Notches in Fatigue

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Stress Concentration Factor

\[ \sigma_{\text{local}} = \sigma_{\text{applied}} \left( 1 + 2 \sqrt{\frac{a}{\rho}} \right) \]

Inglis Solution 1910

Applied stress

Local stress

2a
Notch Rules

Neuber

\[ K_t^2 S_e = \sigma \varepsilon = \frac{\sigma}{E} + \left( \frac{\sigma}{K} \right)^{\frac{1}{n}} \]

Glinka

\[ K_t^2 S_e = \int \sigma \, d\varepsilon = \frac{\sigma}{E} + \frac{1}{1+n} \left( \frac{\sigma}{K} \right)^{\frac{1}{n}} \]

Seeger

\[ K_p^2 S^* e^* = \sigma \varepsilon = \frac{\sigma}{E} + \left( \frac{\sigma}{K} \right)^{\frac{1}{n}} \]

\[ K_p = \frac{S_{\text{Limit}} K_t}{\sigma_y} \quad S^* = \frac{K_t}{K_p} S \]
Define $K_\sigma$ and $K_\varepsilon$ after Yielding

Define: nominal stress, $S$
nominal strain, $e$
notch stress, $\sigma$
notch strain, $\varepsilon$

Stress concentration
$$K_\sigma = \frac{\sigma}{S}$$

Strain concentration
$$K_\varepsilon = \frac{\varepsilon}{e}$$
\[ K_\sigma \] and \[ K_\varepsilon \]

\[ K_\sigma = \frac{\sigma}{S} \]
\[ K_\varepsilon = \frac{\varepsilon}{e} \]
Stress and Strain Concentration

\[ K_\varepsilon \rightarrow K_t^2 \]

First yielding

\[ K_\sigma \rightarrow 1 \]

Nominal Stress

Stress/Strain Concentration
Neuber’s Rule

Stress calculated with elastic assumptions

Actual stress

\[ K_t S K_t e = \sigma \varepsilon \]

Stress calculated with elastic assumptions
Neuber’s Rule for Fatigue

Stress and strain amplitudes

\[ \frac{K_t}{2} \Delta S \frac{K_t}{2} \Delta e = \frac{\Delta \sigma \Delta \varepsilon}{2} \]

Elastic nominal stress

\[ \frac{\Delta e}{2} = \frac{\Delta S}{2E} \]

Substitute and rearrange

\[ K_t \frac{\Delta S}{2} = \sqrt{E \left( \frac{\Delta \sigma}{2} \frac{\Delta \varepsilon}{2} \right)} \]

The product of stress times strain controls fatigue life
SN Materials Data

![Graph showing data for 93 steels and 17 aluminums]
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\[ \varepsilon \text{N Materials Data} \]

- 93 steels
- 17 aluminums

Strain Amplitude

Fatigue Life, Reversals

\[ 10^{-4} \quad 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 1 \quad 10 \quad 10^{2} \quad 10^{3} \quad 10^{4} \quad 10^{5} \quad 10^{6} \quad 10^{7} \]
Outline

1. Notch Rules
2. Fatigue Notch Factor
3. Stress Intensity Factors for Notches
4. Frost Data and $K_f$
5. Small Crack Growth
6. Small Notches
Stress analysis and stress concentration factors are independent of size and are related only to the ratio of the geometric dimensions to the loads.

Fatigue is a size dependant phenomena.

How do you put the two together?
Fatigue of Notches

From Dowling, Mechanical Behavior of Materials, 1999
Notch Size

Large Notch

Small Notch
Microstructure Size

Low Strength

High Strength
Stress Gradient

Low $K_t$  

High $K_t$
$K_t$ vs $K_f$

$K_f = K_t$

Experiments
Peterson’s Equation

\[ \alpha = 0.025 \left( \frac{2070 \text{MPa}}{\sigma_u} \right)^{1.8} \text{mm} \]

\[ K_f = 1 + \frac{K_t - 1}{1 + \frac{\alpha}{\rho}} \]

No effect when \( \rho << \alpha \)

Full effect when \( \rho >> \alpha \)
Pererson’s Constant
Static Strength

- **hole**
  - $K_t = 2.5$

- **slot**
  - $K_t = 5$

- **diamond**
  - $K_t = 20$

- **edge**
  - $K_t = 20$
1018 Steel Test Data

- edge
- diamond
- slot
- hole

load, kN vs. displacement, mm
Notched SN Curve

Stress concentrations are not very important at short lives
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Cracks at Notches

\[ a \ll D \quad K = K_t S \sqrt{\pi a} \]

\[ a \gg D \quad K = S \sqrt{\pi (D + a)} \]
Stress Intensity Factors

\[ K = K_I S \sqrt{\pi a} \]

\[ K = S \sqrt{\pi (D + a)} \]
Once a crack reaches 10% of the hole radius, it behaves as if the hole was part of the crack.
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Crack Growth Data

Nonpropagating cracks

\[ \Delta K_{TH} > \Delta \sigma 1.12 \frac{2}{\pi} \sqrt{\pi a} \]
Frost Data

For $K_t > 4$, the notch acts like a crack with a depth $D$

$$S_{fl} = \frac{\Delta K_{th}}{\sqrt{\pi D}}$$

$K_t$ does not play a role for sharp notches!
Specimens with Similar Geometry

$K_t = 10.7$

$K_t = 2.4$

Ultimate Strength 780 MPa
Yield Strength 660 MPa
Test Results

- **Strength Limited**
- **Crack Growth Dominated**
- **Threshold Stress Intensity Dominated**

- $K_t = 2.4$
- $K_t = 10.7$

Graph: Nominal Stress Amplitude vs. Total Fatigue Life, Cycles
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12 Difference in propagation rates $da/dN$ of short and long fatigue cracks as function of stress intensity factor range $\Delta K$ for 3\%Si iron of yield strength $\sigma_0 = 431$ MN m$^{-2}$ (Ref. 70)
Threshold

16 Variation of threshold stress intensity range $\Delta K_{\text{th}}$ with short crack length $a$ in G40.11 austenitic 0.45%C steel, $\sigma_0 = 550$ MNm$^{-2}$, 0.035%C mild steel, $\sigma_0 = 242$ MNm$^{-2}$, and Al–Zn–Mg alloy, $\sigma_0 = 180$ MNm$^{-2}$ (Ref. 69)
Normalized Thresholds

![Normalized Thresholds Graph]

- Material | $\sigma_e$ (MN m$^{-2}$) |
<table>
<thead>
<tr>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>S20 C</td>
<td>366</td>
</tr>
<tr>
<td>S20 C</td>
<td>194</td>
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<tr>
<td>Mild steel</td>
<td>289</td>
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<tr>
<td>G40 II</td>
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<td>SM 41</td>
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<td>13 Cr cast steel</td>
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<tr>
<td>Copper</td>
<td></td>
</tr>
<tr>
<td>Aluminium</td>
<td>30.4</td>
</tr>
</tbody>
</table>
Growth from Notches

24 Propagation rate \( \frac{da}{dN} \) of cracks emanating from notches as function of maximum stress intensity factor \( K_{\text{max}} \) in 0.15\%C mild steel; \( k_t \) is theoretical elastic stress concentration factor, \( R \) stress ratio, and \( R_\varepsilon \) edge strain ratio.\(^{110} \)
Cracks at Notches

- notch plastic zone
- notch stress field
- crack tip plastic zone
Crack Growth

- Short cracks: Notch plasticity controls
- Long cracks: Crack tip plasticity controls

Crack growth rate vs. $\Delta K$ or $a$
Closure Observations

1026 steel
\[ \Delta \varepsilon_1 / 2 = 0.005 \]
\[ \Delta \varepsilon_2 / 2 = 0.001 \]
Closure Correlation
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Definition of $\sqrt{\text{area}}$

Maximum tensile stress direction
Small Notches

Fatigue limit, $\sigma_w$, MPa

- $0.13\%$ C steel
- $0.46\%$ C steel
- 2017-T4 Al alloy
- 70/30 brass

Diameter of hole, $d$, $\mu$m

$d = h$
Threshold Stress Intensity

\[
\Delta K_{th} \text{ MPa} \cdot \text{m}^{1/2}
\]

\[
\sqrt{\text{area}} \ \mu\text{m}
\]

Higher \( H_v \)
Hardness Correlation

\[ \Delta K_{th} = 3.3 \times 10^{-3} (H_V + 120) \left( \overline{\text{area}} \right)^{1/3} \]
Flaw Sensitivity

Flaw Sensitivity

Vickers Hardness, Hv

Critical Flaw Size, l0 (1e-6m)

reduction of Endurance Limit

no effect on Endurance Limit
Notches in Fatigue