Modelling

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Outline

- Fatigue mechanisms
- Sources of knowledge
- Modelling
  - Crack nucleation
  - Non propagating cracks
  - Crack growth
Stages in the development of a fatigue crack

1. Cyclic slip
2. Persistent slip bands
3. Intrusions and extrusions (surface roughening)
4. Stage I fatigue crack growth
5. Stage II fatigue crack growth
Cyclic slip

First cycle

Many cycles later - cyclic hardening
Cyclic stress-strain behavior

- Development of cell structures (hardening)
- Increase in stress amplitude (under strain control)
- Break down of cell structure to form PSBs
- Localization of slip in PSBs

Cyclic hardening  Cyclic softening

![Graph showing cyclic hardening and softening behavior](image-url)
Slip Band Formation

Cyclically hardened material

Extrusion

Cyclically hardened material

Intrusion

Cyclically hardened material
Intrusions and extrusions on the surface of a Ni specimen
Crack initiation

Fatigue crack initiation at an inclusion
Cyclic slip steps (PSB)
Fatigue crack initiation at a PSB
Cyclic Plastic Zone

Shear band model
Stage I crack growth

Stage I crack growth \((r_c \leq d)\) is strongly affected by slip characteristics, microstructure dimensions, stress level, extent of near tip plasticity.
Stage II crack growth

Stage II crack growth ($r_c >> d$)

Fatigue crack growing in Plexiglas
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The intrinsic fatigue properties of the component’s material are determined using small, smooth or cracked specimens.
Laboratory fatigue test specimens

Smooth specimens
Crack growth specimen

Compact tension specimen (CT):

\[ K_I = \frac{P}{B W^2} \cdot f\left(\frac{a}{W}\right) \]

\[ f\left(\frac{a}{W}\right) = \frac{(2 + \frac{a}{W})[0.886 + 4.64\left(\frac{a}{W}\right) - 13.32\left(\frac{a}{W}\right)^2 + 14.72\left(\frac{a}{W}\right)^3 - 5.6\left(\frac{a}{W}\right)^4]}{(1 - \frac{a}{W})^{\frac{3}{2}}} \]

This information can be used to predict, to compute the answer to……
The three BIG fatigue questions

1. Will a crack nucleate?  
2. Will it grow?  
3. How fast will it grow?
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1. **Will a crack nucleate?**
2. Will it grow?
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Cyclic behavior of materials

Bauschinger effect:
monotonic and cyclic stress-strain different!
Cyclic Deformation

Hysteresis loops

(a) Fully annealed
$\Delta \varepsilon = 0.0084$
$2N_f = 8060$ reversals

(b) Partially annealed
$\Delta \varepsilon = 0.0078$
$2N_f = 4400$ reversals

(c) Cold worked
$\Delta \varepsilon = 0.0099$
$2N_f = 2000$ reversals
MODEL: Cyclic stress-strain curve

\[ \epsilon = \frac{\sigma}{E} + \left( \frac{\sigma}{K'} \right)^{1/n'} \]

\[ \Delta \epsilon = \frac{\Delta \sigma}{E} + 2 \left( \frac{\Delta \sigma}{2K'} \right)^{1/n'} \]

Cyclic stress-strain curve

Hysteresis loops for different levels of applied strain
MODEL: Cyclic stress-strain curve

\[ \varepsilon = \frac{\sigma}{E} + \left( \frac{\sigma}{K'} \right)^{1/n'} \]

\[ \varepsilon = \frac{\sigma}{E} + \left( \frac{\sigma}{K} \right)^{1/n} \]
MODEL USES

- Simulate local cyclic stress-strain behavior at a notch root.
- May need to include cyclic hardening and softening behavior and mean stress relaxation effects.

Notched component
Planar and wavy slip materials

Wavy slip materials

Planar slip materials
Plastic strains cause the accumulation of “fatigue damage.”
MODEL: Strain-life curve

Fatigue test data from strain-controlled tests on smooth specimens.

Elastic component of strain, $\Delta \varepsilon_e$

Plastic component of strain, $\Delta \varepsilon_p$

\[ \Delta \varepsilon_t = \Delta \varepsilon_e + \Delta \varepsilon_p = \frac{\sigma_f'}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \]
Transition Fatigue Life

Transition fatigue life, $N_{tr}$
Elastic strains $= \text{plastic strains}$
Miner's Rule of Cumulative Fatigue Damage is the best and simplest tool available to estimate the likelihood of fatigue failure under variable load histories. Miner's rule hypothesizes that fatigue failure will occur when the sum of the cycle ratios \( \frac{n_i}{N_i} \) is equal to or greater than 1.

\[
\sum_{i} \frac{n_i}{N_i} \geq 1.0
\]

- \( n_i \) = Number of cycles experienced at a given stress or load level.
- \( N_i \) = Expected fatigue life at that stress or load level.

MODEL: Linear cumulative damage

Fatigue Life (N, cycles)

Endurance Limit

2,000,000 cycles

\( n_i \rightarrow \)

\( N_i \rightarrow \)

\( \sum_{i} \frac{n_i}{N_i} \geq 1.0 \)
Mean stress effects

Effects the growth of (small) fatigue cracks.

\[ \log \frac{\Delta \varepsilon}{2} \]

\[ \log 2N_f \]

Compressive mean stress
Zero mean stress
Tensile mean stress

Good!

Bad!
MODEL: Morrow’s mean stress correction

\[ \frac{\Delta \varepsilon}{2} = \frac{\sigma_f' - \sigma_m}{E} \left(2N_f\right)^b + \varepsilon_f' \left(2N_f\right)^c \]
MODEL: Mean stress effects

\[
\frac{\Delta \varepsilon}{2} = \frac{\sigma_f' - \sigma_m}{E} (2N_f)^b + \varepsilon_f'(2N_f)^c
\]

\[
\sigma_{\text{max}} \Delta \varepsilon = \frac{\sigma_f^2}{E} (2N_f)^2 + \sigma_f' \varepsilon_f' (2N_f)^{b+c}
\]

Smith-Watson-Topper
A strain gage mounted in a high stress region of a component indicates a cyclic strain of 0.002. The number of reversals to nucleate a crack ($2N_f$) is estimated to be 86,000 or 43,000 cycles.

\[ \Delta \varepsilon_t = \Delta \varepsilon_e + \Delta \varepsilon_p = \frac{\sigma_f}{E} (2N_f)^b + \varepsilon'_f (2N_f)^c \]
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The three BIG fatigue questions

1. Will a crack nucleate?  
2. **Will it grow?**  
3. How fast will it grow?
Sharp notches may nucleate cracks but the remote stress may not be large enough to allow the crack to leave the notch stress field.
Threshold Stress Intensity

The forces driving a crack forward are related to the stress intensity factor

\[ \Delta K = Y \Delta S \sqrt{\pi a} \]

At or below the threshold value of \( \Delta K \), the crack doesn’t grow.
MODEL: Non-propagating cracks

Failure

\[ a_{th} = \frac{1}{\pi} \left( \frac{\Delta K_{th}}{\Delta S} \right)^2 \]

Stress range, \( \Delta S \)

Crack length

Endurance limit for smooth specimens

Non-propagating cracks
\[ \Delta K = \Delta S \sqrt{\pi a} \]

\[ \Delta K = \frac{Y}{2} \left( \frac{\Delta P}{B \sqrt{\pi a}} + \Delta S \sqrt{\pi a} \right) \]

\[ \Delta P = \Delta S \cdot W \]

\[ W \approx \infty \]

Initial crack length

Groove Welded Butt Joint

Tensile-Shear Spot Weld

Cycles, \( N_{P2} \)

Cycles, \( N_{P2} \)
MODEL USES

- Will the defect of length \( a \) grow under the applied stress range \( \Delta S \)?
- How sensitive should our NDT techniques be to ensure safety?

\[
a_{th} = \frac{1}{\pi} \left( \frac{\Delta K_{th}}{\Delta S} \right)^2
\]
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How fast will it grow???

Measuring Crack Growth - Everything we know about crack growth begins here!

Cracks grow faster as their length increases.
Growth of fatigue cracks is correlated with the range in stress intensity factor: $\Delta K$.

$$\Delta K = Y \Delta S \sqrt{\pi a}$$

$$\frac{da}{dN} = C(\Delta K)^n$$
Fatigue fracture surface

Scanning electron microscope image - striations clearly visible

Schematic drawing of a fatigue fracture surface
Crack growth rate \( \frac{da}{dN} \) is related to the crack tip stress field and is thus strongly correlated with the range of stress intensity factor:

\[
\Delta K = Y \Delta S \sqrt{\pi a}
\]

Paris power law

\[
\frac{da}{dN} = C(\Delta K)^n
\]
Crack Growth Data

All ferritic-pearlitic, that is, plain carbon steels behave about the same irrespective of strength or grain size!
The fatigue crack growth rates for Al and Ti are much more rapid than steel for a given $\Delta K$. However, when normalized by Young’s Modulus all metals exhibit about the same behavior.
MODEL: Mean stresses (R ratio)

\[
\frac{da}{dN} = \frac{C \Delta K^m}{(1-R) K_c - \Delta K}
\]

Forman’s equation which includes the effects of mean stress.

\[
\frac{da}{dN} = \frac{C \Delta K^m}{(1-R)^\gamma}
\]

Walker’s equation which includes the effect of mean stress.
While some applications are actually constant amplitude, many or most applications involve variable amplitude loads (VAL) or variable load histories (VLH).

Aircraft load histories
FIX: Root mean cube method

\[
\Delta K = \frac{\sum (\Delta K_i)^m n_i}{\sum n_i}
\]
PROBLEM: Overloads, under-loads?

Overloads retard crack growth, under-loads accelerate crack growth.
Infrequent overloads help, but more frequent may be even better. Too frequent overloads very damaging.
FIX: MODEL for crack tip stress fields

Monotonic plastic zone size
Cyclic plastic zone size

\[ r_y = \frac{1}{\beta \pi} \left( \frac{K}{\sigma_y} \right) \]

\( \beta \) is 2 for (a) plane stress and 6 for (b)
MODEL: Overload plastic zone size

Crack growth retardation ceases when the plastic zone of ith cycle reaches the boundary of the prior overload plastic zone.

\[ r_y = \frac{1}{\beta \pi} \left( \frac{K}{\sigma_y} \right) \]

\( \beta \) is 2 for plane stress and 6 for plain strain.
MODEL: Retarded crack growth

Wheeler model

\[
\frac{da}{dN} = \beta \left( \frac{da}{dN} \right)_{CA}
\]

\[
\beta = \left( \frac{2 r_y}{a_p - a_i} \right)^k
\]

\[
r_y = \frac{1}{2 \pi} \left( \frac{K}{\sigma_{ys}} \right)^2
\]
Infrequent overloads help, but more frequent may be even better. Too frequent overloads very damaging.
PROBLEM: Periodic overloads?

Effects of Periodic Overloads

Infrequent overloads help, but more frequent may be even better. Too frequent overloads very damaging.
Sequence Effects

Last one controls!

Compressive overloads accelerate crack growth by reducing roughness of fracture surfaces.

Compressive followed by tensile overloads…retardation!

Tensile followed by compressive, little retardation
Sequence effects

Overloads retard crack growth, under-loads accelerate crack growth.
PROBLEMS: Small Crack Growth?

Small cracks don’t behave like long cracks! Why?
PROBLEMS: Environment?

A. Dissolution of crack tip.

B. Dissolution plus H+ acceleration.

C. H+ acceleration

D. Corrosion products may retard crack growth at low ΔK.
FIX: Crack closure

Crack open

Crack closed

Remote Stress, $S$

Time, $t$

$S = S_{\text{max}}$

$S_{\text{open}}$, $S_{\text{close}}$

$S = 0$

$S_{\text{max}}$

$S_{\text{open}}$, $S_{\text{close}}$

Plastic wake

New plastic deformation
CAUSE: Plastic Wake

Short cracks can’t do this so they behave differently!
“Effective” part of load history

- Damaging portion of loading history
- Nondamaging portion of loading history
- Opening load
MODEL: Crack closure

\[
\frac{da}{dn} = C (\Delta K)^m (K_{\text{max}})^p
\]
Other sources of crack closure

Roughness induced crack closure (RICC)

Oxide induced crack closure (OICC)
MODEL: Crack closure

\[ \Delta K_{\text{eff}} = U \Delta K \]

\[ U = \frac{\Delta K_{\text{eff}}}{\Delta K} = \frac{S_{\text{max}} - S_{\text{open}}}{S_{\text{max}} - S_{\text{min}}} = \frac{1}{1 - R} \left( 1 - \frac{S_{\text{open}}}{S_{\text{max}}} \right) \]

\[ U \approx 0.5 + 0.4R \] (sometimes)

Plasticity induced crack closure (PICC)

\( A, A', A'' \) Crack tip positions

\( \) Plastic zones for crack positions \( A...A'' \)

\( \) Plastic wake
The use of the effective stress intensity factor accounts for the effects of mean stress.

\[
\frac{da}{dN} = C(\Delta K)^n
\]

\[
\frac{da}{dN} = C' (\Delta K_{\text{eff}})^n = C' (U\Delta K)^n
\]
MODEL USE

Use of $\Delta K_{\text{eff}}$

Crack closure at a notch model.
Information about $U(a)$ becomes the “sticking point.”

$$N_F = \int_{a_i}^{a_f} \left( \frac{da}{dN} \right)^{-1} \, da$$

Bilinear Paris Law
PROBLEM: Fully plastic conditions?

Correlate crack growth rate with strain intensity?

\[ \Delta K = E \, \Delta \varepsilon \, Y(a) \sqrt{a} \]

Correlate crack growth rate with values of the J integral?

\[ \Delta K^2 = \Delta J \, E = 1.6 \, \Delta \sigma^2 \, a + \frac{5.0 \, \Delta \sigma^2 \, \Delta \varepsilon_p \, a \, E}{1 + n'} \]
PROBLEM: Fully plastic conditions?

Correlate crack growth rate with values of the crack tip opening displacement (COD)?