Sequence Effects in Fatigue

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Outline

1. Mean Stress
2. Nonlinear Damage Models
3. Crack Growth Interactions
4. Crack Tip Plasticity Models
5. Crack Closure Models
Mean Stresses

\[ S_{\text{mean}} = \frac{S_{\text{max}} + S_{\text{min}}}{2} \]

\[ R = \frac{S_{\text{min}}}{S_{\text{max}}} \]
General Observations

- Tensile mean stresses reduce the fatigue life or decrease the allowable stress range.
- Compressive mean stresses increase the fatigue life or increase the allowable stress range.
Fatigue damage is a shear process

Tensile mean stresses open microcracks and make sliding easier
.. whether the assumptions of the theory are justifiable or not .... We adopt it simply because it is the easiest to use, and for all practical purposes, represents Wöhlers data.

\[ S_{\text{ultimate}} = S_{\text{min}} + 2 \Delta S \]
Goodman Diagram

\[ \frac{\Delta S}{2} = \left( \frac{\Delta S}{2} \right)_{R=-1} \left( 1 - \frac{S_{\text{mean}}}{S_{\text{ultimate}}} \right) \]
Test Data (1941)

J.O. Smith, The Effect of Range of Stress on the Fatigue Strength of Metals, Engineering Experiment Station Bulletin 334, University of Illinois, 1941
Compression

beneficial

no influence

detrimental

$-S_u \quad R = -\infty$

$0 \quad R = -1$

$S_u \quad R = 1$
Modified Goodman (no yielding)

\[ R = -\infty \quad S_{ys} \]
\[ R = -1 \quad \frac{\Delta S}{2} + S_{mean} < S_{ys} \]
\[ R = 1 \quad S_{ys} \]
\[ -S_u \quad -S_{ys} \]

\[ S_{ys} \]
Mean Stress Influence on Life

Graph showing the relationship between Mean Stress and Relative Fatigue Life.

- Mean Stress is measured from -0.6 to 0.6 (Ultimate Strength).
- Relative Fatigue Life is measured from 1000 to 0.01.

The graph illustrates a downward trend indicating that as Mean Stress increases, Relative Fatigue Life decreases.
Stress Concentrations

The elastic material surrounding the plastic zone around a stress concentration forces the material to deform in strain control.
Mean Stresses at Notches

Nominal mean stress is less than notch mean stress

Nominal mean stress is greater than notch mean stress
Morrow Mean Stress Correction

\[ \frac{\Delta \varepsilon}{2} = \frac{\sigma'}{E} \left( \frac{2N_f}{\varepsilon_f} \right)^b + \varepsilon_f' \left( \frac{2N_f}{\varepsilon_f} \right)^c \]

\[ \frac{\sigma_{\text{mean}}}{E} \]

Strain Amplitude

Reversals, \(2N_f\)
Smith Watson Topper

\[ \sigma_{\text{max}} \frac{\Delta \varepsilon}{2} = \frac{\sigma'_f}{E} \left(2N_f\right)^{2b} + \sigma'_f\varepsilon'_f \left(2N_f\right)^{b+c} \]
Mean Stress Relaxation

FIG. 7—Cyclic softening and relaxation of mean stress under Neuber control (Ti-8Al-1Mo-IV, $K_t = 1.75$).

Stadnick and Morrow, “Techniques for Smooth Specimen Simulation of Fatigue Behavior of Notched Members” ASTM STP 515, 1972, 229-252
Loading Histories

Load History A

Load History B
Test Results

- Notched Specimen
  Constant Amplitude at $\Delta S_1$

- Smooth Specimen Simulation
  Load History A (See Fig. 4)

- Notched Specimen
  Load History A

- Notched Specimen
  Load History B

- Smooth Specimen Simulation
  Load History B

Diagram: 2 $N_1$, Reversals To Failure vs. $\Delta S_2$ ksi
Sources of Mean/Residual Stress

- Loading History
- Fabrication
- Shot Peening
- Heat Treating
Loading History

Tension overloads produce favorable compressive residual stress

Compressive overloads produce unfavorable tensile residual stress
Fabrication
Cold Expansion

1965 Basic Cx process conceptualized (Boeing)

The split sleeve is slipped onto the mandrel, which is attached to the hydraulic puller unit.

The mandrel and sleeve are inserted into the hole with the nosecap held firmly against the workpiece.

When the puller is activated, the mandrel is drawn through the sleeve radially expanding the hole.

Courtesy of Fatigue Technology Inc.
Theory of Cold Expansion

Courtesy of Fatigue Technology Inc.
Fatigue Life Improvement

![Graph showing Fatigue Life Improvement](image)

Nominal Stress vs. Fatigue Life

- Cold Expanded

Courtesy of Fatigue Technology Inc.
Shot Peening

Residual stress in a shot peened leaf spring
Shot Peening Results

![Graph showing the effect of shot peening on fatigue strength and ultimate tensile strength.](www.metalimprovement.com)

$$S_{FL} = \frac{S_u}{2}$$
50 mm diameter induction hardened 1045 steel shaft
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Cumulative Damage

High - Low

Low - High

$\Delta S_L$

$\Delta S_H$
Linear Damage

Miners Rule:

\[
\text{Damage} = \sum \frac{n}{N_F} = \frac{n_H}{N_f H} + \frac{n_L}{N_f L}
\]
Nonlinear Damage

\[ \Delta \varepsilon = 0.020 \]

\[ \Delta \varepsilon = 0.004 \]

\[ \Sigma D = 0.7 \]

\[ \Sigma D = 1.3 \]

\[ \Sigma D \sim 1 \]
The fatigue limit is reduced by a factor of 3 when a few large cycles are applied.

Bonnen and Topper, “The Effects of Periodic Overloads on Biaxial Fatigue of Normalized SAE 1045 Steel” ASTM STP 1387, 2000, 213-231
Relative Miner’s Rule

\[ \sum D = \sum \frac{n}{N_f} = 0.3 \]

Set Miner’s damage sum to a number less than 1
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Crack Growth Physics

Maximum load

Minimum load

Mean stresses in plastic zone are small
Mean Stress Effects

\[ \frac{da}{dN} = \frac{C \Delta K^m}{(1-R)\gamma} \]

0 < \gamma < 0.5

From Dowling, Mechanical Behavior of Materials, 1999

\[ \gamma = \frac{\Delta \sigma - R}{\Delta \sigma + R} \]

\[ R = \frac{\sigma_{\max}}{\sigma_{\max} + \sigma_{\min}} \]

\[ \Delta \sigma = \sigma_{\max} - \sigma_{\min} \]

\[ \sigma_{\max} = \sigma_{\text{max}} \]

\[ \sigma_{\min} = \sigma_{\text{min}} \]

\[ \Delta K = \sqrt{\frac{2}{E}} \Delta \sigma \sqrt{a} \]

\[ E = \text{Young's Modulus} \]

\[ a = \text{ Crack Length} \]

\[ \Delta \sigma = \text{Maximum Stress} - \text{Minimum Stress} \]

\[ \sigma_{\text{max}} = \text{Maximum Stress} \]

\[ \sigma_{\text{min}} = \text{Minimum Stress} \]
Sequences

a

Load

+  -

Time

c

Load

+  -

Time

b

Load

+  -

Time

d

Load

+  -

Time
Crack Growth

![Graph showing crack growth over cycles.](image)

- **No overload**
- Curves labeled a, b, c, d.
Crack Growth Rate

Growth rate, mm/cycle

Distance from peak load, mm

10^{-5} to 10^{-3}

10^{-4}
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Stress Fields After Overload

**Before**
- monotonic plastic zone
- cyclic plastic zone

**After**
- overload monotonic plastic zone
- overload compressive plastic zone
Crack Growth Retardation

![Graph showing crack growth retardation](image)

- Growth rate, mm/cycle
- Distance from peak load, mm

- Logarithmic scale for growth rate
- Linear scale for distance from peak load
Wheeler Model

\[
\frac{da}{dN} = \left( \frac{r_{yi}}{a_p - a_i} \right)^\gamma \left( \frac{da}{dN} \right)_{CA}
\]
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Compression

Crack open

Crack closed
Compressive Stresses

Compressive stresses are not very damaging in crack growth

Crack opening level
\[ \Delta K_{\text{eff}} = \left( 0.5 + 0.4R \right) \Delta K \]

\[ \frac{da}{dN} = C (\Delta K_{\text{eff}})^m \]
Maximum Stress Dependence

\[ S_{\text{max}}, \text{ MPa} \]

- ○ 55
- □ 138
- ▲ 276

\[ \frac{S_0}{S_{\text{max}}} \]

- ● 32
- ▲ Compact [16]

2219-T851
\[ t = 6.35 \text{ mm} \]

\[ S_{\text{max}}, \text{ MPa} \]

- ● 55
- ○ 138
- ▲ 276

- --- \( \alpha = 1.9 \)
Newman Model

\[ \Delta K_{\text{eff}} = \left[ 1 - \frac{S_0}{S_{\text{max}}} \right] \frac{1 - R}{1 - R} \Delta K \]

Newman

\[ \frac{S_0}{S_{\text{max}}} = A_0 + A_1 R + A_2 R^2 + A_3 R^3 \]

\[ A_0 = (0.825 - 0.34 \alpha + 0.05 \alpha^2) \cos \left( \frac{\pi S_{\text{max}}}{2 \sigma_0} \right) \left[ \frac{1}{\alpha} \right] \]

\[ A_1 = (0.415 - 0.071 \alpha) \frac{S_{\text{max}}}{\sigma_0} \]

\[ A_2 = 1 - A_0 - A_1 - A_3 \]

\[ A_3 = 2 A_0 + A_1 - 1 \]
Closure Observations

![Image a](image-a.png)
![Image b](image-b.png)
![Image c](image-c.png)
![Image d](image-d.png)

Diagram:
- a: 1st subcycle
- b: 500th subcycle
- c: 1026 steel
- d: Δε₁/2 = 0.005
- d: Δε₂/2 = 0.001

1026 steel

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Spectrum Loading

Strain ($\mu\varepsilon$)

Non Damaging Cycles
Sequence Effects in Fatigue