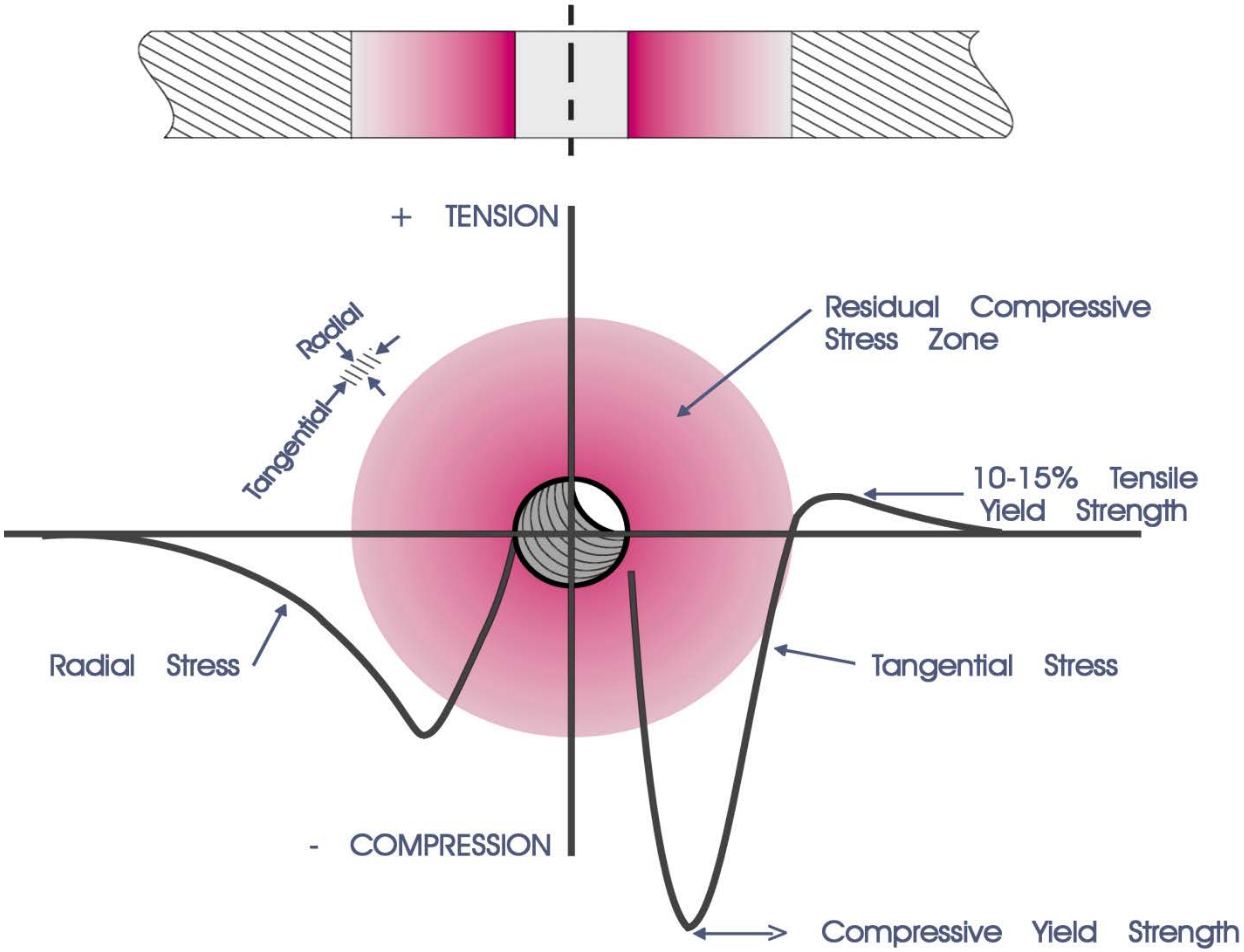
The split sleeve is slipped onto the mandrel, which is attached to the hydraulic puller unit.

The mandrel and sleeve are inserted into the hole with the nosecap held firmly against the workpiece.

When the puller is activated, the mandrel is drawn through the sleeve radially expanding the hole.

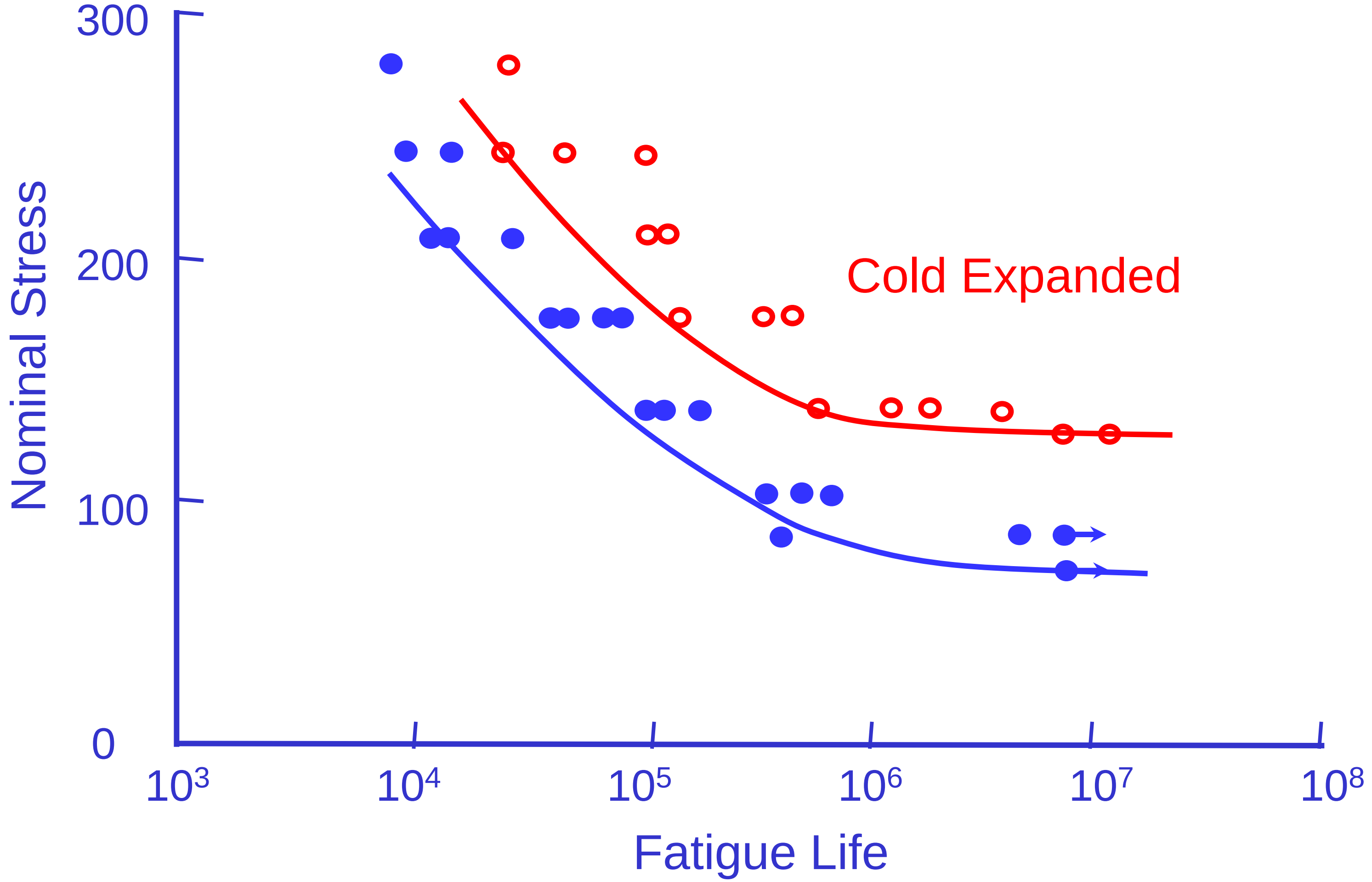
Courtesy of Fatigue Technology Inc.

Theory of Cold Expansion



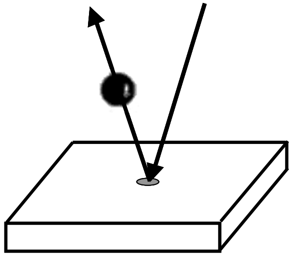
Courtesy of Fatigue Technology Inc.

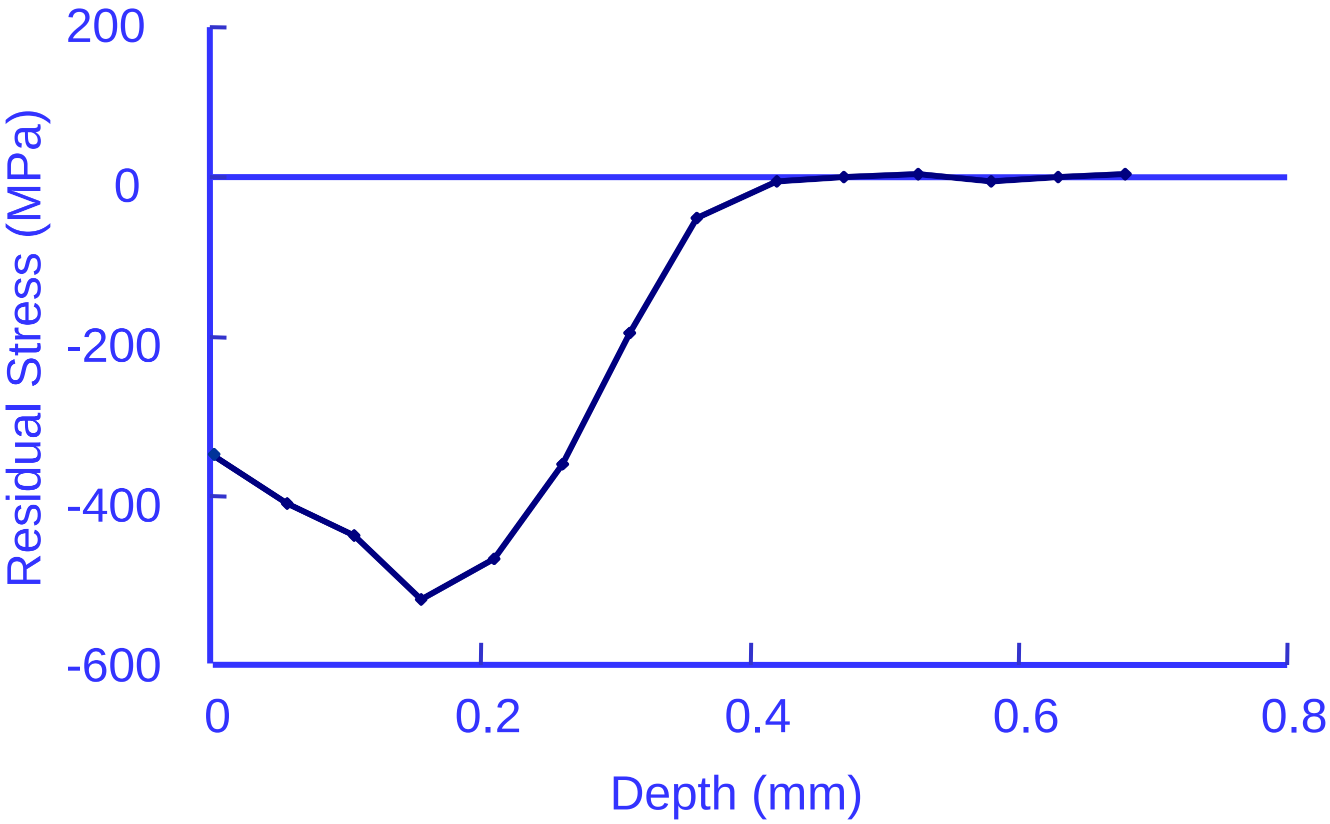
Fatigue Life Improvement



Courtesy of Fatigue Technology Inc.

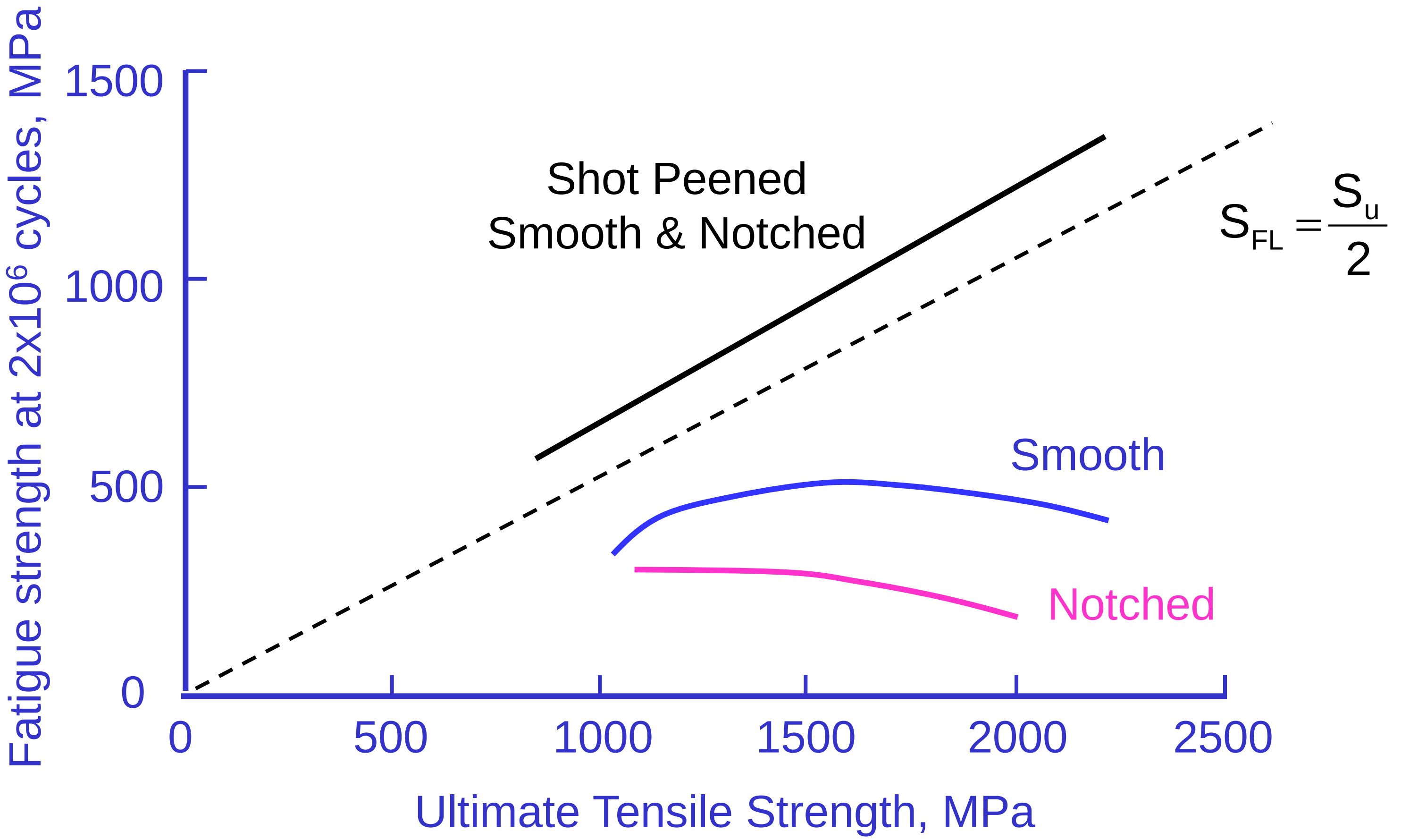
Shot Peening





Residual stress in a shot peened leaf spring

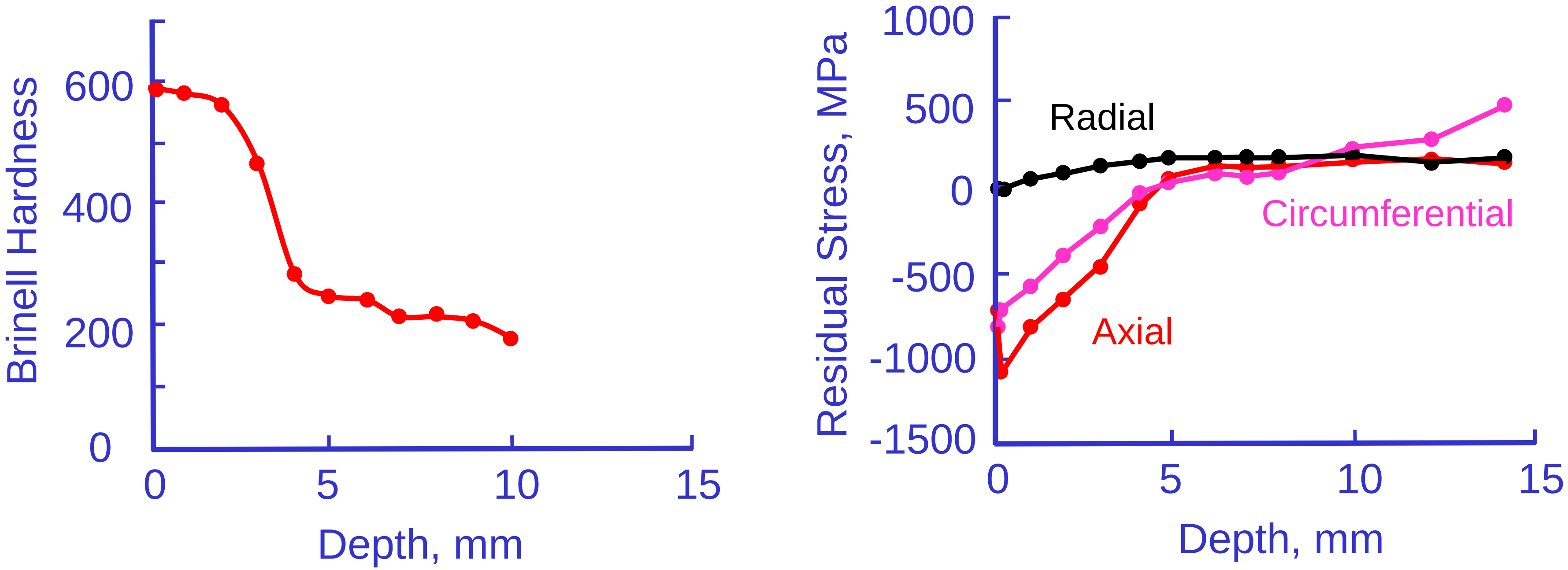
Shot Peening Results



$$S_{FL} = \frac{S_u}{2}$$

www.metalimprovement.com

Heat Treating



50 mm diameter induction hardened 1045 steel shaft

Things Worth Remembering

- Local mean stress rather than the nominal mean stress governs the fatigue life
- Mean stress has the greatest effect on crack nucleation

Factors Influencing Fatigue

- Mean Stress
- **Variable Amplitude**
- Stress Concentrations
- Surface Finish

Variable Amplitude Loading

How to you identify cycles ?

How do you assess fatigue damage for a cycle ?

Rainflow Cycle Counting

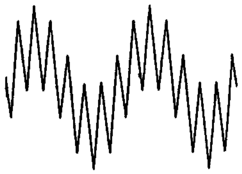


図6. 重畳波尖頭値の系列値

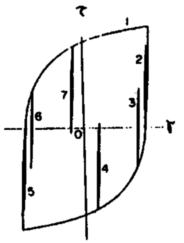


図7(a) 重畳波履歴曲線

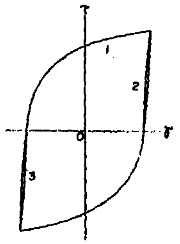


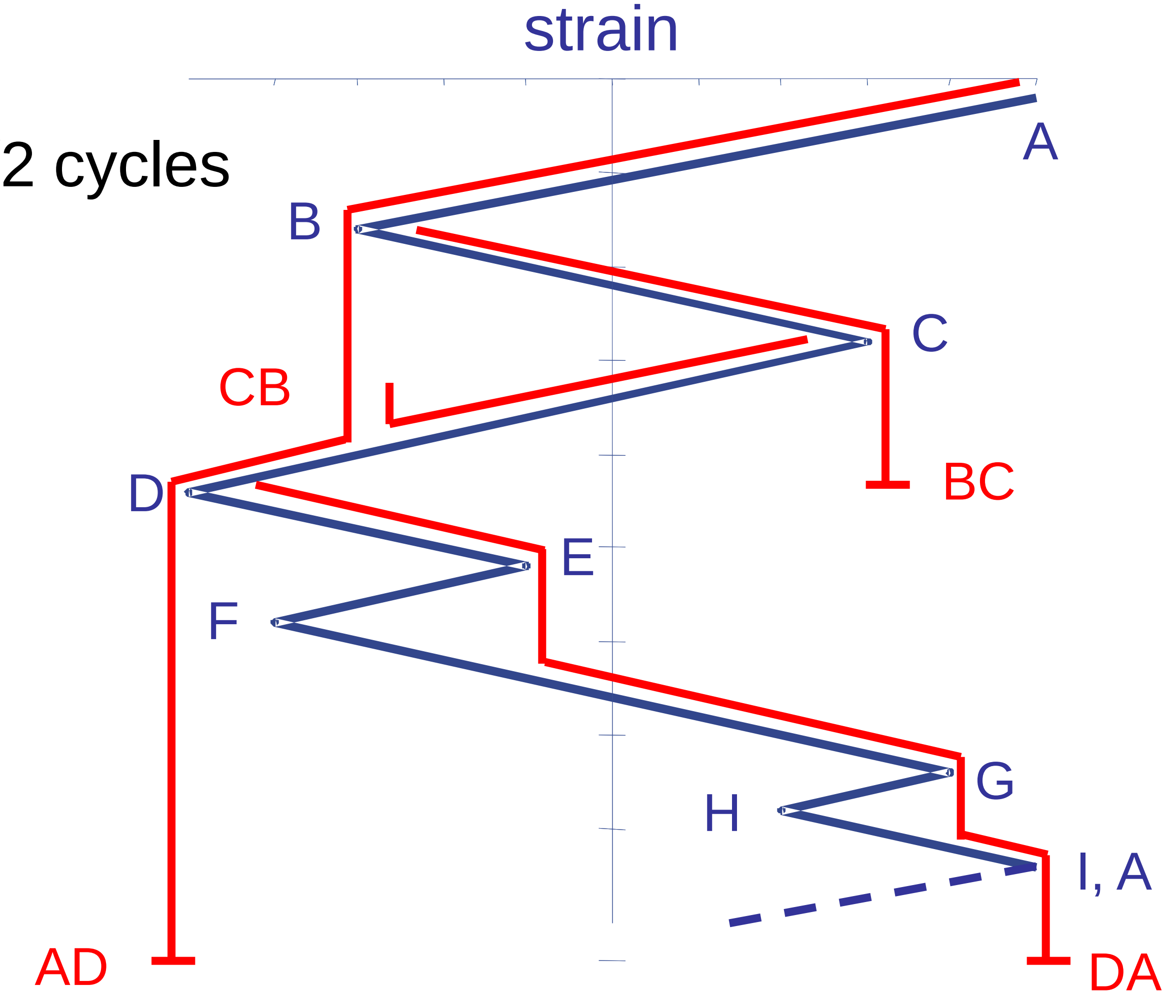
図7(b) 双子波履歴曲線

What could be more basic than learning to count correctly?

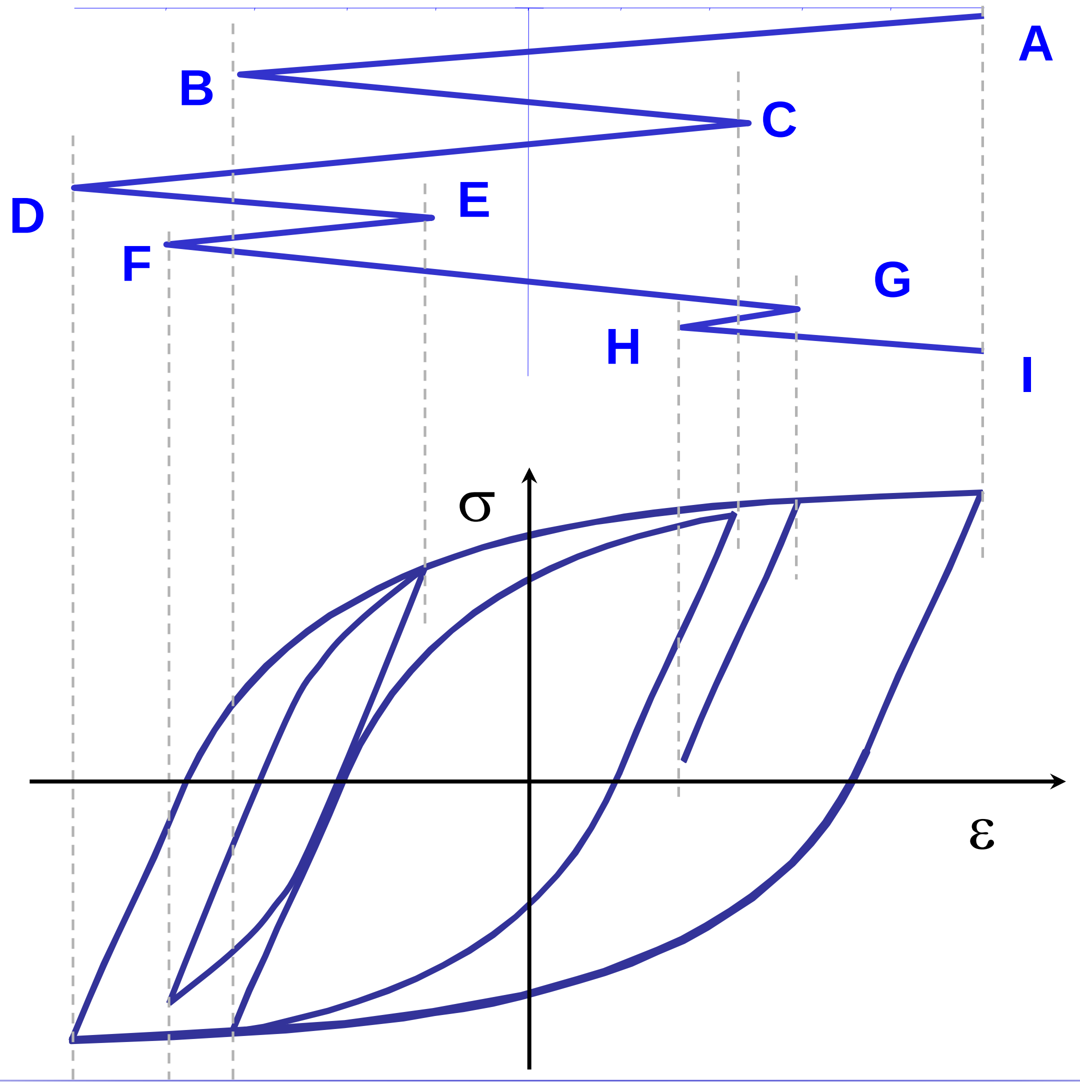
Matsuishi and Endo (1968) Fatigue of Metals Subjected to Varying Stress – Fatigue Lives Under Random Loading, Proceedings of the Kyushu District Meeting, JSME, 37-40

Rainflow

Counts 1/2 cycles

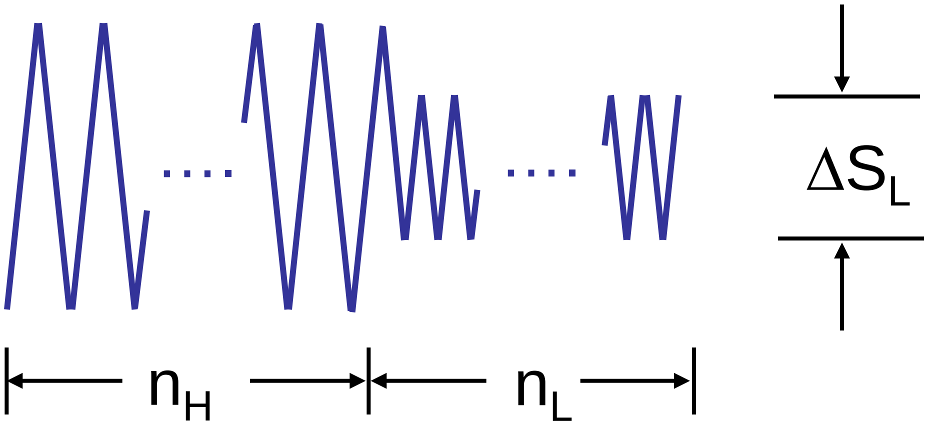


Rainflow and Hysteresis

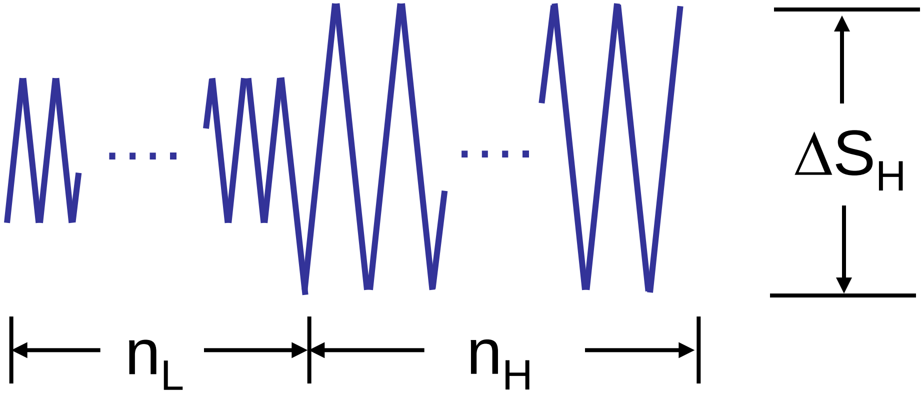


Cumulative Damage

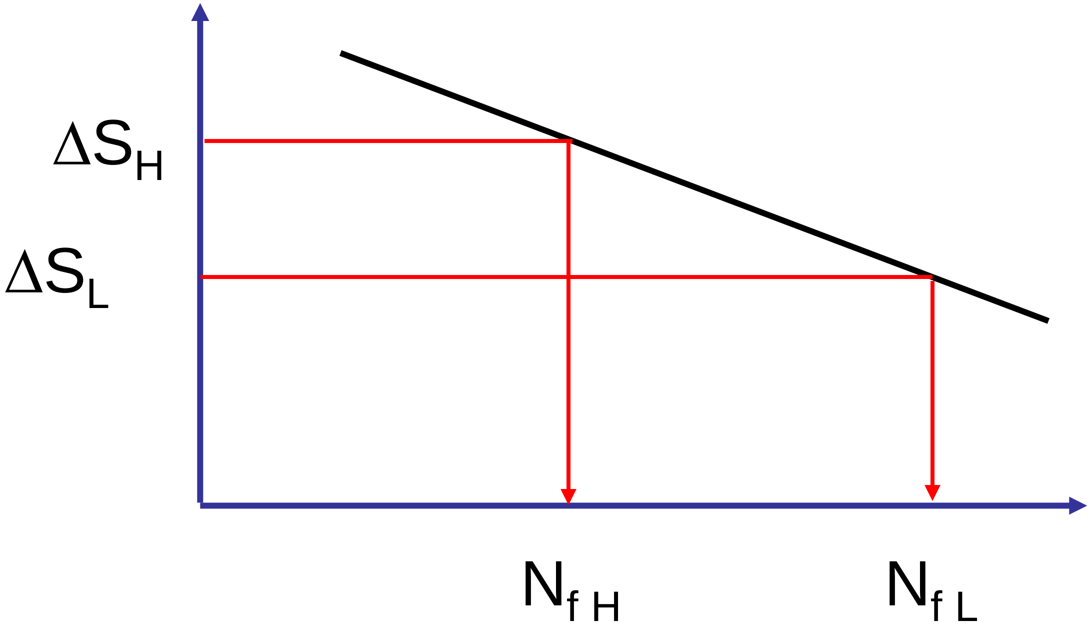
High - Low



Low - High



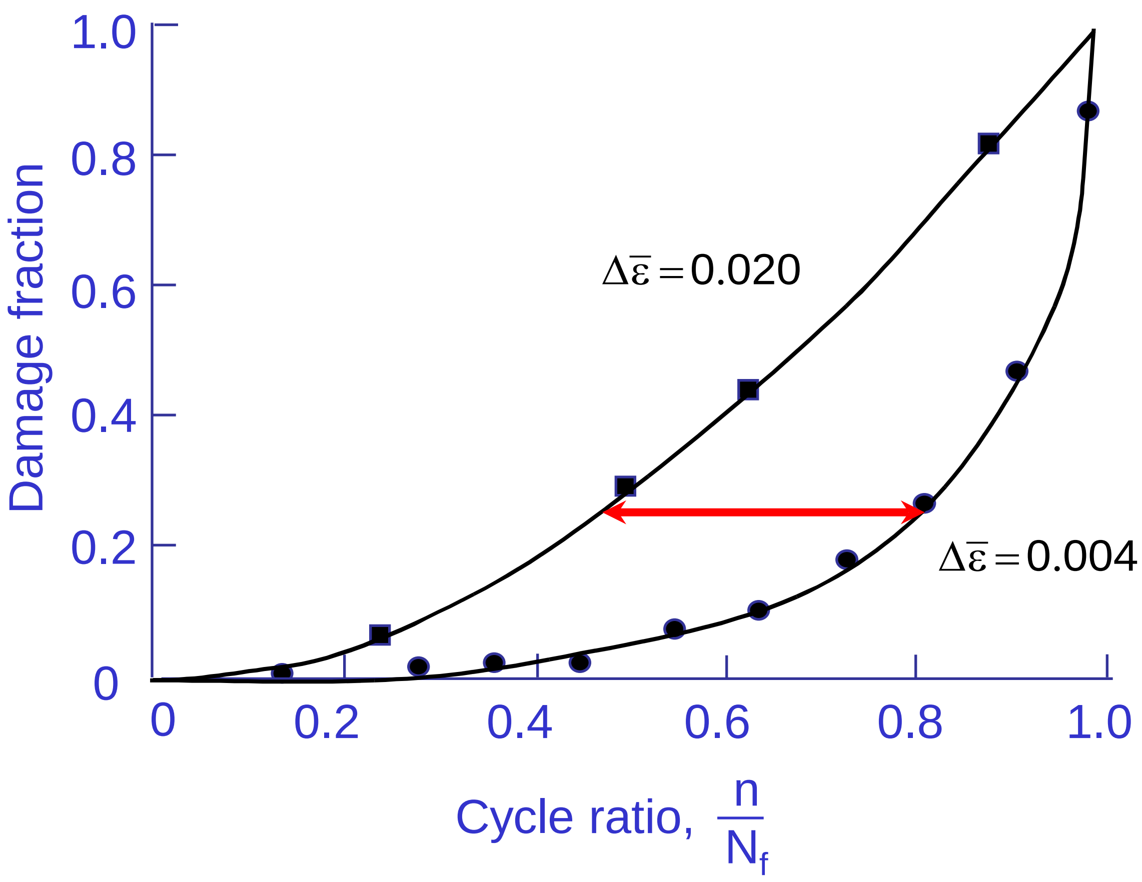
Linear Damage

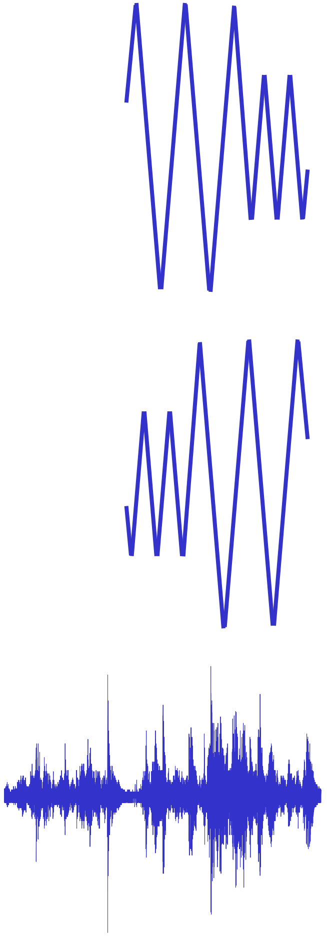


Miner's Rule:

$$\text{Damage} = \sum \frac{n}{N_F} = \frac{n_H}{N_{f\ H}} + \frac{n_L}{N_{f\ L}}$$

Nonlinear Damage



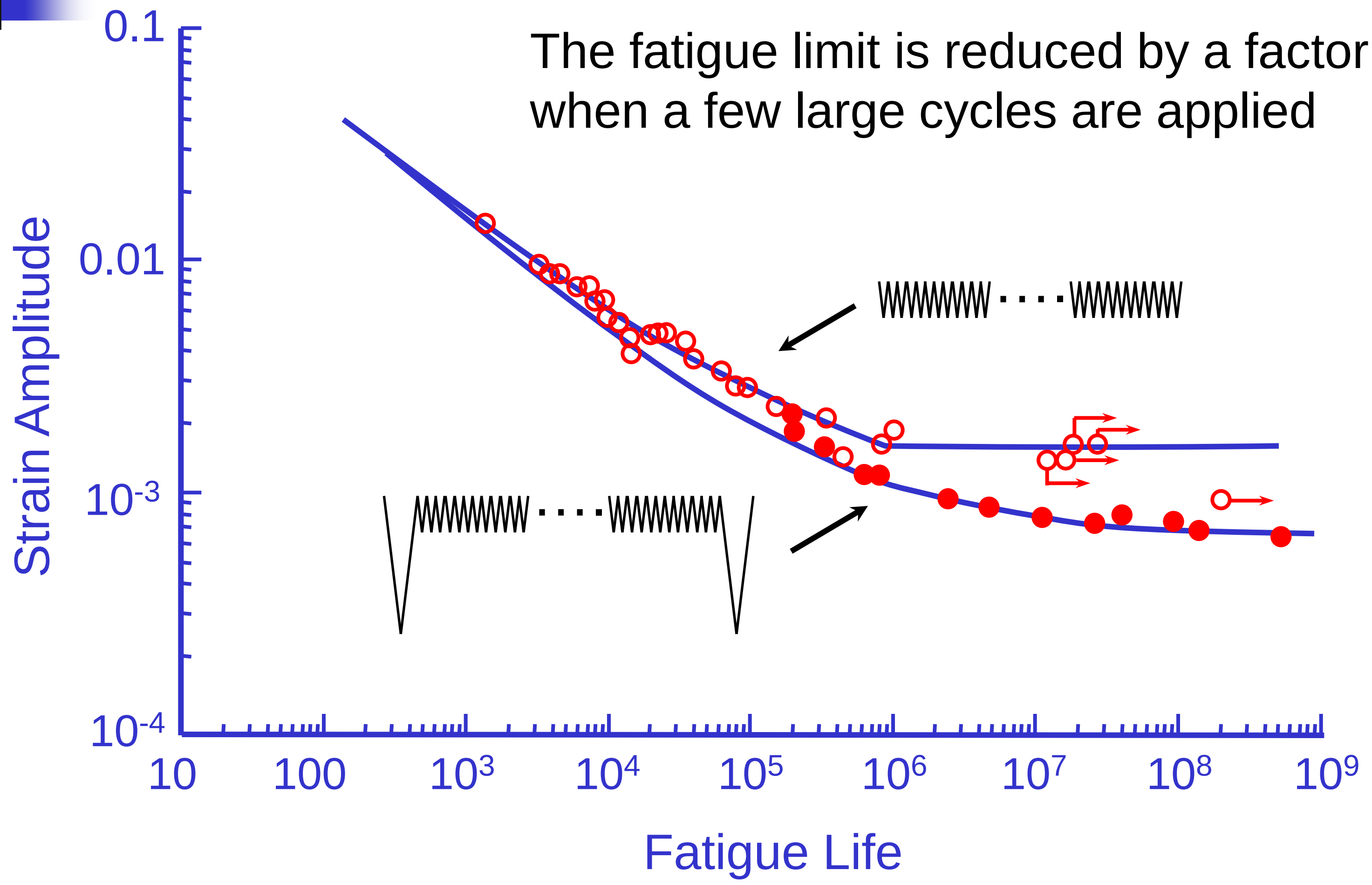


$\Sigma D = 0.7$

$\Sigma D = 1.3$

$\Sigma D \sim 1$

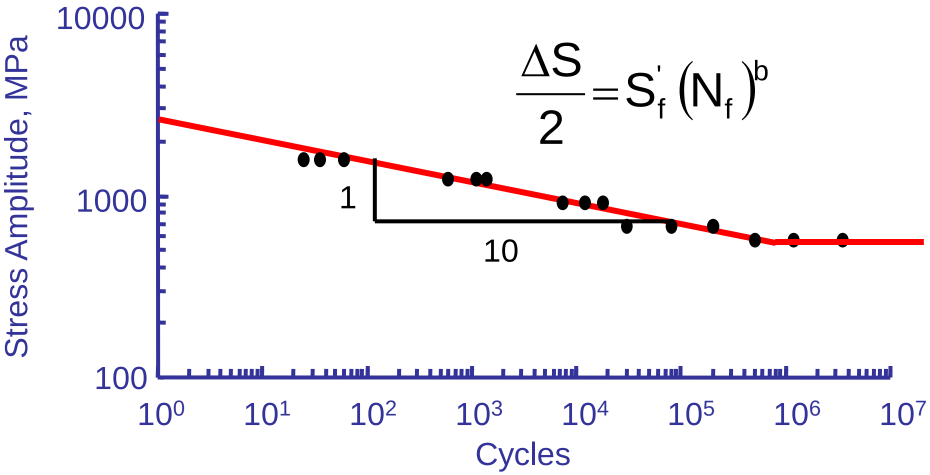
Periodic Overload Results



The fatigue limit is reduced by a factor of 3 when a few large cycles are applied

Bonnen and Topper, “The Effects of Periodic Overloads on Biaxial Fatigue of Normalized SAE 1045 Steel” ASTM STP 1387, 2000, 213-231

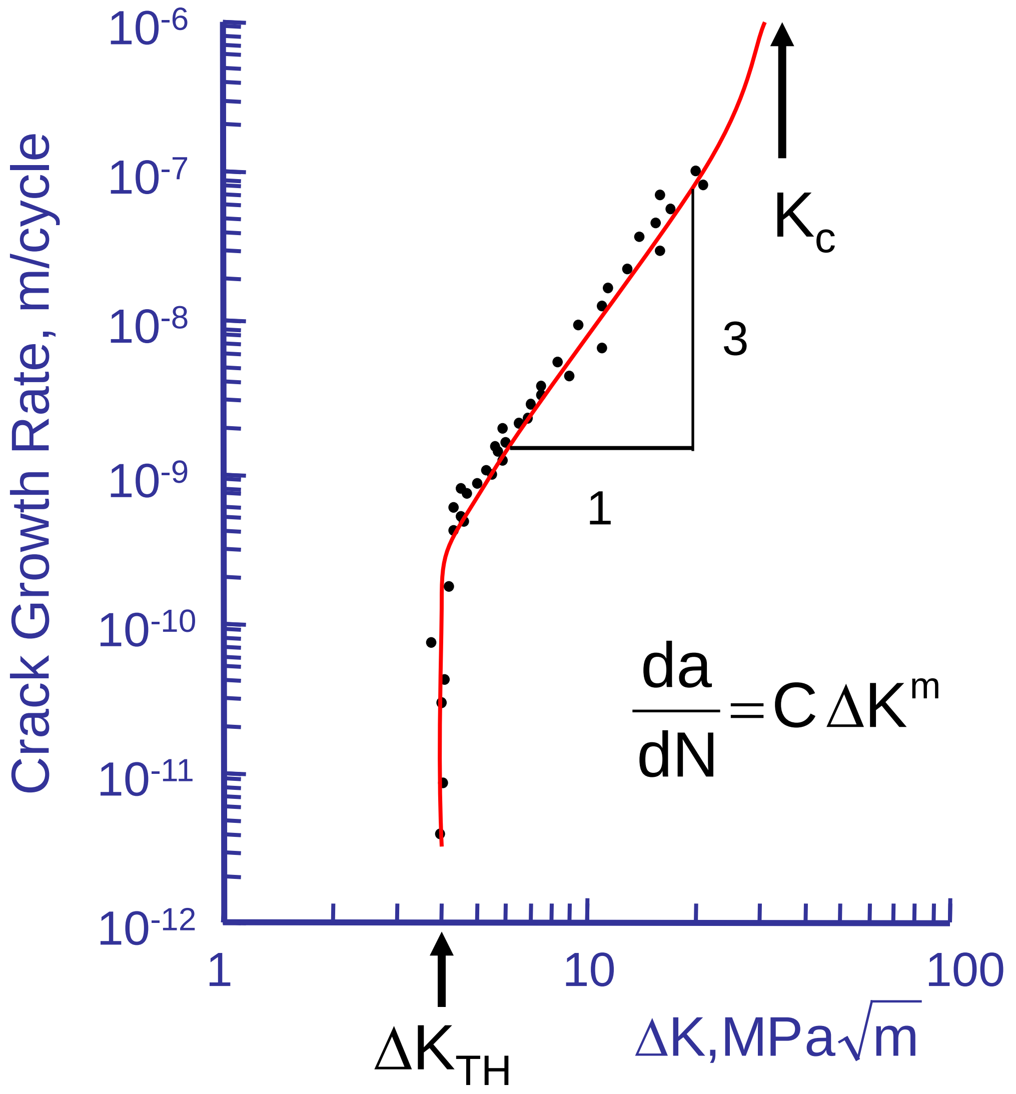
Fatigue Damage Calculations



$$N_f = \left(\frac{\Delta S}{2S_f'}\right)^{\frac{1}{b}}$$

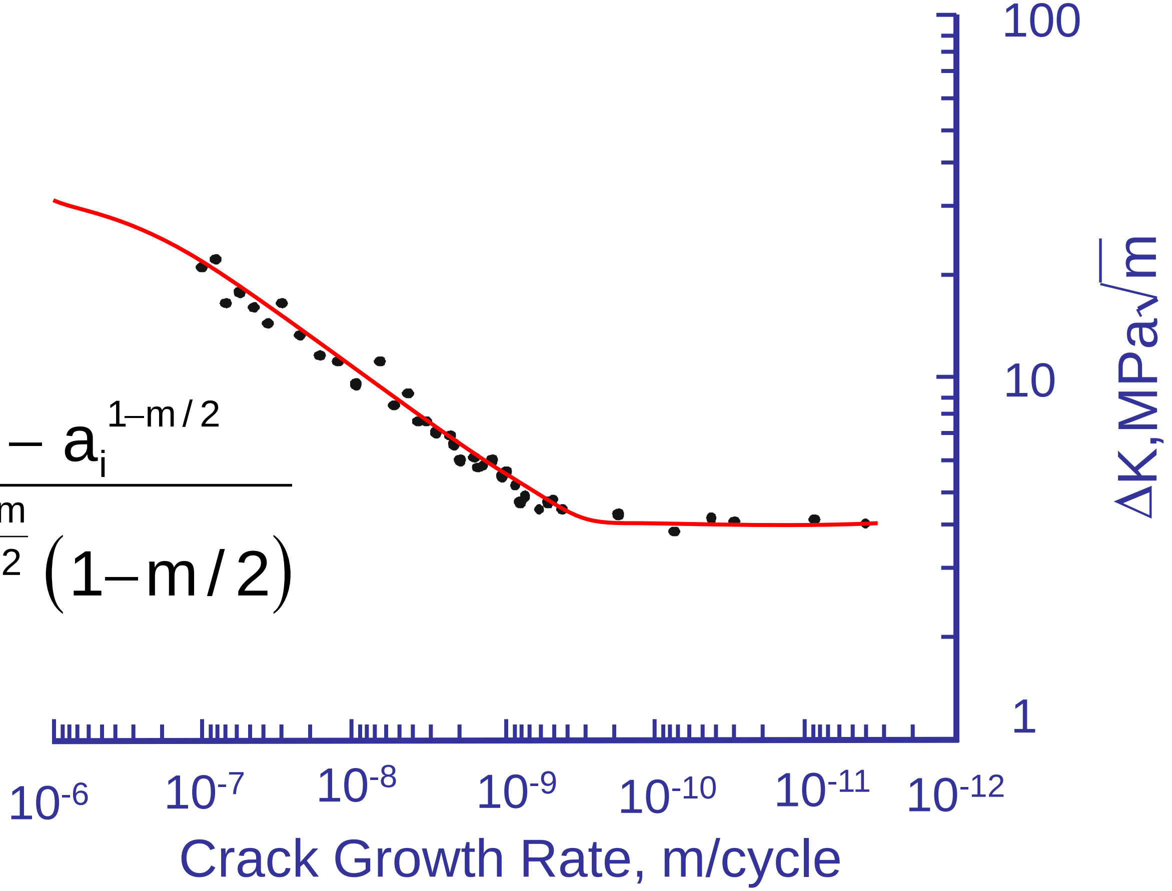
$$\text{Damage} \propto \Delta S^{10}$$

Crack Growth Data



$$\frac{da}{dN} = C\,\Delta K^m$$

Crack Growth Data



$$N_f = \frac{a_f^{1-m/2} - a_i^{1-m/2}}{C\,\Delta S^m \pi^{\frac{m}{2}} \left(1-m/2\right)}$$

$$\text{Damage} \propto \Delta S^3$$

Multiple Choice

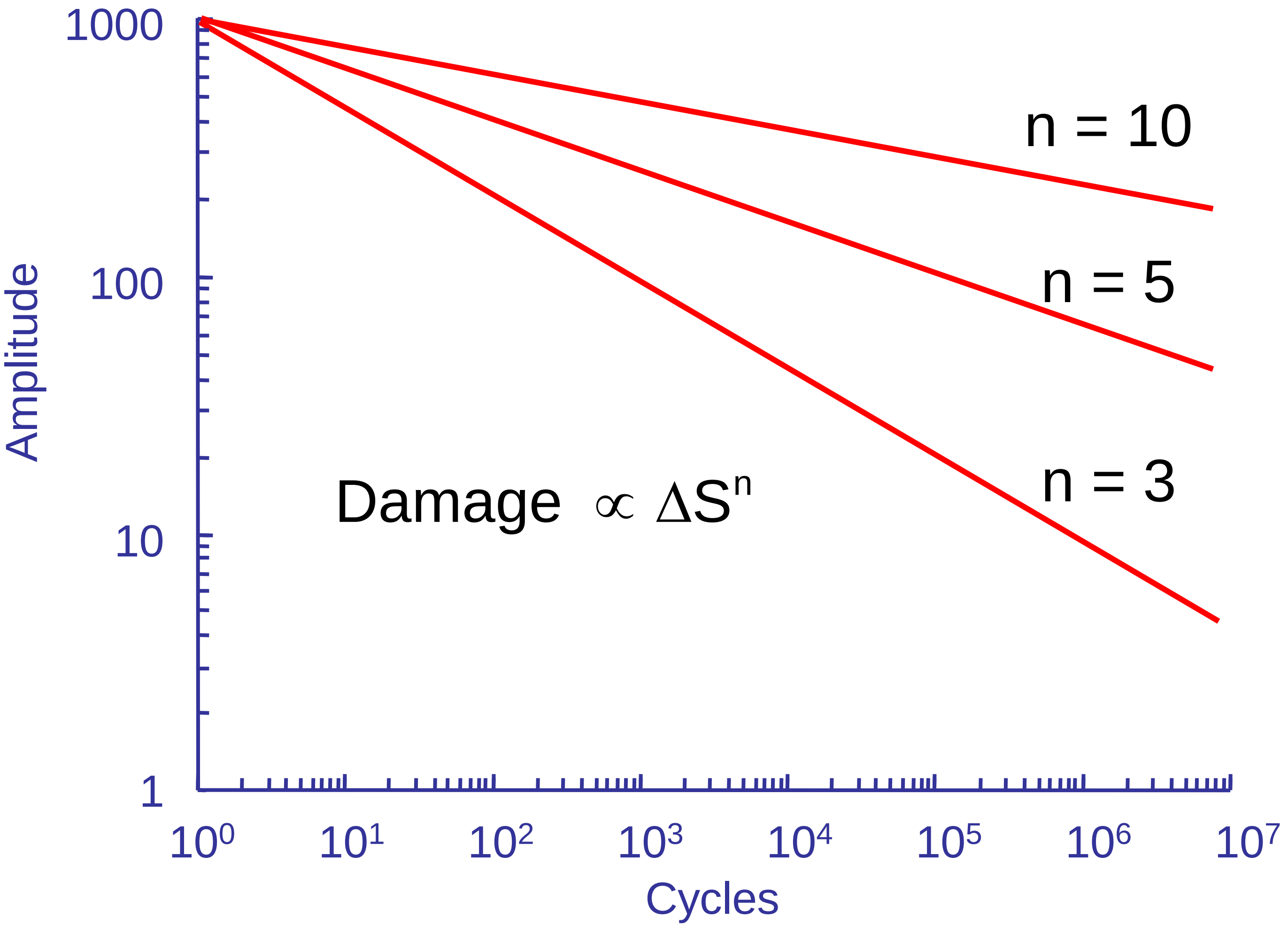
Which cycles do the most fatigue damage ?

(a) a few large cycles

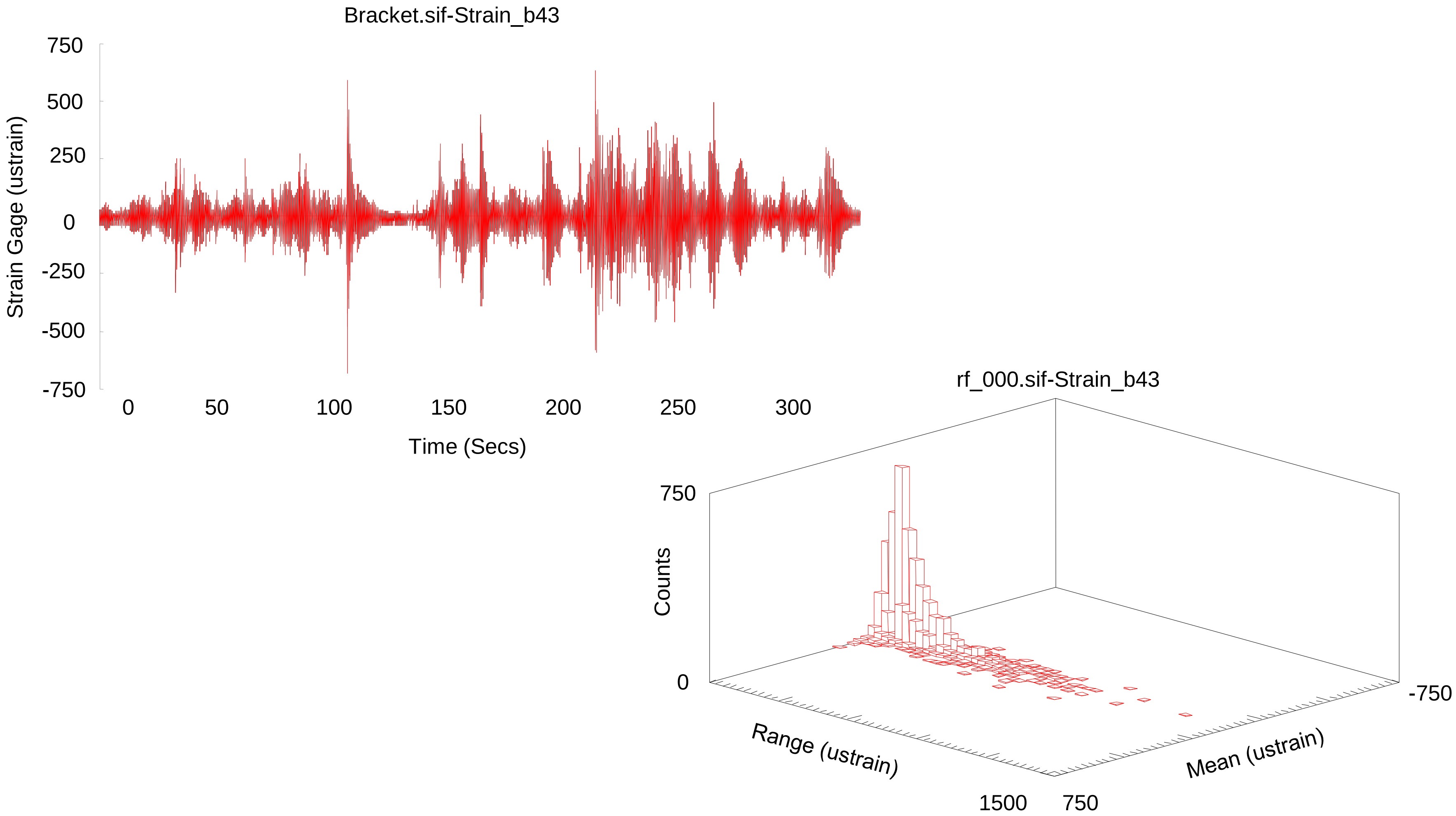
(b) a moderate number of intermediate cycles

(c) a large number of small cycles

Fatigue Data

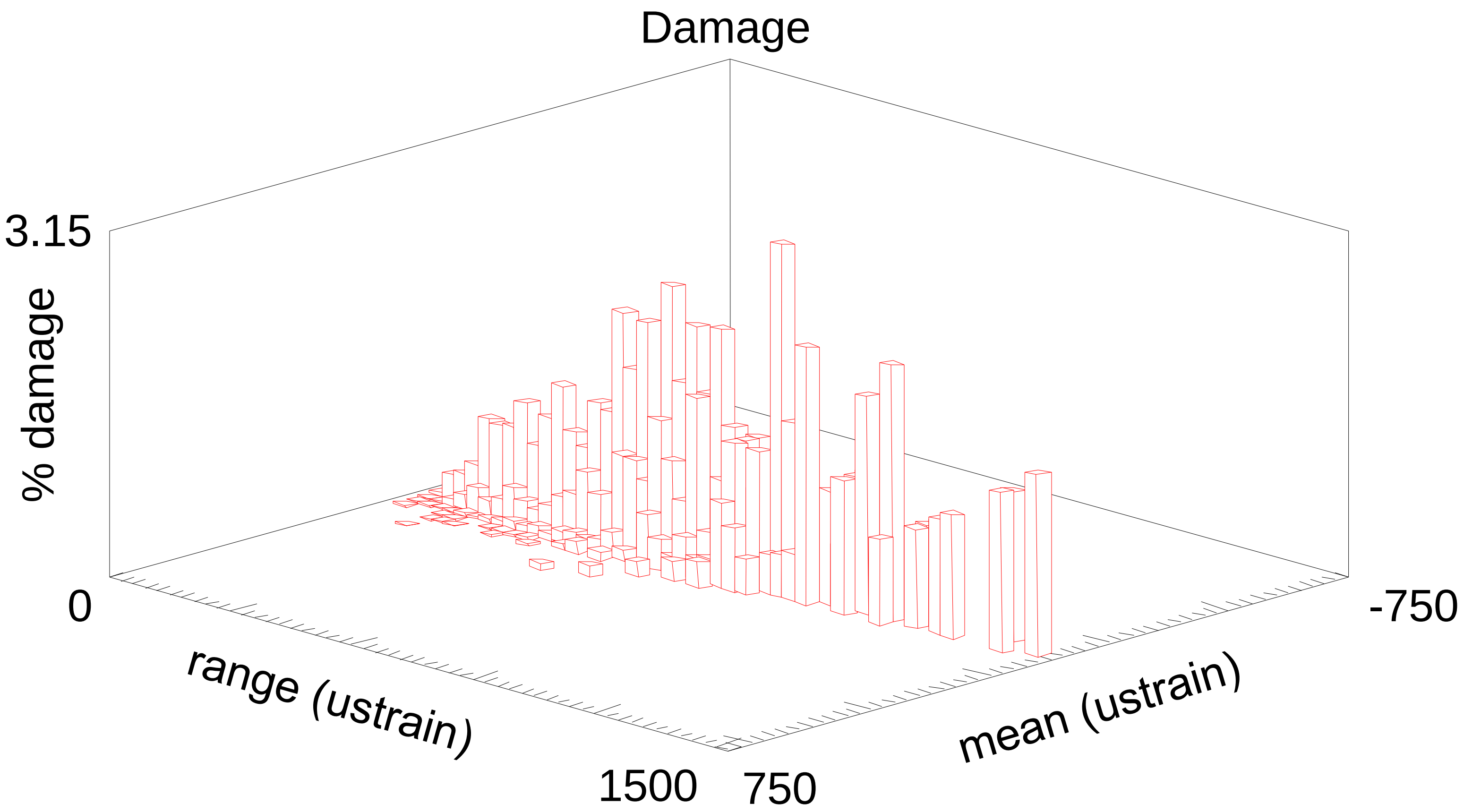


Loading History

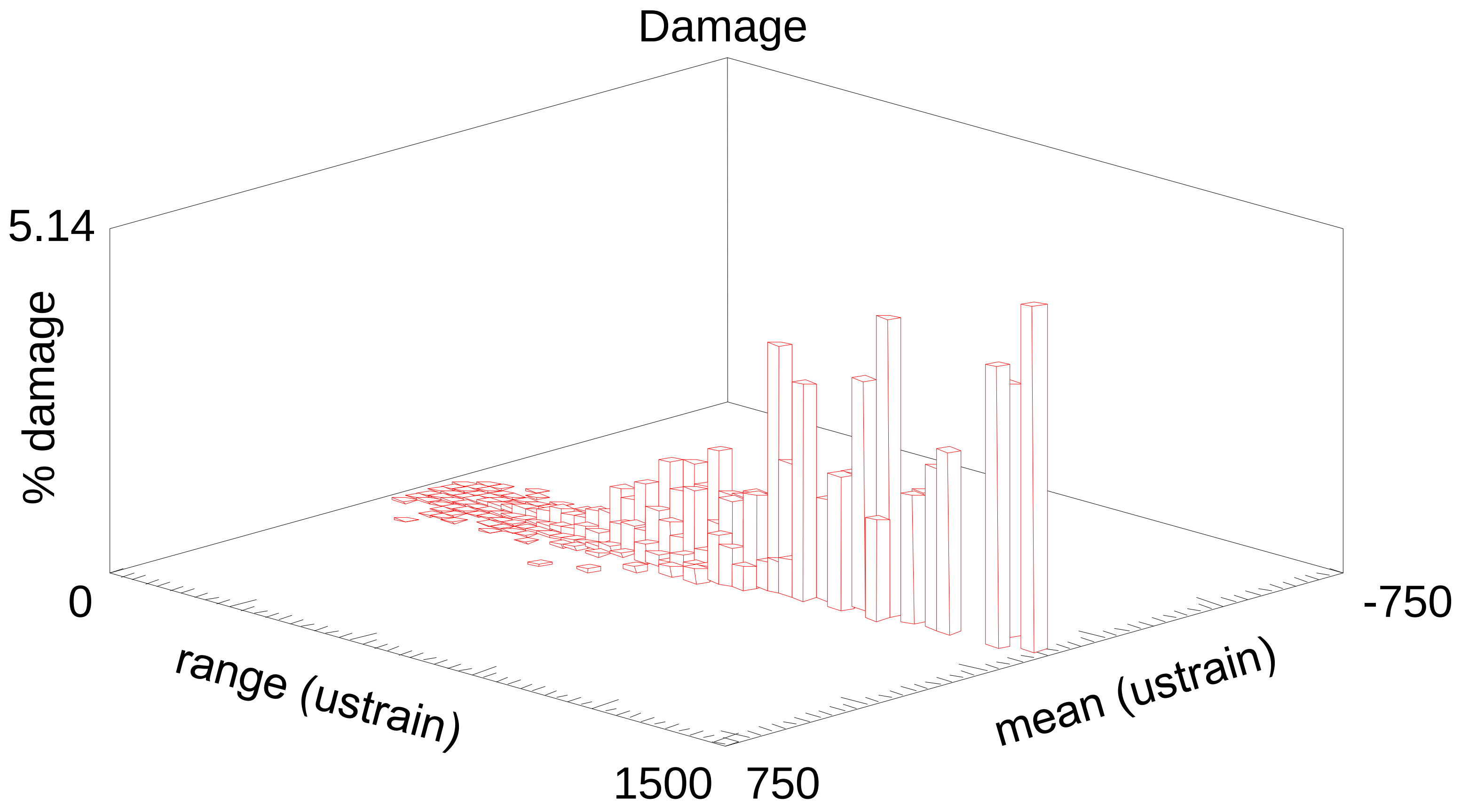

Bracket.sif-Strain_b43

rf_000.sif-Strain_b43

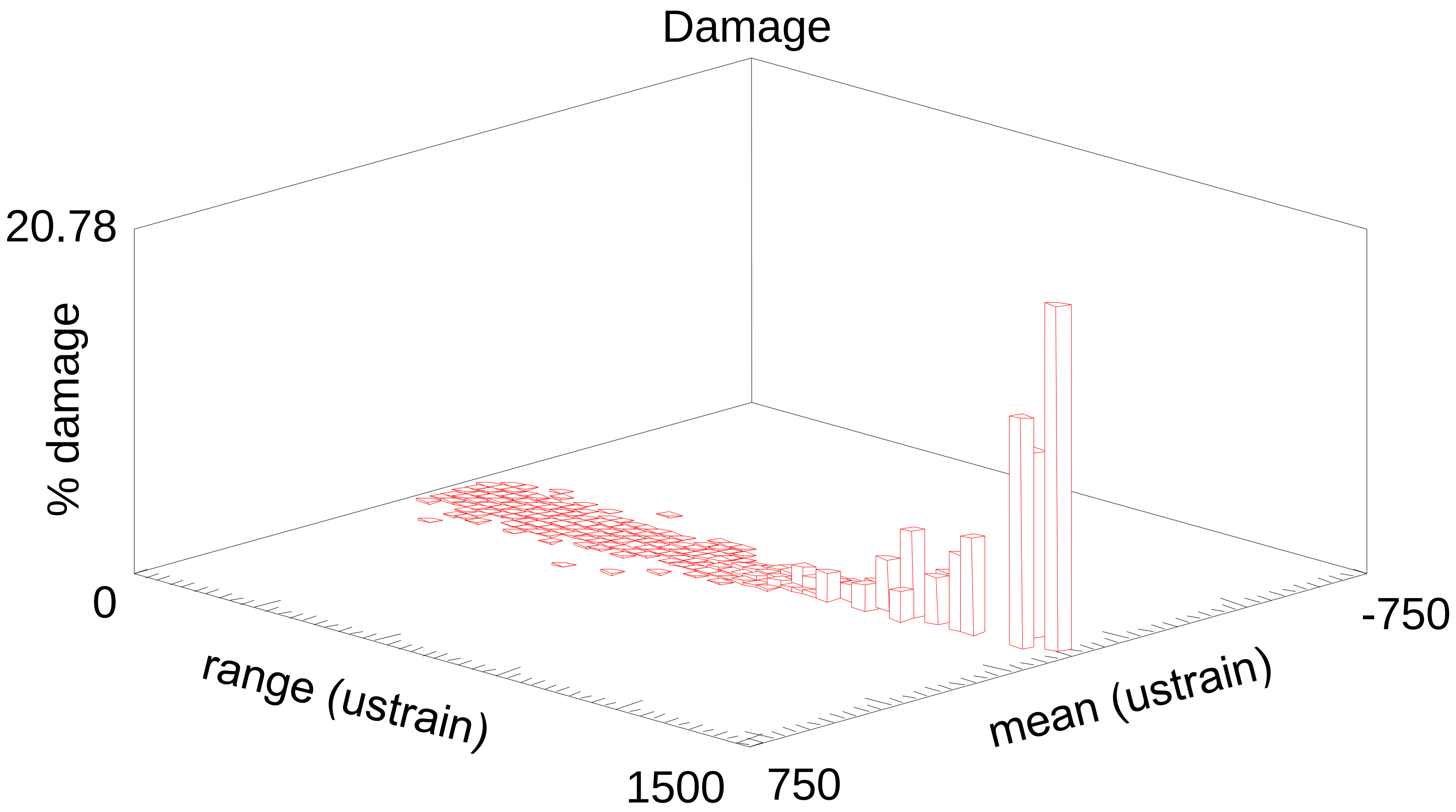
Slope = 3



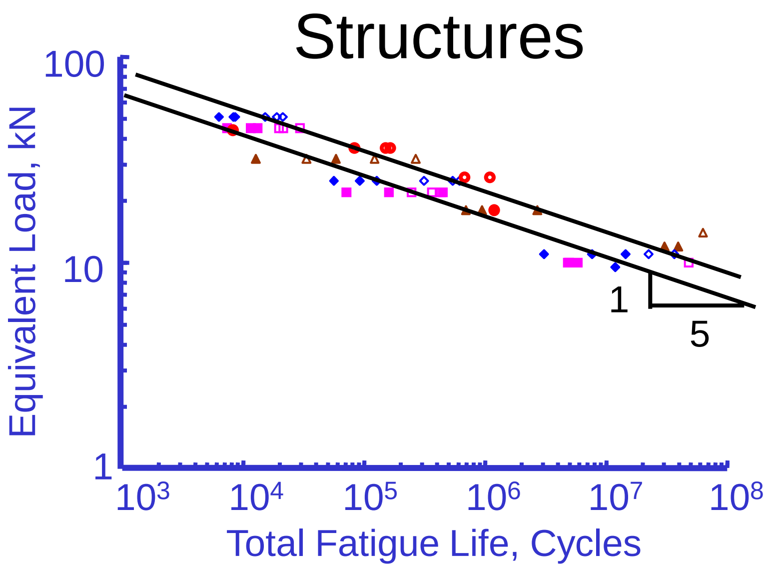
Slope = 5

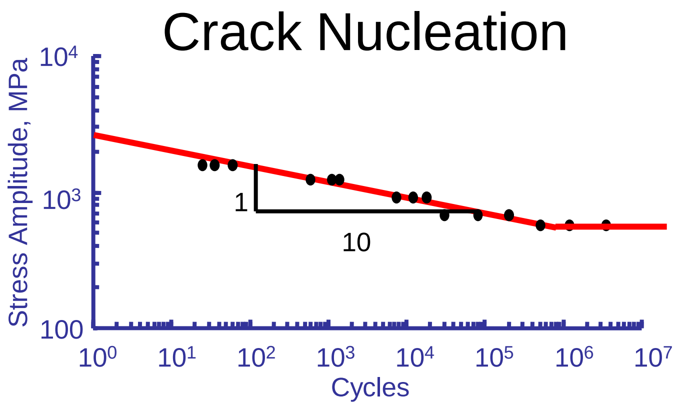


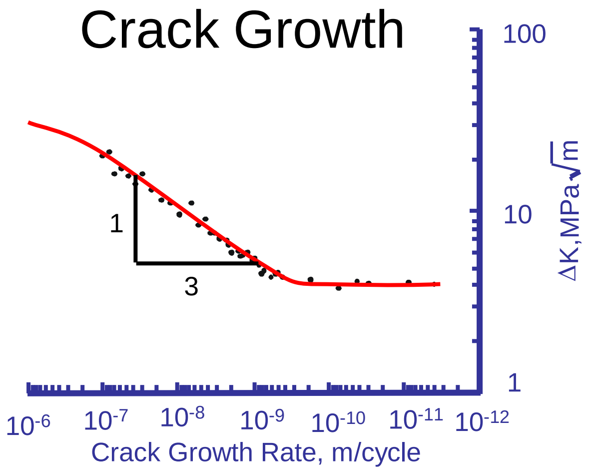
Slope = 10


Damage
20.78
% damage
0
range (ustrain)
1500
750
mean (ustrain)
-750

Mechanisms and Slopes


Structures


Crack Nucleation


Crack Growth

A combination of nucleation and growth

Equivalent Load

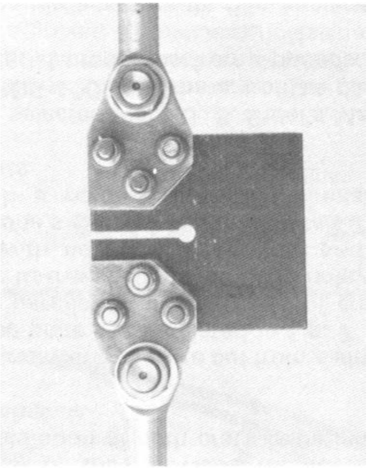
Equivalent constant amplitude loading

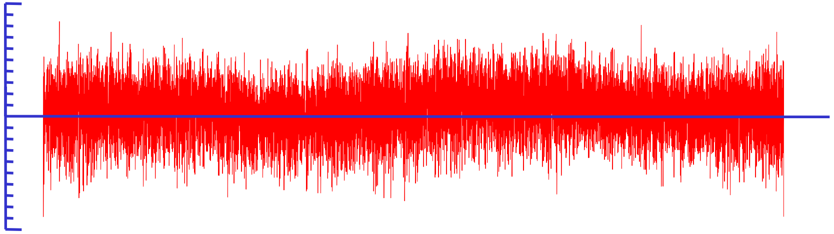
$$\Delta\overline{S} = \sqrt[n]{\frac{\sum_{i=1}^{N} \Delta S_i^{\,n}}{N}}$$

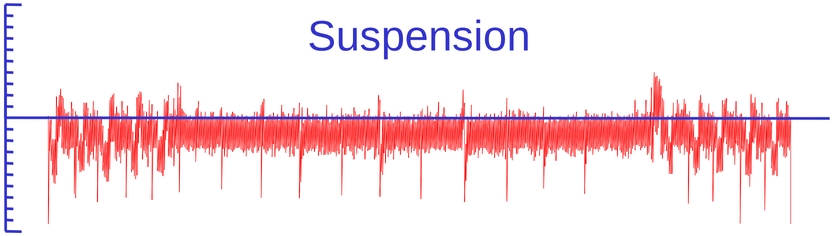
Typically n ranges from 4 to 6 for structures

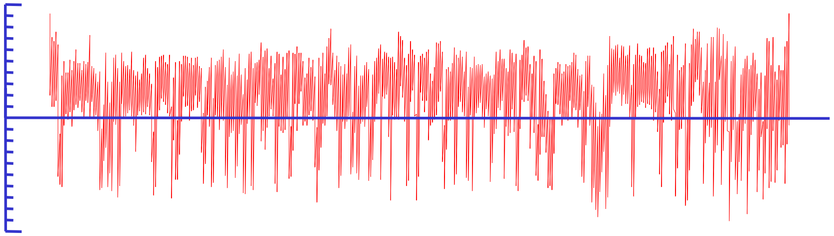
N cycles at an amplitude of $\Delta\overline{S}$ does as much damage as the entire loading history

SAE Keyhole Specimen

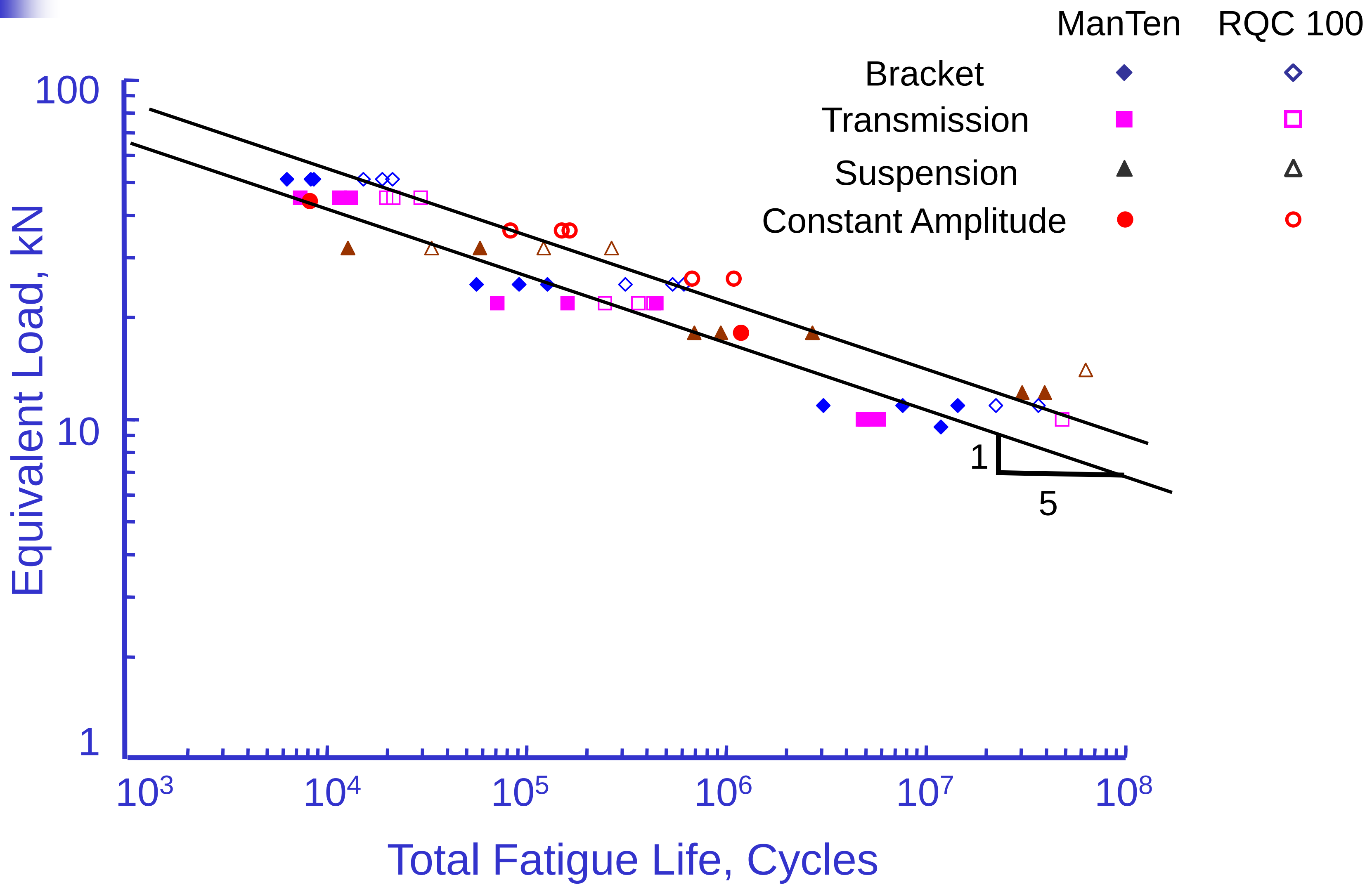



Bracket


Suspension


Transmission

SAE Keyhole Test Data



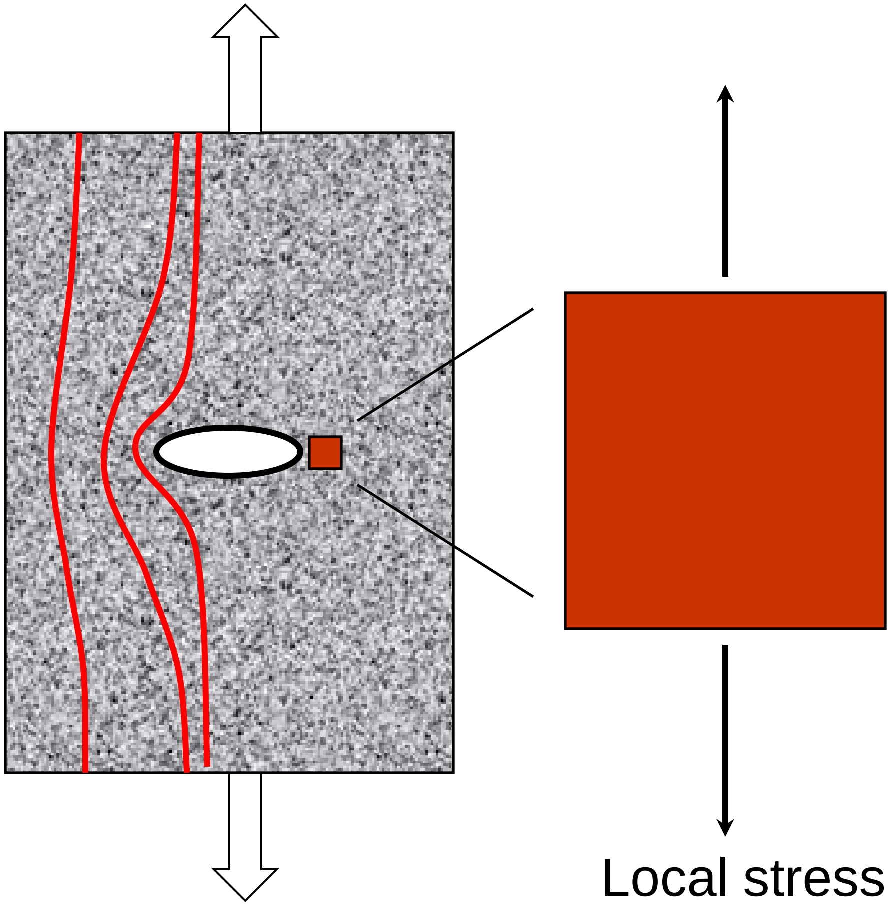
Things Worth Remembering

- Rainflow counting is employed to identify cycles
- The slope of the fatigue curve (damage mechanism) has a large influence on how much damage is caused by smaller cycles

Factors Influencing Fatigue

- Mean Stress
- Variable Amplitude
- **Stress Concentrations**
- Surface Finish

Stress Concentration Factor

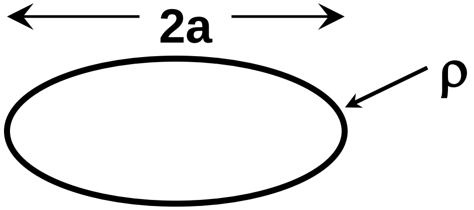


Local stress

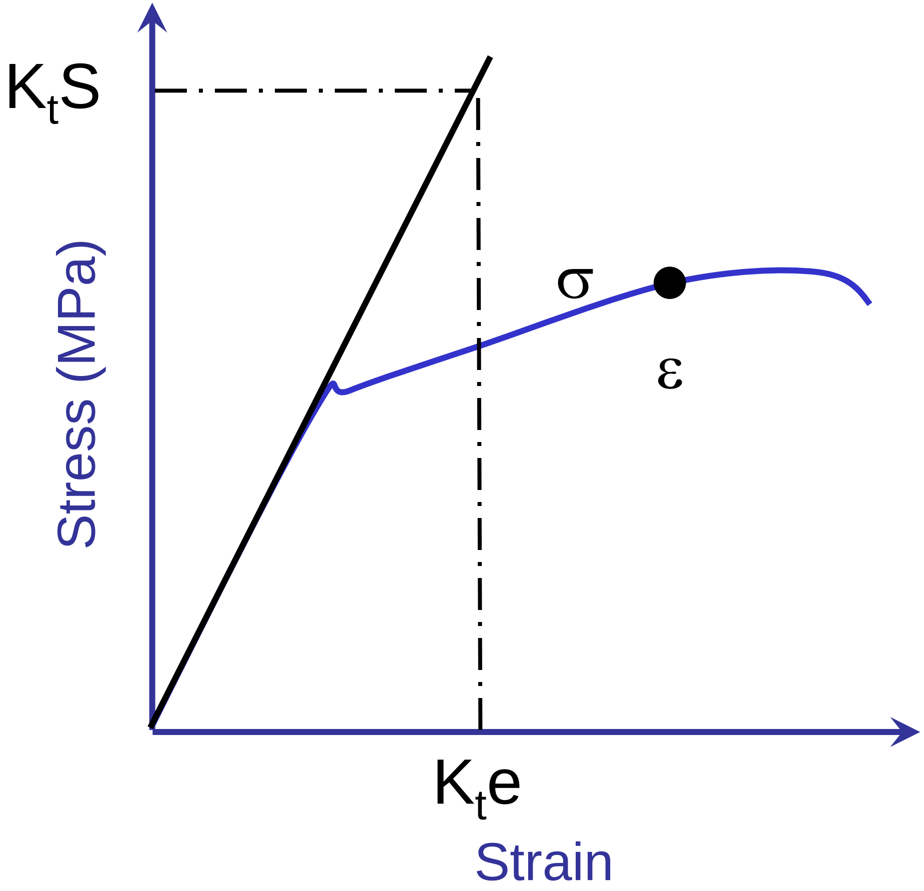
Applied stress

$$\sigma_{\text{local}} = \sigma_{\text{applied}}\left(1 + 2\sqrt{\frac{a}{\rho}}\right)$$

Inglis Solution 1910



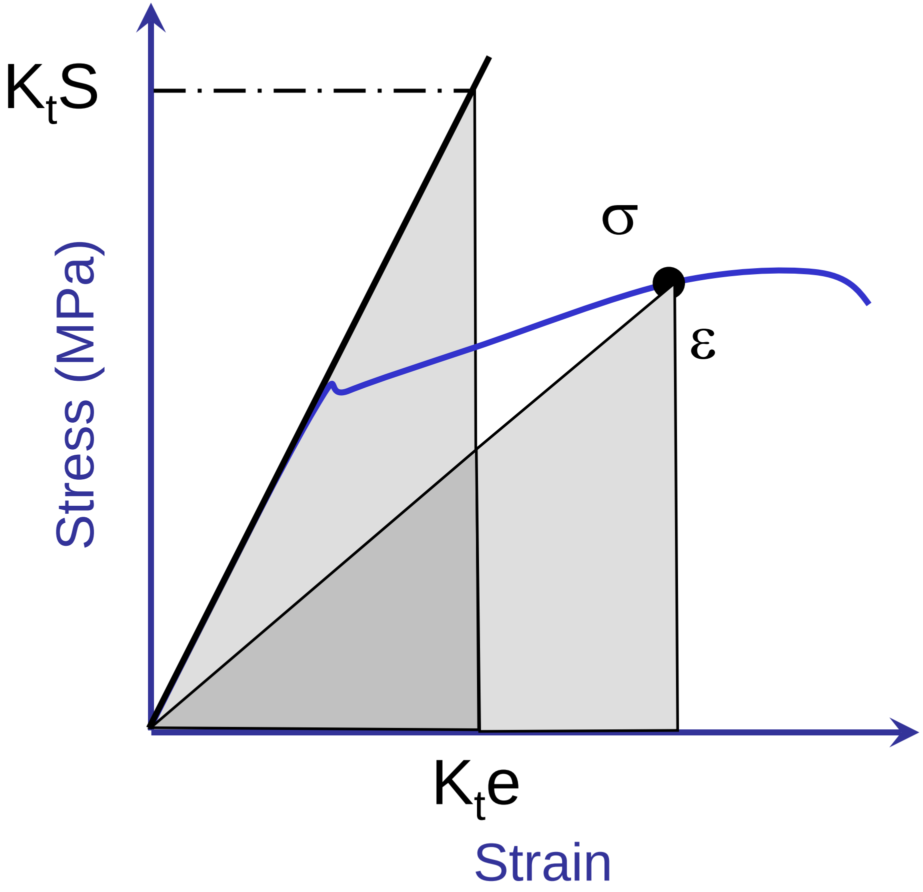
K_σ and K_ε



$$K_\sigma = \frac{\sigma}{S}$$

$$K_\varepsilon = \frac{\varepsilon}{e}$$

Neuber's Rule



Actual stress

$$\underbrace{K_t S}\, K_t e = \sigma\varepsilon$$

Stress calculated with elastic assumptions

Neuber’s Rule for Fatigue

Stress and strain amplitudes

$$\frac{K_t\,\Delta S\;K_t\,\Delta e}{2\qquad\;\;2} = \frac{\Delta\sigma\,\Delta\varepsilon}{2\;\;2}$$

Elastic nominal stress

$$\frac{\Delta e}{2} = \frac{\Delta S}{2E}$$

Substitute and rearrange

$$K_t\frac{\Delta S}{2} = \sqrt{E\,\frac{\Delta\sigma}{2}\,\frac{\Delta\varepsilon}{2}}$$

The product of stress times strain controls fatigue life

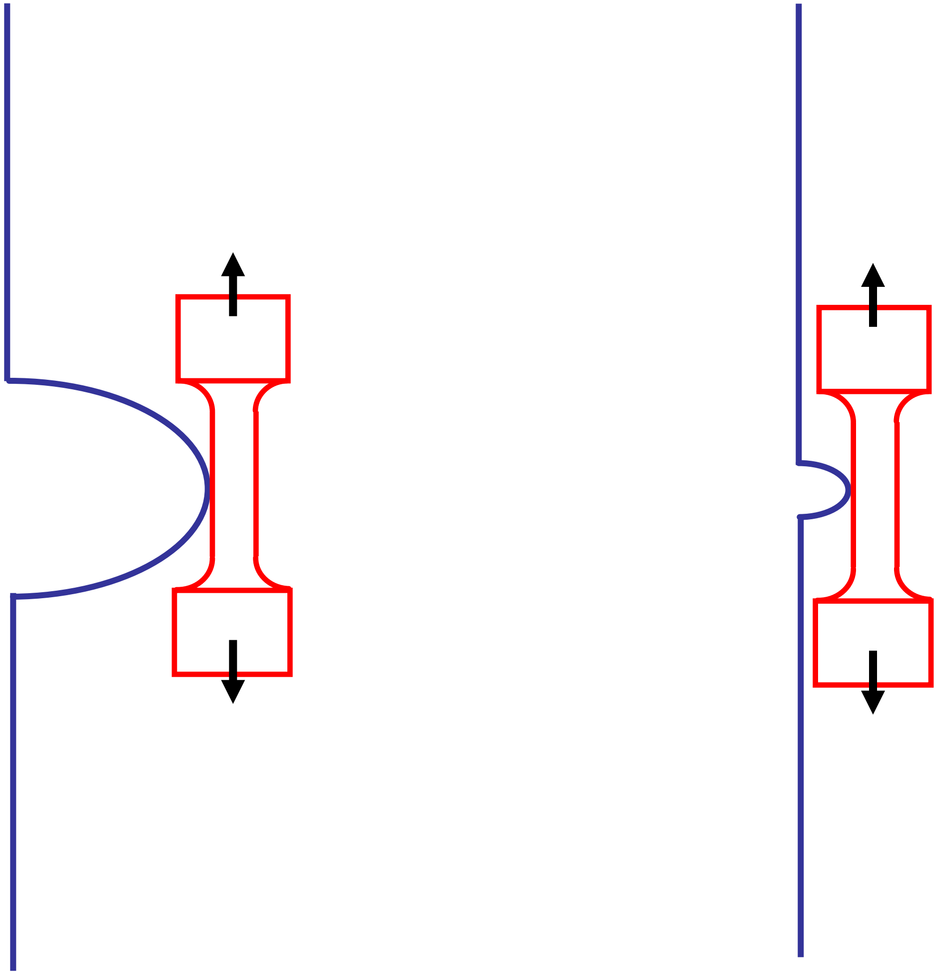
A Dilemma

Stress analysis and stress concentration factors are independent of size and are related only to the ratio of the geometric dimensions to the loads

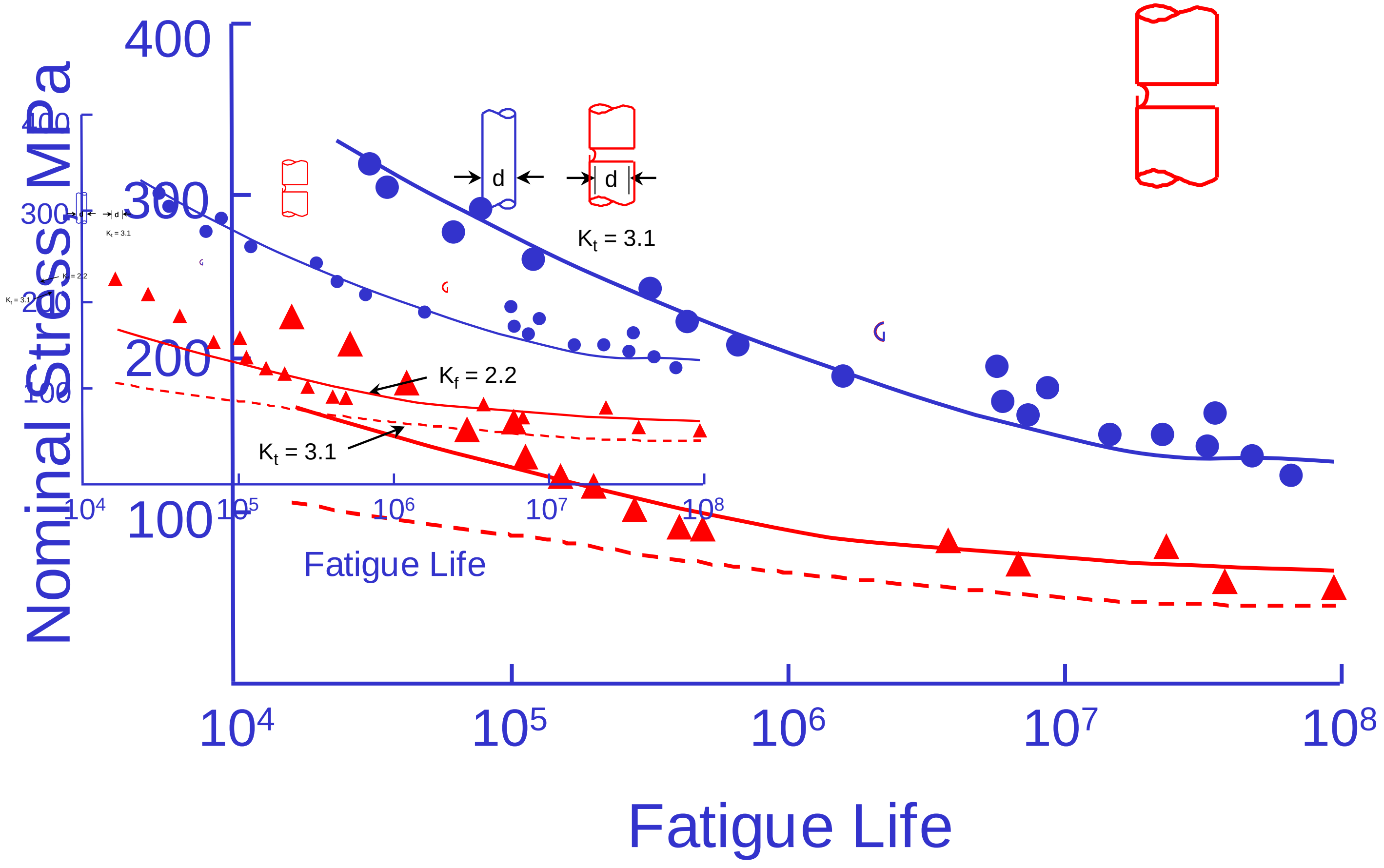
Fatigue is a size dependent phenomenon

How do you put the two together ?

Similitude

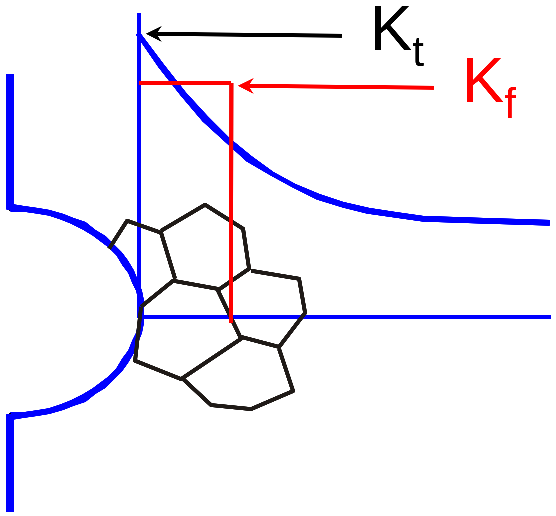


Fatigue of Notches

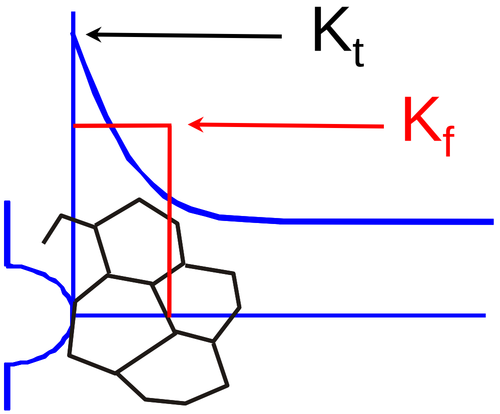


From Dowling, Mechanical Behavior of Materials, 1999

Notch Size

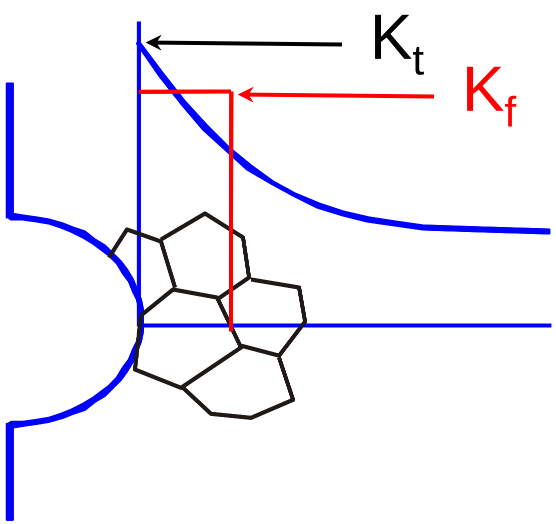


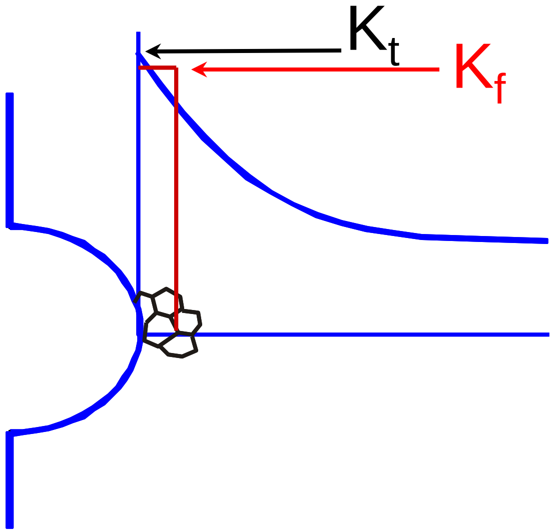
Large Notch



Small Notch

Microstructure Size

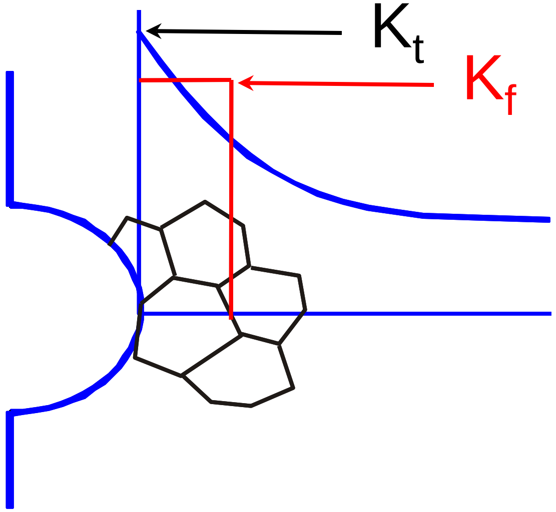




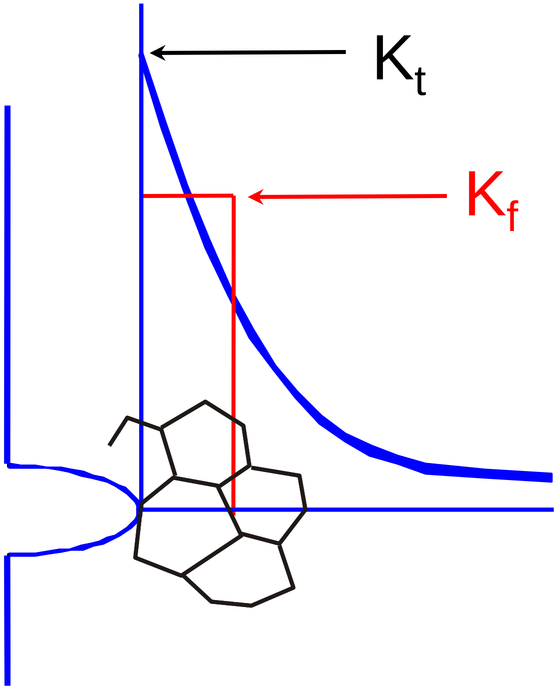
Low Strength

High Strength

Stress Gradient

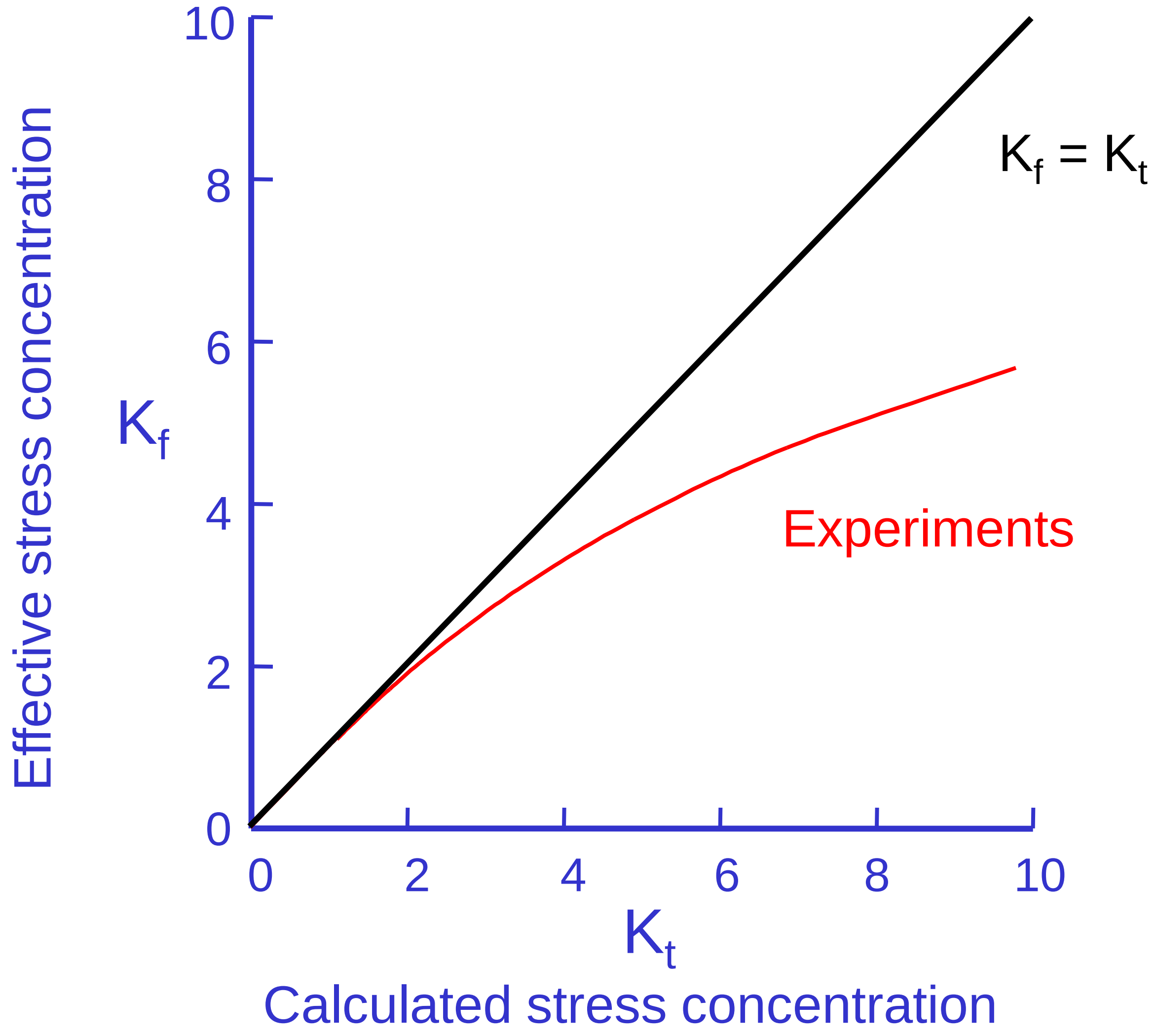


Low K_t

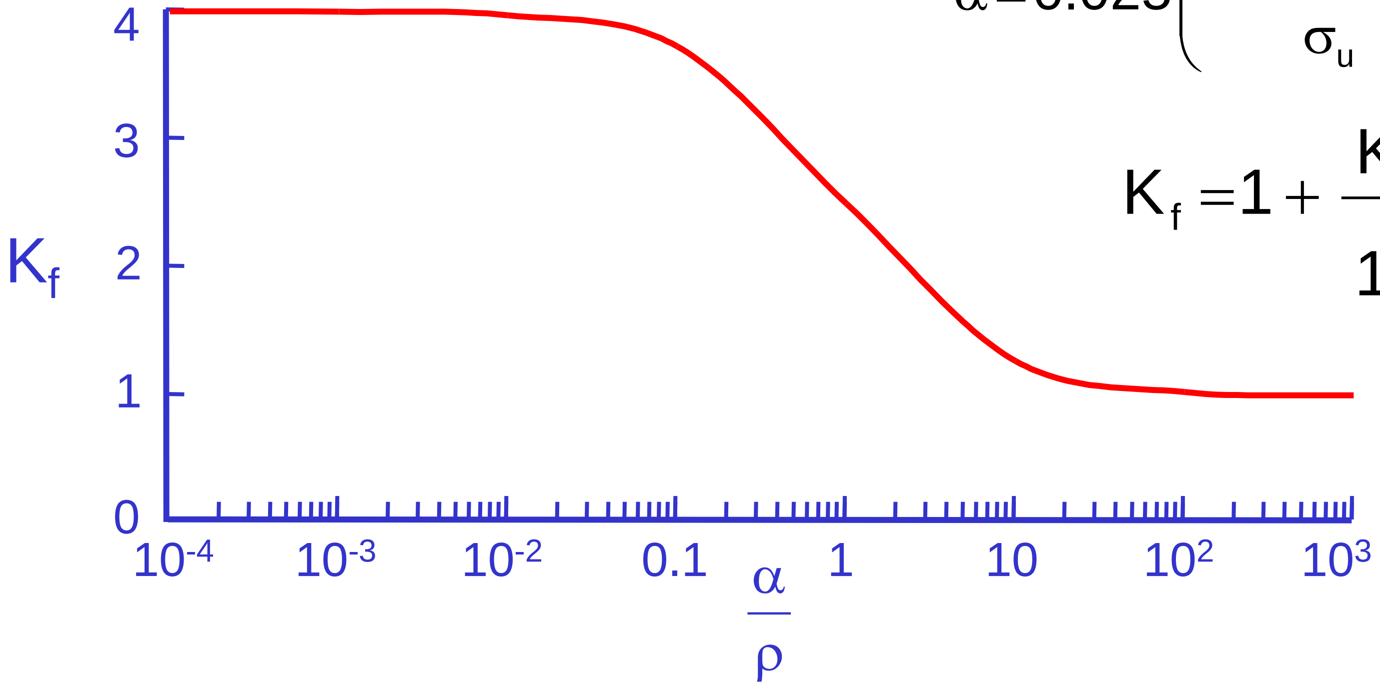


High K_t

K_t vs K_f


10
8
6
4
2
0
Effective stress concentration
K_f
0
2
4
6
8
10
K_t
Calculated stress concentration
$K_f = K_t$
Experiments

Peterson's Equation



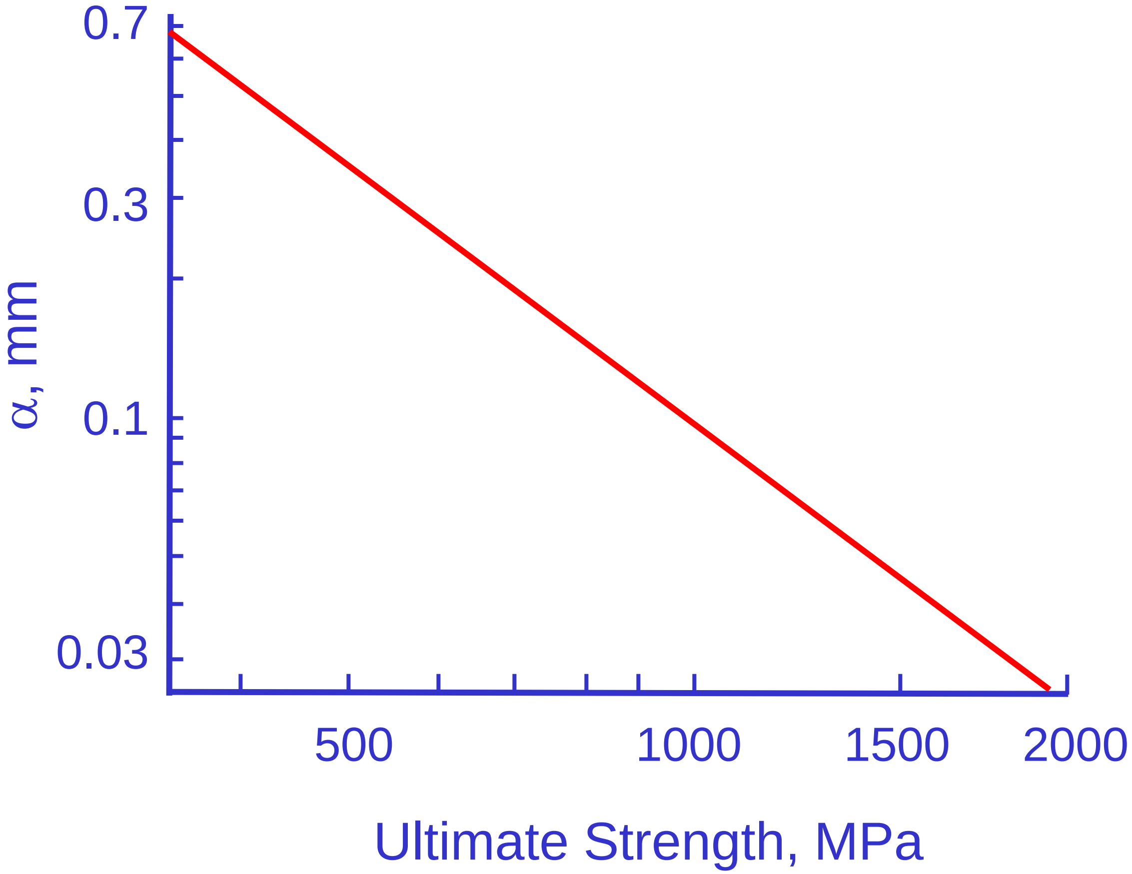
$$\alpha = 0.025\left(\frac{2070\text{MPa}}{\sigma_u}\right)^{1.8}\ \text{mm}$$

$$K_f = 1 + \frac{K_t - 1}{1 + \frac{\alpha}{\rho}}$$

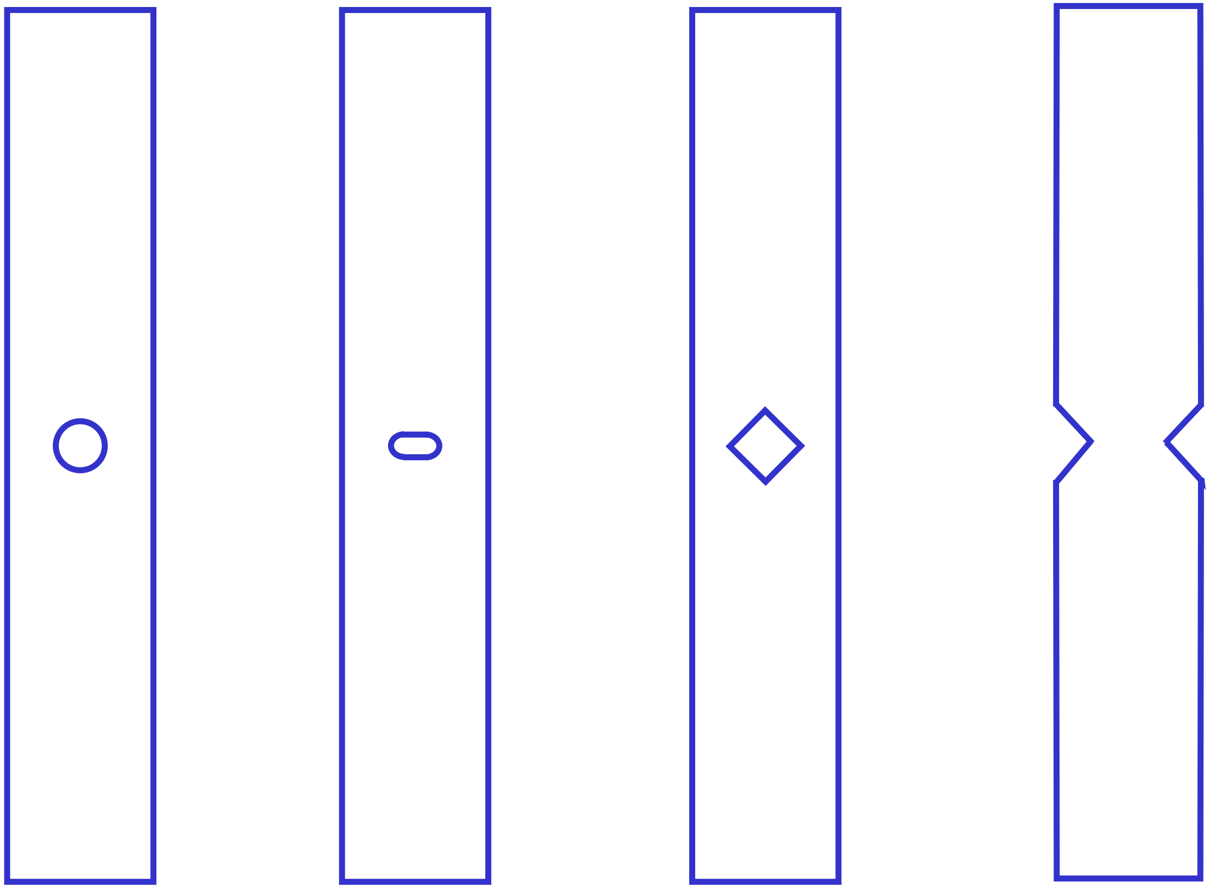
No effect when $\rho << \alpha$

Full effect when $\rho >> \alpha$

Peterson’s Constant



Static Strength



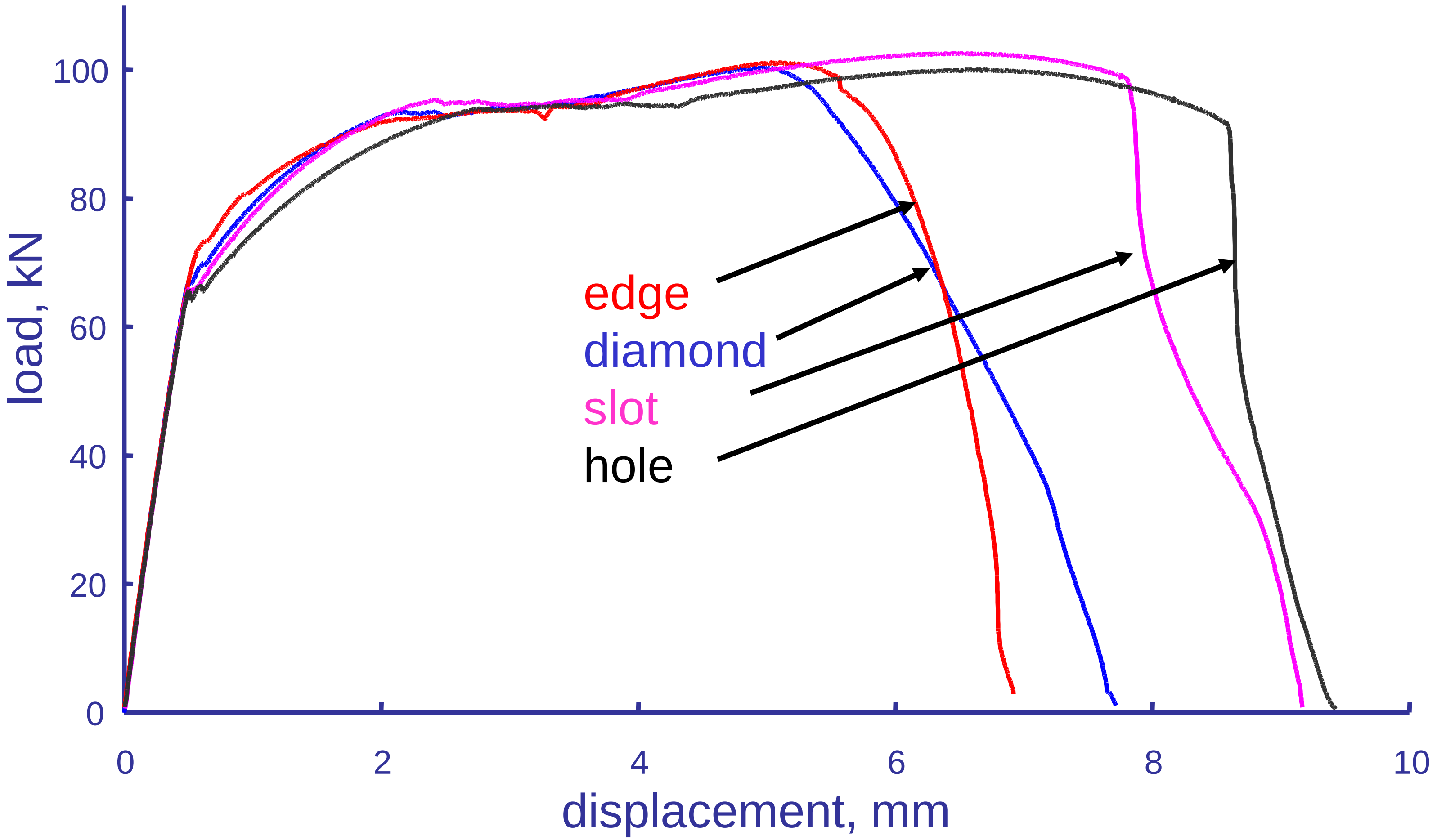
hole
$K_t = 2.5$

slot
$K_t = 5$

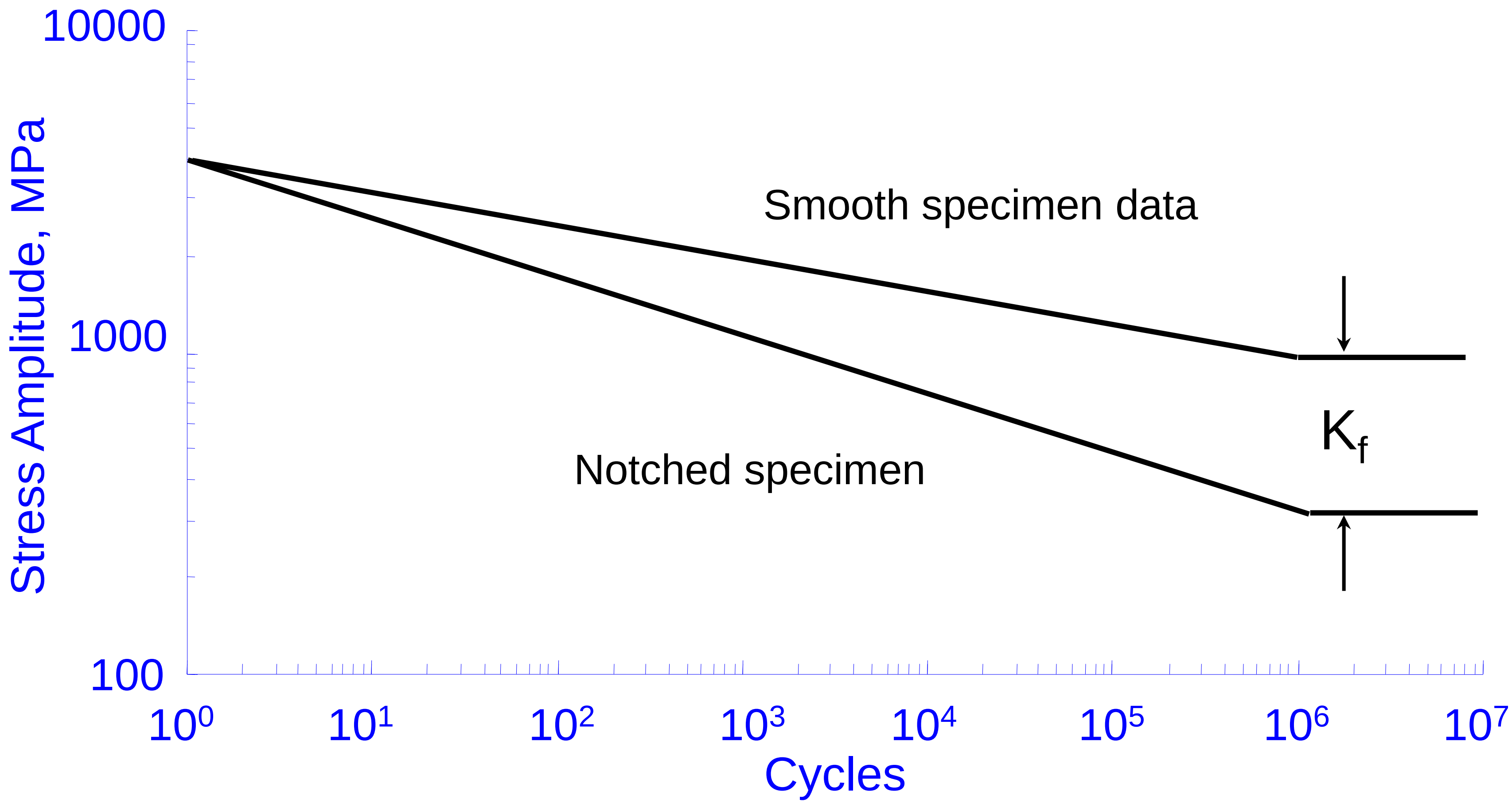
diamond
$K_t = 20$

edge
$K_t = 20$

1018 Steel Test Data

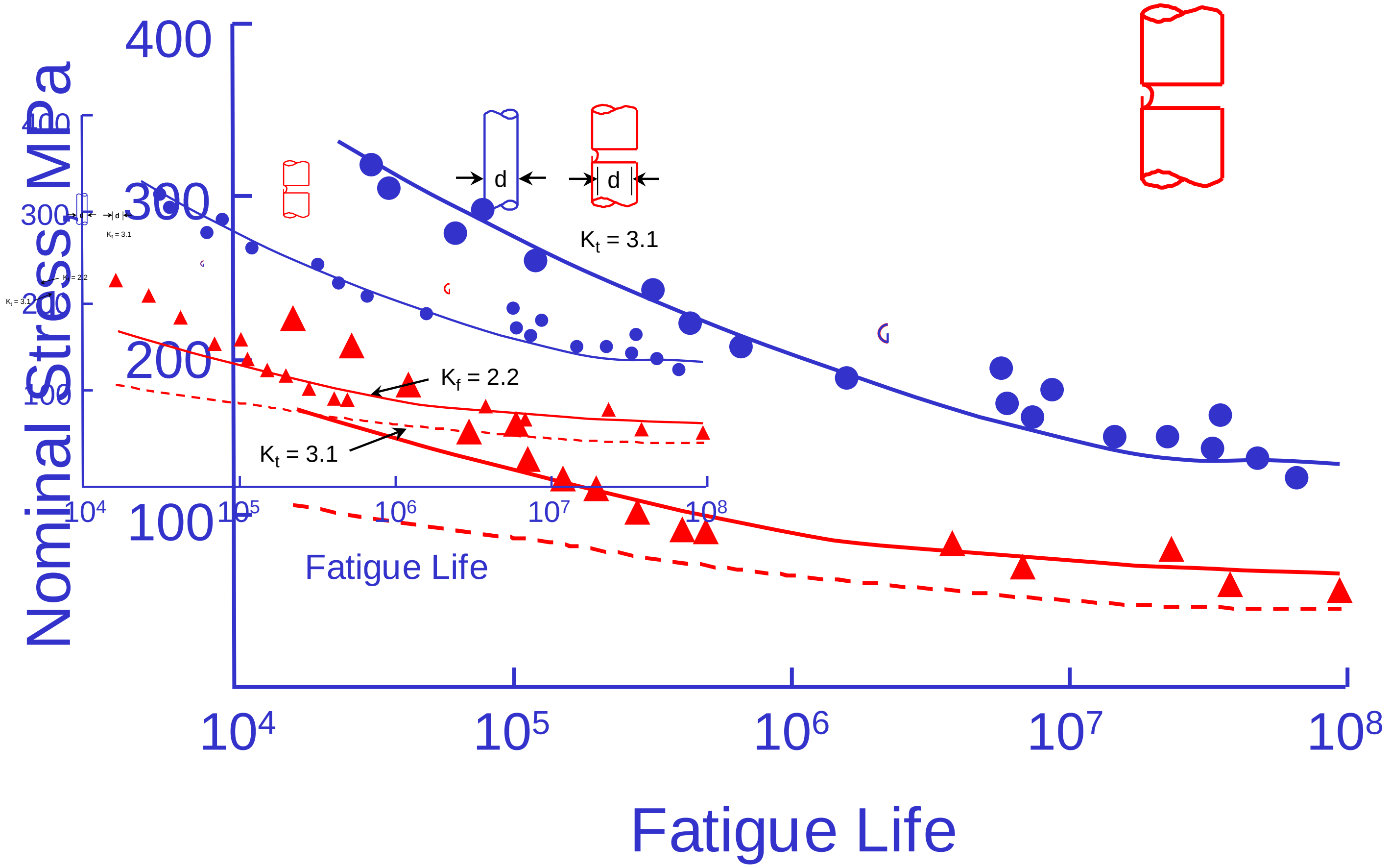


Notched SN Curve



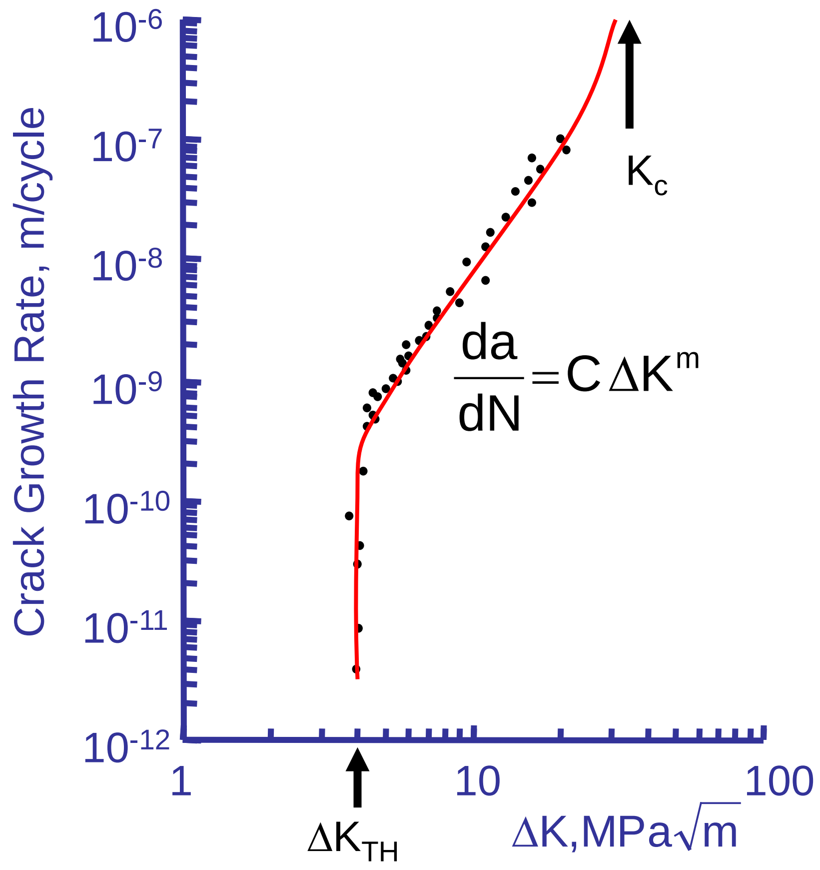
Stress concentrations are not very important at short lives

Fatigue of Notches



From Dowling, Mechanical Behavior of Materials, 1999

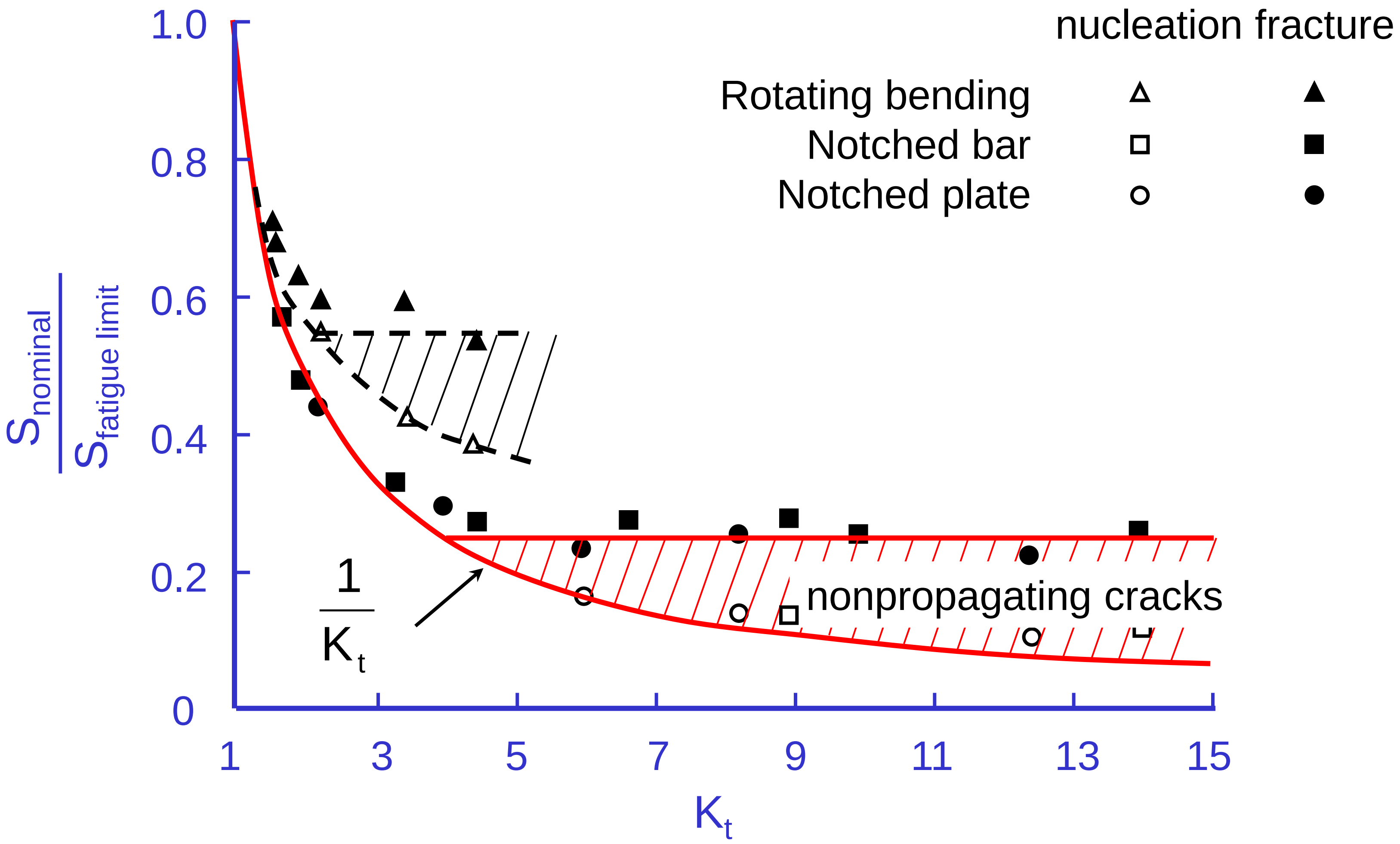
Crack Growth Data



Nonpropagating cracks

$$\Delta K_{TH} > \Delta\sigma 1.12 \frac{2}{\pi}\sqrt{\pi a}$$

Frost Data



Frost, “A Relation Between the Critical Alternating Propagation Stress and Crack Length for Mild Steel” Proceedings of the Institute for Mechanical Engineers, Vol. 173, No. 35, 1959, 811-836

Significance

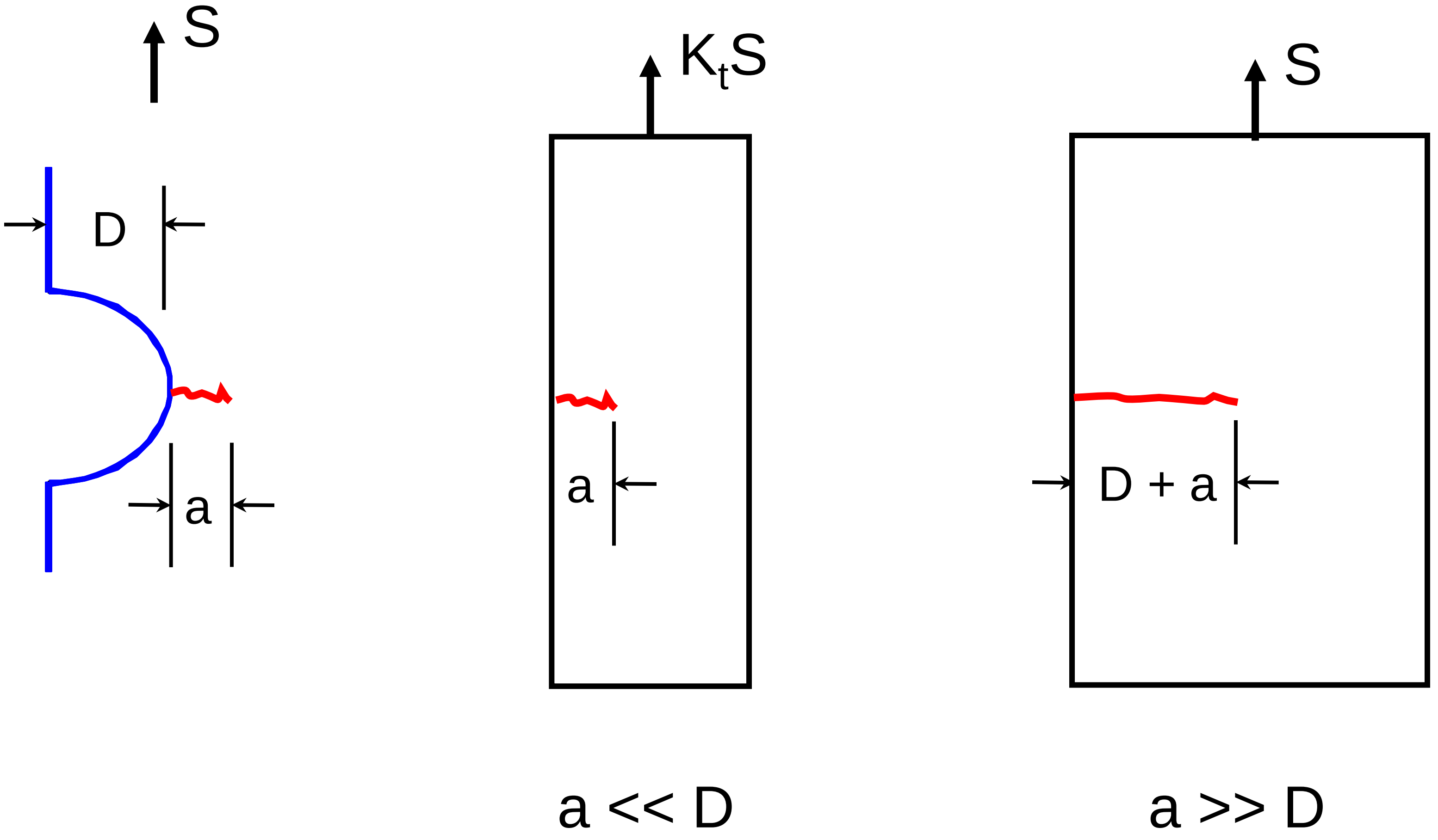
For $K_t > 4$, the notch acts like a crack with a depth D

$$S_{fl} = \frac{\Delta K_{th}}{\sqrt{\pi D}}$$

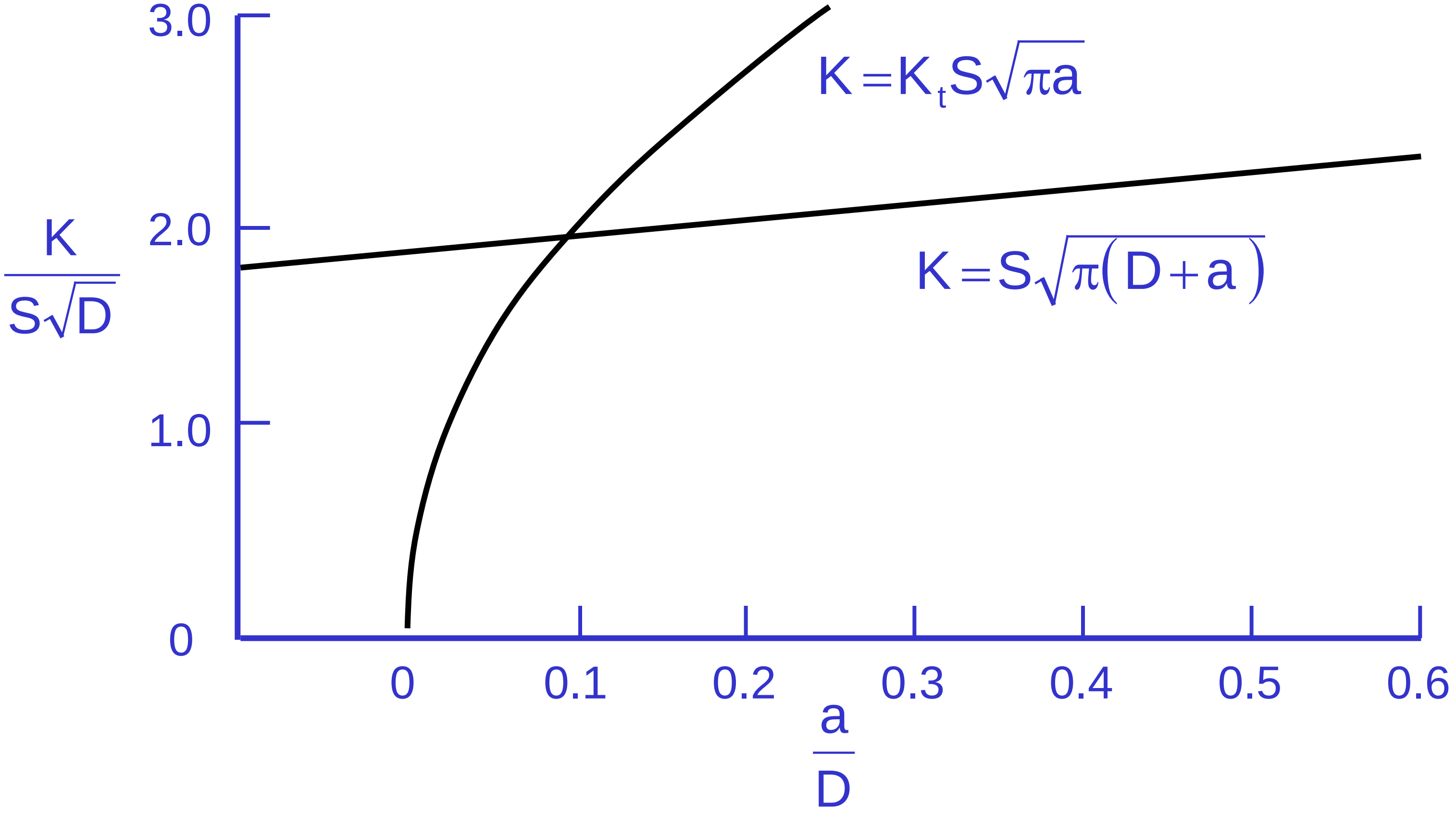
K_t does not play a role for sharp notches !

A stress concentration behaves like a crack
once a stress concentration becomes large (Kt > 4)

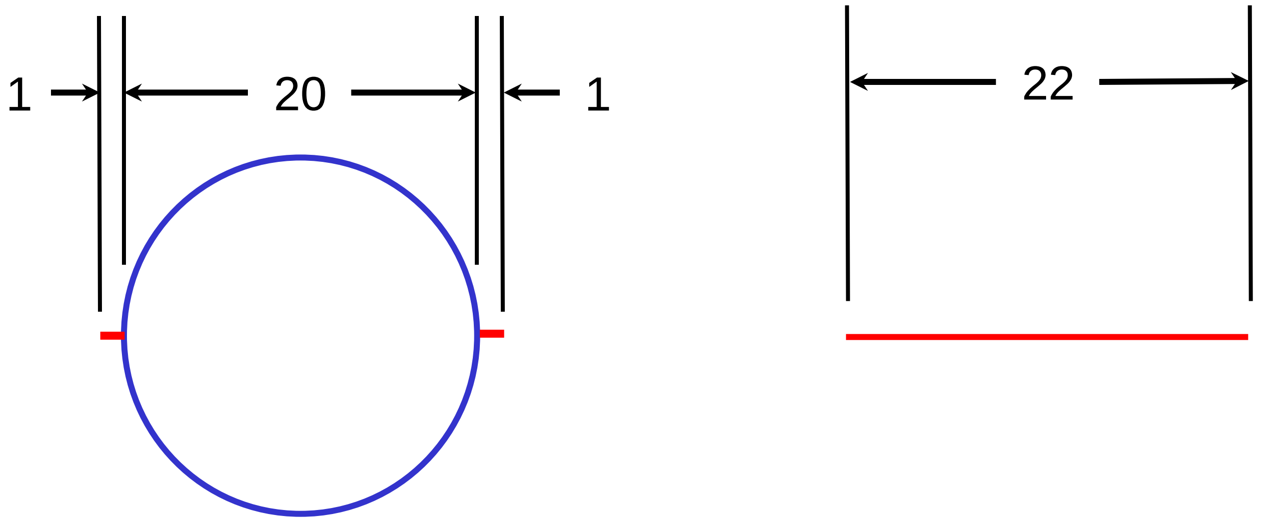
Cracks at Notches



Stress Intensity Factors

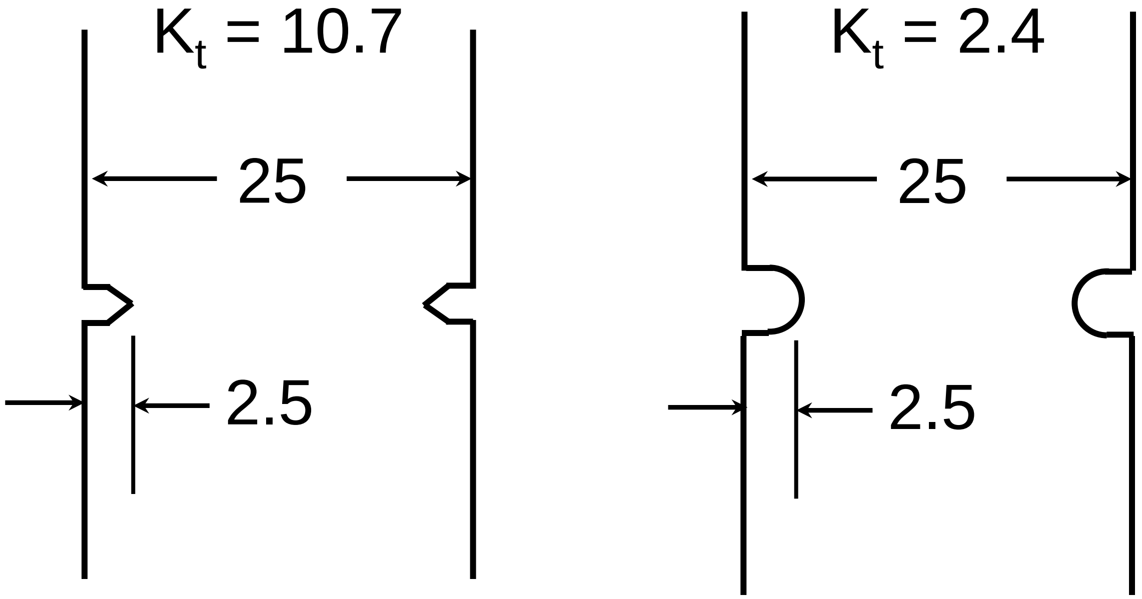


Cracks at Holes



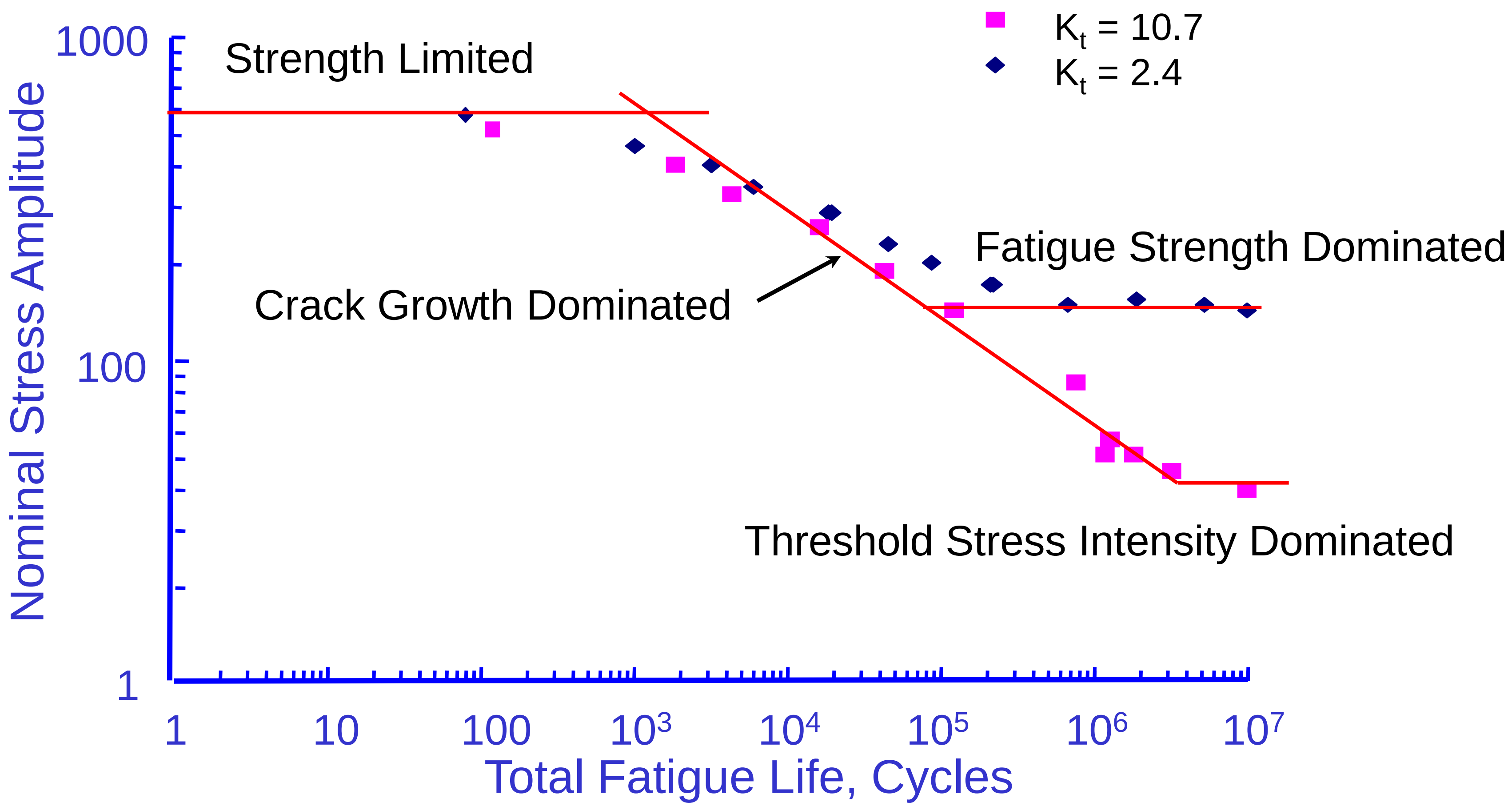
Once a crack reaches 10% of the hole radius, it behaves as if the hole was part of the crack

Specimens with Similar Geometry



Ultimate Strength 780 MPa
Yield Strength 660 MPa

Test Results



Things Worth Remembering

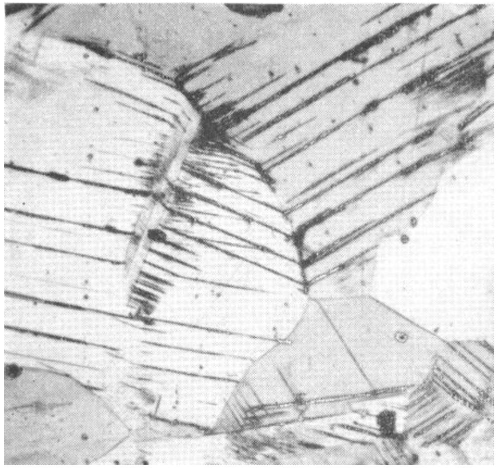
- Fatigue may be thought of as a failure of the average stress concept, consequently, fatigue usually begins at stress concentrators which are most frequently located on the surface
- The severity of a stress concentrator in fatigue is size dependent
- Small stress concentrators are more effective in high strength materials

Factors Influencing Fatigue

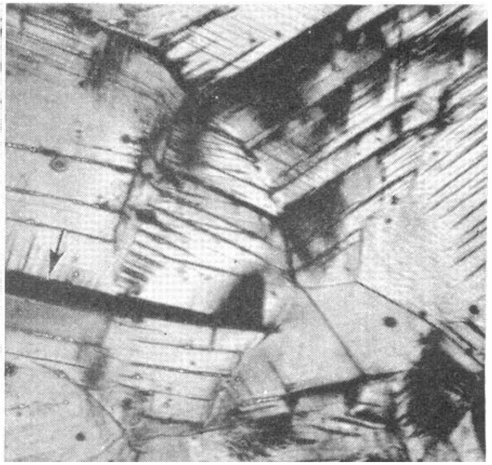
- Mean Stress
- Variable Amplitude
- Stress Concentrations
- **Surface Finish**

Modern View of the Fatigue Limit

The fatigue limit is the stress where a crack may nucleate but will not grow through the first microstructural barrier such as the grain size, pearlite colony size, prior austenite grain size, eutectic cell size or precipitate spacing.

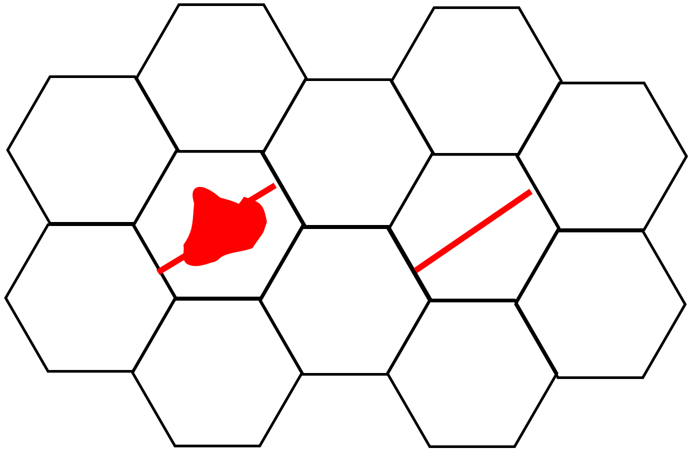


Slip Bands

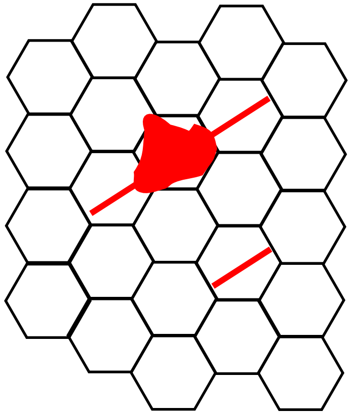


Crack

Intrinsic Flaws



Little effect of surface pit because
it is smaller than the grain size



Large effect of defect because
it is larger than the grain size

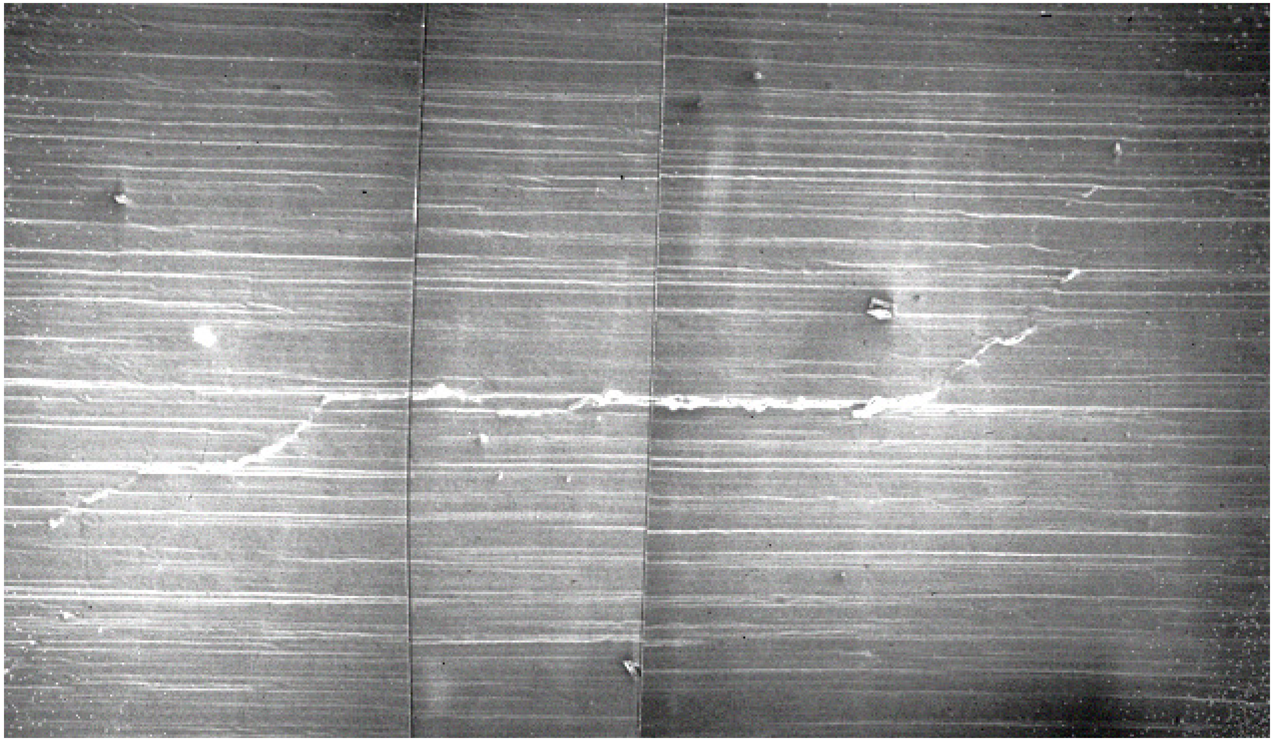
Surface Finish Influence

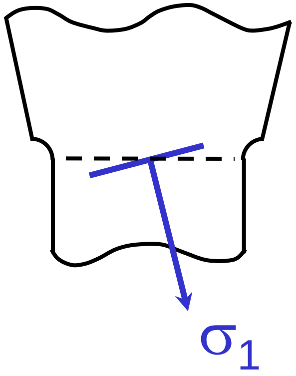
Method	Physics	Size	Influence of Surface Finish
Stress-Life	Crack Nucleation	0.01 mm	Strong
Strain-Life	Microcrack Growth	0.1 - 1 mm	Moderate
Crack Growth	Macrocrack Growth	> 1mm	None

Sources of Surface Effects

- Machining
  - Cutting
  - Grinding
- Corrosion
  - General
  - Pitting
- Processing
  - Cutting/Shearing
  - Casting
  - Forging
  - Plating
- Foreign Object Damage
  - Nicks
  - Scratches

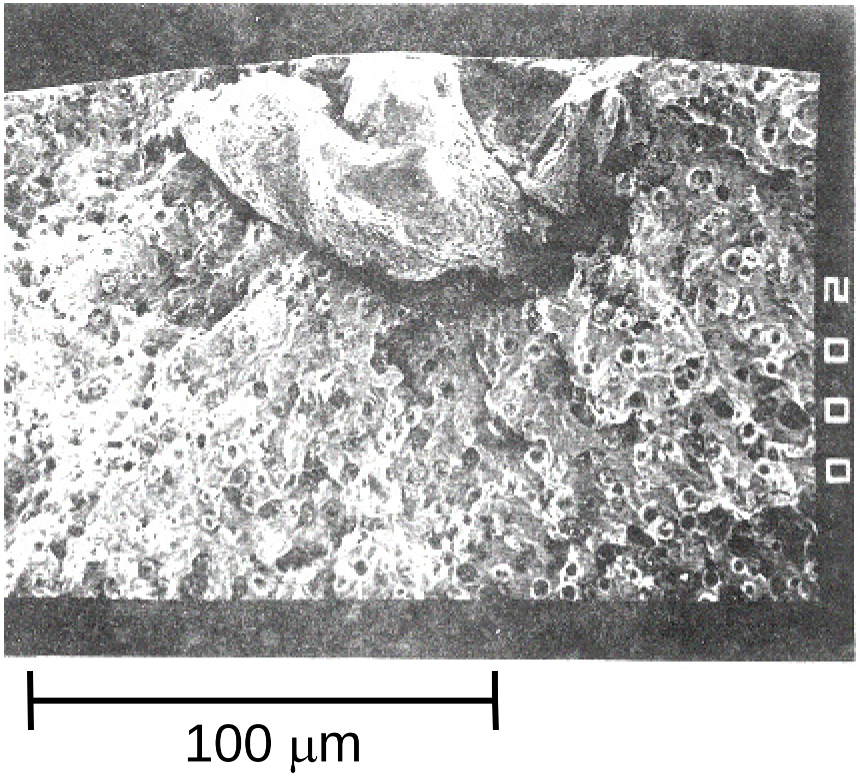
Machining





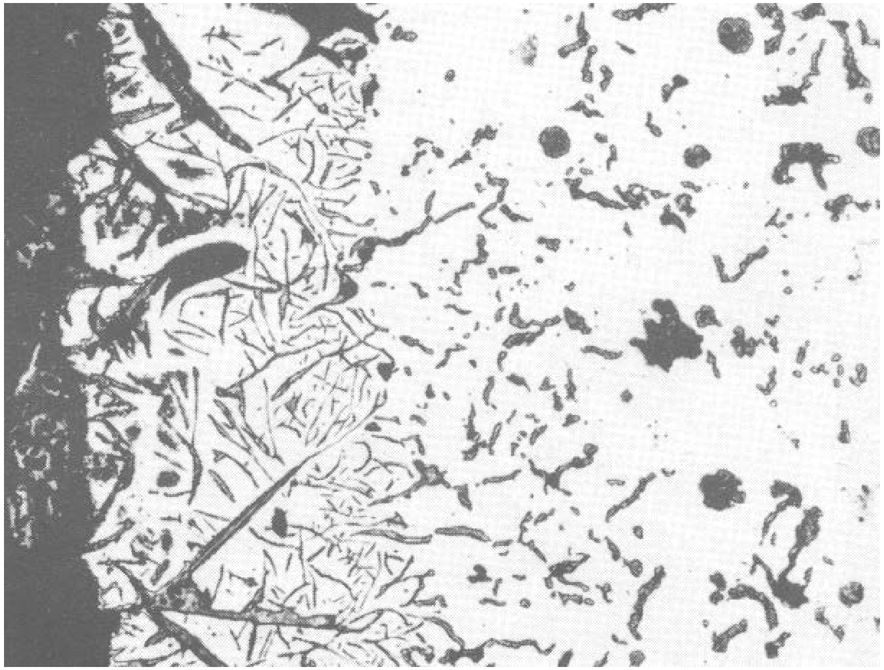
Cracks start in machining marks not in the direction of the maximum principal stress

Casting



Surface flaw in gray cast iron

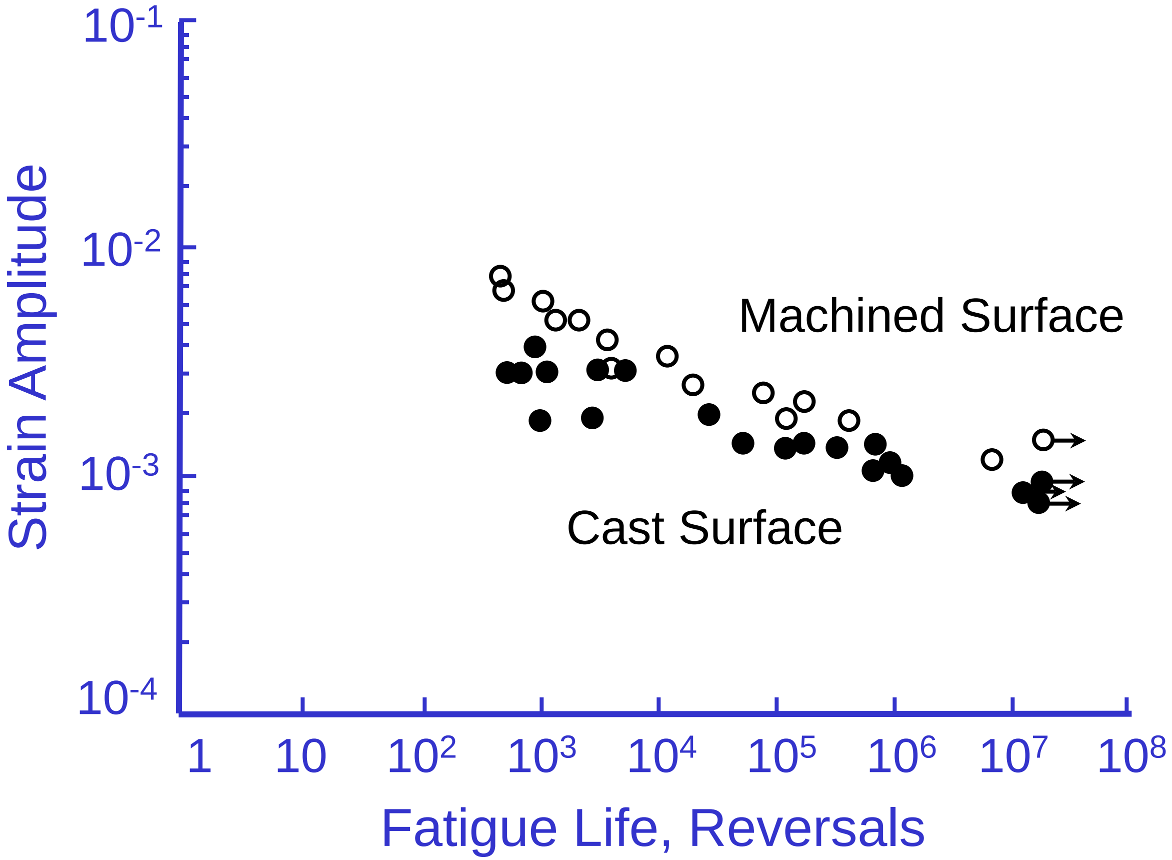
Nodular Iron Surface



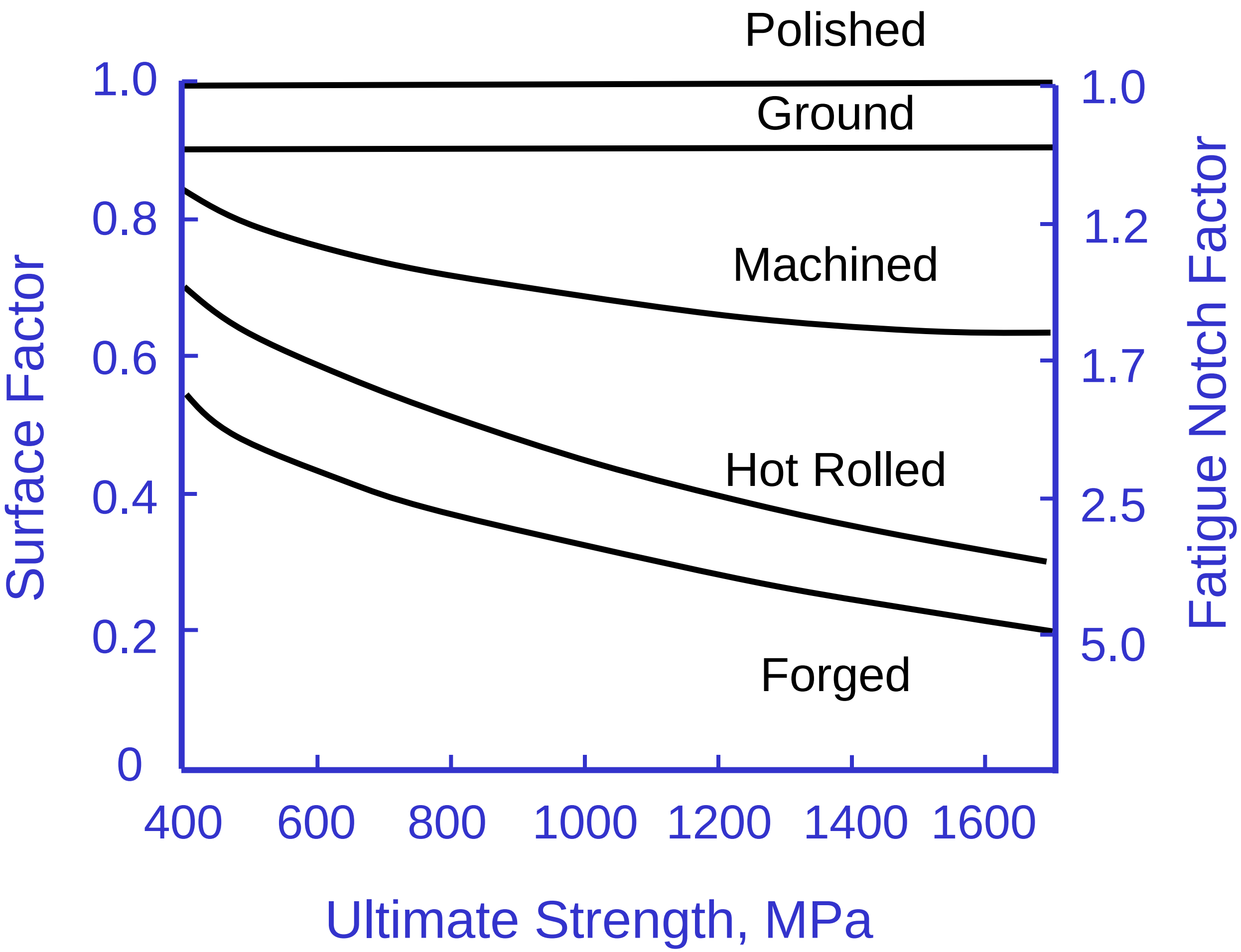
Flake graphite formed on the surface of a nodular iron casting

Starkey and Irving, “A Comparison of the Fatigue Strength of Machined and As-cast Surfaces of SG Iron” International Journal of Fatigue, July, 1982, 129-136

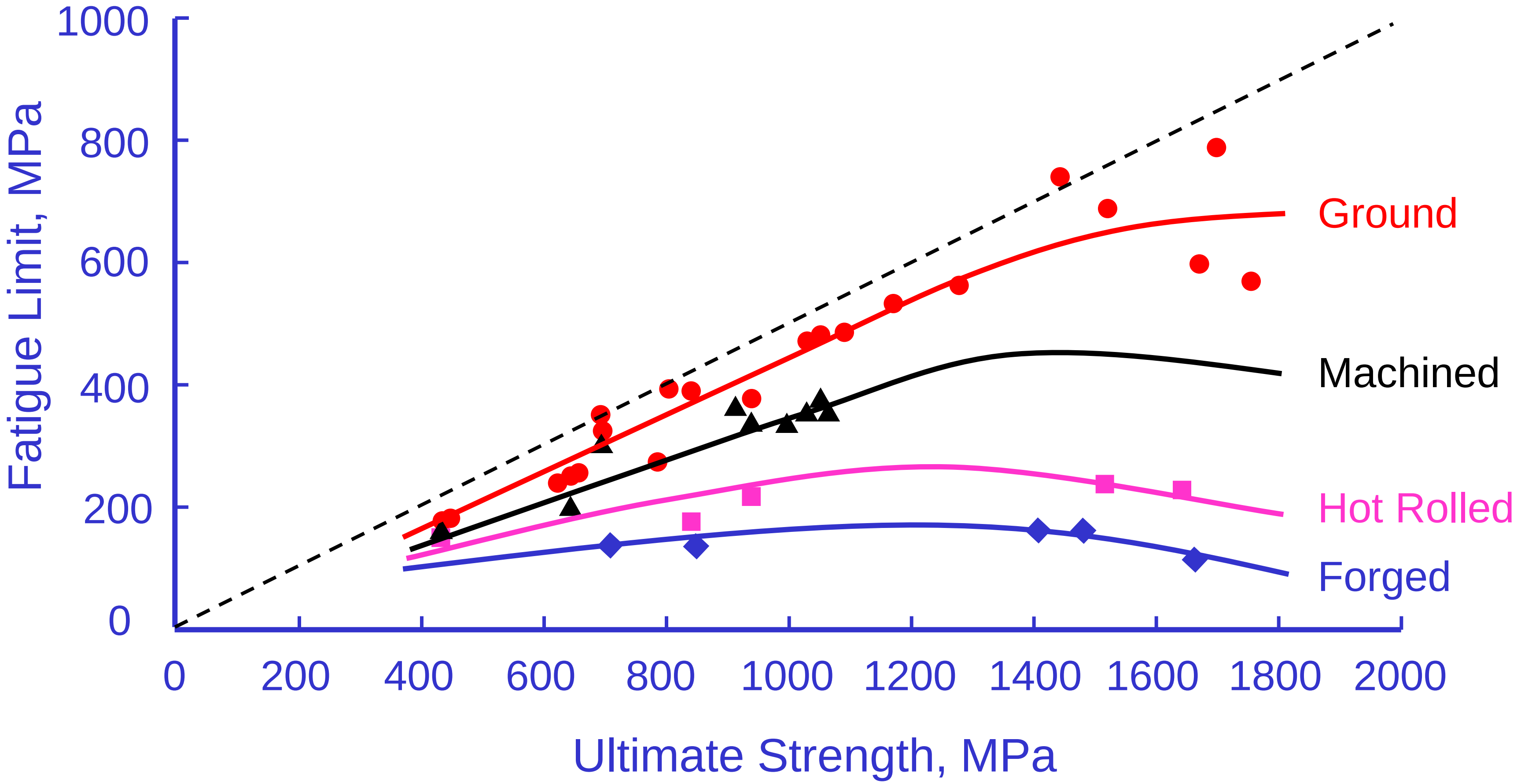
Test Data


Strain Amplitude
10-1
10-2
10-3
10-4
Machined Surface
Cast Surface
1
10
10²
10³
10⁴
10⁵
10⁶
10⁷
10⁸
Fatigue Life, Reversals

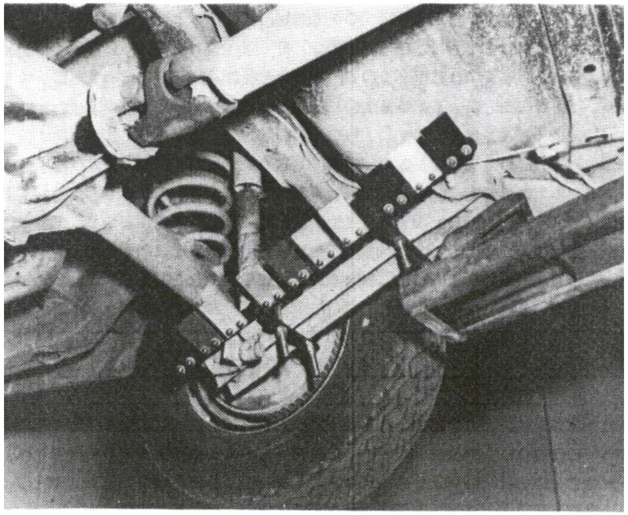
Surface Reduction Factors



Noll and Lipson 1945



Hiam and Pietrowski 1978



Driven for 1 or 2 years
in Southern Ontario
before making specimens
to evaluate corrosion effects

Strain controlled fatigue testing

Hiam and Pietrowski, “The Influence of Forming and Corrosion on the Fatigue Behavior of Automotive Steels”, SAE Paper 780040, 1978

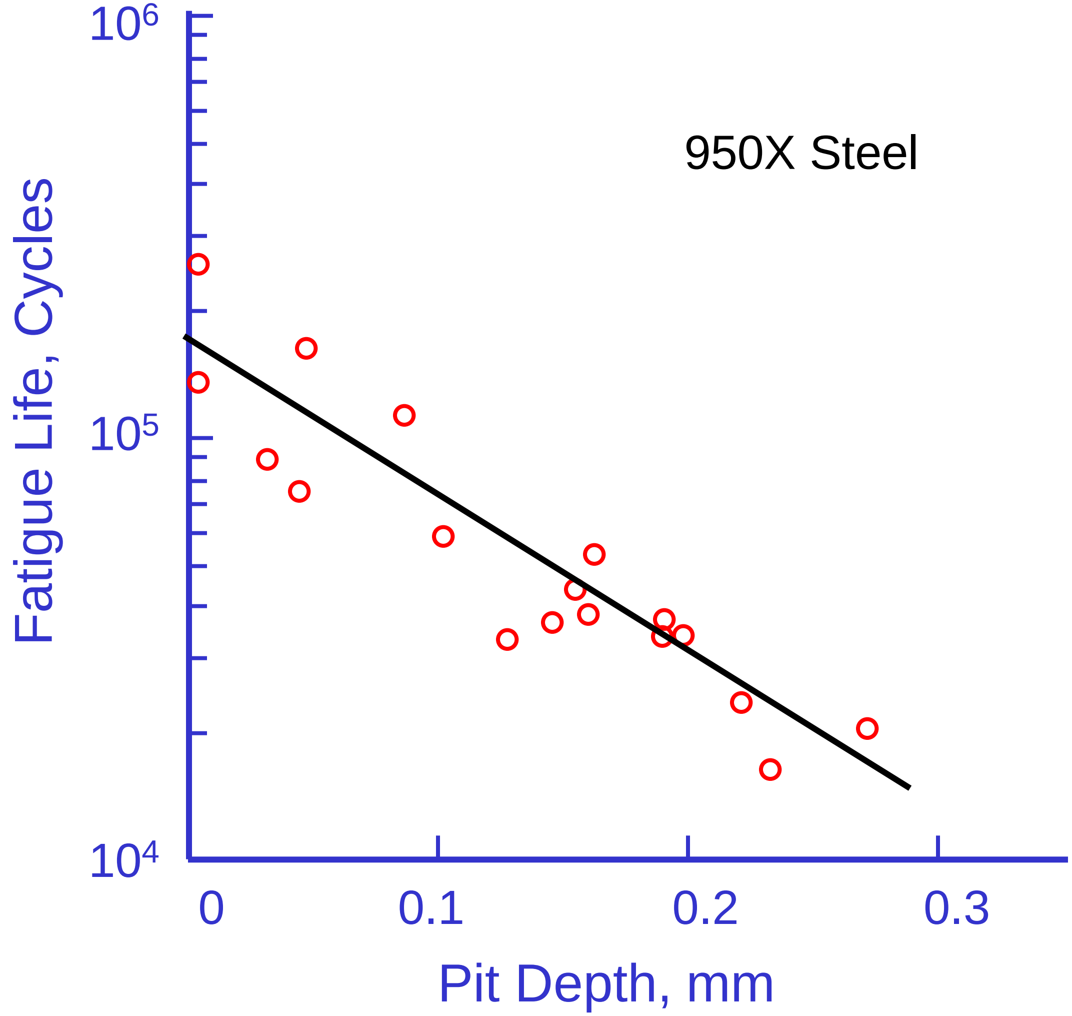
K_f for pitting

	Hot Rolled Surface	Corroded Surface
950X	1.12	1.49
0.06% C HSLA	1.18	1.65
0.18% C HSLA		1.90

Surface finish factor predicts K_f = 1.6 for a Hot Rolled Surface

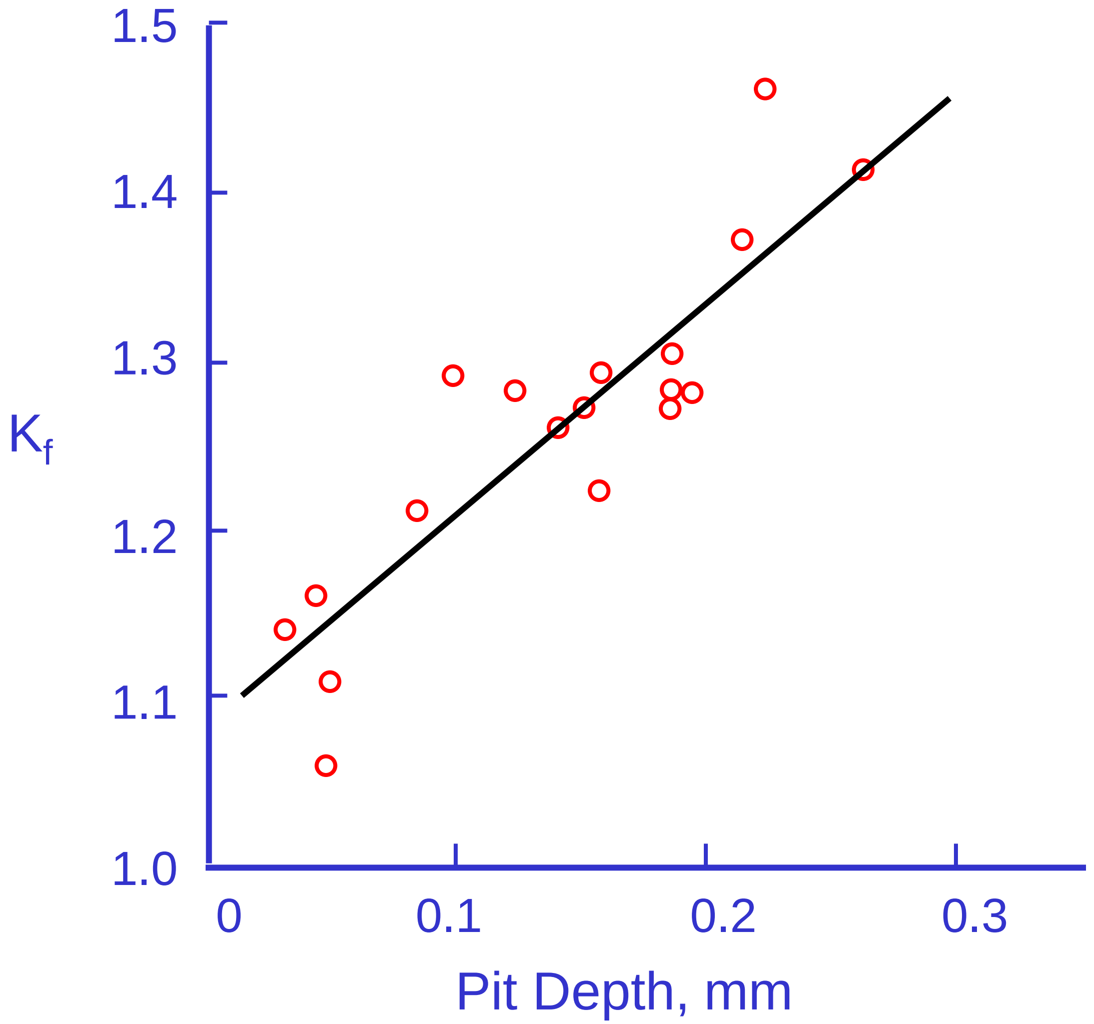
from Hiam and Pietrowski

Pit Depth Effects on Life

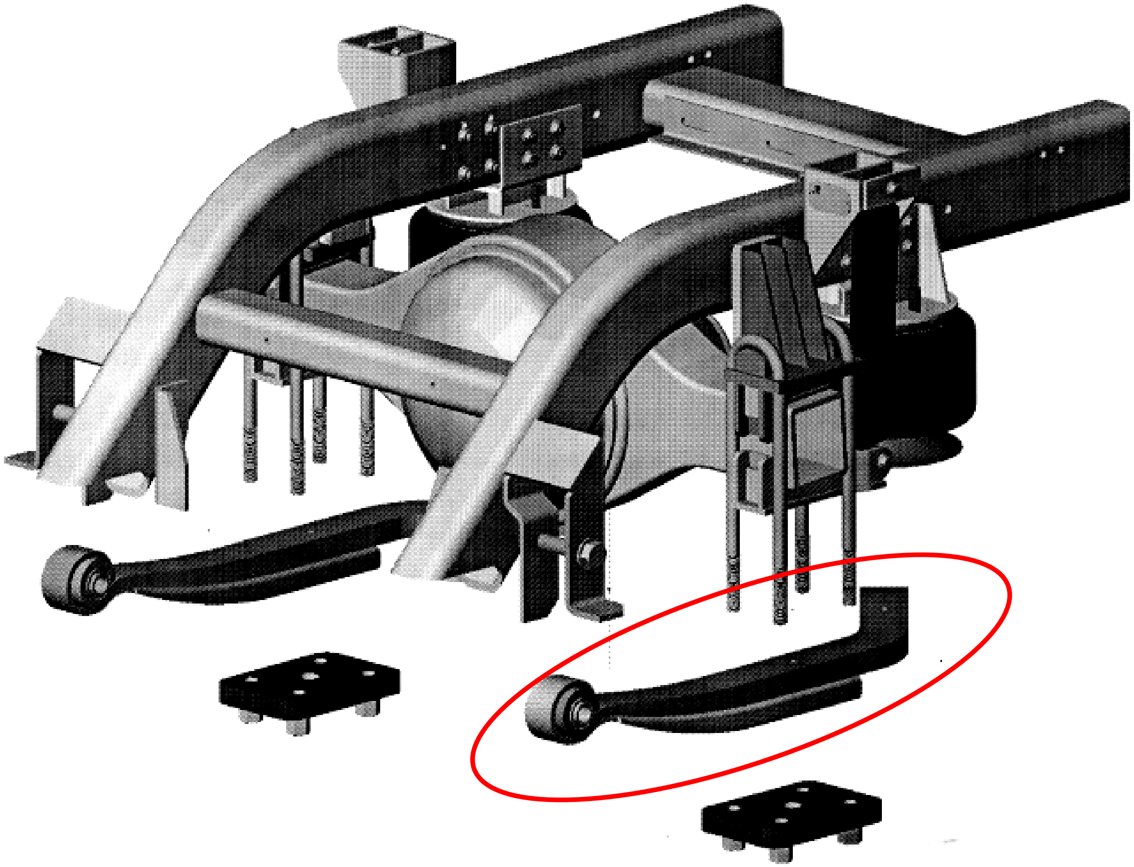


from Hiam and Pietrowski

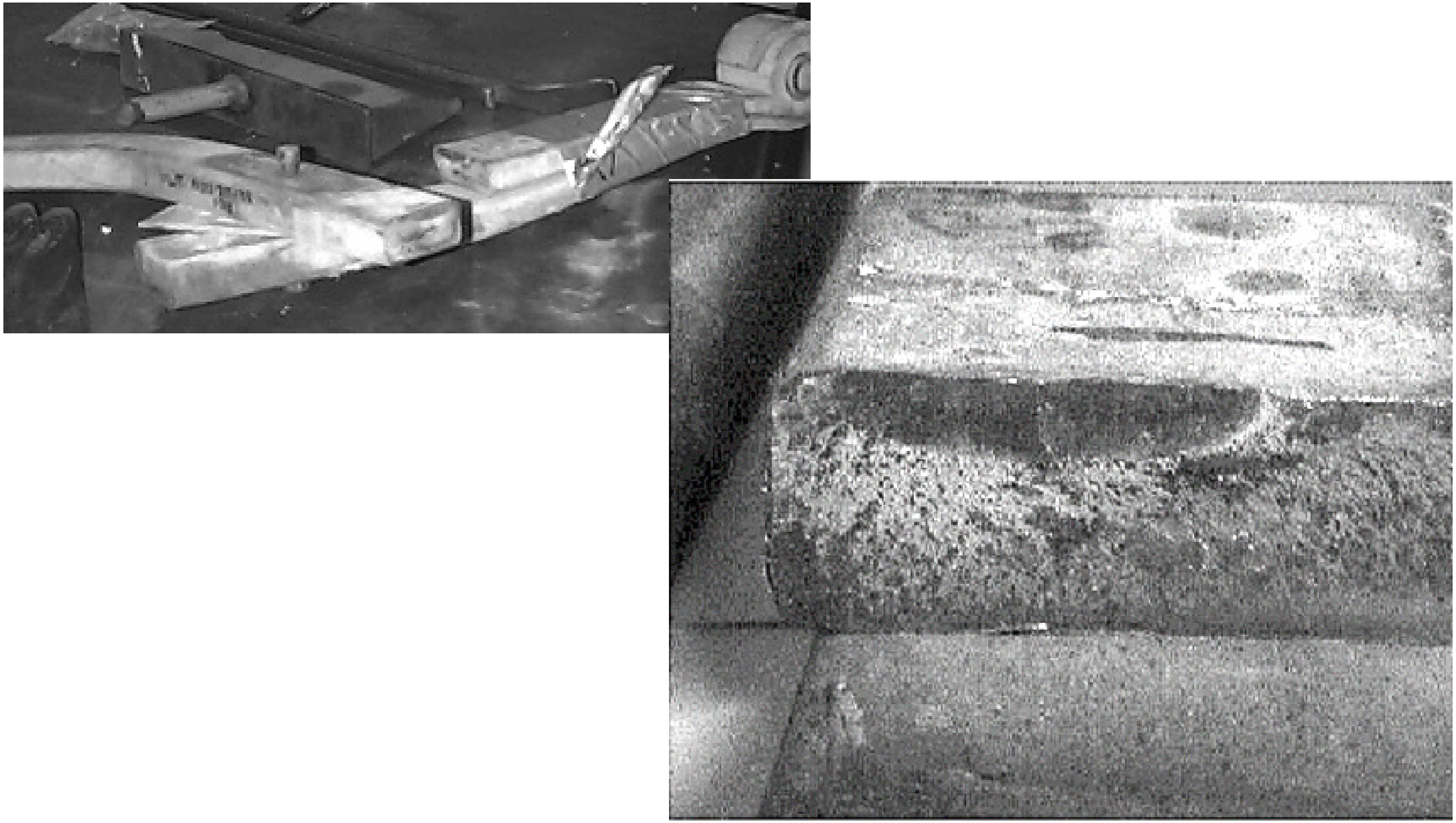
Fatigue Notch Factor for Pits



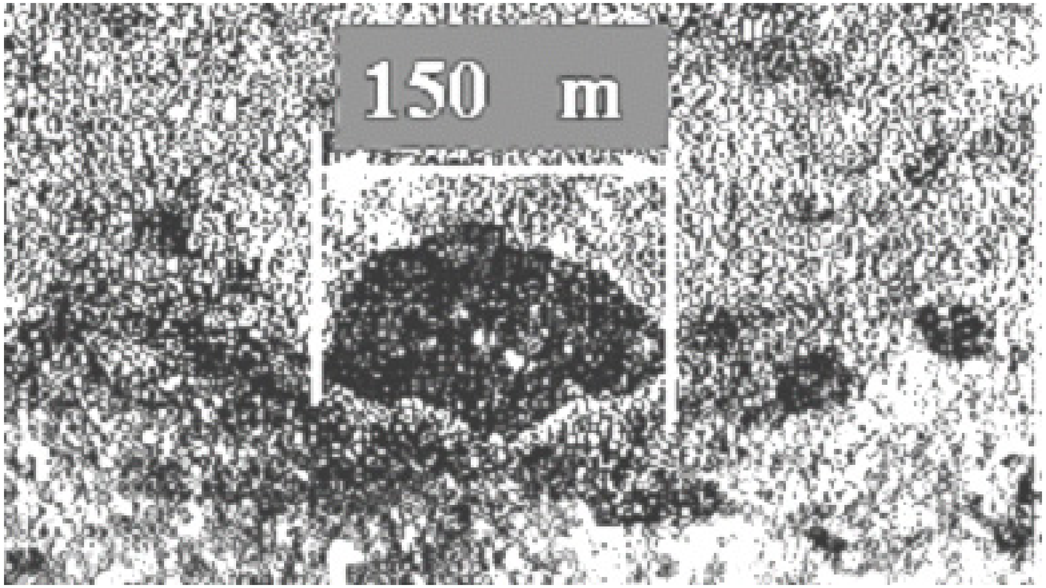
Suspension



Spring Failures

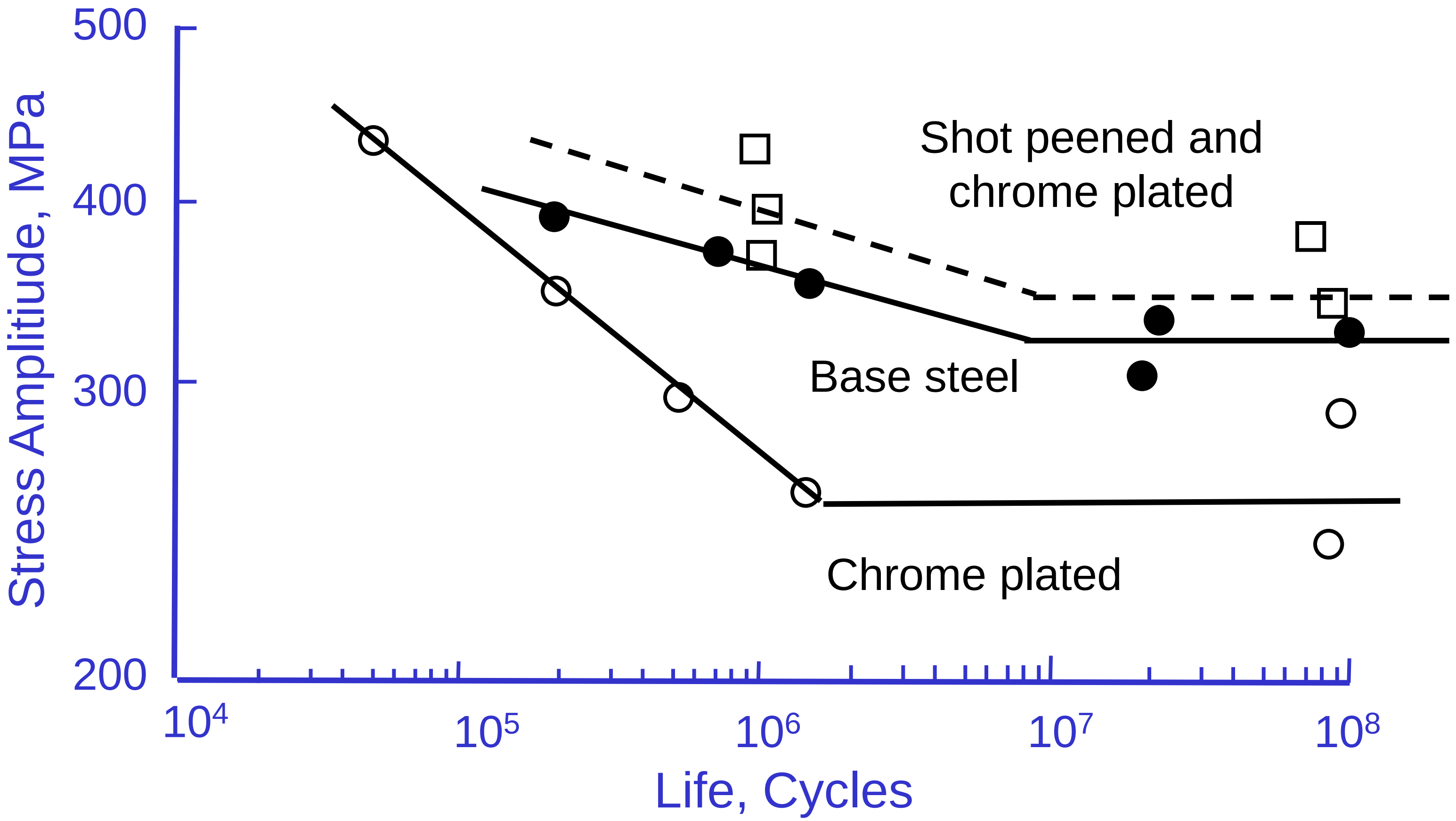


Microscopic Examination



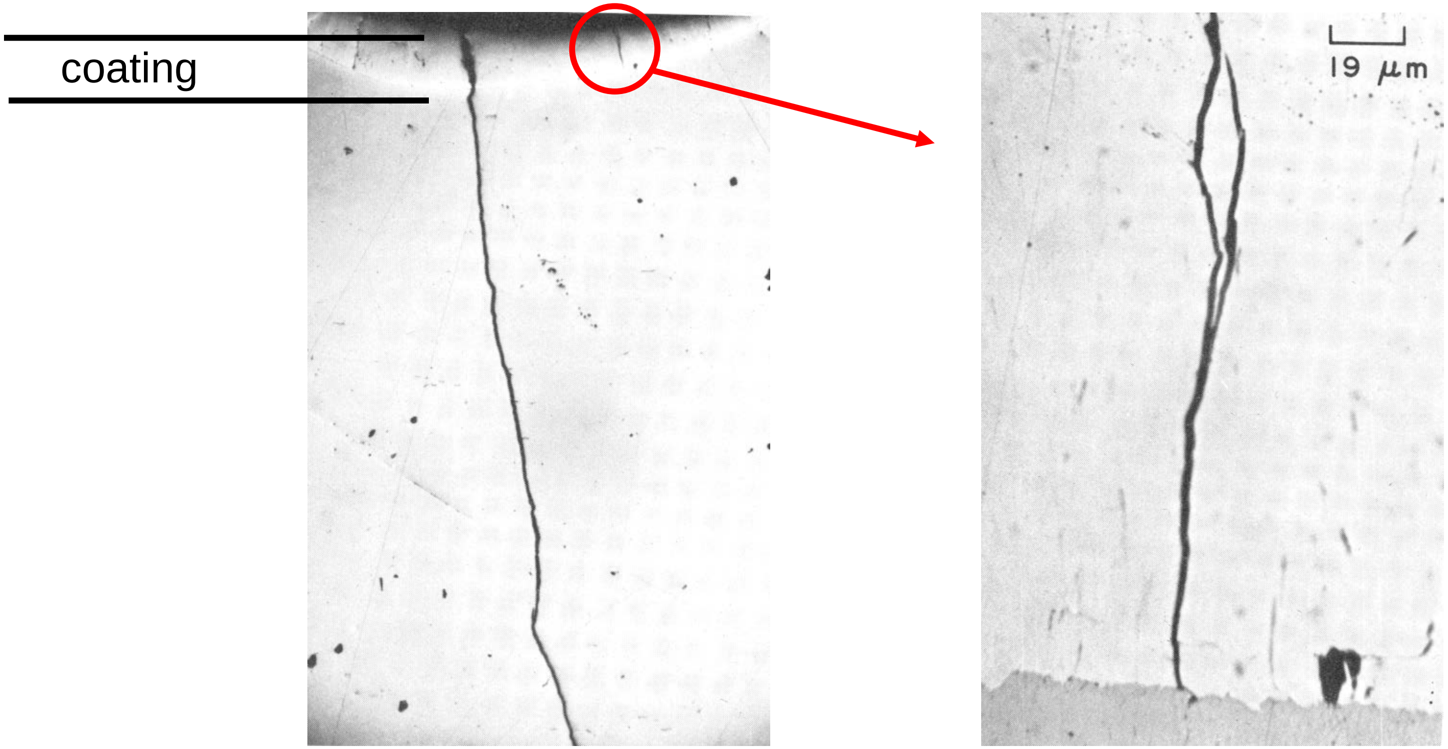
Corrosion Pits

Chrome Plating



Almen, “Fatigue Loss and Gain by Electroplating” , Product Engineering, Vol. 22, No. 5, 1951, 109-116

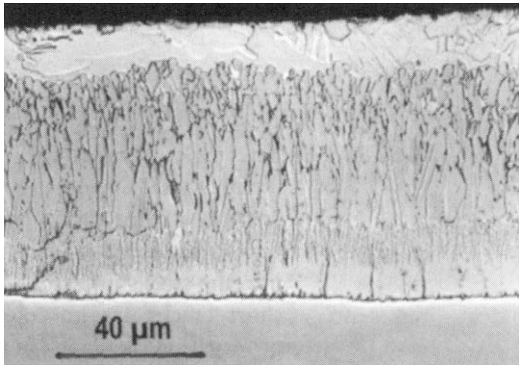
Hard Chrome Plating

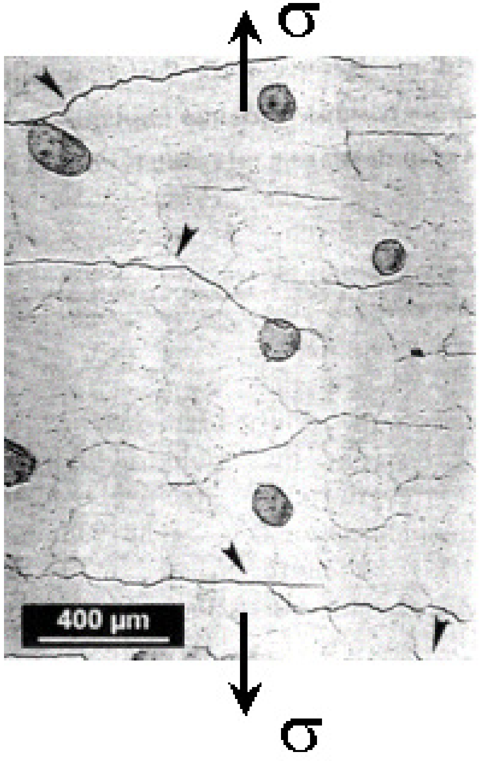


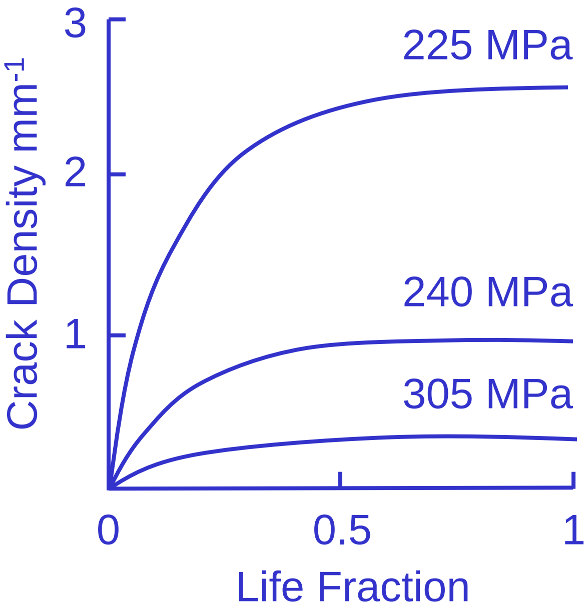
In addition to cracks, coatings frequently have high tensile residual stresses

Metals Handbook, Volume 9, Fractography and Atlas of Fractographs

Galvanized Steel

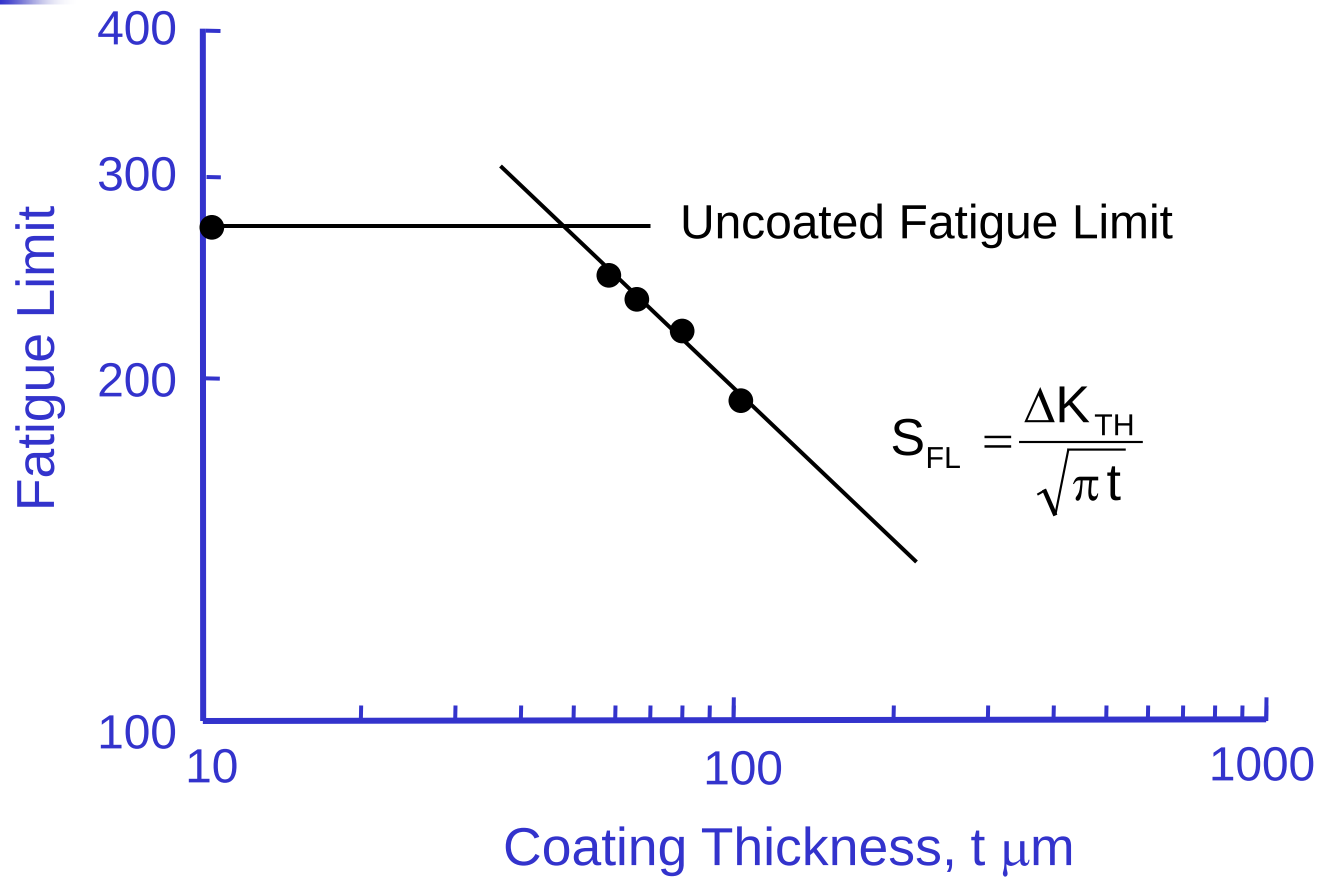






Vogt, Boussac, Foct, “Prediction of Fatigue Resistance of a Hot-dip Galvanized Steel” Fatigue and Fracture of Engineering Materials and Structures, Vol. 23, No. 1, 2001,33-40

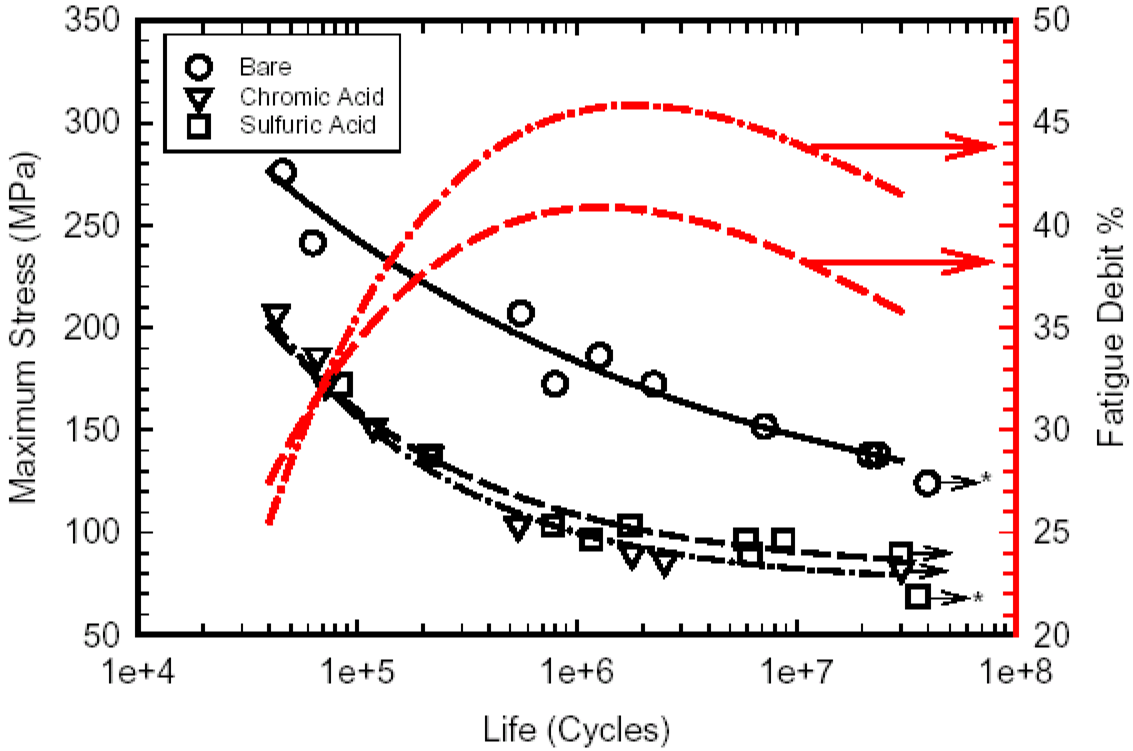
Fatigue Limit for Galvanized Steel



$$S_{FL} = \frac{\Delta K_{TH}}{\sqrt{\pi t}}$$

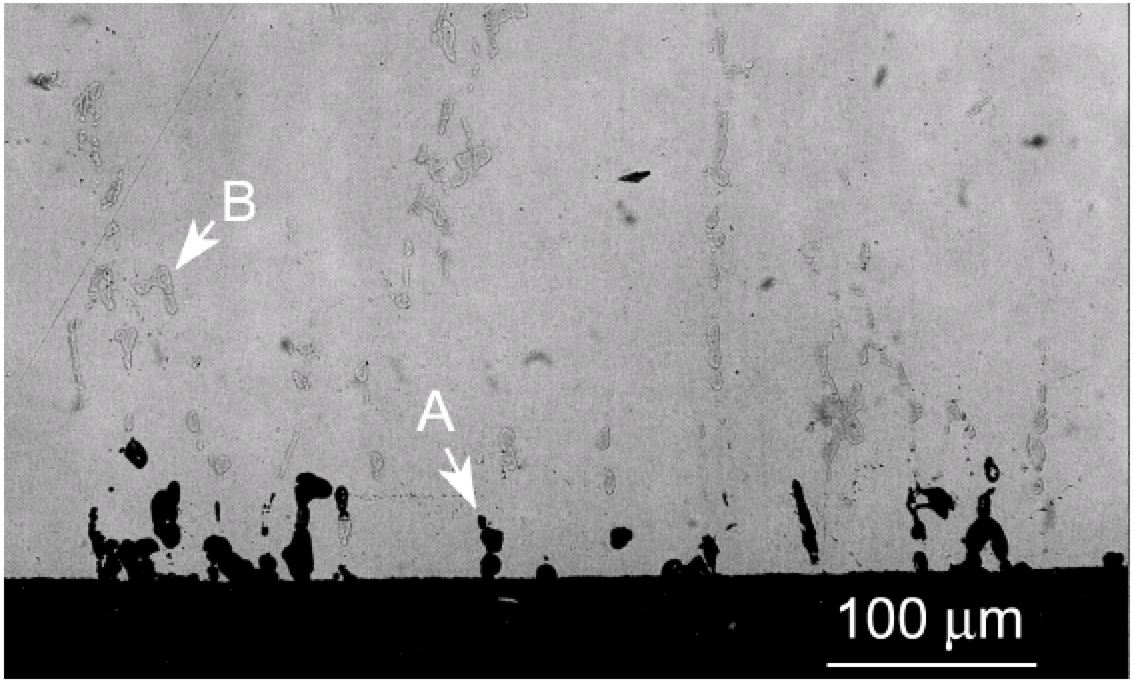
Coatings can be modeled with a crack equal to the coating thickness

Anodized Aluminum



Rateick et. al. “Relationshipp of Microstructure to Fatigue Strength Loss in Anodized Aluminum-Copper Alloys” Aeromet 2004, June 2004

Pitting at Cu Rich Constituent



- AA2219-T851 plate cross sectioned immediately after anodizing
- A: Pits
- B: Cu rich constituent

Upper Control Arm

Serial Number

Things Worth Remembering

- Fatigue crack nucleation is a surface phenomena and everything about the surface affects the fatigue life
- Most of the design rules are conservative having been developed for materials of the 1950’s

Fatigue and Fracture
(Basic Course)